SF2972 GAME THEORY
Lecture 1: Introduction and simple examples

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1 What is game theory?

A mathematically formalized theory of strategic interaction between

— countries at war and peace, in federations and international negotiations

— political candidates and parties competing for power

— firms in markets, owners and managers, employers and trade-unions

— members of communities with a common pool of resources

— family members and generations who care about each other's well-being

— animals within the same species, from different species, plants, cells

— agents in networks: computers, cell phones, vehicles in traffic systems

2 A brief history of game theory

- Emile Borel (1920s): Small two-player zero-sum games
- John von Neumann (1928): Two player zero-sum games of arbitrary size. Maximin Theorem
- von Neumann & Morgenstern (1944): Games and Economic Behavior, The Expected Utility Hypothesis
- John Nash (1950s): Non-cooperative equilibrium [Film: “A Beautiful Mind”]
- John Harsanyi (1960s-1980s): Incomplete information

- Reinhard Selten (1970s-today): Rationality as the limit of bounded rationality

- John Maynard Smith (1970s-1990s): Evolutionary stability of behaviors in large populations

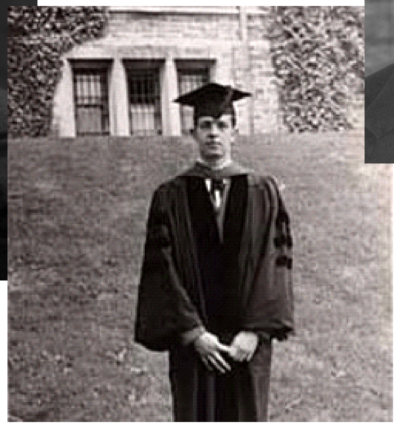
- Robert Aumann (1959-today), Thomas Schelling (1960-1990s), and many current game theorists

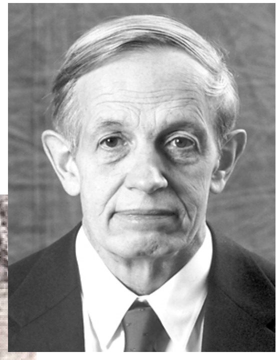
3 Two interpretations

- In 1950 **John Nash** (then Ph D student at Princeton Math Dept) invented *normal-form games* and an *equilibrium concept* for these, Nash equilibrium.
  - A *normal-form game* specifies a strategy set and a payoff function for each player, where each such function's domain is the set of all *strategy profiles*.
  - A *Nash equilibrium* is a strategy profile such that no player can increase his or her payoff by a unilateral deviation. Equivalently: a Nash equilibrium is a strategy profile that is a *best reply* to itself.

John Nash

(born 1928, PhD 1950)





Nash suggested two interpretations:

1. The rationalistic (or epistemic) interpretation [the by far most well-known and the foundation of modern economics]

2. The "mass action" (or evolutionary) interpretation [discovered in 1994 and more in line with sociology, biology and computer science]

3.1 The rationalistic interpretation

1. The players have never interacted before and will never interact in the future

2. The players are *rational* [Nowadays usually interpreted in the sense of Savage, *The Foundations of Statistics* (1954)]

3. Each player *knows* the game in question (knows all players' strategy sets and payoff functions)

- This does not imply that they will play a Nash equilibrium (For example, player i may believe that player j does not know i's payoffs)

- One nowadays assumes *common knowledge* (in the sense of Lewis, *Convention: A Philosophical Study* (1969), and as formalized in Aumann, *Agreeing to Disagree* (1976)] of the game and of all players' rationality

Coordination game:

	A	B
A	2, 2	0, 0
B	0, 0	1, 1

Zero-sum game:

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

Game with a unique NE:

	L	C	R
T	7, 0	2, 5	0, 7
M	5, 2	3, 3	5, 2
B	0, 7	2, 5	7, 0

3.2 The mass-action interpretation

1. For each player role in the game: a large population of identical individuals

2. The game is recurrently played, in time periods $t = 0, 1, 2, 3, ...$ by randomly drawn individuals, one from each player population

3. Individuals learn from experience (own and/or others) to avoid suboptimal actions

- A mixed strategy for a player role is a statistical distribution over the actions available in that role

- If all individuals avoid suboptimal actions, and the population distribution of action profiles is stationary, then it constitutes a Nash equilibrium

- Reconsider the above examples in this interpretation!

