SF 2972 GAME THEORY

Lecture 1

Introduction and simple examples

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1 What is game theory?

A mathematically formalized theory of strategic interaction between

— countries at war and peace, in federations and international negotiations

— political candidates and parties competing for power

— firms in markets, owners and managers, employers and trade-unions

— members of communities with a common pool of resources

— family members and generations who care about each other's well-being

— animals within the same species, from different species, plants, cells

— agents in networks: computers, cell phones, vehicles in traffic systems

2 A brief history of game theory

- Emile Borel (1920s): Small zero-sum games
- John von Neumann (1928): the Minimax Theorem
- von Neumann & Morgenstern (1944): Games and Economic Behavior
- John Nash (1950): Non-cooperative equilibrium [“A Beautiful Mind”]
- John Harsanyi (1960s): Incomplete information
- Reinhard Selten (1970s): Rationality as the limit of bounded rationality
- John Maynard Smith (1970s): Evolutionary stability

3 Two interpretations

In the Ph D thesis of John Nash (Princeton, 1950).

Definition: A *Nash equilibrium* is a strategy profile such that no player can unilaterally increase his or her payoff.

1. The rationalistic interpretation

2. The "mass action" interpretation

3.1 The rationalistic interpretation

1. The players have never interacted before and they will never interact in the future.

2. The players are *rational* in the sense of Savage (1954)

3. Each player *knows* the game in question

- However, this does not imply that they will play an equilibrium

- Common knowledge (CK)

Coordination:

	A	B
A	$2, 2$	$0, 0$
B	$0, 0$	$1, 1$

Matching pennies:

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

A game with a unique NE:

	L	C	R
T	$7, 0$	$2, 5$	$0, 7$
M	$5, 2$	$3, 3$	$5, 2$
B	$0, 7$	$2, 5$	$7, 0$

3.2 The mass-action interpretation

1. For each player role in the game: a large population of identical individuals

2. The game is recurrently played, in time periods $t = 0, 1, 2, 3, ...$ by randomly drawn individuals, one from each player population

3. Individuals learn from experience (own and/or others) to avoid suboptimal actions

- A mixed strategy for a player role is a statistical distribution over the actions available in that role

- Suppose that (a) individuals avoid suboptimal actions and (b) the population distribution of action profiles is stationary

- Reconsider the above examples in this interpretation!

4 Examples

4.1 A prisoners' dilemma

- Two fishermen, fishing in the same area

- Each fisherman can either fish modestly, M, or aggressively, A. The profits are

	M	A
M	3, 3	1, 4
A	4, 1	2, 2

- Both prefer (M, M), and both dislike (A, A)

- If each of them strives to maximize his or her profit, and they are both rational: (A, A)

- The First Welfare Theorem does not hold: competition leads to over-exploitation, not welfare maximum

- Would monopoly be better?

- What if the competitive interaction is repeated over time?

