On some stochastic competition models

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Joint work with Henrik Fagerholm, Mats Gyllenberg and Brita Jung.

In his recent review, Schreiber (2012) describes the state of the art for stochastic competition models of the general form

$$X_{t+1}^{i} = f_i(X_t, \xi_t) X_t^i, \;\; i = 1, \dots k, \;\; t = 0, 1, 2 \dots$$

where the state space $\mathbf{S}$ is a subset of $\Re_+^k$ and the union of the coordinate axes in $\Re_+^k$ forms the *extinction set* $\mathbf{S}_0$; the ξ's represent a randomly evolving environment.

I will look at some particular cases and variants of this model. Specifically, we will limit ourselves to two populations, i. e., $k = 2$, and the functions f_i are chosen to be of the *Ricker type*

$$\exp(r_t^i - K_t^i(X_t^i + \alpha_j X_t^j)), \;\; i, j = 1, 2, \;\; i \neq j.$$

Here the r_t^i's model the average intrinsic per capita growth rate at time t. The growth is attenuated by the negative term where the factor K_t^i describes the intra-specific competition at time t and a_j (assumed constant over time) the relative importance of the inter-specific competition.

Another variant of the model is concerned with *demographic* stochasticity. Here our aim is to study the evolution of two finite populations as size-dependent branching processes, which on average follow the above Ricker-type model (with non-random r^i, K^i). We want to describe the long-term behavior and compare it with that of the corresponding deterministic model. The size-dependent branching processes will necessarily have finite life-times. Can anything be said about those?

Main reference:

Sebastian J. Schreiber: Persistence for stochastic difference equations: A minireview, *Journal of Difference Equations and Applications* (2012), now available via the author's web site
`www-eve.ucdavis.edu/sschreiber/pubs.shtml`