Inter-Risk Correlation within Economic Capital

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Abstract

Economic Capital consists of internally defined capital the purpose of which is to protect shareholders from insolvency up to a given level and over a given time horizon. When calculating its Economic Capital, a bank is allowed to take diversification benefits into account, i.e., take into account the correlation between risk types in its portfolio. The higher the correlation between risks, the lower diversification effects; hence a higher amount of Economic Capital is needed. While it is of importance for a bank to protect its shareholders from insolvency, a bank should also strive to maintain as little Economic Capital as needed as this enables investment of capital at a higher rate of return.

The aim of this thesis is to study the inter-risk correlation between a Bank's risks within Economic Capital, i.e., credit, market, business, operational and life risk when explicitly considering a Bank's risk characteristics. Correlation calculation is based on actual risk figures and historical simulations of risks. The historical simulations are based on relationships determined by a linear regression between macroeconomic and market risk drivers and the actual risk figures being available.

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Contents

1. Introduction

1.1. Background

In order to avoid insolvency a bank needs to hold a certain amount of capital to cover its unexpected losses and changes in asset values. This required capital can be divided into Regulatory and Economic Capital. While Regulatory Capital is the capital required by the National Supervisory Authorities to protect the banking system and its depositors, Economic Capital is an internally defined capital the purpose of which is to protect shareholders from insolvency to a given level over a time given horizon. Economic Capital reflects a bank's risk appetite and is allocated to the business areas and is an important measure as it is used in pricing decisions and as key performance indicators.

The Basel II regulations, under which Economic Capital is controlled, convey new demands as well as possibilities for risk modelling. One possibility is that a bank can take diversification benefits into account when calculating Economic Capital, i.e., take into account for dependency (i.e. correlation) between risk types of its portfolio. The higher the correlation between risks, the lower the diversification effect, hence a higher amount of Economic Capital is needed. While it is of importance for a bank to protect its shareholders from insolvency, it also strives to maintain as little Economic Capital as needed as this enables investment of capital at a higher rate of return. Hence, as correlation between risk types will explicitly decide the level of Economic Capital, it is of importance that this correlation well corresponds to the specific characteristics of a bank's risk exposure. Today, many banks rely on a third party provided benchmark correlation matrix in order to calculate their diversified Economic Capital. In addition to the importance of an internal correlation matrix, the Basel II regulations are in progress and are toward an incorporation of diversification benefits within Regulatory Capital, which is why dependency modelling is highly relevant.

1.2. Aim

The aim of this thesis is to study the inter-risk correlation between the Bank's[1] risks within Economic Capital, i.e., credit, market, business, operational and life risk when explicitly considering the Bank's risk characteristics.

1.3. Scope

There exist limited amounts of loss data for credit, market, business, operational and life risk that corresponds to the Bank's current risk exposure. The least common denominator, in which losses exist, is 18 quarters (i.e. 18 data points), which is not enough in absolute terms, nor does it capture enough economic cycles.

In order to create loss data that include several economic cycles and correspond to the Bank's risk portfolio (i.e. today's organisational structure and geographical exposure), historical simulations of risks will be performed with macroeconomic variables. Simulations of risks will only be made in credit, market and business risk due to the lack of explanation between macro economy and operational and life risk (see Chapter 2

[1] The name "the Bank" is used in order not to disclose the identity of the bank in question

for definitions of those risks). Moreover, credit, market and business risk are the major risks; they correspond to 92% of total risks within Economic Capital[2]. Hence the correlations between them have the main impact on the level of diversified Economic Capital, which determines the diversification effects (see Chapter 2 for a further explanation of the relation between correlation and diversification). Correlation between operational and life risk vis-à-vis credit, market and business risks will be approached differently (see Chapter 5).

1.4. Method

In order to provide the reader with an overview of the processes involved in correlation calculation, this section will present the major parts of this thesis.

First of all, the Bank's definitions of each risk needs to be studied in order to quantify each risk in a closed form expression.

Next step consists of identifying and collecting macroeconomic variables that are likely to act as drivers for credit, market and business risk. This will be done by studying literature describing how financial institutions have investigated the predicting power of macroeconomic variables on each risk. Moreover, internal material from the Bank will be studied in order to identify relevant risk drivers with respect to the Bank's geographical exposure and other specific characteristics. Macroeconomic variables will be gathered from Ecowin[3], statistics from Nordic government agencies providing statistics and European statistics agencies. The final preparation of data before regression can be performed consists of studying the optimal time lag between each risk and the relevant macroeconomic variable.