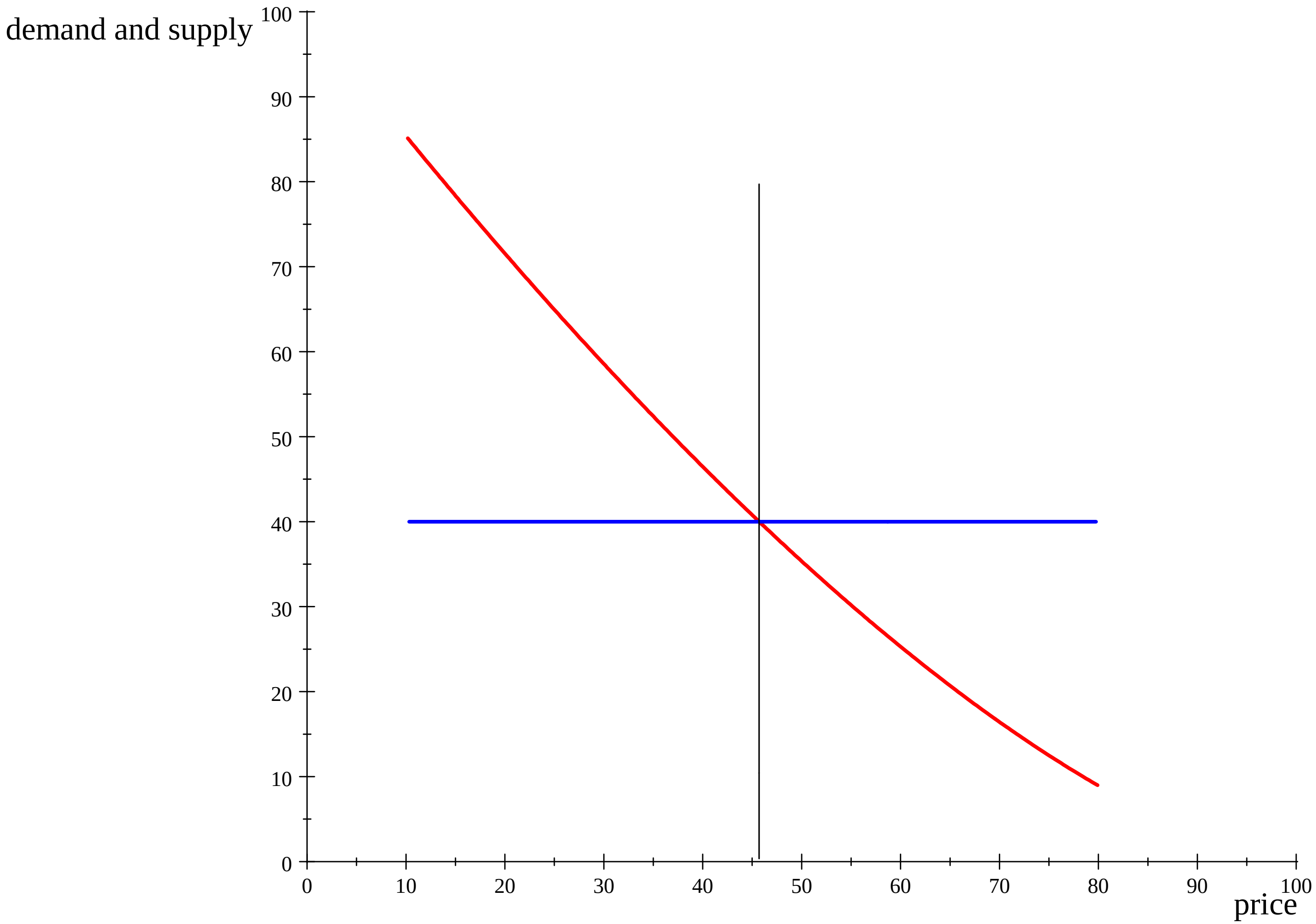
4.2 Market competition à la Cournot

- n firms competing in a homogeneous product market

Stage 1: simultaneously select output levels q_1, q_2, ..., q_n

Stage 2: market clearing: the price p given by $D(p) = Q$, where Q is aggregate supply,

$$Q = q_1 + ...q_n$$

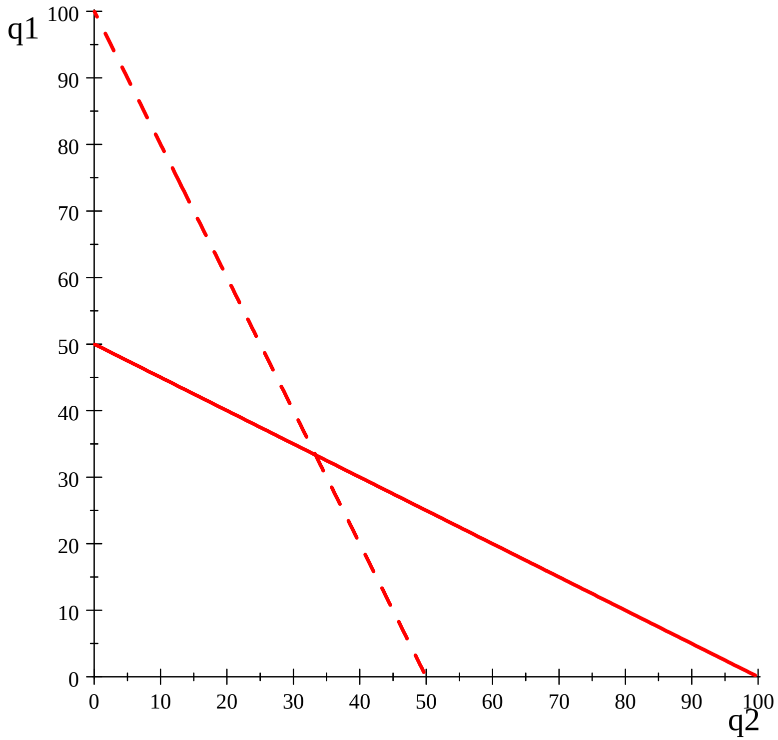


demand and supply
100
90
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
90
100
price

- Suppose *you are the manager of firm* i, that all firms have the same production costs, and the managers of all firms are rational profit maximizers who know that the others are rational too.

- What level of output, q_i, would you choose?

- Solution in the case of duopoly, $n = 2$:



- Cournot (1839)

- Suppose you are player 1 and you are rational, but you suspect that player 2 is not rational. What will you do?

- Suppose you know that the other player is not rational and will choose $q_2 = 50$. What will you do? Who will earn the highest profit of the two of you?

- Suppose both players are rational and can write a contract that dictates what quantity each shall produce (a cartel). Can they both do better than under competition à la Cournot? Will their aggregate output be larger or smaller than under competition? Will the resulting market price be lower or higher? [Compare with OPEC.]

Example: Partnerships (or "hawk-dove")

- Small start-up businesses, or pairs of students writing an essay

- Each partner has to choose between "contribute" ("work") and "free-ride" ("shirk")

  - If both choose C: *gain to both*

  - If one chooses C and the other F: *loss* to the first and *large gain* to the second

  - If both choose F: *heavy loss to both*

	C	F
C	$3, 3$	$-1, 4$
F	$4, -1$	$-2, -2$

- This is **not** a Prisoners' Dilemma: F does not dominate C: better to "work" if the other "shirks"

- Consider a large pool of potential partners, and random pairwise matching

- What do you think would happen? A tendency to play C? To play F?

- Could some population frequency of C (and thus also of F) be "stationary" or even "stable" in some sense?