KTH Matematik

EXAMINATION IN SF2942 PORTFOLIO THEORY AND RISK MANAGEMENT, 2013-01-09.

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Allowed technical aids: calculator.

Any notation introduced must be explained and defined. Arguments and computations must be detailed so that they are easy to follow.

GOOD LUCK!

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Problem 1

Consider a financial asset, with value S_T at a future time T, on which there are call and put options written with several different strikes, all of which maturity T. Explain the notion of an *implied forward distribution.* In particular you must explain:

(a) what assumptions that guarantee the existence of an implied forward distribution,

(b) how an implied forward distribution can be obtained from call option prices. (10 p)

Problem 2

Consider a time period from 0 to $T > 0$ and suppose there is a risk free asset with return R_0 over that period. A portfolio has known value V_0 at time 0 and random value V_T at time T. For the portfolio you must

(a) define the risk measure *Value-at-Risk* in terms of V_0, V_T, and R_0, (2 p)

(b) define the risk measure *Expected Shortfall* in terms of the Value-at-Risk, (2 p)

(c) express the *Expected Shortfall* in terms of the quantile function for the discounted loss $L = V_0 - V_T/R_0$, (3 p)

(d) show that *Expected Shortfall* is a positively homogeneous risk measure. (3 p)

Problem 3

Consider a car owner whose car is worth SEK 300000. The car owner has the opportunity to purchase a full coverage insurance for the duration of one year at the price SEK 6000. The probability of an accident during the year is 0.035. In case of an accident the value of the car is estimated to be reduced to SEK 150000 if the owner does not have insurance and the value of the car is fully recovered if the owner has insurance. The car owner is assumed be an expected-utility maximizer with a utility function of the form $u(x) = x^\beta$, $x > 0$, $\beta \in (0, 1]$. The car owner decides not to buy the full coverage insurance. Determine numerically the range of β-values that are consistent with that decision? (10 p)

Problem 4

Consider an investor who can invest in assets with uncorrelated returns, $R_1, \ldots, R_n$, each having identical means μ_0 but different variances, $\sigma_1^2, \ldots, \sigma_n^2$. The investor invests according to the optimal solution to the trade-off problem with trade-off parameter $c > 0$ and initial capital V_0.

(a) Determine the optimal portfolio $(w_1, \ldots, w_n)^T$ explicitly in the parameters V_0, c, μ_0, and $\sigma_1, \ldots, \sigma_n$.

(b) Determine the mean and the variance of the optimal portfolio.

(c) What is the efficient frontier in this case?

(10 p)

Problem 5

Let the time unit be months, that is, time k refers to k months from today. Let $B(k, n)$ denote the price at time k of a zero-coupon bond with face value 1 which matures at time n. Today's discount factors, $B(0, n)$, $n = 1, \ldots, 18$ are listed in Table 1.

Consider the following model (called Ho-Lee's model) for the one-month zero-rates: the zero-rate r_k from time $k-1$ to time k is given by

$$r_k = \log\left(\frac{B(0, k-1)}{B(0, k)}\right) + \frac{\sigma^2}{2}(k-1)^2 + \sigma(Z_1 + \cdots + Z_{k-1}),$$

where $Z_1, Z_2, \ldots$ are independent $\mathrm{N}(0, 1)$-distributed and Z_k is known at time k. The resulting model for the zero-coupon bond prices is

$$B(k, n) = \frac{B(0, n)}{B(0, k)} e^{-\frac{\sigma^2}{2}(n-1)(n-k)k} e^{-\sigma(n-k)(Z_1 + \cdots + Z_k)}.$$

An investor has a liability to deliver, in 7 months from today, a zero-coupon bond with face value 1 and maturity in 12 months. The investor wants to hedge the liability using a zero-coupon bond with face value 1 and maturity in 18 months and a cash deposit that does not pay interest. The volatility parameter is $\sigma = 0.00012$.

(a) Determine the quadratic hedge.

(b) Determine the price of the quadratic hedge and compare it to the current value of the liability.

(10 p)

Maturity (months)	1	2	3	4	5	6
Discount factor	0.9991	0.9980	0.9967	0.9953	0.9937	0.9921
Maturity (months)	7	8	9	10	11	12
Discount factor	0.9903	0.9885	0.9867	0.9848	0.9829	0.9810
Maturity (months)	13	14	15	16	17	18
Discount factor	0.9791	0.9772	0.9754	0.9737	0.9721	0.9705

Table 1: Discount factors.