



KTH Matematik

EXAMINATION IN SF2701 FINANCIAL MATHEMATICS, 2013-05-24, 14:00–19:00.

Examiner: Henrik Hult, tel. 790 6911, e-mail: hult@kth.se

Allowed technical aids: calculator.

Any notation introduced must be explained and defined. Assumptions must be clearly stated. Arguments and computations must be detailed so that they are easy to follow.

Interest rates are compounded continuously.

GOOD LUCK!

General information:

We use the notation $(a)_+ = \max(a, 0)$.

Black's formula for European call and put options:

$$\begin{aligned}c &= Z_t(G_0^{(t)}\Phi(d_1) - K\Phi(d_2)), \\p &= Z_t(K\Phi(-d_2) - G_0^{(t)}\Phi(-d_1)), \\d_1 &= \frac{\ln(G_0^{(t)}/K)}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t}, \\d_2 &= d_1 - \sigma\sqrt{t}\end{aligned}$$

A table of the standard Normal distribution is given at the end of the exam.

Problem 1

Let Z_t denote the price of a zero-coupon bond with face value 1 and maturity t . Let S_t denote the spot price of a share of a stock at time t . Suppose the stock pays a dividend of size dS_{t_0} at time t_0 . **Derive** the forward price $G_0^{(t_1)}[S_{t_1}]$ for receiving one share of the stock at time $t_1 > t_0$. For full points you must give a complete derivation of the forward price from the law of one price. It is not sufficient just to state the formula. (10 p)

Problem 2

Today the futures price on crude oil for delivery in three months is 92.65 US dollars per barrel. Compute the price of an American (futures) put option with maturity in three months to sell 1 barrel of crude oil for 93 US dollars. Use a binomial model with time step one month, where the forward tree is given in Table 1. You may assume that the interest rate is deterministic and equal to 1% per month. (10 p)

Table 1: Forward tree for crude oil (USD)

time (months)	0	1	2	3
	93.0	95.8	98.7	101.6
		88.6	91.3	94.0
			85.5	88.5
				83.4

Problem 3

A European firm has entered into a five year fixed-for-fixed currency swap with a US financial institution, which enabled the European firm to borrow 20 million USD. The currency swap is constructed so that once per year the European firm receives the amount 789000 EUR and pays the amount 1.34 million USD. At the termination of the contract, the European firm receives 16.17 million EUR and pays 21.34 million USD.

At the present time, just over two years has passed since the initiation of the currency swap (and just under three years remains). The current exchange rate is 0.769 EUR/USD and the European and US interest rates are $r_{EU} = 5.0\%$ per year and $r_{US} = 6.5\%$ per year, respectively (flat term structure). Compute the current value in EUR of the remaining cash flow from the swap faced by the European firm. (10 p)

Problem 4

Consider Ho-Lee's binomial model for the short rate. The Ho-Lee interest tree for the short rate observed at times $t_1 = 1$ year, $t_2 = 2$ years, $t_3 = 3$ years is given in Table 2. Determine the price of an option to sell, in two years from now at price 93

Table 2: Ho-Lee interest rate tree (%)

time	t_0	t_1	t_2	t_3
	2.0	3.1	4.8	4.7
		2.5	4.2	4.1
			3.6	3.5
				2.9
rate	r_1	r_2	r_3	r_4

SEK, a zero coupon bond with face value 100 SEK and maturity in four years. (10 p)

Problem 5

The forward price for delivery of 1 EUR in six months is 8.45 SEK. The forward price for delivery of 1 USD in six months is 6.50 SEK. Compute the price in SEK of an option that gives you the right (but not obligation) to buy 1 million EUR for 1.3 million USD in six months. The volatility of the euro relative to the US dollar is 3% per year. The swedish interest rate is 2% per year. Use Black's model. (10 p)

