SF2972 Game Theory
Problem set on extensive form games

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There are five exercises, to be handed in at the final lecture (March 10). For a bonus point, please answer all questions; at least half of your answers must be correct. Do not hesitate to contact me with questions. If you want to draw game trees, I recommend using the ps-tree commands in the PSTricks package for LaTeX . Good luck! It was fun teaching you!

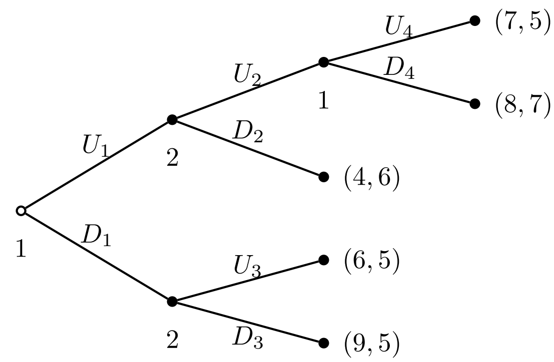
Exercise 1.

(a) Make exercise 103.2 from Osborne and Rubinstein.

(b) Find the subgame perfect equilibria if there is an upper bound $M \in \mathbb{N}$ on the numbers that the players can announce in the second round.

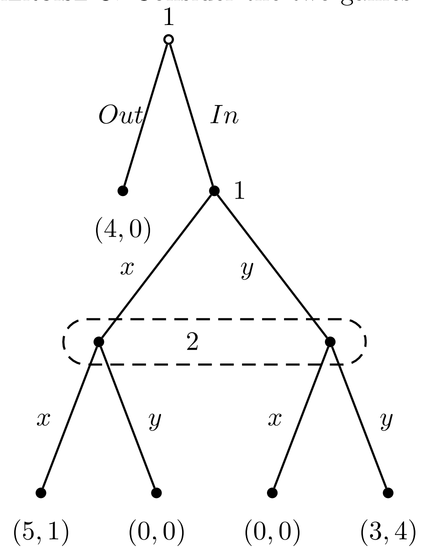
Exercise 2.

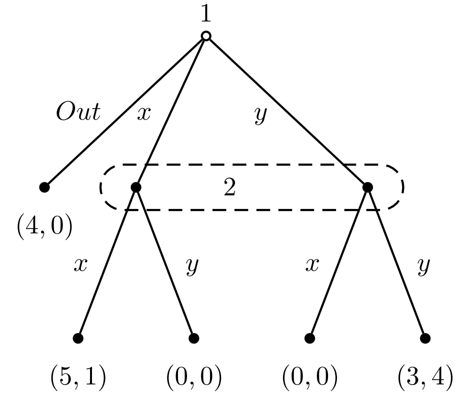
(a) Find all (pure) Nash and subgame perfect equilibria of the game below.



(b) Determine the corresponding strategic game. For each subgame perfect equilibrium, specify an order of iterated elimination of weakly dominated strategies that preserves the corresponding equilibrium. Can the same be done for the game's Nash equilibria?

EXERCISE 3. Consider the two games below.





(a) Do the games have the same *(i)* strategic form, *(ii)* reduced strategic form, *(iii)* agent strategic form?

(b) For both games, compute the set of sequential equilibria.

(c) Discuss the difference between the two games. What equilibrium (if any) do you find most appealing?

Exercise 4. In this exercise, we compare consistency with two other requirements on beliefs which I discussed informally during lecture 11.

A belief system μ in an extensive form game with perfect recall is ***structurally consistent***[1] if for each information set I there is a strategy profile β such that:

- I is reached with positive probability under β,
- $\mu(I)$ is derived from β using Bayes' rule.

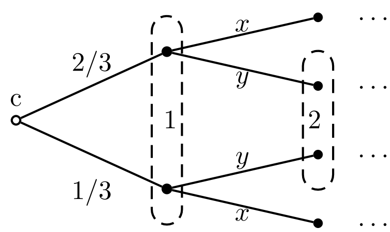
(a) In Osborne and Rubinstein, Figure 228.1, specify the set of structurally consistent belief systems.

