KTH Matematik

Avd. Matematisk statistik

EXAM IN SF2972 GAME THEORY, Friday March 17, 2017, 08.00-13.00.

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Tillåtna hjälpmedel/Permitted means of assistance: Inga/non.

Define all game-theoretic concepts you use and motivate your answers carefully.

Answers to the exam questions will be available at the course's homepage.

Good luck!

Exercise 1

Consider two firms in price competition in a market for a homogeneous good. The firms simultaneously set their prices p_i. All customers buy from the firm with the lowest price. If both firms set the same price, then each firm receives half the market demand at that price. Both firms produce, upon demand, at the same unit cost, $c = 1$ (and have no fixed costs). Let market demand be $D(p) = 5 - p$ for all prices $p \in [0, 3]$. Hence, the profit for a firm with a higher price than the competitor is zero, the profit to a firm with a lower price than the competitor is market demand times the profit per unit sold, $p - c$, and the profit for a firm with the same price as the competitor is half the market demand times the profit per unit sold, $p - c$.

(a) Define the associated normal-form game $G = \langle I, S, \pi \rangle$. (Please be careful since the rest of the task will depend on this.) Are the payoff functions π_i continuous? Do best replies always exist? (2 p)

(b) Find all Nash equilibria in pure strategies in game G. (2 p)

(c) Does there exist any undominated Nash equilibrium in pure strategies in game G? (2 p)

Exercise 2

Suppose that prices in the preceding exercise are constrained to integers, more precisely: $p_i \in P = \{0, 1, 2, 3\}$, for players $i = 1, 2$.

(a) Write up the associated normal-form game $\hat{G}$ as a bimatrix. (Please be careful since the rest will depend on this!) (3 p)

(b) Find all strictly dominated pure strategies, and all weakly dominated pure strategies in $\hat{G}$. (2 p)

(c) Find all rationalizable pure strategies in $\hat{G}$. (1 p)

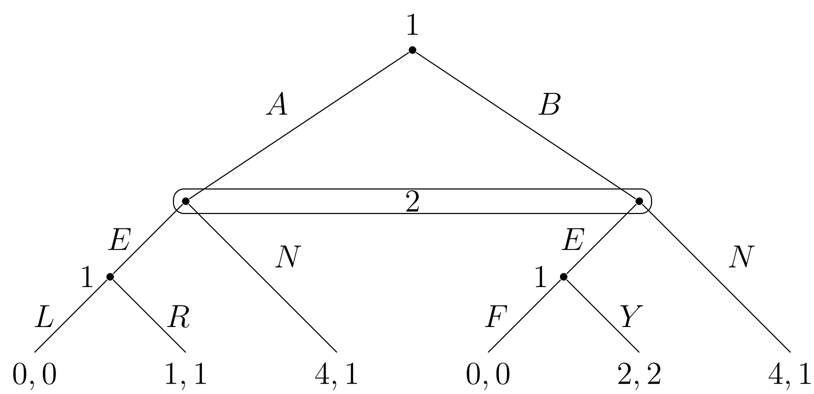
(d) Find all pure-strategy Nash equilibria and all pure-strategy perfect equilibria in $\hat{G}$. (2 p)

(e) Suppose that the game $\hat{G}$ is played exactly twice, in time periods 1 and 2, and that each firm strives to maximize the sum of its profits in the two periods. Suppose that both firms can observe each other's first-period prices before they set their second-period prices (they act simultaneously in each time period). What is the highest total profit each firm can obtain in subgame perfect equilibrium? (2 p)

(f) Suppose that the game $\hat{G}$ is infinitely repeated, and that each firm strives to maximize the present value of its profit stream, when both firms discount future profits by the same factor $\delta \in (0, 1)$. What does the one-shot deviation principle say? What does the Fudenberg-Maskin folk theorem say? Draw a picture illustrating the theorem, define minmax strategies and the minmax point of the game $\hat{G}$, and describe the strategies used in the proof of the Fudenberg-Maskin theorem. (2 p)

Exercise 3

Consider the following extensive form game:

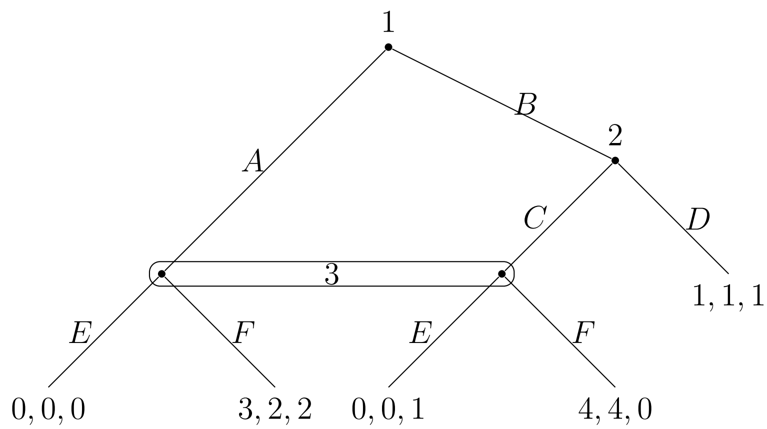


(a) How many pure strategies does player 1 have? And player 2? Is the pure-strategy profile where 1 plays (B, L, Y) and 2 plays E a Nash equilibrium? Is it a subgame-perfect equilibrium? (3 p)

(b) Find all subgame-perfect equilibria in pure strategies. (3 p)

Exercise 4

In the three-player extensive-form game below, find (if any) the sequential equilibria where player 2 chooses D with probability one. (5 p)



Exercise 5

Consider the marriage problem with four men, four women, and the following ranking matrix:

	w_1	w_2	w_3	w_4
m_1	$3,2$	$4,4$	$1,2$	$2,1$
m_2	$4,1$	$2,3$	$1,1$	$3,3$
m_3	$2,3$	$3,1$	$1,3$	$4,4$
m_4	$2,4$	$3,2$	$4,4$	$1,2$

(a) Show: matching $\{(m_1, w_1), (m_2, w_3), (m_3, w_2), (m_4, w_4)\}$ is not stable. (1 p)

(b) Find a stable matching using the men-proposing deferred acceptance algorithm. (1 p)

(c) If the men-proposing deferred acceptance algorithm is used, can man m_4 find a better partner by lying about his preferences? Can woman w_1?