4 More examples

4.1 Hawk-dove game

- Start-up business with two partners, or a pair of students who are to write a joint thesis

- To *work* or *shirk*?

$$\begin{array}{ccc} & W & S \\ W & 3,3 & 0,4 \\ S & 4,0 & -1,-1 \end{array}$$

- Nash equilibrium? Your prediction under each prediction?

4.2 Prisoners' dilemma

- Two fishing companies, fishing in the same North Sea area

- Each company can either fish modestly, a, or aggressively, b. The profits are

$$\begin{array}{ccc} & a & b \\ a & 3,3 & 1,4 \\ b & 4,1 & 2,2 \end{array}$$

- If each company strives to maximize its profit, (b, b) will result (irrespective of what they believe about each other's goal function or rationality)

- Would monopoly be better? An agreement on fishing quota?

- The First Welfare Theorem does not hold: Competition leads to over-exploitation

- What if the interaction is repeated over time? One hundred periods? Infinitely many periods?

4.3 An even worse welfare failure

- Traffic (or network) equilibrium under congestion. Infinitely many players, a continuum, and nevertheless equilibrium may results in the worst possible outcome
  - All wish to travel from location A to location B. There are two roads from A to B.
  - Road 1 takes one hour, independent of traffic. Road 2 takes longer the more traffic there is. If everybody takes road 2, it takes 1 hour.
  - Each individual wishes to minimize his or her travel time. They make their choices simultaneously and leave A at the same time.
  - Define this as a game. What will they do in Nash equilibrium? Is there any worse outcome, in terms of total travel time?

- Suppose the travel time on road 2 is x when the population share x takes this road (for any given $x \in [0, 1]$).
  - In Nash equilibrium: $x = 1$.
  - What x-value would minimize total travel time?

