KTH Matematik

Avd. Matematisk statistik

EXAM IN SF2972 GAME THEORY, Thursday June 8, 2017, 14.00-19.00.

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Tillåtna hjälpmedel/Permitted means of assistance: Inga/non.

Define all game-theoretic concepts you use and motivate your answers carefully.

Answers to the exam questions will be available at the course's homepage. Each exercise gives 10 points maximum.

Good luck!

Exercise 1

Consider the following two-player game G:

	A	B	C	D
A	$4,4$	$8,3$	$0,0$	$0,0$
B	$3,8$	$7,7$	$0,0$	$0,0$
C	$0,0$	$0,0$	$2,2$	$1,1$
D	$0,0$	$0,0$	$1,1$	$1,1$

1. Find all pure strategies that are strictly dominated (by a pure or mixed strategy), and for each such strategy (if such exist) specify the (pure or mixed) strategy by which it is dominated. (3 p)

2. Find all rationalizable pure strategies for each player. (3 p)

3. Find all pure-strategy Nash equilibria. (2 p)

4. Find all pure-strategy perfect equilibria. (2 p)

Exercise 2

Two individuals, $i = 1, 2$ each has to choose an effort level $x_i \in [0, 1]$, resulting in provision $x_1 + x_2$ of a public good, and in utilities

$$u_i(x_1, x_2) = (x_1 + x_2)\sqrt{1 - x_i}.$$

1. Suppose x_1 and x_2 have to be chosen simultaneously. Specify the associated normal-form (pure-strategy) game $G = (N, S, u)$. Find each player's best-reply correspondence, find the set of pure-strategy Nash equilibria, and illustrate your results in a diagram with x_1 on the horizontal axis and x_2 on the vertical axis. (3 p)

2. Suppose individual 1 has to select x_1 before individual 2 selects x_2, and assume that individual 2 observes x_1 without error, before 2 selects x_2. Find a subgame perfect equilibrium in pure strategies. Does individual 1 make more or less effort in this equilibrium than in equilibrium in (a)? (3 p)

3. Find the common effort level, $\hat{x} = x_1 = x_2$, that maximizes the sum of the individuals utility. Call this the *socially optimal* effort level. Suppose the simultaneous-move game G is repeated infinitely, with common discount factor $\delta \in (0, 1)$. Find a range of δ such that the socially optimal effort level is sustainable in subgame perfect equilibrium, and define a repeated-game strategy profile, based upon one period of mutual min-maxing, that constitute such an equilibrium. (4 p)

Exercise 3

Two pub owners, player 1 and player 2, must choose a location for their pubs in a long, straight street. Player 1 wants to open two new pubs in the street, but player 2 only one. The possible locations are A, B, and C, where A is at the beginning of the street, C at the end, and B somewhere in between. Between A and B there are 200 potential customers (uniformly distributed) and between B and C there are 300 potential customers (uniformly distributed). Each customer will go to the pub closest to his or her house.
First, player 1 chooses a location for his first pub. Next, player 2 observes player 1's choice and chooses a location for his pub. He must choose a different location than player 1. Finally, player 1's second pub is located at the remaining location. The objective for each pub owner is to maximize his or her number of customers.

(a) Formulate this problem as an extensive form game. How many pure strategies does player 1 have? And player 2? (5 p)

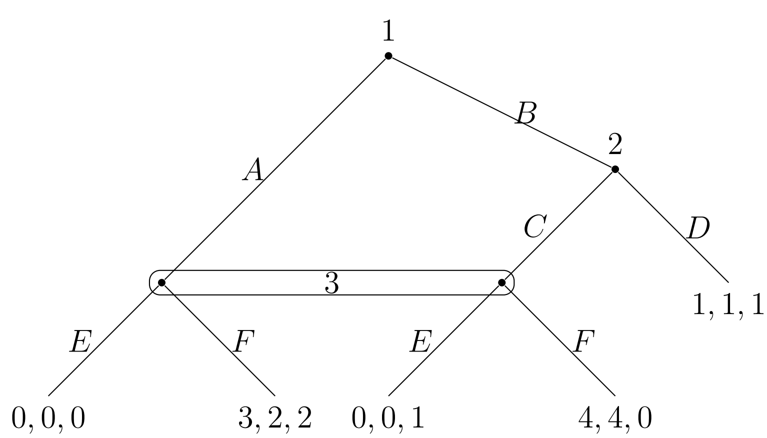
(b) Find all subgame perfect Nash equilibria in pure strategies. (5 p)

Exercise 4

In the three-player extensive-form game below, find (if any) the sequential equilibria where:

(a) player 2 chooses C with probability one. (5 p)

(b) player 2 chooses both C and D with strictly positive probability. (5 p)



Exercise 5

Consider a marriage problem with four men and four women.

