EXAMINATION IN SF2701 FINANCIAL MATHEMATICS, 2011-05-28, 09:00–14:00.

*Examiner*: Henrik Hult, tel. 790 6911, e-mail: hult@kth.se

*Allowed technical aids*: calculator.

Any notation introduced must be explained and defined. Arguments and computations must be detailed so that they are easy to follow.

Interest rates are compounded continuously.

### Good luck!

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## Problem 1

Today is time 0. Let $X$ be a random payment which will occur $T$ days in the future. Let $e^{R(0,T)}$ be the value of the money market account $T$ days in the future. Prove, using the law of one price, that the present price of $X$ for delivery at $T$ is equal to the futures price of a contract which pays $X/e^{R(0,T)}$ at $T$. (10 p)

## Problem 2

The spot price to buy one US dollar today is 45.87 Indian Rupees. The forward price to buy one US dollar for delivery in six months is 47.37 Indian Rupees. Is the Indian six month interest rate higher or lower than the US six month interest rate? How much higher/lower? (express interest rates in % per year). State your answer with four significant digits. (10 p)

## Problem 3

Consider Ho-Lee's binomial model where the interest rate between time $t-1$ and $t$ is given by

$$r_t = \ln\left(\frac{Z(0,t-1)}{Z(0,t)}\right) + \ln(\cosh((t-1)\sigma)) + \sigma(b_2 + \cdots + b_t),$$

where $Z(0,t)$ is the price of a zero coupon bond with maturity $t$, and $b_1, b_2, \ldots$ are independent taking values 1 and $-1$ with probability 0.5 each, under the futures distribution. The interest rates are represented in a binomial tree as shown in Table 1, where the interest is given in % per time-step.

(a) Determine the binomial tree at times $t = 1, 2$, and 3 for the price of a bond which pays a coupon of SEK 10 at time 2 and the face value SEK 100 at time 3.

(b) Determine the price at time 0 of a European call option written on the bond in (a). The option has strike 80 and matures at time 1.
State your answer with four significant digits. (10 p)

Table 1: Interest rates

| period | 0 | 1 | 2 |
|--------|-----|-----|-----|
|        | 4.0 | 4.5 | 5.0 |
|        |     | 3.5 | 4.0 |
|        |     |     | 3.0 |

## Problem 4

Today the spot price of one share of the Erixon stock is 92.35. The forward prices for delivery in one month, two months etc are given in Table 2. There is a noticable drop in the forward price between 3 and 4 months.
(a) Give a plausible reason for the price drop between month 3 and 4.

Table 2: Forward prices for Erixon in Problem 4

| months | 1 | 2 | 3 | 4 | 5 |
|--------|-------|-------|-------|-------|-------|
| forward price | 92.56 | 92.75 | 92.95 | 89.52 | 89.72 |

(b) Determine the forward (interest) rate between months 4 and 5 expressed in % per year. State your answer with four significant digits.
(c) The spot price for Noxia is 53.25 and the forward prices are given in Table 3. Determine the price of a zero coupon bond with face value 100 for delivery in five months. (10 p)

Table 3: Forward prices for Noxia in Problem 4

| months | 1 | 2 | 3 | 4 | 5 |
|--------|-------|-------|-------|-------|-------|
| forward price | 53.37 | 53.48 | 51.56 | 51.69 | 51.81 |

## Problem 5

In Ho-Lee's model the short rate from day $t-1$ to $t$ is modeled by

$$r_t = \ln\left(\frac{Z(0,t-1)}{Z(0,t)}\right) + \frac{\sigma^2}{2}(t-1)^2 + \sigma(z_1 + \cdots + z_{t-1}),$$

and the price on day $t$ of a zero coupon bond with face value 1 maturing on day $T$ has the representation

$$Z(t,T) = \frac{Z(0,T)}{Z(0,t)}e^{-\frac{\sigma^2}{2}(T-1)(T-t)t}e^{-\sigma(T-t)(z_1+\cdots+z_t)},$$

where $z_1, z_2, \ldots,$ are independent $N(0,1)$ under the futures distribution.

Determine an explicit expression for the futures price $F_0 = F_0^{(t)}[Z(t,T)]$. That is, the futures price at time zero for delivery at time $t$ of a zero coupon bond with maturity $T$.

(10 p)