SF2972 GAME THEORY
Introduction

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1 What is game theory?

A mathematically formalized theory of strategic interaction between

— countries at war and peace, in federations and international negotiations

— political candidates and parties competing for power

— firms in markets, owners and managers, employers and trade-unions

— members of communities with a common pool of resources

— family members and generations who care about each other's well-being

— animals within the same species, from different species, plants, cells

— agents in networks: computers, cell phones, vehicles in traffic systems

2 A brief history of game theory

- Emile Borel (1920s): Small two-player zero-sum games

- John von Neumann (1928): Two player zero-sum games of arbitrary size

- von Neumann & Morgenstern (1944): Games and Economic Behavior

- John Nash (1950s): Non-cooperative equilibrium [Film: “A Beautiful Mind”]

- John Harsanyi (1960s-1980s): Incomplete information

- Reinhard Selten (1970s-today): Rationality as the limit of bounded rationality

- John Maynard Smith (1970s-1990s): Evolutionary stability of strategies

- Robert Aumann (1959-today), Thomas Schelling (1960-1990s), Erik Maskin (1980s-today), Roger Myerson (1978-today)

3 Two interpretations

In the Ph. D. thesis of John Nash (Princeton, 1950).

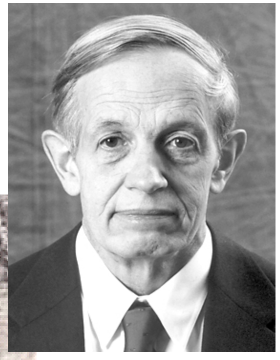
> **Definition**: A *Nash equilibrium* is a strategy profile such that no player can unilaterally increase his or her payoff.
>
> Equivalently: *a strategy profile that is a best reply to itself*

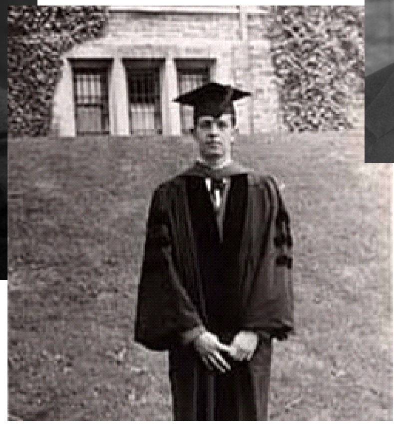
1. The rationalistic interpretation (prevalent in economics)

2. The "mass action" interpretation (more in line with sociology and biology)

John Nash

(born 1928, PhD 1950)





3.1 The rationalistic interpretation

1. The players have never interacted before and they will never interact in the future

2. The players are *rational* in the sense of Savage (1954)

3. Each player *knows* the game in question (knows all players' strategy sets and preferences or goal functions)

- However, this does not imply that they will play a Nash equilibrium

- Assume *common knowledge* (Lewis, 1969, Aumann, 1976) of the game and of all players' rationality. Contrary to what is often believed, this still does not imply equilibrium

Coordination game:

	A	B
A	2, 2	0, 0
B	0, 0	1, 1

Zero-sum game:

	H	T
H	1, −1	−1, 1
T	−1, 1	1, −1

Game with a unique NE:

	L	C	R
T	7, 0	2, 5	0, 7
M	5, 2	3, 3	5, 2
B	0, 7	2, 5	7, 0

3.2 The mass-action interpretation

1. For each player role in the game: a large population of identical individuals

2. The game is recurrently played, in time periods $t = 0, 1, 2, 3, ...$ by randomly drawn individuals, one from each player population

3. Individuals learn from experience (own and/or others) to avoid suboptimal actions

- A mixed strategy for a player role is a statistical distribution over the actions available in that role

- If all individuals avoid suboptimal actions, and the population distribution of action profiles is stationary, then it constitutes a Nash equilibrium

- Reconsider the above examples in this interpretation!