(a) Suppose all women have the same preferences over men: m_1 is best, then m_2, then m_3, finally m_4. Describe the stable matching(s). (4 p)

In the remainder of the exercise, suppose the ranking matrix is:

	w_1	w_2	w_3	w_4
m_1	$3,2$	$4,4$	$1,2$	$2,1$
m_2	$4,1$	$2,3$	$1,1$	$3,3$
m_3	$2,3$	$3,1$	$1,3$	$4,4$
m_4	$2,4$	$3,2$	$4,4$	$1,2$

(b) Find stable matchings using the men-proposing and women-proposing deferred acceptance algorithms. (3 p)

(c) Is there a stable matching where m_1 and w_2 are coupled? (3 p)

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Suggested answers to the exam in SF2972 Game Theory, Thursday June 8, 2017, 14.00-19.00.

Exercise 1

1. The game is symmetric, so any strategy that is dominated for one player is also dominated for the other player. Strategy B is strictly dominated by many mixed strategies that put much probability on A and some on C or D, for example, $x_1^* = (9/10, 0, 1, 0)$.
2. After strategies B have been eliminated, no remaining strategy is strictly dominated. Hence, the iterated elimination of strictly dominated strategies results in the subset $\{A, C, D\}$ for each player. Since the game is a two-player game, this is also each player's set of rationalizable strategies.
3. (A, A), (C, C) and (D, D).
4. The first two NE are strict, hence perfect. The third is using a weakly dominated strategy and is hence not perfect.

Exercise 2

1. Each player's payoff function is strictly concave in his/her own strategy.

   Necessary FOC for player 1:

   $$\frac{\partial u_1(x)}{\partial x_1} = \sqrt{1-x_1} - \frac{x_1 + x_2}{2\sqrt{1-x_1}} = 0$$

   or

   $$2\left(1 - x_1\right) = x_1 + x_2 \Leftrightarrow x_1 = \frac{2 - x_2}{3}$$

   Hence, unique NE $x_1^* = x_2^* = 1/2$.

2. Individual 2 has to play her best response after any choice of x_1, so SPE is obtained by choosing x_1 so that is maximizes

   $$(x_1 + \frac{2 - x_1}{3})\sqrt{1 - x_1} = \frac{2}{3}\left(1 + x_1\right)\sqrt{1 - x_1}$$

   which yields $x_1 = 1/3$ and $x_2 = 5/9$. The public good in sequential SPE is lower than in simultaneous-move NE.

3. A socially optimally effort level $\hat{x}$ has to meet the first-order condition

$$2\sqrt{1-x} - \frac{x+x}{2\sqrt{1-x}} = 0$$

and hence $\hat{x} = 2/3$, resulting in individual utility $\hat{u} = 4/\left(3\sqrt{3}\right) \approx 0.77$, which is higher than the equilibrium utility, $u^* = 1/\sqrt{2} \approx 0.70$. Each player's minmax strategy against the other is to do nothing, $x^o = 0$, resulting in the other party's utility $v^o = 2/\left(3\sqrt{3}\right) \approx 0.38$. Mutual minmaxing results in utility zero to both. Consider play of $(\hat{x}, \hat{x})$, supported by the threat of mutual minmaxing for $L = 1$ period, followed by return to business as usual". This strategy pair is SPE iff

$$\hat{u} \geq (1-\delta)\, v^o + \delta\hat{u} \quad \text{and} \quad 0 + \delta\hat{u} \geq (1-\delta)\, v^o + 0 + \delta^2\hat{u}$$

or, equivalently, iff $\delta\hat{u} \geq v^o$, or, equivalently, $\delta \geq 1/2$.

Exercise 3

See review problem 10 in the course book.

Exercise 4

This exercise complements a similar question from the main exam, where you were asked for sequential equilibria where D was chosen with probability one. There are no other equilibria:

(a) C is optimal for player 2 if 3 chooses F with probability $p \geq 1/4$. Given such strategies of 2 and 3, player 1's unique best reply is B. So player 3 must assign probability 1 to being in the right node of the information set, where F is not a best reply, contradicting that it is chosen with positive probability.

(b) Player 2 must be indifferent between C and D, so player 3 chooses E with probability 3/4. Player 3 must be indifferent between E and F, so he believes to be in the left node of the information set with probability 1/3. Since both nodes are assigned positive probability, player 1 must use both A and B with positive probability. However, given the strategies of 2 and 3, A gives expected payoff 3/4, and B gives expected payoff 1: A is not a best reply.

Exercise 5

See review problem 47 in the course book.