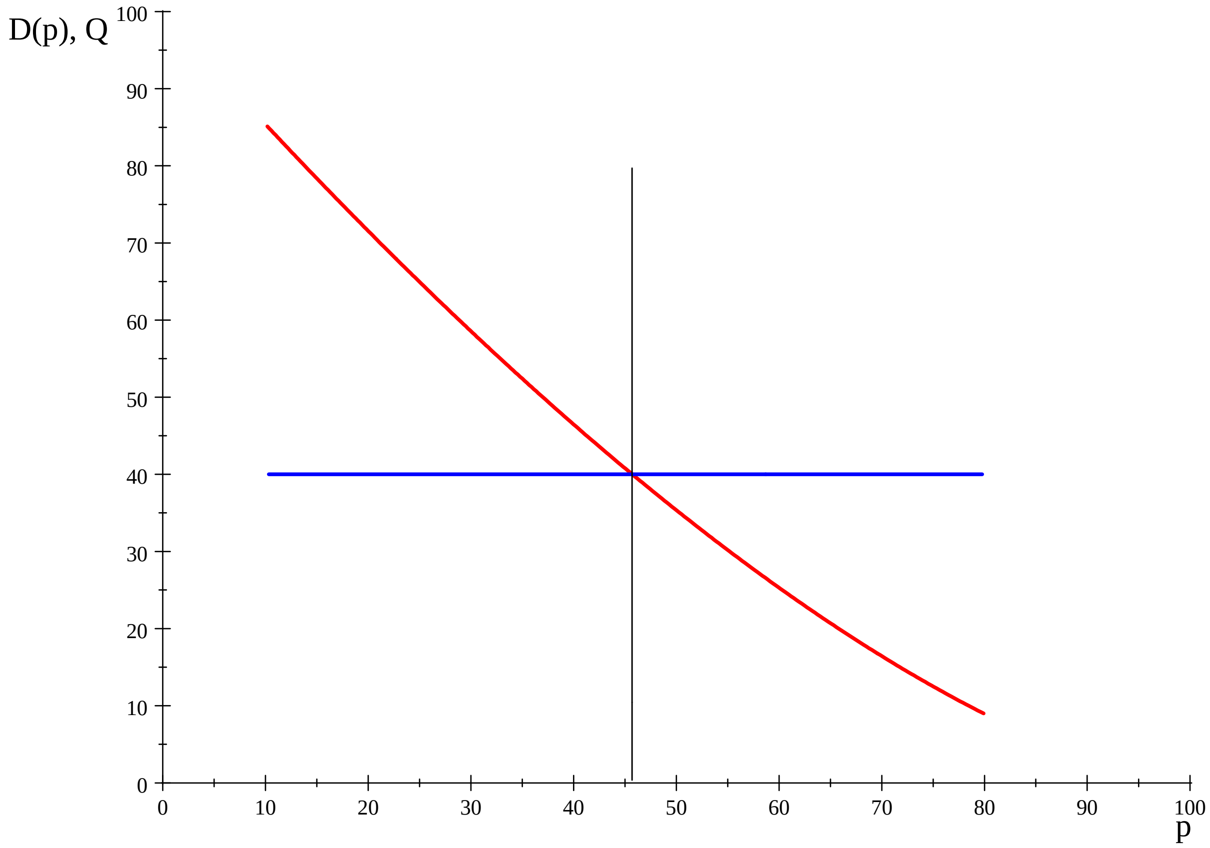
$$T(x) = x \cdot x + (1 - x) \cdot 1$$

4.4 Cournot market competition

- n firms competing in a homogeneous product market

Stage 1: Simultaneous choice of output levels q_1, q_2, ..., $q_n \geq 0$. Aggregate supply $Q = q_1 + ...q_n$

Stage 2: Market clearing price p, determined by $D(p) = Q$, where $D(p)$ is demand at price p


D(p), Q
100
90
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
90
100
p

- How represent this as a game? Players? Strategy sets? Payoff functions?

- Suppose *you are the manager of firm* i, that all firms have the same production costs, and it is common knowledge among the managers that they all are rational profit maximizers and know the game

- What level of output, q_i, would you choose if you were the manager of firm i?

- Consider the special case when $n = 2$, demand is linear, constant unit cost, c, of production!

5 Strategic interactions over time

In many, if not most, real-life interactions there is a time element; players may move after each other, acquire information etc. Then a strategy is a complex object, a "rule" that specifies what to do in any contingency that may arise. Hence, strategy sets may be very big.

5.1 Example: Sequential Cournot duopoly

- Suppose that the previous Cournot duopoly example takes place sequentially:
  - Stage 1: Firm 1 chooses q_1
  - Stage 2: Firm 2 observes q_1 and then chooses q_2
  - Stage 3: The market clears, $D(p) = q_1 + q_2$
- What are now the strategy sets of the two players? [Player 2 may now condition her quantity choice on 1's quantity choice]

- If you were manager of firm 1, how would you reason and what would you do?

- If you were manager of firm 2, how would you reason and what would you do?

- Will firm 1 (2) earn a higher or equal profit than in the case of simultaneous moves? Or the same profit as before?

- Such sequential Cournot duopoly games are called *Stackelberg leadership* games

6 Next lecture

- We will make precise the notions of *normal-form games* and *Nash equilibrium*, and we will establish a general existence theorem for such equilibria, and consider examples

- In that lecture, you are supposed to know the following properties of sets in Euclidean spaces: finite, open, closed, bounded, compact and convex. And you need to know what is a continuous function

- Please read ahead in the text-book!