4 More examples

4.1 Hawk-dove games

- Start-up business with two partners

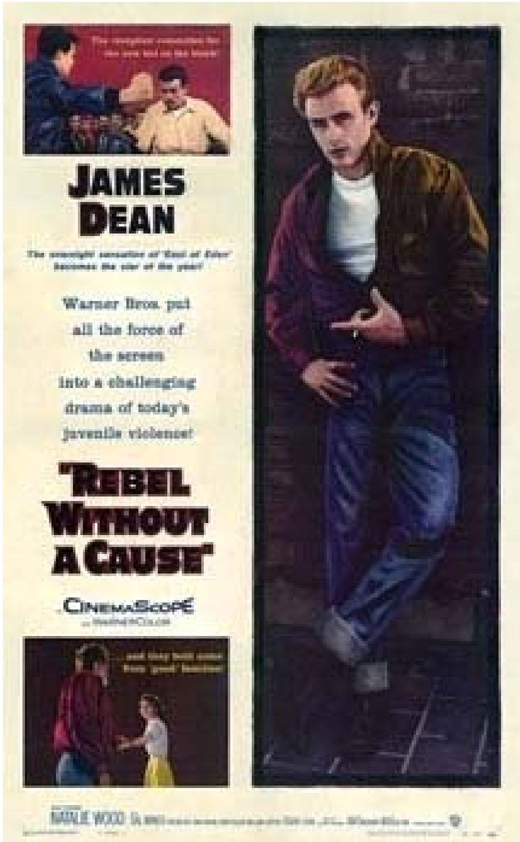
- Pairs of researchers or workers assigned a common task

To *work* or *shirk*?

	W	S
W	$3, 3$	$0, 4$
S	$4, 0$	$-1, -1$

What will happen?

- This game is also sometimes called "Chicken" [Film: "Rebel without a Cause"] or "Brinkmanship" (Bertrand Russell, about the cold war),



The Hawk-Dove game played in a large population of randomly matched pairs, with *no player roles assigned* to the participants

- A unique strategy that is a *best reply to itself*: randomize 50/50 between "work" and "shirk"

- This is an *evolutionarily stable strategy*, an *ESS* (Maynard Smith and Price, 1973)

4.2 Prisoners' dilemma

- Two fishing companies, fishing in the same area

- Each company can either fish modestly, a, or aggressively, b. The profits are

	a	b
a	$3, 3$	$1, 4$
b	$4, 1$	$2, 2$

- If each company strives to maximize its profit, (b, b) will result (irrespective of what they believe about each other's goal function or rationality)

- Would monopoly be better? An agreement on fishing quota?

- The First Welfare Theorem does not hold: Competition leads to over-exploitation

- What if the interaction is repeated over time? One hundred periods? Infinitely many periods?

4.3 An even worse welfare failure

- Infinitely many players, a continuum in which a individual's action has no influence on the aggregate, and nevertheless equilibrium play may result in the worst possible outcome

- Example in class: each individual faces a binary choice, to take road A or road B to a given destination. The travel time on road A is 1 hour. The travel time on road B is longer the more people take that road.

  – Let $x \in [0,1]$ be the population share that take road B, and let $T(x)$ be their (individual) travel time

  – Assume that $T : [0,1] \rightarrow \mathbb{R}$ is continuous and strictly increasing with $T(1) \geq 1$

