BOOTSTRAP FOR TIME-SERIES

EX. AR(1)

$$Z_t = \beta Z_{t-1} + \varepsilon_t \qquad t = 1, 2, \dots$$

Z_0 GIVEN $\quad \boxed{|\beta| < 1}$

$\{\varepsilon_t\}$ WHITE NOISE $\quad E[\varepsilon_t \varepsilon_s] = \delta_{s,t} = \begin{cases} \sigma^2 & t = s \\ 0 & t \neq s \end{cases}$

$\varepsilon_t \in F$ (= UNKNOWN DISTRIBUTION)

OBSERVED TIME-SERIES $z_0, z_1, \dots, z_n$

LEAST SQUARES: MINIMIZE $Q(\beta)$

$$Q(\beta) = \sum_{t=1}^{n} (z_t - \beta z_{t-1})^2$$

$$\hat{\beta} = \frac{\sum_{t=1}^{n-1} (z_t - \bar{z})(z_{t+1} - \bar{z})}{\sum_{t=1}^{n} (z_t - \bar{z})^2} \quad \text{(yulwalk)}$$

$$\bar{z} = \frac{1}{n} \sum_{t=1}^{n} z_t$$

Q: 1) IS $\hat{\beta}$ BIASED?

2) WHAT IS $D(\hat{\beta})$ (= STANDARD ERROR)

A BOOTSTRAP TECHNIQUE:

$$\hat{\varepsilon}_t = z_t - \hat{\beta} z_{t-1} \quad t = 2, 3, \dots, n$$

$\hat{F}$: THE EMPIRICAL DIST'N OF THE $n-1$ RESIDUALS $\hat{\varepsilon}_t$

(OR OF ($\bar{\varepsilon} = \frac{1}{n-1} \sum_{t=2}^{n} \varepsilon_t$) $\hat{\varepsilon}_t - \bar{\varepsilon}$)

GENERATE BOOTSTRAP TIME SERIES

$$z_t^* = \hat{\beta} z_{t-1}^* + \varepsilon_t^* \quad , t = 2, 3, \dots, n$$

REPEAT B TIMES FOR $t = 2, \dots, n$

AND ESTIMATE $\hat{\beta}^*$ FOR EACH BOOTSTRAP TIME SERIES

- $\widehat{se}_{BOOT,B}$: estimate of $D(\hat{\beta})$