Portfolio theory and risk management
Homework set 3

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General information

The homework set gives at most 3 points which are added to your result on the exam. You may work individually or in groups of at most two persons. To obtain the points you/the group must present the solution nicely in a report which clearly shows how the problems were solved. I will consider both correctness and the quality of the report important when I evaluate your work. The report, printed on paper, must be handed in on time in order to be accepted.

The solutions to the homework set must be handed in no later than Friday 15/10 10:15.

Exercises

Exercise 1. Consider the option prices specified in Table 1. The option prices were listed 2010-08-27. The options are the actively traded European call and put options that day on the value of a stock market index on 2010-10-15.

Strike	980	990	1000	1020	1040
Call price	63	56	49.5	37.25	27
Strike	1060	1080	1100	1120	1140
Call price	18.25	11.75	6.75	3.65	1.65

Table 1: Prices 2010-08-27 of options maturing 2010-10-15.

Suppose that now is 2010-08-27. Let S be the random value of the index on 2010-10-15. You believe that $\mathrm{P}(S < 1000) \approx 0.30$ and $\mathrm{P}(S > 1045) \approx 0.45$. Suppose also that S can be considered lognormally distributed according to your view.

(a) You have $V_0 = 10000$ SEK that you want to invest by taking long positions (or no positions) in the call options listed above. You may also put some of your money on a cash account that pays no interest. You invest

according to the investment criterion that says that you should maximize the expected value of the value V_1 of your position at the maturity time of the options subject to the risk constraint that $\mathrm{ES}_{0.05}(V_1 - V_0) \leq c$. Choose c, motivate your choice, and solve the investment problem. Assume that you can buy fractions of options.

(b) Do the same thing as in (a) but now with a different investment criterion. Here you want to minimize $\mathrm{ES}_{0.05}(V_1 - V_0)$ subject to the constraint $\mathrm{E}[V_1] \geq \mu_0$. Choose μ_0, motivate your choice, and solve the investment problem.

(c) Suppose now that you may also invest (long positions) in the index. One share of the index costs 1020 SEK and the index pays no dividends until 2010-10-15. Invest optimally according to one of the criteria in (a) and (b).