KTH Matematik

EXAMINATION IN SF2942 PORTFOLIO THEORY AND RISK MANAGEMENT, 2011-10-18.

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Allowed technical aids: calculator.

Any notation introduced must be explained and defined. Arguments and computations must be detailed so that they are easy to follow.

GOOD LUCK!

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Problem 1

Explain how Principal Component Analysis can be used in the context of immunization of cash flows. Your explanation should include, but is not limited to, the following topics:

- a description of how to determine the principal components,
- a discussion of which, and how many, principal components to use,
- an explanation of where the principal components enter in the immunization procedure.

(10 p)

Problem 2

Explain the notion of "the efficient frontier" in the context of quadratic investment. Your explanation should include, but is not limited to, the following topics:

- a precise statement of what the efficient frontier is,
- a description of how the points on the efficient frontier, and the corresponding portfolios, can be determined,
- a picture of what the efficient frontier may look like.

(10 p)

Problem 3

A credit rating agency gives a credit rating to every large company and country. Suppose the credit rating can be: "Excellent", "Good", "Poor", or "Default". Consider a bond issued by the United States. The United States has credit rating "Good". In six months the United States will receive an updated credit rating. You have access to a market where you can buy the following contracts:

1. A contract which pays 10,000 EUR in six months if the US credit rating at that time is "Excellent". The price of the contract today is 1150 EUR.
2. A contract which pays 10,000 EUR in six months if the US credit rating at that time is "Very good". The price of the contract today is 8100 EUR.
3. A contract which pays 10,000 EUR in six months if the US credit rating at that time is "Good". The price of the contract today is 700 EUR.
4. A contract which pays 10,000 EUR in six months if the US credit rating at that time is "Poor". The price of the contract today is 50 EUR.

Your subjective probabilities for the US credit rating in six months are

$$P(\text{"Excellent"}) = 0.11, \quad P(\text{"Good"}) = 0.80,$$
$$P(\text{"Poor"}) = 0.08, \quad P(\text{"Default"}) = 0.01.$$

Determine how to optimally invest an initial capital of size 10,000 EUR in the four contracts if the objective is to maximize expected utility of your capital in six months, using a HARA utility function of the form

$$u(x) = \frac{1}{\gamma - 1}(\tau + \gamma x)^{1-1/\gamma},$$

with $\tau = 0$ and $\gamma = 2.5$. (10 p)

Problem 4

Consider a market where there is a risk free asset with return $R_0 = 1.05$ between time 0 and time 1. On the market there are two defaultable bonds. The issuers of the bonds can be assumed to default independently of each other. The bonds all have face value 100,000 EUR which is paid if the issuer of the bond does not default before time 1. The prices of the two bonds are equal

$$P_k = \frac{100000}{R_0}(1 - q), \quad k = 1, 2,$$

where $q = 0.025$ can be interpreted as the markets implied default probability. You believe that the market is overestimating the default probability. Your subjective default probability is $p = 0.024$. You have 1 million EUR that you can invest in the bonds and in the risk free asset.

(a) Determine the portfolio which maximizes your expected return given that the standard deviation of your portfolio does not exceed 25,000 EUR. You are not allowed to take short positions in the bonds or in the risk free asset.

(b) Determine the expected value and the standard deviation of the value at time 1 of the optimal portfolio in (a). (10 p)

Problem 5

Suppose you invest the amount $100,000$ EUR in each of the bonds in Problem 4 and you place the remaining $800,000$ EUR in the risk free asset.
(a) Compute (under your subjective probability) the Value-at-Risk at level 5% of the net worth $(V_1 - V_0 R_0)$ of your portfolio.
(b) Compute the Expected Shortfall at level 5% of the net worth $(V_1 - V_0 R_0)$ of your portfolio.
(c) Shortly after buying the portfolio a financial crisis breaks out and you realize that one of the bonds in your portfolio is a Greek government bond. You update the default probability to 0.91 for the Greek bond. The other bond is unaffected by the crisis and the default probability remains at 0.024. You can assume that the default events are independent. Compute Value-at-Risk and Expected Shortfall of your portfolio, at the level 5%. (10 p)