

Ex 3 sect 5.8.3 p 159 (**)

$X_1, X_2, \dots \in N(0,1)$ i.i.d. and N is an independent random variable valued in \mathbb{N} .

Set $S_N = X_1 + \dots + X_N$. We know that the characteristic function of S_N is given by

$$\varphi_{S_N}(t) = g_N(\varphi_{N(0,1)}(t)) = g_N(e^{-t^2/2}) \quad (1)$$

Where g_N is the probability generating function of N .

Assume that $S_N \in N(0, \sigma)$ for some variance $\sigma > 0$. for the sake of finding a contradiction.

↑
it is straightforward to check that $E[S_N] = 0$
using the law of total probability

The assumption is equivalent to $\varphi_{S_N}(t) = e^{-\sigma t^2/2}$

$$(1) \Leftrightarrow u^\sigma = g_N(u) \quad (1)$$

• For $\sigma \notin \mathbb{N}_0$, equation (2) has no solution since the function $u \mapsto u^\sigma$ is not analytic at 0.

• If $\sigma \in \mathbb{N}_0$, (2) $\Leftrightarrow P[N=\sigma] = 1$
which contradicts the assumptions of the exercise.

cc) The assumption $S_N \in N(0, \sigma)$ for some $\sigma > 0$ is wrong.

□