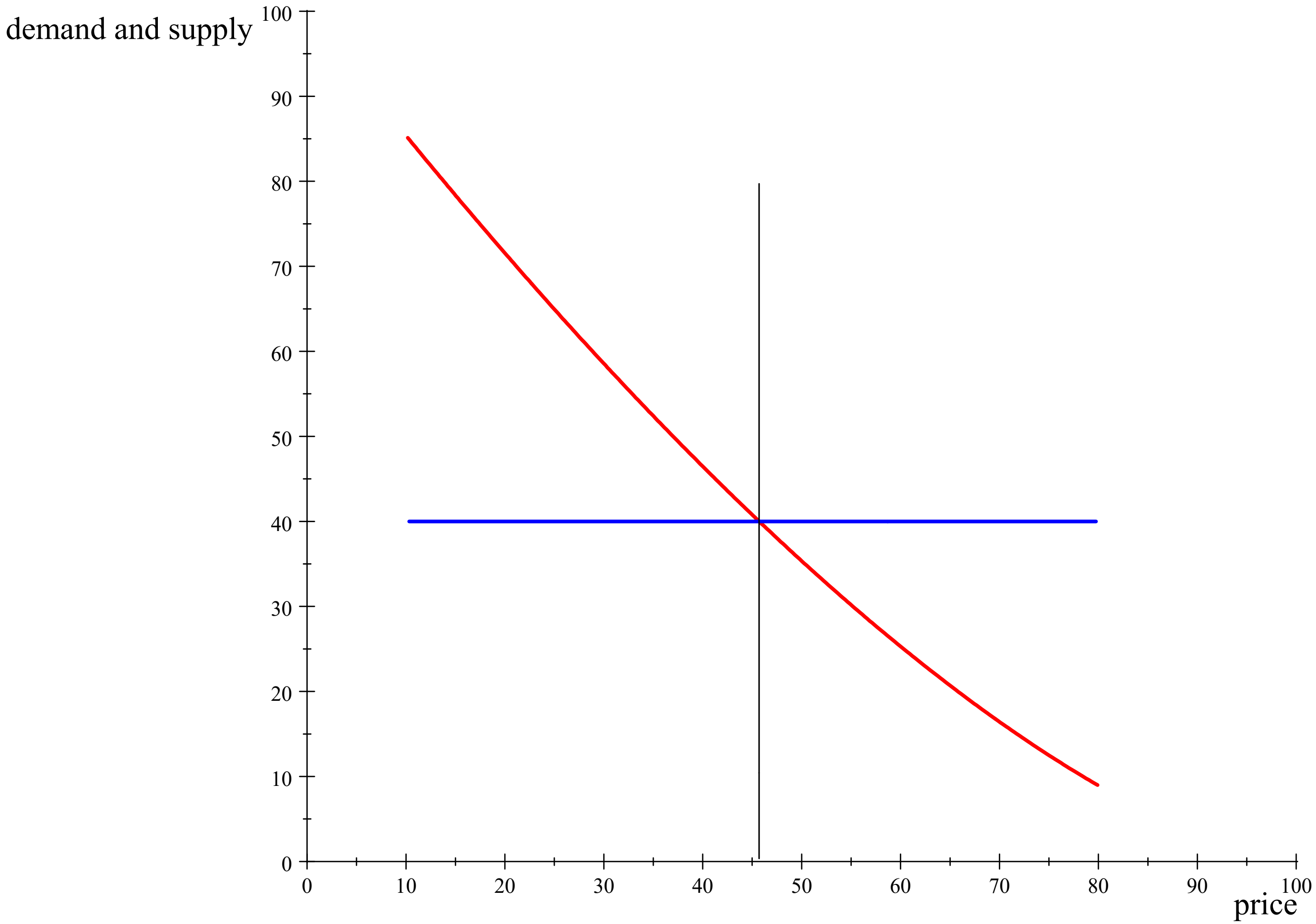
5 Cournot (1839) market competition

- n firms competing in a homogeneous product market

Stage 1: simultaneously select output levels q_1, q_2, ..., q_n

Stage 2: market clearing: the price p given by

$$D(p) = Q = q_1 + ...q_n$$


demand and supply
100
90
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
90
100
price

- Suppose *you are the manager of firm* i, that all firms have the same production costs, and the managers of all firms are rational profit maximizers who know that the others are rational too.

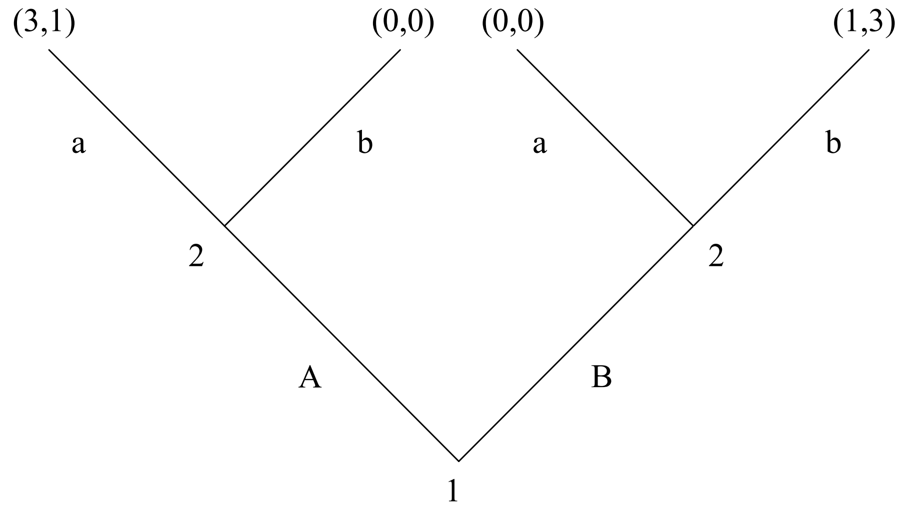
- What level of output, q_i, would you choose?

- Solution based on CK[game&rationality] in the case of duopoly, $n = 2$?

6 Informally about the extensive form

- Is more information always better?

- If you are one of the players in a two-player game, would you like to be informed about the other player's move before you make your move, or would you instead prefer that the other player is informed about your move before making his?



