

EXAMINATION IN SF2943 TIME SERIES ANALYSIS

Date: 2016-05-30, 08:00-13:00

Lecturer: Fredrik Armerin, tel. 070-251 75 55, email: armerin@math.kth.se

Allowed technical aids: Calculator and "Formulas and survey, Time series analysis" by Jan Grandell, without notes

Any notation must be explained and defined. Arguments and computations must be detailed so that they are easy to follow. Write only on one side of the page.

Good luck!

-----

**Problem 1**

The sample

0.0679, -1.5562, -1.3285, -0.3145, 0.0569

is from the causal AR(1) process

$$X_t - \varphi_1 X_{t-1} = Z_t, \{Z_t\} \sim \text{WN}(0, \sigma^2).$$

Estimate  $\varphi_1$  and  $\sigma^2$ . (10 p)

**Problem 2**

Consider the time series  $\{X_t\}$  defined by

$$X_t - (0.2 + \alpha)X_{t-1} + 0.2\alpha X_{t-2} = Z_t - 0.8Z_{t-1}, \text{WN}(0, \sigma^2).$$

- (a) For which values on the parameter  $\alpha \in \mathbb{R}$  is  $\{X_t\}$  a well defined ARMA(2, 1) process? (5 p)
- (b) For which values of  $\alpha \in \mathbb{R}$  is  $\{X_t\}$  a causal ARMA(2, 1) process? (5 p)

### Problem 3

The following sample ACF's and PACF's are observed for two different time series:

Lag	ACF Serie 1	PACF Serie 1	ACF Serie 2	PACF Serie 2
1	0.8102	0.8103	0.8922	0.8925
2	0.4940	-0.4731	0.8001	0.0204
3	0.2861	0.2994	0.7289	0.0552
4	0.1793	-0.1436	0.6582	-0.0242
5	0.1516	0.2102	0.5914	-0.0126
6	0.1531	-0.1114	0.5264	-0.0281
7	0.1521	0.1270	0.4678	-0.0057
8	0.1431	-0.0670	0.4056	-0.0507
9	0.1286	0.0826	0.3509	-0.0031
10	0.1064	-0.0758	0.3024	-0.0086
11	0.0784	0.0489	0.2535	-0.0305

Which of the models WN, AR( $p$ ) for some  $p = 1, \dots, 5$ , MA( $q$ ) for some  $q = 1, \dots, 5$  or ARMA(1, 1) best fits the two observed ACF's and PACF's for Serie 1 and Serie 2 respectively?

If you answer is AR( $p$ ) or MA( $q$ ) to one or both of the questions, then you must also specify the order  $p$  and  $q$  respectively. (10 p)

### Problem 4

Consider the ARCH(1) process

$$\begin{aligned} X_t &= \sigma_t Z_t, \{Z_t\} \sim \text{IID } N(0, 1) \\ \sigma_t^2 &= 1.24 + 0.75X_{t-1}^2. \end{aligned}$$

Derive  $E[X_t]$  and  $\text{Var}(X_t)$  for this ARCH(1) process (10 p)

### Problem 5

Consider the MA(2) process

$$X_t = Z_t - 1.5Z_{t-1} + 0.56Z_{t-2}, \{Z_t\} \sim \text{WN}(0, 0.16).$$

(a) Determine the ACVF  $\gamma(h)$  of  $\{X_t\}$  for  $h = 0, \pm 1, \pm 2, \dots$  (5 p)

(b) Determine the spectral density  $f(\lambda)$  of  $\{X_t\}$ . (5 p)