KTH Matematik

SOLUTION TO EXAMINATION IN SF2701 FINANCIAL MATHEMATICS 2013-05-24.

Problem 1

Consider the following strategy. At time 0, buy one share. At t_0 collect the dividend and reinvest it in shares of the stock. Then sell all shares at time t_1.
By the law of one price the price S_{t_0+} just after the dividend has been paid must equal $S_{t_0} - dS_{t_0} = S_{t_0}(1-d)$. Therefore the number of new shares purchased at t_0 is

$$\frac{dS_{t_0}}{S_{t_0+}} = \frac{d}{1-d}.$$

The total number of shares at time t_1 is then

$$1 + \frac{d}{1-d} = \frac{1}{1-d}.$$

The cash flow of the strategy is $-S_0$ at time 0 and $\frac{1}{1-d}S_{t_1}$.
The same cash flow can be generated by, at time 0, entering into $\frac{1}{1-d}$ forward contracts on S_{t_1} and a zero coupon bond with maturity t_1 and face value $\frac{1}{1-d}G_0$. By the law of one price the initial cash flow at time 0 must be identical:

$$S_0 = \frac{1}{1-d}G_0.$$

Therefore

$$G_0 = S_0(1-d).$$

Problem 2

The option tree is given in Table 2

Table 1: Forward tree for crude oil (USD)

time (months)	0	1	2	3
	93.0	95.8	98.7	101.6
		88.6	91.3	94.0
			85.5	88.5
				83.4

Table 2: Option tree

time (months)	0	1	2	3
	2.93	1.10	0	0
		4.82	2.23	0
			7.5	4.5
				9.6

Problem 3

The present value in EUR of the EUR cash flow is

$$P_{EU} = 789000e^{-r_{EU}} + 789000e^{-2r_{EU}} + 16.17 \cdot 10^6 e^{-3r_{EU}} = 15'382'085 \text{ EUR}.$$

Similarly, the present value in USD of the USD cash flow is

$$P_{US} = 1.34 \cdot 10^6 e^{-r_{US}} + 1.34 \cdot 10^6 e^{-2r_{US}} + 21.34 \cdot 10^6 e^{-3r_{US}} = 19'991'610 \text{ USD}.$$

The present value in EUR of the USD cash flow is therefore

$$0.769 \cdot 19'991'610 = 15'373'548 USD.$$

The European firm pays the USD cash flow and receives the EUR cash flow. Thus the present value of the swap is

$$15'382'085 - 15'373'548 = 8'537 \text{ EUR}$$

Problem 4

The binomial tree for the short rate is given in Table 3. The bond tree is given in Table 4 and the options tree in Table 5.

Table 3: Ho-Lee interest rate tree (%)

time	t_0	t_1	t_2	t_3
	2.0	3.1	4.8	4.7
		2.5	4.2	4.1
			3.6	3.5
				2.9
rate	r_1	r_2	r_3	r_4

Problem 5

Let X be the exchange rate EUR/SEK at $T = 6$ months and Y be the exchange rate USD/SEK. The payoff of the option at T is $\max(X - 1.3Y, 0)$. Then, using Y as the numeraire we have

$$10^6 P_0^{(T)}[\max(X - 1.3Y, 0)] = 10^6 P_0^{(T)}[Y] E^Y[\max(X/Y - 1.3, 0)]$$

Table 4: Bond tree

time	t_0	t_1	t_2	t_3	t_4
	87.99	88.96	91.21	95.40	100
		90.58	92.31	95.98	100
			93.43	96.56	100
				97.14	100
					100

Table 5: Options tree

time	t_0	t_1	t_2
	0.75	1.20	1.79
		0.34	0.69
			0

Note that X/Y is the USD/EUR exchange rate and

$$G_0^{(T)}[X/Y] = \frac{G_0^{(T)}[X]}{G_0^{(T)}[Y]} = \frac{8.45}{6.50} = 1.3.$$

Assume Black's model: $X/Y = Ae^{\sigma\sqrt{T}Z}$, where $Z \sim N(0,1)$. Then,

$$A = G_0^{(T)}[X/Y]e^{-\sigma^2 T/2} = \frac{G_0^{(T)}[X]}{G_0^{(T)}[Y]}e^{-\sigma^2 T/2}.$$

By Black's formula the price is given by

$$\begin{aligned} 10^6 P_0^{(T)}[\max(X - 1.3Y, 0)] &= 10^6 P_0^{(T)}[Y]E^Y[\max(X/Y - 1.3, 0)] \\ &= 10^6 Z_T G_0^{(T)}[Y]\left(\frac{G_0^{(T)}[X]}{G_0^{(T)}[Y]}\Phi(d_1) - 1.3\Phi(d_2)\right) \\ &= 10^6 Z_T G_0^{(T)}[X](\Phi(d_1) - \Phi(d_2)), \end{aligned}$$

where

$$\begin{aligned} d_1 &= \frac{\ln(1)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} = \frac{1}{2}\sigma\sqrt{T}, \\ d_2 &= -\frac{1}{2}\sigma\sqrt{T} = -d_1. \end{aligned}$$

Putting in numerical values gives the price $70'505.5$ SEK