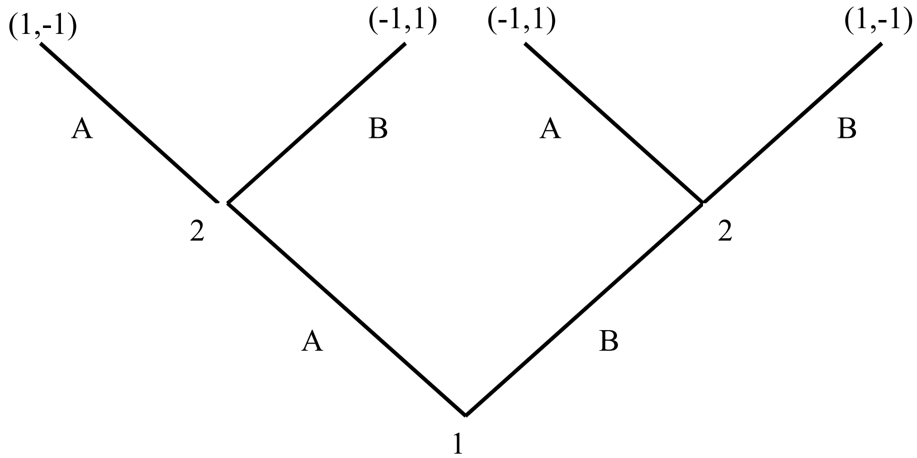
- Player 2 is informed about player 1's move, before 2 makes her move
- A game of perfect information
- Better to be uninformed, to be the first mover (a first-mover advantage)

- Are there games in which it is better to be informed, that is, with a *second*-mover advantage?

- A fox (player 1) and a rabbit (player 2), each choosing between two locations, A and B.

- If you were the rabbit (fox), would you like to choose first or second?

- If the fox chooses first:



7 Incomplete information

- In many strategic interactions, the actors know the "rules of the game" but not each others' preferences

- Such situations of *incomplete* information are modelled as games of *imperfect* information [Harsanyi (1967-8)]

- Create a "metagame" by introducing a neutral player, "nature", or "player 0", who makes a random draw from the set of possible games, one for each possible combination of preferences

- An extensive-form game with an initial random move by "nature".

Example 1: Product-market competition. Two competing firms, who know the *prior* probabilities, μ, for the possible cost constellations, (c_L, c_L), (c_L, c_H), (c_H, c_L), (c_H, c_H), and se Bayes' law to infer the *posterior* probability distribution for the other firm's cost, given their own cost,

$$\text{Pr}\left[\text{Firm 2's cost is } c_L \mid \text{1's cost is } c_L\right] = \frac{\mu\left(c_L, c_L\right)}{\mu\left(c_L, c_L\right) + \mu\left(c_L, c_H\right)}$$

Example 2: Signalling and coordination. Two equally likely states of nature, $\omega = A$ and $\omega = B$. Player 1 observes the state of nature and sends one of two messages, a or b, to player 2. Having received 1's message, 2 takes one of two actions, α or β. Both players receive payoff 2 if action α (β) is taken in state A (B), and otherwise both receive zero. Let "nature" ("player 0") first choose the state of nature. Then each player has 4 pure strategies. The normal form of this game is

	$\alpha\alpha$	$\alpha\beta$	$\beta\alpha$	$\beta\beta$
aa	$1, 1$	$1, 1$	$1, 1$	$1, 1$
ab	$1, 1$	$2, 2$	$0, 0$	$1, 1$
ba	$1, 1$	$0, 0$	$2, 2$	$1, 1$
bb	$1, 1$	$1, 1$	$1, 1$	$1, 1$

8 Social preferences

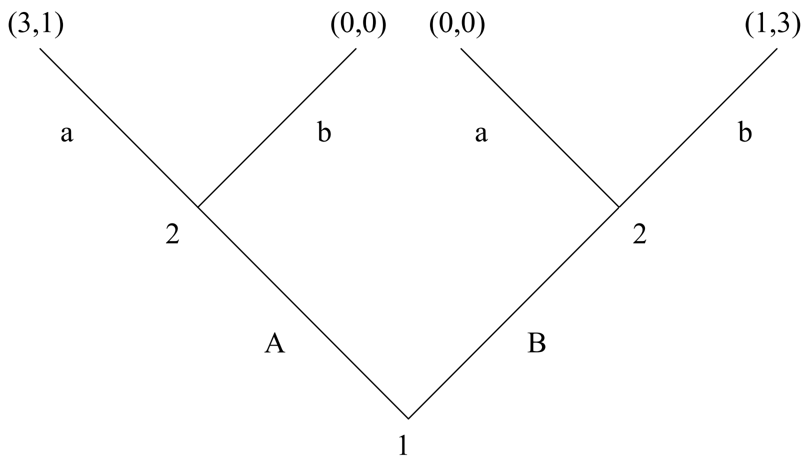
- Game theory does not presume that players are selfish

- It presumes they have *some* goal functions. For example: they may be
  - selfish
  - altruistic or spiteful
  - fairness-concerned or inequity averse
  - morally motivated, wanting to" do the right thing" (Immanuel Kant's categorical imperative)

9 Informally about the normal form

- For finite two-player games, the normal form can be summarized in the form of a bimatrix

- Reconsider the first extensive-form example!



