Probability (SF2940) - Exercises

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1 Information

This is a list of exercises for the course SF2940. A subset of these will be solved during the exercise sessions and one or two exam questions might be chosen from here, so the student is recommended to try to solve all of the problems. Solutions will **not** be handed out per request. This list will be updated throughout the course, and the format is not complete at this time.

When referring to the course literature in this document, we write [K] for *Lecture Notes: Probability and Random Processes at KTH* by Timo Koski and [G] for *An Intermediate Course in Probability* by Allan Gut. If there is a difference in the numbering of the problem between different versions of [K], then the numbering of the 2014 edition will be written in a parenthesis.

2 Exercise session 1

1. Let Ω be countable and let

$$\mathcal{A} = \{A \subset \Omega : A \text{ is finite or } A^c \text{ is finite}\}.$$

   Show that $\mathcal{A}$ is an algebra. Show that it need not be a σ-algebra.

2. Let Ω be infinite and let $\mathscr{F}$ be a σ-algebra of its subsets. For $A \in \mathscr{F}$, define the set function

$$\mathbb{P}(A) = \begin{cases} 0 & \text{if } A \text{ is finite,} \\ \infty & \text{if } A \text{ is infinite.} \end{cases}$$

   Show that $\mathbb{P}$ is finitely additive but not countably additive.

3. Exercise 1.12.2.6 in [K].

4. Exercise 1.12.2.7 in [K].

5. Exercise 1.12.2.9 in [K].

6. Exercise 1.12.3.6 in [K].

7. Exercise 1.12.3.10 in [K].

3 Exercise session 2

3.1 Prioritized during session

1. Exercise 2.6.2.4 in [K].
2. Exercise 2.6.3.6 in [K].
3. Exercise 2.6.3.15 in [K].
4. Exercise 2.6.3.17 in [K]. **ERROR:** the answer to (b) should be
$$f_{X-Y}(v) = \frac{1}{2(1+|v|)^2},\ -\infty < v < \infty.$$
5. Exercise 2.6.3.20 in [K].

3.2 Recommended

1. Exercise 2.6.2.1 in [K].
2. Exercise 2.6.2.2 in [K].
3. Exercise 2.6.2.5 in [K].
4. Exercise 2.6.3.1 in [K].
5. Exercise 2.6.3.4 in [K].
6. Exercise 2.6.3.5 in [K].
7. Exercise 2.6.3.10 in [K].
8. Exercise 2.6.3.21 in [K].
9. Problem 1.24 in [G].

4 Exercise session 3

4.1 Prioritized during session

1. Exercise 2.6.5.2 in [K].
2. Exercise 3.8.3.5 in [K].
3. Exercise 3.8.3.6 in [K].
4. Exercise 3.8.3.10 in [K].
5. Exercise 3.8.3.14 in [K].

4.2 Recommended

1. Exercise 3.8.3.11 in [K].
2. Exercise 3.8.4.8 in [K].

5 Exercise session 4

5.1 Prioritized during session

1. Exercise 3.8.5.1 in [K].
2. Exercise 3.8.5.3 in [K].
3. Exercise 3.8.5.4 in [K].
4. Exercise 3.8.5.5 in [K].
5. Exercise 3.8.5.6(a) in [K].

5.2 Recommended

1. Exercise 3.8.5.2 in [K].

6 Exercise session 5

6.1 Prioritized during session

1. Exercise 4.7.1.2 in [K].
2. Exercise 4.7.1.3 in [K].
3. Exercise 4.7.1.6(a) in [K].
4. Exercise 4.7.1.7 in [K].
5. Let $X \sim \text{Ske}(\mu_1, \mu_2)$, that is, $X \stackrel{d}{=} N_1 - N_2$, where $N_1 \sim \text{Po}(\mu_1)$ and $N_2 \sim \text{Po}(\mu_2)$ are independent.

   (a) Show that

   $$\varphi_X(t) = e^{-(\mu_1+\mu_2)+\mu_1 e^{it}+\mu_2 e^{-it}}.$$

   (b) Find $\mathbb{E}[X]$ and $\text{Var}(X)$ using $\varphi_X(t)$.

   (c) Show that if $X \sim \text{Ske}(\mu_1, \mu_2)$ and $Y \sim \text{Ske}(\lambda_1, \lambda_2)$ are independent, then $X + Y \sim \text{Ske}(s_1, s_2)$ and $X - Y \sim \text{Ske}(t_1, t_2)$ for some parameters s_1, s_2, t_1, t_2 (no need to find them). Hint: use the characterization of Ske as the difference of Poisson random variables. (Reformulation of Exercise 4.7.1.12 in [K].)
6. Exercise 4.7.2.1 in [K].

6.2 Recommended

1. Exercise 4.7.1.1 in [K].
2. Exercise 4.7.1.5 in [K]. Hint: use the result of Exercise 4.7.1.4 in [K].
3. Exercise 4.7.1.6(b) in [K].
4. Exercise 4.7.1.8 in [K].

7 Exercise session 6

7.1 Prioritized during session

1. Exercise 5.8.1.4 in [K].
2. Exercise 5.8.1.5 in [K].
3. Exercise 5.8.2.6 in [K] (Exercise 5.8.2.5 in [K]).
4. Exercise 5.8.2.7 in [K] (Exercise 5.8.2.6 in [K]).
5. Exercise 5.8.2.8 in [K] (Exercise 5.8.2.7 in [K]).

7.2 Recommended

1. Exercise 5.8.1.2 in [K]. (Focus on parts (a) and (b). Exercise 5.8.1.1 gives the necessary information about the Pascal distribution.)
2. Problem 3.8.6 in [G].
3. Problem 3.8.8 in [G].
4. Problem 3.8.19 in [G].
5. Problem 3.8.21 in [G].

8 Exercise session 7

8.1 Prioritized during class

1. Exercise 6.8.1.3 in [K].
2. Exercise 6.8.1.15 in [K].
3. Exercise 6.8.1.16 in [K].
4. Exercise 6.8.1.17 in [K].
5. Exercise 6.8.2.1(a) in [K].
6. Exercise 6.8.2.7 in [K].
7. Exercise 6.8.3.1 in [K].

8.2 Recommended

1. Exercise 6.8.1.2 in [K].
2. Exercise 6.8.1.6 in [K].
3. Exercise 6.8.1.7 in [K].
4. Exercise 6.8.1.8 in [K].
5. Exercise 6.8.1.12 in [K].
6. Problem 6.8.13 in [G].

9 Exercise session 8

9.1 Prioritized during class

1. Exercise 8.5.1.3 in [K].
2. Exercise 8.5.1.10 in [K].
3. Exercise 8.5.1.13 in [K].
4. Exercise 8.5.1.15 in [K].
5. Exercise 8.5.1.16 in [K].
6. Exercise 8.5.1.17 in [K].

9.2 Recommended

1. Problem 5.10.3 in [G].
2. Problem 5.10.9 in [G].
3. Problem 5.10.10 in [G].