KTH Matematik

EXAMINATION IN SF2942 PORTFOLIO THEORY AND RISK MANAGEMENT, 2012-10-19.

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Allowed technical aids: calculator.

Any notation introduced must be explained and defined. Arguments and computations must be detailed so that they are easy to follow.

GOOD LUCK!

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- **All interest rates are given as yearly interest rate with continuous compounding.**
- **Black's formula for European call options:**

$$C_0 = B_0(G_0\Phi(d_1) - K\Phi(d_2)), \quad d_1 = \frac{\log(G_0/K)}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}, \quad d_2 = d_1 - \sigma\sqrt{T}.$$

Problem 1

Let V_0 and V_1 denote the value of a portfolio at times 0 and 1, respectively, and R_0 be the return of a risk free asset. Let $X = V_1 - V_0 R_0$ be the future net worth of the portfolio. For a monetary risk measure ρ the risk $\rho(X)$ can be interpreted as the amount of capital that needs to be added to the portfolio and invested in the risk free asset at time 0 to make the portfolio acceptable. Define, in mathematical terms, the following properties of a risk measure. In addition you should give a brief interpretation in words of each property. The properties are:

(a) Translation invariance

(b) Monotonicity

(c) Convexity

(d) Positive homogeneity

(e) Subadditivity

Bond	A	B	C
Price (SEK)	1035573	1055748	1061677
Maturity (months)	8	20	44
Annual coupon (%)	4.25	4.00	3.25
Face value (SEK)	1000000	1000000	1000000

Table 1: Bonds.

(10 p)

Problem 2

The bootstraping procedure is useful for determining the zero rates (the zero-rate curve) from prices of traded bonds. Explain how the bootstrapping procedure works and illustrate it by deriving the zero rates for the maturity times $8, 20, 32, 44$ months, using the bonds listed in Table 1. The bonds pay an annual coupon. (10 p)

Problem 3

Consider a futures contract and a forward contract on the value X of one barrel of crude oil at time T. Suppose there is a known interest rate that applies to any loans and deposits until time T.
Derive the delta hedge of a European put option on X with maturity T in terms of an amount on a risk-free bank account and a position in a futures contract on X. Use Black's formula. (10 p)

Problem 4

An expected-utility-maximizing investor has the opportunity to invest a capital of \$100 in the following digital options written on the value of the five-month zero rate in three months from now, $r_{3,8}$. In other words $e^{-r_{3,8}(5/12)}$ is the price, in three months from now, of a zero-coupon bond with face value 1 maturing in eight months from now. All the digital options mature in three months from now and pays \$100 if $r_{3,8}$ lies in the indicated range. The prices and ranges of the digital options are given in Table 2. The current price of a zero-coupon bond with maturity in three months and face value 1 is 0.9975.
The investor believes that $r_{3,8}$ follows a normal distribution with mean 1.00% and standard deviation 0.25%. Determine the investor's optimal portfolio if the investor uses a HARA utility with $\tau = 0$ and $\gamma = 4$. Recall that the HARA utility function can be written as:

$$u(x) = \frac{1}{\gamma - 1}(\tau + \gamma x)^{1-1/\gamma}.$$

A table of the normal distribution is given at the end of the exam. (10 p)

Option name	Price (\$)	Range
A	6.64	$r_{3,8} \leq 0.7$
B	49.25	$0.5 < r_{3,8} \leq 1.0$
C	77.28	$0.7 < r_{3,8} \leq 1.2$
D	34.05	$1.0 < r_{3,8} \leq 1.2$
E	49.26	$1.0 < r_{3,8} \leq 1.5$
F	15.83	$1.2 < r_{3,8}$

Table 2: Digital options.

Problem 5

A large Swedish bank lends $1.75 \cdot 10^9$ SEK to Swedish home owners. The lending rate that the home owners pay is usually changed every three months. Therefore the banks costs for financing the loans are usually determined on a three months horizon. To finance the loans the bank issues (sells) the bonds A and B in Table 1. This problem is to find out how much of each bond A and B to sell.

Let B_k, $k \geq 1$, be the price today of a zero-coupon bond with face value 1 which matures in k months from today. Let R_t be the three-month return associated with the t month zero-rate. That is, if $r_{3,3+t}$ is the t-months zero-rate in three months, then

$$R_t = \frac{e^{-r_{3,3+t}(t/12)}}{B_t}.$$

Suppose the vector $(R_5, R_{17})^{\mathrm{T}}$ has covariance matrix

$$10^{-5} \cdot \begin{pmatrix} 2.59 & 2.55 \\ 2.55 & 3.09 \end{pmatrix}.$$

Determine the amounts of bonds of type A and B to be sold to minimize the variance of the resulting portfolio subject to the constraint that the bank must sell bonds for at least $1.75 \cdot 10^9$ SEK.

Figure 1: Standard Normal distribution

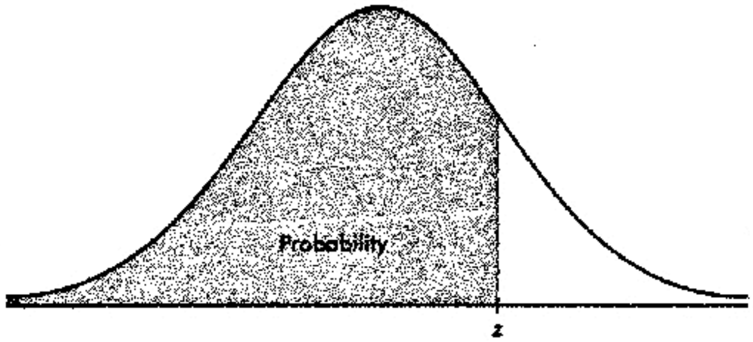

TABLE A: STANDARD NORMAL PROBABILITIES (CONTINUED)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998