- Its normal-form representation:

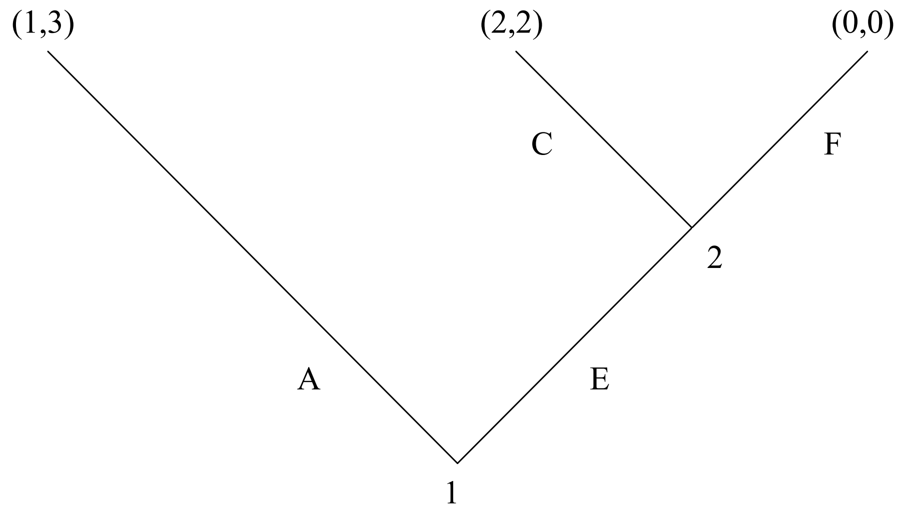
	aa	ab	ba	bb
A	3, 1	3, 1	0, 0	0, 0
B	0, 0	1, 3	0, 0	1, 3

- Player 1 has only 2 pure strategies while player 2 has 4. (And yet player 2 is worse off...)

10 Extensive forms with the same normal form

- Sometimes different extensive-form games have the same normal form

- An *entry-deterrence* game: A potential entrant (player 1) in a monopolist's (player 2) market

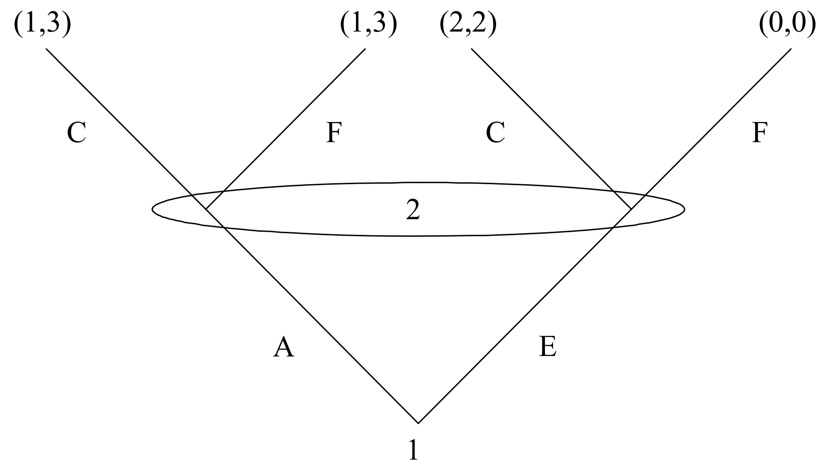


- Its normal form:

	C	F
A	1, 3	1, 3
E	2, 2	0, 0

  Two pure-strategy NE in this game: (A, F) and (E, C), but only the latter satisfies "backward induction" in the game tree

- Another extensive-form game with the same normal form:



- If players are rational, should the two extensive forms be deemed strategically equivalent? Lead to the same predictions?

11 What properties do we want a solution concept to have?

Q1: Should solutions only depend on the normal form?

Q2: Should solutions have some form of dynamic consistency and optimality in the extensive form?

Q3: What other invariance properties should solutions have?

Q4: Should solutions be robust to small amounts of irrationality and/or strategic uncertainty?

In this course we will examine a number of solution concepts with a keen eye on these questions!