KTH Matematik

EXAMINATION IN SF2701 FINANCIAL MATHEMATICS, 2013-08-19, 08:00–13:00.

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Allowed technical aids: calculator.

Any notation introduced must be explained and defined. Assumptions must be clearly stated. Arguments and computations must be detailed so that they are easy to follow.

Interest rates are compounded continuously.

Good luck!

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General information:

We use the notation $(a)_+ = \max(a, 0)$.

Black's formula for European call and put options:

$$
\begin{aligned}
c &= Z_t(G_0^{(t)}\Phi(d_1) - K\Phi(d_2)),\\
p &= Z_t(K\Phi(-d_2) - G_0^{(t)}\Phi(-d_1)),\\
d_1 &= \frac{\ln(G_0^{(t)}/K)}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t},\\
d_2 &= d_1 - \sigma\sqrt{t}
\end{aligned}
$$

A table of the standard Normal distribution is given at the end of the exam.

Problem 1

The spot price of one share of the ABB stock is 145.40 SEK. A dividend of unknown size is paid in roughly 8 months. The price of a European call option with strike 150 and maturity in 10 months is 7.75 SEK. The price of a European put option with strike 150 and maturity in 10 months is 14.00 SEK. The interest rate is 1.0% per year. The dividend payment is assumed to be dS_t where t is the time of the dividend payment and S_t is the spot price at that time. Determine d.

Problem 2

The current spot price for a share of the Finnish stock Sampo A is 30.15 EUR. In 10 days, a dividend of 1.35 EUR per share will be paid. The current exchange rate is 8.684 SEK/EUR and the Swedish and Finnish interest rates are 0.90% and 0.21%, respectively. The volatility of the share can be assumed to be 40% per year.
(a) Use Black's model to compute the value, in SEK, of a European call option on one share of Sampo A with strike price 29 EUR and maturity in two months from today. (5 p)

(b) Use a binomial tree to compute the value, in SEK, of an American call option on one share of Sampo A with strike price 29 EUR and maturity in two months from today. Use a time step of one month. (5 p)

Hint: $u = 1 + \tanh(0.4/12) = 1.066568$ and $d = 1 - \tanh(0.4/12) = 0.933432$.

Problem 3

The current market interest rates are as follows:

- the European one year interest rate is 0.58% per year, the two-year rate is 0.62% per year and the three year rate is 0.66% per year,
- the US one year interest rate is 1.10% per year, the two year rate is 1.20% per year and the three year rate is 1.40% per year.

A European firm is about to enter into a three year fixed-for-floating currency swap with a US financial institution. The currency swap is constructed so that the European firm receives the amount x EUR at the end of years one and two and 16 $\textit{million} + x$ EUR at the end of the third year. In addition the European firm must pay the principal amount 20 million USD at the end of the third year as well as floating interest payments in USD at the end of years $1, 2$ and 3. The floating interest payments are computed as the annual interest earned on the principal and are computed using the continuously compounded floating one-year rates r_k^{US}, $k = 1, 2, 3$. Here r_k^{US} denotes the floating US one-year rate between year $k-1$ and k. The current exchange rate is 0.75 EUR/USD. Determine x such that the initial value of the currency swap is zero. (10 p)

Problem 4

In Ho-Lee's model the short interest rate r_k between time $k-1$ and k is modeled as

$$r_k = \theta_k + \sigma(z_1 + \cdots + z_{k-1}), \quad k \geq 1,$$

where $\theta_1, \theta_2, \ldots$ are parameters, σ is the volatility and $z_1, z_2, \ldots$ are independent standard normal random variables under the futures distribution. Let Z_k denote the price at time 0 of a zero-coupon bond, with face value 1, maturing at time k. Suppose $Z_1, Z_2, \ldots$ can be observed on the market and that there is no arbitrage. Show that, in order for the Ho-Lee model to be consistent with the market, the following relation must hold:

$$\theta_k = \ln\left(\frac{Z_{k-1}}{Z_k}\right) + \frac{\sigma^2}{2}(k-1)^2.$$

(10 p)

Problem 5

Consider a market that is free of arbitrage where all relevant contracts are traded. Let X be the price of one barrel of crude oil at a future time $T > 0$ and let N be the value of one kg of gold at time T. The gold distribution, P^{gold}, is such that

$$P^{\text{gold}}(A) = \frac{P_0^{(T)}[I_A N]}{P_0^{(T)}[N]},$$

for all events A observed at T. In particular

$$E^{\text{gold}}[Y] = \frac{P_0^{(T)}[YN]}{P_0^{(T)}[N]},$$

for all random variables Y observed at T. Show that $X_t = \frac{P_t^{(T)}[X]}{P_t^{(T}[N]}$, $t \geq 0$, is a P^{gold}-martingale. (10 p)

Figure 1: Standard Normal distribution

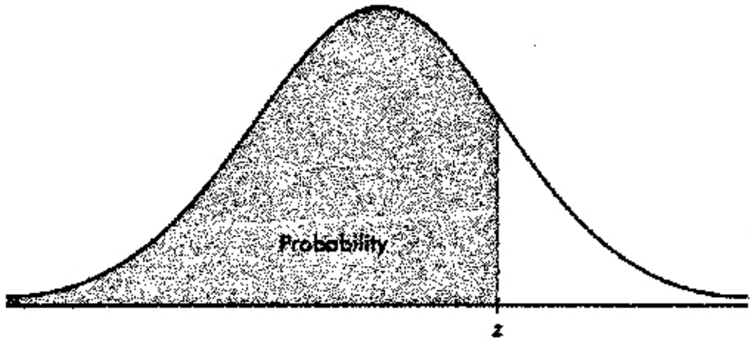

TABLE A: STANDARD NORMAL PROBABILITIES (CONTINUED)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998