Ex 6 section 5.8.2 p 158

Assume that $\mathbb{E}[X^n] = \frac{1}{n+1}$ $\forall\, n = 1, 2, \dots$

What is the law of the random variable X?

$$\Psi_X(t) = \sum_{n=0}^{\infty} \mathbb{E}[X^n] \frac{t^n}{n!} \quad \text{(moment generating fct)}$$

$$= \sum_{n=0}^{\infty} \frac{t^n}{(n+1)!}$$

show that!

$$= \frac{1}{t}\left\{ e^t - 1 \right\}$$

$$= \int_0^1 e^{st}\, ds$$

it shows that $X \in U(0,1)$ $\square$

Ex 7 section 5.8.2 p 158

Assume that $\mathbb{E}[X^n] = c$ $\forall\, n = 1, 2, \dots$

What is the law of the random variable X?

$$\Psi_X(t) = 1 + \sum_{n=1}^{\infty} c \frac{t^n}{n!} = (1-c) + c\, e^t$$

We can directly see that $c \in [0,1]$ since $\Psi_X(t) \geq 0$ and that $X \in \text{Bernoulli}(c)$. We can check that:

$$\Psi_{\text{Bernoulli}(c)}(t) = \int e^{tx} \left\{ (1-c)\,\delta_{x=0} + c\,\delta_{x=1} \right\} dx$$

$$= (1-c) + c\, e^t$$

$\square$