KTH Teknikvetenskap

SF2972 Game Theory
Exam
March 19, 2015

Time: 8.00-13.00
No permitted aids
Examiner: Boualem Djehiche

The exam is divided in two: Part A on classical game theory and Part B on combinatorial game theory. The maximal score is 20 points from part A and 15 points from part B, and your grade is calculated as follows:

Minimal score:	17	18	21	24	27	30
Grade:	Fx	E	D	C	B	A

Write clearly and concisely, give precise definitions of game-theoretic concepts and precise statements of game-theoretic results that you refer to. Provide precise derivations and motivations for your answers.

Part A – Classical Game Theory
Jörgen Weibull and Mark Voorneveld

1. Consider the following finite two-player game G, where player 1 chooses row and player 2 chooses column:

	a'	b'	c'	d'	e'
a	$-1,-1$	$-2,0$	$-2,-2$	$-1,0$	$0,-1$
b	$0,-1$	$0,0$	$0,0$	$0,0$	$0,0$
c	$0,0$	$0,0$	$3,3$	$6,0$	$6,0$
d	$0,0$	$0,0$	$0,6$	$4,4$	$7,0$

(a) Find all pure strategies that are *weakly* dominated (by a pure or mixed strategy). **[1.5 pts]**

(b) Find all pure strategies that are *strictly* dominated (by a pure or mixed strategy). **[1.5 pts]**

(c) Find all pure-strategy *Nash* equilibria. **[1.5 pts]**

(d) Find all pure-strategy *perfect* equilibria. **[1 pt]**

(e) Find all pure-strategy *proper* equilibria **[1 pt]**

2. There are $n \geq 1$ individuals who together maintain a forest. Each individual i makes effort $x_i \geq 0$, resulting in utility

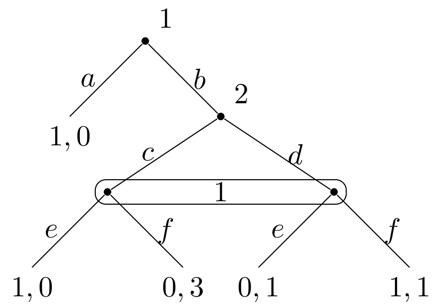
$$u_i(x_1, ..., x_n) = \frac{100y}{1+y} - x_i$$

for the individual, where $y = x_1 + \cdots + x_n$ is the total amount of efforts to maintain the forest. Hence, everybody gets more utility from the forest the better it is maintained. All individuals choose their efforts simultaneously. Each individual strives to maximize his or her utility.

(a) Show that this game has exactly one symmetric Nash equilibrium (in pure strategies), that is, an equilibrium in which all individuals make the same effort, x^*. Find x^* and the associated total amount of effort, $y^* = nx^*$. **[2 pts]**

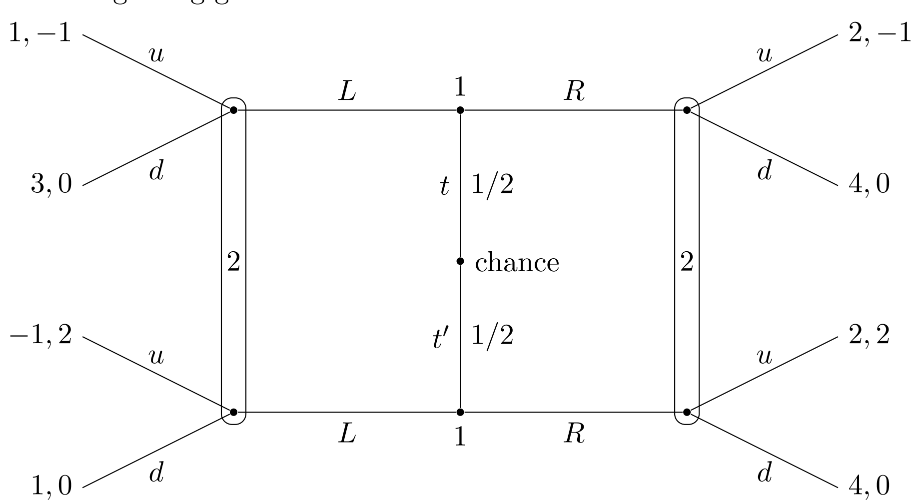
(b) Suppose that the individuals can pre-commit to a common effort level, the same for all. Let $\hat{x}$ be the common effort level that maximizes the sum of all individuals' utilities. Find $\hat{x}$ and the associated total amount of effort, $\hat{y} = n\hat{x}$. Compare these (socially optimal) levels with those in Nash equilibrium. Is there a difference when $n = 1$? When $n > 1$? Explain! **[1.5 pts]**

3. Consider the following extensive form game:



(a) Find the corresponding strategic (i.e., normal form) game. **[1 pt]**
(b) Find all subgame perfect equilibria in behavioral strategies. **[2 pts]**
(c) Find all sequential equilibria. **[3 pts]**

4. Consider the signaling game below:



(a) Find, if any, the separating equilibria where 1 chooses R if chance selects t, but L if chance selects t'. **[2 pts]**
(b) Find, if any, the pooling equilibria where 1 chooses R both if chance selects t and if chance selects t'. **[2 pts]**

Part B – Combinatorial Game Theory

Jonas Sjöstrand

5. For any game G, we define the game $+_G := \{\,0 \parallel 0 \mid -G\,\}$.
(a) Show that $+_{+_{+_G}} = \{\,0 \mid *\,\}$ for any game G. **[2 pts]**
(b) Find all games G such that $+_G = G$. **[1 pt]**

6. The game *Toppling dominoes* is played with a row of black and white dominoes. In each move, the player chooses a domino and topples it either left or right. Every domino in that direction also topples and is removed from the game. Left may only choose a black domino and Right may only choose a white one. Here is an example of a game of Toppling dominoes:

$$\blacksquare\square\blacksquare\blacksquare\blacksquare\square \xrightarrow{L} \blacksquare\square\blacksquare\blacksquare \xrightarrow{R} \blacksquare \xrightarrow{L} \varnothing$$

Prove that the following equalities hold for any nonnegative integers m and n.

$$\underbrace{\blacksquare\cdots\blacksquare}_{m+1}\underbrace{\square\cdots\square}_{n+1} = \{\,m \mid -n\,\}$$

$$\underbrace{\blacksquare\square\blacksquare\cdots\square}_{2n} = *n$$

$$\underbrace{\blacksquare\square\blacksquare\cdots\square\blacksquare}_{2n+1} = \frac{1}{2^n}$$

$$\blacksquare\underbrace{\square\square\cdots\square}_{n+2}\blacksquare = \{\,0 \parallel 0 \mid -n\,\}$$

[4 pts]

7. Let $G = \{\,5 \mid 3 \parallel 0\,\}$ and $H = \{\,0 \mid -4\,\}$. What are the temperatures of G, H and $G + H$? **[4 pts]**

8. The impartial game Kayles is played with a row of pins, possible containing some gaps. A move consists in throwing a ball that removes either one or two adjacent pins. Example:

$$\underline{\dagger\,\dagger\,\dagger\,\dagger\,\dagger} \to \underline{\dagger\quad\dagger\,\dagger} \to \underline{\dagger\qquad} \to \underline{\qquad\quad}$$

Compute the Grundy value of K_5, a row of 5 pins: $\underline{\dagger\,\dagger\,\dagger\,\dagger\,\dagger}$. **[2 pts]**

9. Compute the value of the following Blue-Red Hackenbush position. (Solid edges are blue and dashed edges are red.) **[2 pts]**