

EXAMINATION IN SF2942
PORTFOLIO THEORY AND RISK MANAGEMENT
2014-01-09, 14:00-19:00

The exam consist of 5 questions, with maximum 4 points for each question:
Already passed homework can replace question 1).
Passed Project gives you maximum 3 credit extra (to be added).

Credit scale:

A= at least 18 points, B= at least 16 points,
C= at least 13 points, D= at least 11 points,
E= at least 9 points, Fx= at least 8 points.

Total possible points are $23 = 20$ (p. from Exam) + 3 (p. from project).

Examiner: Henrik Shahgholian, with assistance of Sadna Sajadini.

Allowed technical aids: CALCULATOR.

1) Consider a market consisting of the five risk-free bonds shown in the table below. Show the market is free of arbitrage and determine the zero rates, or construct an arbitrage portfolio.

Bond	A	B	C	D	E
Price (\$)	98.51	100.71	188.03	111.55	198.96
Maturity (years)	0.5	1	1.5	1.5	2
Annual coupon (\$)	0	4	0	12	8
Face value (\$)	100	100	200	100	200

(Half of the annual coupon is paid every 6 months from today and including the time of maturity; the first payment is in 6 months.)

2) A life insurance policy pays 1 at the random time τ (at the end of the year of death of the policy holder) Suppose the current time is 0 and that τ takes values 1 or 2. Suppose also there are 1- and 2-year risk-free zero-coupon bonds available on market.

a) What is the value L of the insurance policy at time 1?

b) Determine the portfolio formed at time 0 that at time 1 best approximates L in the quadratic hedging sense.

3) A bank has written a contract that requires the bank to pay \$10,000 in 6 months from today. In return, the bank receives \$9,700 today and wants to invest this amount to manage the liability. There are two investment opportunities available: a long position in a 9-month zero-coupon bond and a deposit in an account that does not pay interest. A 9-month zero-coupon bond with a face value of \$10,000 costs \$9,510 today. The bank believes that the 3-month zero rate, per year, in 6 months from today is normally distributed with mean 6% and standard deviation 1.5%.

Determine the portfolio, among those whose initial value does not exceed \$9,700, that minimizes the variance of the value of the assets minus that of your liability 6 months from today, subject to the constraint that the expectation of the portfolio value in 6 months from today is nonnegative. Determine the expected value and the standard deviation of the value in 6 months of the optimal portfolio.

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4) Given are n uncorrelated assets with returns R_j , (identical) mean μ_0 and variances σ_j^2 ($j = 1, \dots, n$). Consider now an investment in these assets according to the trade-off-problem with trade-off-parameter $c > 0$ and initial capital V_0 .

- a) Determine the optimal monetary portfolio weights in terms of V_0 , c , μ_0 , and $\sigma_1, \dots, \sigma_n$.
- b) Determine the mean and variance of the optimal portfolio value.

5) Consider a future portfolio value X with finite variance and a risk measure $\rho(X) = -E[X/R_0] + 2\sqrt{Var(X/R_0)}$, where R_0 denotes the percentage return on a risk-free asset. A coherent risk measure satisfies the properties (T) translation invariance, (PH) positive homogeneity, (M) monotonicity and (S) subadditivity. For each of these properties, show that ρ satisfies the property or give a counterexample showing that it does not.

GOOD LUCK