At the exam you should, among other things, be ready to give accounts of the following topics.

Random number generation

- Explain how to generate pseudo-random numbers using the inversion method. Prove that the output has the desired distribution. What are the pros and cons of this method?
- Explain how to generate pseudo-random numbers using rejection sampling. Prove that the output has the desired distribution. What are the pros and cons of this method?

Monte Carlo integration

- Explain—in full detail—the basic Monte Carlo sampler, the importance sampling algorithm, and self-normalized importance sampling.
- Prove that the basic Monte Carlo sampler satisfies a central limit theorem and provide an expression of the asymptotic variance. How does this apply to importance sampling? How can this be used for constructing confidence intervals? Explain how the variance of the estimator can be used for optimal design of the instrumental distribution for an importance sampling problem.

Variance reduction for Monte Carlo methods

- Describe in detail how the variance of the standard Monte Carlo sampler can be reduced using control variates. How can we design optimally the key parameter of the method?
- Describe the principle of antithetic sampling. In addition, prove a theorem that is useful in this context and give an example of an application.

Sequential Monte Carlo methods

- Formulate the sequential Monte Carlo problem and give two examples of applications. In addition, derive—in full detail—the sequential importance sampling (SIS) algorithm.
- Explain the pros and cons of the basic SIS algorithm and describe how SIS can be improved by means of an additional selection step (yielding the SISR algorithm). Prove that selection does not add bias to the estimator.

Markov Chain Monte Carlo (MCMC) methods

- Give accounts of the following: Markov chain, stationary distribution, ergodicity, and geometric ergodicity. Sketch the proof of a law of large numbers for ergodic Markov chains and state a central limit theorem.
- Explain the Metropolis-Hastings algorithm and prove that it allows the target distribution as stationary distribution.
- Explain the Gibbs sampler and the hybrid sampler. Motivate that the latter is a valid MCMC algorithm. Why is good mixing for MCMC samplers? What can be said in general when it concerns mixing for the MCMC samplers treated in the course?

Bootstrap

- Give account of parametric, semi-parametric, and non-parametric bootstrap.

The EM algorithm

- Give account of how to perform maximum likelihood estimation in latent data models using the EM algorithm. Motivate the method by establishing the EM inequality.