Portfolio theory and risk management
Homework set 1

Filip Lindskog

General information

The homework set gives at most 3 points which are added to your result on the exam. You may work individually or in groups of at most two persons. To obtain the points you/the group must present the solution nicely in a report which clearly shows how the problems were solved. I will consider both correctness and the quality of the report important when I evaluate your work. The report, printed on paper, must be handed in on time in order to be accepted.

Continuous compounding of interest rates is used.

The solutions to homework set 1 must be handed in no later than Tuesday 14/9 15:15.

Exercises

Exercise 1. Consider the option prices specified in Table 1. The option prices were listed 2010-08-27. The options are the actively traded European call and put options that day on the value of a stock market index on 2010-10-15.

Strike	980	990	1000	1020	1040
Bid price call	63	56	49.5	37.25	27
Ask price call	64.25	57.25	50.5	38	27.5
Bid price put	23.5	26.5	30	37.75	47.25
Ask price put	24.25	27.25	30.75	38.5	48
Strike	1060	1080	1100	1120	1140
Bid price call	18.25	11.75	6.75	3.65	1.65
Ask price call	18.75	12.25	7.5	4	2.1

Table 1: Prices 2010-08-27 of options maturing 2010-10-15.

The bid price is the price you would sell or short options at. If you own options and wish to sell them immediately, you do so at the bid price. The ask price is the price you would buy options at. If you wish to buy options immediately, you would do so at the ask price.

(a) Determine the implied volatilities relative to Black's model. Use the bid prices and interpolate linearly between implied volatilities and then use the ask prices and interpolate linearly between implied volatilities and plot the two curves in one plot.

(b) You can create a syntetic derivative contract with payoff S by taking positions in the options and setting aside cash (no interest is earned on this cash). How much do you need to pay for this derivative?

Let S be the random value of the index on 2010-10-15. You belive that $\mathrm{P}(S < 1000) \approx 0.30$ and $\mathrm{P}(S > 1045) \approx 0.30$.

Suppose that S can be considered lognormal both according to your view and according to the markets view (the one reflected in the option prices, with some suitable average implied volatility).

(c) Plot the two lognormal densities in the same plot. Is your view more optimistic than that of the market?

(d) Use the capital 1000 to take a derivative position on S with payoff $h(S)$ for some increasing function h. Choose the position you would like to purchase and motivate your choice.

Assume that you may trade fractions of options. You are allowed to take a position in the derivative with payoff S and put cash aside (take a long position in a zero-coupon bond that pays no interest).

If you are ambitious, plot the density of $h(S)$ together with the density of S (according to your probability views).

Exercise 2. Consider the bond data in Table 2. Note that in this case the coupon rate corresponds to the coupon amount paid each coupon date. For each coupon paying bond, it is one year between two consecutive coupon dates, and the last coupon date is also the maturity date (coupon plus face value is paid).

Use the bootstrap method to determine the spot rates and plot the spot rate curve. Use linear interpolation between discount factors.

(a) Consider only the first four bonds.

(b) Consider all bonds.

If you are ambitious, do this once more using linear interpolation between spot rates instead of between discount factors.

Maturity	Price	Coupon rate (%)
2010-12-16	99.96	0
2011-03-15	99.86	0
2011-09-15	104.7	5.25
2013-04-08	112.2	5.5
2014-11-05	123.8	6.75
2016-02-12	112.7	4.5
2017-01-12	103.8	3
2018-02-12	107.9	3.75
2019-09-12	109.1	4.25
2021-06-01	117.6	5
2039-09-30	95.9	3.5

Table 2: Bond market data 2010-09-01: The first two bonds are zero coupon bonds. The other bonds have yearly coupons. All bonds have face values equal to 100.