KTH Teknikvetenskap

SF2972 Game Theory
Exam
June 12, 2015

Time: 8.00-13.00
No permitted aids
Examiner: Boualem Djehiche

The exam is divided in two: Part A on classical game theory and Part B on combinatorial game theory. The maximal score is 20 points from part A and 15 points from part B, and your grade is calculated as follows:

Minimal score:	17	18	21	24	27	30
Grade:	Fx	E	D	C	B	A

Write clearly and concisely, give precise definitions of game-theoretic concepts and precise statements of game-theoretic results that you refer to. Provide precise derivations and motivations for your answers.

PART A – CLASSICAL GAME THEORY

Jörgen Weibull and Mark Voorneveld

1. Consider the two-player normal-form game

	a_2	b_2	c_2
a_1	$6, 2$	$1, 0$	$0, 3$
b_1	$0, 2$	$4, 3$	$0, 0$
c_1	$7, 0$	$4, 0$	$1, 1$

(a) Find all pure strategies that are weakly dominated (by a pure or mixed strategy). **[1 pt]**

(b) Find all pure strategies that are strictly dominated (by a pure or mixed strategy). **[1 pt]**

(c) Find all rationalizable pure strategies. **[1 pt]**

(d) Find all pure-strategy Nash equilibria. **[1 pt]**

(e) Find all pure-strategy perfect equilibria. **[1 pt]**

2. Consider two ice-cream vendors, A and B, who sell the same ice-cream (say, Magnum Classic) to consumers who are uniformly distributed on a 1 kilometers long beach. Let $X = [0, 1000]$ represent the beach. The vendors have no fixed costs but each vendor has a unit cost of c euros per ice-cream and sells each ice-cream at a fixed and given price $p > c$. Each vendor has to choose a location, x_A and x_B, respectively, in X. Each consumer buys exactly one ice-cream from the seller nearest to his or her location (and with equal probability from either A or B in case they happen to be at the same distance from the consumer).

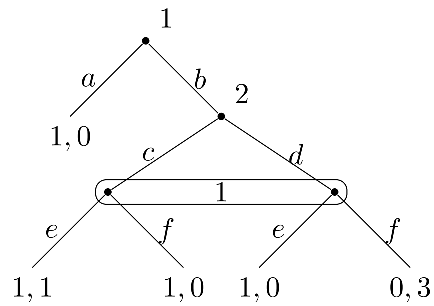
(a) Draw a picture of the set X, indicate two (arbitrary) distinct locations, $x_A < x_B$, for the ice-cream vendors, and indicate in the diagram, and define also algebraically, which consumers will buy from which ice-cream vendor. **[1 pt]**

(b) For any given locations, $x_A < x_B$, define each vendor's profit (units sold times net revenue on each ice-cream) as a function of x_A and x_B. **[1 pt]**

(c) Suppose you could choose the locations for the ice-cream vendors, and your goal was to minimize the average distance for consumers to their nearest ice-cream vendor. What locations would you then choose? **[1 pt]**

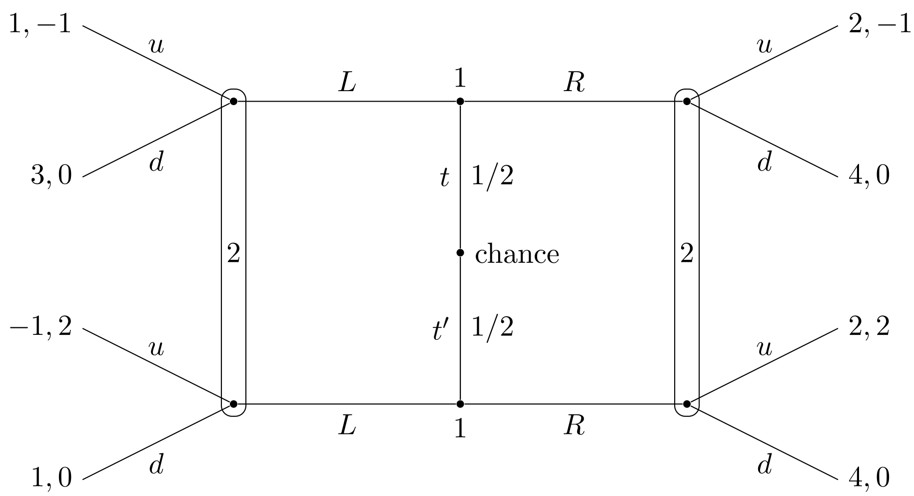
(d) Suppose instead the two vendors are free to choose their locations in X, and that they would do so simultaneously, that is independently of each other, without knowing what location the other vendor chooses. (They may happen to choose the same location, in which case they will sell equally many units.) Define this as a normal-form game, that is, the players, their (pure) strategy sets, and their payoff functions. Show that these payoff functions are discontinuous, that the players do not always have a best reply to each other's strategy, but that there nevertheless exists a unique Nash equilibrium. Find this! Do the vendors earn higher or lower profits than under your proposal in (c)? **[2 pts]**

3. Consider the following extensive form game:



(a) Find the corresponding strategic (i.e., normal form) game. **[1 pt]**
(b) Find all subgame perfect equilibria in behavioral strategies. **[2 pts]**
(c) Find all sequential equilibria. **[3 pts]**

4. Consider the signaling game below:



(a) Find, if any, the separating equilibria where 1 chooses L if chance selects t, but R if chance selects t'. **[2 pts]**
(b) Find, if any, the pooling equilibria where 1 chooses L both if chance selects t and if chance selects t'. **[2 pts]**

Part B – Combinatorial Game Theory

Jonas Sjöstrand

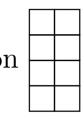
5. A game G is said to be *small* if $-x < G < x$ for any positive number x, and it is said to be *all small* if all its positions are small.

Let $G = \{\, 0 \;\|\; 0 \,|\, -1 \,\}$.

(a) Show that G is positive. **[1 pt]**

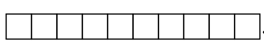
(b) Show that G is small. **[1 pt]**

(c) Show that G is smaller than any positive all small game. **[2 pts]**

6. Compute the value of the Domineering position  and write it in canonical form. **[4 pts]**

7. (a) Show that $t(G) > t(H) \Rightarrow t(G + H) = t(G)$. **[2 pts]**

(b) Find games G and H such that $t(G + H) < \max\{t(G), t(H)\}$. (You don't have to find *all* such games — one example is enough.) **[1 pt]**

8. Consider the impartial variant of the game Domineering where both players are allowed to remove horizontal dominoes. Compute the Grundy value of the position . **[2 pts]**

9. Compute the value of the following Blue-Red Hackenbush position. (Solid edges are blue and dashed edges are red.) **[2 pts]**