(b) One could define a new equilibrium notion: an assessment (β, μ) that is sequentially rational and structurally consistent. Would that make sense?

An assessment (β, μ) is ***weakly consistent*** if for each information set I that is reached with positive probability under β, the beliefs $\mu(I)$ over its histories are derived from β using Bayes' rule.

Notice the difference with consistency: weak consistency imposes no constraints on beliefs over information sets that are reached with probability zero.

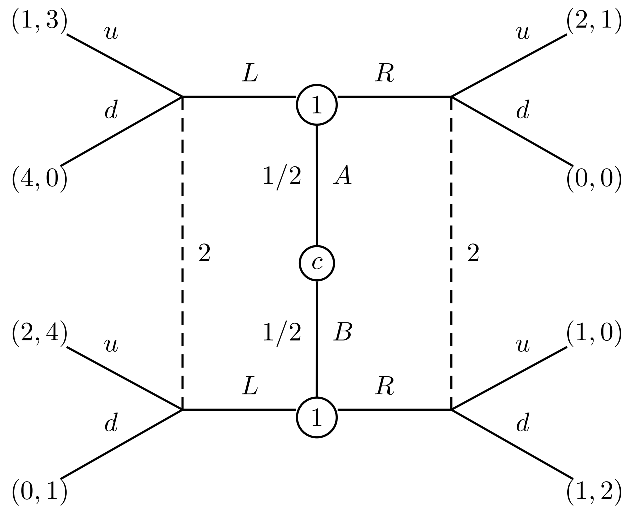
(c) Consider the information set I of player 2 in the game below. The "$\cdots$" are meant to indicate that whatever else happens in the game tree is irrelevant.



Which beliefs $\mu(I)$ are feasible under *(i)* consistency, *(ii)* structural consistency, *(iii)* weak consistency. Do you find all beliefs under weak consistency reasonable?

[1] See Osborne and Rubinstein, def. 228.1

Exercise 5. I skipped Section 12.3: it introduces unnecessary notation for special cases of extensive form games. This exercise goes through some of the essential parts.



A ***signalling game*** is played as follows (see the game above for a special case):

- Chance starts by assigning to player 1, the ***sender***, a type. In our example, the set of types is $\{A, B\}$ and each type has equal probability.
- Player 1 observes his type and sends a message to player 2, the ***receiver***. In our example, the set of messages is $\{L, R\}$.
- Player 2 observes the message of player 1 (but not 1's type) and chooses an action. In our example, the set of actions is $\{u, d\}$.
- The game ends. Payoffs can depend on types, messages, and actions.

This models a range of economic situations in which one player controls the information, but another player controls the actions. Sometimes the sender might benefit from trying to inform the receiver about his type, sometimes it is more beneficial to try to mislead the receiver.

Rather than searching for all sequential equilibria of a signalling game, it is common to make a number of simplifications: firstly, it is common to look at weakly perfect Bayesian equilibria in pure strategies (or, more precisely, behavioral strategy profiles where each player chooses in each information set a feasible action with probability one). Formally, an assessment (β, μ) is a ***weakly perfect Bayesian equilibrium*** if it is sequentially rational and weakly consistent.

Secondly, the sender may or may not be interested in sending information about his type: a weakly perfect Bayesian equilibrium in pure strategies is called a ***pooling equilibrium*** if he assigns to each type the same message and a ***separating equilibrium*** if he assigns to each type a different message. For instance, in a pooling equilibrium of our example, $\beta_1(A) = \beta_1(B)$, whereas in a separating equilibrium, $\beta_1(A) \neq \beta_1(B)$.

(a) Find all separating and pooling equilibria of the game above.

(b) Which of these equilibria are sequential equilibria?