KTH Matematik

Avd. Matematisk statistik

ELECTIVE HOMEWORK 1 in SF2940 PROBABILITY THEORY

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Write your solutions on only one page of each sheet. You should define and explain your notation. Your computations and your line of reasoning should be written down so that they are easy to follow. You will not gain points by submitting an answer without corresponding computations. No copier prints will be accepted.

Staple your sheets of solutions together, with the homework cover sheet (downloadable in the course homepage) as uppermost. There can be only one student name on each submitted set of solutions.

The deadline of submission is Tuesday September 19, 2017 at 12.00 hours. It should be handed in the Box 'Matematik (SF)' to be found in Teknikringen 8D.

No electronic submission is permitted.

The homework will be graded and the graded solutions will be handed back no later than Friday October 14, 2016.

There are FOUR assignments in Homework 1. The maximum number of points awarded by each assignment is conferred next to it.

The bonus points gained will be valid **in the exam 25th of October, 2016, AND in the Re-exam 19th of December 2016.**

THE SCALE:
Bonus points in the exam −− graded points in the Homework 1.

0 for 0 – 9 points,
5 for 10 – 20 points.

Bonus points from Homework 2 will be added to the bonus points gained in Homework 1. Those who got 10 bonus points in the homeworks may skip Problem 1, whereas those who got 5 points may skip part (a) of Problem 1 in the exam.

Good luck!

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Problem 1 Let $(\Omega, \mathcal{F}, P)$ be a probability space and $\mathcal{A}$ a sub-σ-algebra of $\mathcal{F}$. Show that if X and Y are random variables with $E[Y|\mathcal{A}] = X$ and $E[X^2] = E[Y^2]$, then $X = Y$ almost surely. (5p)

Hint: show that $E[(X-Y)^2] = 0$.

Problem 2 Let X and Y be jointly distributed random variables such that

$$Y \mid X = x \in Bin(n, x) \quad \text{with} \quad X \in U(0,1),$$

where $Bin(n, x)$ stands for the Binomial distribution with parameters p, x and $U(0,1)$ is the Uniform distribution on the interval $(0,1)$.

Compute the characteristic function of Y and $Cov(X, Y)$ (without using the explicit distribution of Y). (5p)

Problem 3 Let X_1, X_2, X_3 and X_4 be independent and exponentially distributed with parameter λ random variables. Show that

$$Y_1 := X_1 + X_2 + X_3 + X_4,$$

$$Y_2 := \tfrac{X_1}{X_1+X_2},$$

$$Y_3 := \tfrac{X_1+X_2}{X_1+X_2+X_3},$$

$$Y_4 := \tfrac{X_1+X_2+X_3}{X_1+X_2+X_3+X_4}$$

are independent and determine joint distribution of (Y_1, Y_2, Y_3, Y_4). (5p)

Problem 4

(a) Show that if X is a positive random variable, then $E[X] = \int_0^{+\infty} P(X > t)dt$. (1p)

(b) If X takes values in $\{0, 1, 2, \ldots\}$, show that $E[X] = \sum_{j=0}^{+\infty} P(X > j)$. (1p)

Let F_X denote the probability distribution function of a continuous random variable X.

(c) Show that $E[X] = \int_0^{+\infty}(1 - F_X(t))dt - \int_{-\infty}^0 F_X(t)dt$. (2p)

(d) Derive a similar formula for $E[X^p]$, $p > 1$, when X is a positive random variable. (1p)

Hint: Note that $X = X^+ - X^-$, *where* $X^+ = max(X, 0)$, $X^- = max(-X, 0)$.