(2 p)

(d) How many stable matchings are there in this marriage problem? (2 p)

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Suggested answers to the exam in SF2972 Game Theory, Friday March 17, 2017, 08.00-13.00.

1. (a) The answer to both questions is no". Here $I = \{1, 2\}$, $S_1 = S_2 = [0, 3]$ and, for $i = 1, 2$ and $j \neq i$ the payoff functions are

$$\pi_i(p_1, p_2) = \begin{cases} 0 & \text{if } p_i > p_j \\ \frac{1}{2}(5 - p_i)(p_i - 1) & \text{if } p_i = p_j \\ (5 - p_i)(p_i - 1) & \text{if } p_i < p_j \end{cases}$$

   (b) The game has a unique NE (Nash equilibrium), in which both firms price at marginal cost, $p_1 = p_2 = c = 1$

   (c) The answer is no", the strategies used in the unique NE are weakly dominated

2. (a)

	0	1	2	3
0	$-5/2, -5/2$	$-5, 0$	$-5, 0$	$-5, 0$
1	$0, -5$	$0, 0$	$0, 0$	$0, 0$
2	$0, -5$	$0, 0$	$3/2, 3/2$	$3, 0$
3	$0, -5$	$0, 0$	$0, 3$	$2, 2$

   (b) Strategy 0 is strictly dominated by strategy 1. Strategy 1 is weakly dominated by strategy 2. Strategy 3 is weakly dominated by strategy 2.

   (c) Strategies 1, 2 and 3 are rationalizable, and strategy 0 is not.

   (d) There are two NE: $(1, 1)$ and $(2, 2)$. The first is weakly dominated and hence not perfect, the second is strict and hence perfect.

   (e) The answer is: $7/2$. This total payoff is obtained in SPE if both players use the following strategy: first play 3, then play 2 if both players played 3 in the first period, otherwise play 1. The equilibrium payoffs for a firm are then 2 in period 1, and $3/2$ in period 2.

   (f) See Lecture Notes. This game $\hat{G}$ has two minmax strategies: strategy 0 and strategy 1, both forcing the other player down to payoff zero, so the minmax payoff vector is $v^0 = (0, 0)$. Mutual minmaxing can be done by using either strategy 0 or 1, where strategy 1 is the least costly to use.

3(a.) Player 1 has 8, player 2 has 2 pure strategies. Pure-strategy profile $((B, L, Y), E)$ is a Nash equilibrium. Both players receive payoff two. Player 2 cannot profitably deviate since she receives her highest payoff 2. Likewise, if 2 plays E, player 1 can get a payoff in $\{0, 1, 2\}$, so the strategy (B, L, Y) leading to payoff 2 is indeed a best reply. Since player 1 does not choose an optimal action in the subgame after actions (A, E), it is not a subgame-perfect equilibrium.

3(b). Player 1 must choose R and Y in their respective subgames. Consequences of $\{A, B\} \times \{E, N\}$ are then

	E	N
A	$1, 1$	$4, 1$
B	$2, 2$	$4, 1$

with pure Nash equilibria (A, N) and (B, E). This gives two subgame-perfect equilibria in pure strategies: $((A, R, Y), N)$ and $((B, R, Y), E)$.

4. Player 1 plays B with probability one; player 2 players D with probability one; player 3 plays E with probability in $[3/4, 1]$.

   If player 3 chooses both actions with positive probability, the belief system must assign probability $1/3$ to the left node of the information set. If player 3 chooses E with probability one, the belief system must assign probability in $[0, 1/3]$ to the left node.

   By careful limit arguments, all these assessments are indeed consistent.

5(a). Blocking pair: (m_1, w_4).

5(b). $\{(m_1, w_4), (m_2, w_3), (m_3, w_1), (m_4, w_2)\}$.

5(c). Man m_4 cannot benefit from lying: the men-proposing DA algorithm is strategy-proof for men. Woman w_1 cannot benefit either: only m_2 and m_1 would be better partners. Any matching including (m_2, w_1) has blocking pair (m_2, w_3). Any matching including (m_1, w_1) has blocking pair (m_1, w_4).

5(d) One. In any stable matching, m_2 and w_3 must be paired, since they are each other's top choice. Given this, m_1 and w_4 must be paired. Since all partners are acceptable, no person remains single. So there are two candidates for a stable matching:

$$\{(m_1, w_4), (m_2, w_3), (m_3, w_1), (m_4, w_2)\} \quad \text{and} \quad \{(m_1, w_4), (m_2, w_3), (m_3, w_2), (m_4, w_1)\}.$$

The former is stable by (b), the latter is not stable: (m_3, w_1) is a blocking pair. Alternatively, note that both the man- and the woman-proposing variant of deferred acceptance lead to the same outcome, which implies that there is only one stable matching.