

### GAME THEORY — PROBLEM SET 3

#### PROBLEM 1

We are playing a game of Nim (with the normal play convention) and my last move resulted in a position with five piles of sizes 26, 19, 10, 9, 7. Do you have a winning move from this position? If so, what is the maximum number of sticks you can remove in a winning move?

#### PROBLEM 2

Let  $G = \{\{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$  be an impartial game as represented in the lecture notes.

- (a) How many positions does  $G$  have?
- (b) Compute the Grundy value  $g(G)$ .
- (c) Who will win the game?

#### PROBLEM 3

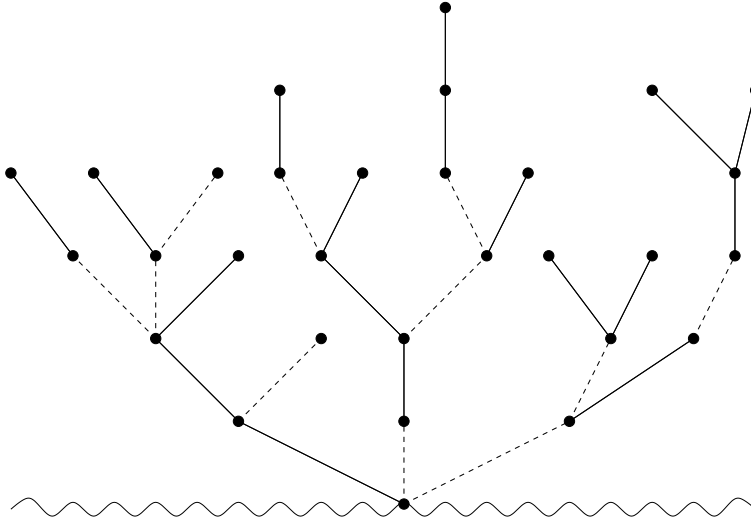
Show that the assumption that the game is short is necessary in the translation theorem.

#### PROBLEM 4

Show that the game  $\{0|*\}$  is positive but less than any positive number!

## PROBLEM 5

Compute the value of the following Blue-Red Hackenbush position. Who will win the game?



## PROBLEM 6

Let  $G = \{ \{7|5\}, \{10 || 5|3\} | \{1|0\} - \{7|0\} \}$ .

- Draw the thermograph of  $G$ .
- What is the temperature of  $G$ ?
- What is the mean value of  $G$ ?
- Who will win  $G$ ?
- Who will win the game  $7G - 20$ ?

## PROBLEM 7

Write the following short games on canonical form:

- $*2$ ,
- $\{ \{ *|0 \}, 0 || \{ *|0 \} | * \}$ .

## PROBLEM 8

Determine the value of  $\{ \frac{1}{8} | \frac{5}{8} \} + \{ -\frac{91}{64} | -\frac{41}{32} \}$ .