KTH Mathematics

Exam in SF2701 Financial Mathematics.
Monday June 5 2017 8.00-13.00.

Examiner: Camilla Landén, tel 070-719 3938.

Aids: Calculator.

General instructions: The solutions should be legible, easy to follow and not lacking in explanations. In particular, all notation used should be explained.

N.B. Number the pages and write your name on each sheet. Write only one exercise per sheet and state the number of the exercise on the sheet.

To pass the exam you need 25 points. A chance to take a supplementary examination is given if you have 23 or 24 points (to take the supplementary examination you must contact Camilla Landén within two weeks from the posting of the results of the exam).

The Black-Scholes formula and a table of the cumulative distribution function of a standard normal can be found at the end of the exam.

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1. (a) A stock price is currently \$100. Over each of the next two three-month periods it is expected to go up by 5% or down by 5%. The risk-free interest rate is 3% per annum with continuous compounding. What is the value of a six-month European put option with a strike of \$100? You should use a two period binomial model to price the option. .... (5p)

(b) A compound option is an option written on an option. The payoff function of a European call on a call with strike K and maturity T is

$$X_{calloncall} = \max\{C(T, S_T) - K, 0\},$$

where $C(T, S_T)$ denotes the price of the underlying call option at time T (the maturity of this option is $T_1 > T$ and the strike is K_1). Similarly the payoff function of a European put on a call with strike K and maturity T is

$$X_{putoncall} = \max\{K - C(T, S_T), 0\}.$$

What put-call parity relationship exists between the price of a European call on a call and a European put on the same call? .... (5p)

2. (a) Compute the price of an American call option written on the futures price of crude oil. The current futures price of crude oil is \$50 for one barrel, and the volatility of the futures price is 15%. The maturity of the option is in four months and the strike price is \$48. The risk free interest rate with continuous compounding is 5% per annum. You should use a two period binomial model to price the option. . . . . (8p)

(b) An index futures is a futures contract written on a stock index (or some other index). Would it change anything in exercise 2a if the option was written on an index futures instead of the futures price of crude oil? If so, what? Please motivate your answer! . . . . (2p)

3. In this exercise all interest rates are quoted per annum with continuous compounding.

(a) Suppose that the spot price of the U.S. dollar is currently 0.89 EUR and that the EUR/U.S. dollar exchange rate has a volatility of 10% per annum. The risk-free interest rates in the Euro region and the United States are 4% and 6%, respectively.

i. Compute the forward exchange rate in eight months expressed as EUR/U.S. dollar (this is the same as the forward price of the exchange rate). . . (3p)

ii. Compute the price of a European call option to buy one U.S. dollar for 0.88 EUR in eight months. . . . . (3p)

(b) Suppose that you have just sold a European put option with a strike price of 225 SEK, and exercise date in six months, written on the stock Hennes & Mauritz series B (HMB). The stock currently trades at 230 SEK. The volatility of HMB is 20%, and the risk free interest rate is 1% per annum.

How many HMB stocks should you add to your portfolio in order to make it delta neutral? . . . . (4p)

4. (a) Suppose that the following bonds are currently trading in the market:

- A zero coupon bond with face value 100 and maturity one year trades at 99.00
- A coupon bond with face value 100, maturity two years, and a coupon of 3% per annum paid annually trades at 102.92
- A coupon bond with face value 100, maturity in three years, and a coupon of 2% per annum paid annually trades at 99.98

Suppose that two parties want to set up a forward rate agreement for the third year on \$1 million. What should they set the borrowing (lending) rate to? Quote the rate per annum with annual compounding. (5p)

(b) Suppose that a little more than 12 months ago a two year interest rate swap was set up. Under the terms of the swap 1.7% per annum (compounded semi-annually) is exchanged for six-month LIBOR on a principal of \$1.000.000 (payments are thus made every six months). Today's term structure is as given in exercise 4a, and by the fact that the six-month LIBOR/swap rate (with semi-annual compounding) is 1% per annum.

Given that the remaining life of the swap is (just under) 12 months compute the value of the swap today to the party paying fixed (and consequently receiving floating). .... (5p)

5. (a) Consider the following portfolio: a newly entered-into long forward contract with delivery date T on a stock (the forward contract has just been set up and if you own the portfolio, you own the forward contract), and a long position in a European put option on the stock with maturity T and a strike price that is equal to the forward price of the stock when the portfolio is set up (if you own the portfolio, you own the European put option).

i. Determine the payoff at time T of the portfolio just described. ...... (3p)

ii. Show that the price of the European put option described above is the same as that of a European call option with the same strike price and maturity, when the portfolio is set up. .... (2p)

(b) For this exercise consider the standard Black-Scholes model. A *gap option* is a T-claim X, such that $X = \phi(S_T)$, where the payoff function ϕ is defined by

$$\phi(s) = \begin{cases} s - K_1, & \text{if } s > K_2, \\ 0, & \text{otherwise.} \end{cases}$$

Here K_1 and K_2 are non-negative constants.

Determine the price of X today. Your answer may contain density functions and cumulative distribution functions of known distributions and the parameters of the problem: μ (local mean rate of return of the stock under P), σ, r, T, S_0, K_1, and K_2. .... (5p)

Hint: You may find it helpful to compare the payoff function to that of a standard option.

Good luck!

Hints:

You are free to use the following in any of the above exercises.

- The density function of a normally distributed random variable with expectation m and variance σ^2 is given by

$$\varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-m)^2/(2\sigma^2)}.$$

- Let Φ denote the cumulative distribution function for the $\mathrm{N}(0,1)$ distribution. Then

$$\Phi(-x) = 1 - \Phi(x).$$

- The standard Black-Scholes formula for the price $\Pi(t)$ of a European call option with strike price K and time of maturity T is $\Pi(t) = C(t, S(t))$, where

  $$C(t,s) = s\Phi[d_1(t,s)] - e^{-r(T-t)}K\Phi[d_2(t,s)].$$

  Here Φ is the cumulative distribution function for the $N(0,1)$ distribution and

  $$\begin{aligned} d_1(t,s) &= \frac{1}{\sigma\sqrt{T-t}}\left\{\ln\left(\frac{s}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)\right\}, \\ d_2(t,s) &= d_1(t,s) - \sigma\sqrt{T-t}. \end{aligned}$$

The table on the next page shows values of $\Phi(x) = P(X \leq x)$ for $X \in \mathrm{N}(0,1)$, i.e. Φ is the cumulative distribution function of the standard normal distribution. For negative values of x use that

$$\Phi(-x) = 1 - \Phi(x).$$