As macroeconomic drivers for each risk have been gathered, a closed form expression will be established through a linear regression. The regression will be performed in SAS, a statistical computer software which is used by the Bank. As relationships between credit, market and business risk have been identified by the regression, historical simulations will be performed with the gathered macroeconomic variables.

When a sufficient number or risk data have been obtained, linear correlation will be calculated between credit, market and business risk by calculating two linear Pearson-correlation matrices. One matrix will be calculated by using all risk data created back to 1983 and one by using risk data from extreme events, i.e., when all risks peak at once. Those extreme events will be chosen visually. The last step is to incorporate operational and life risk in the correlation calculation. Correlation between operational risk versus all other risk types will be based on the actual observations available, rather than on simulated data, whereas correlation with life risk will be set by analysing how the unique underlying factor correlates with the other risk drivers.

[2] Internal Material from the Bank

[3] EcoWin is a provider of economic and financial market data. For more information about EcoWin, the reader is referred to http://www.ecowin.com/.

1.5. Outline

This thesis consists of five major parts; a theoretical field (Chapter 2), data analysis (Chapter 3), mathematical methods (Chapter 4), conclusions (Chapter 5) and improvements (Chapter 6).

Chapter 2 begins with some basic economic and mathematical definitions in order to provide the reader with the definitions necessary for continued reading. This chapter will also give an account for the currently used benchmark correlation matrix.

In Chapter 3, the process of gathering and analysing relevant data is described. It starts with collection and analysis of historical loss data for each of the specified risks and thereafter, identification and collection of relevant macroeconomic variables that are likely to explain the dynamics of each risk. Finally, the optimal time lag is studied in order to capture the maximum level of explanation behind each macroeconomic variable's impact on the specific risk.

In Chapter 4, the mathematical models, i.e., linear regression and correlation calculations will be described. First of all, linear regression is discussed and is used in order to find a closed form expression between macroeconomic variables and each risk. The associated statistics and tests performed are presented and discussed further in Appendix C. Thereafter, it is described how risks are simulated back to 1983 with the macroeconomic variables identified by the linear regression. Finally, it is described how correlation is calculated and two correlation matrices are presented.

In Chapter 5, conclusions are presented and improvements are discussed in Chapter 6.

2. Theoretical background

2.1. Basic Economic Terminology

2.1.1. Regulatory and Economic Capital

A bank is obliged to hold a certain amount of capital in order to avoid insolvency, to insure its responsibilities to debt holders and depositors and to protect the financial system. Each country is authorised and supervised by its National Supervisory Authorities. In Sweden, the bank and credit sector is authorised by the Swedish Financial Supervisory Authority (i.e. Finansinspektionen). The capital needed to avoid insolvency can be divided into Regulatory Capital and Economic Capital. Regulatory Capital is the capital required to protect the banking sector and the depositors.

Economic Capital is the internally defined capital, the aim of which is to limit the probability of insolvency to a given level over a given horizon. Though there are no requirements on the level of Economic Capital from regulators as in the case of Regulatory Capital, guidelines within the Economic Capital is provided by the Committee of European Central Supervisors (CEBS). Economic Capital should reflect the true risk sensitivity for a bank and is an important dialogue with the regulator within the Regulatory Capital. When calculating the Economic Capital diversification effects are taken into account. This will be discussed further in the section for mathematical definitions.

2.1.2. Credit Risk

Credit risk is defined as the risk that counterparties fail to fulfil their agreed obligations and that the pledged collateral does not cover the Bank's claims.

Within Economic Capital, credit risk refers to the capital required to cover unexpected losses, based on the anticipated volatility of the annual loss rate and is estimated over a one-year time horizon given a specific confidence interval.

2.1.3. Market Risk

Market risk is defined as the risk of loss in market value as a result of movements in financial market variables such as interest rates, foreign exchange rates, equity prices and commodity prices.

- The Interest Rate Risk is the risk for capital loss (interest rate price risk) or lower net interest income (net interest income risk) due to changes in interest rates.
- The Foreign Exchange Risk is the risk that value of assets, liabilities and future payments change due to changes in exchange rates.
- Equity Risk is the risk that market value of equities and equity related instruments changes due to changes in commodity prices.
- Commodity Risk is the risk that market value of commodity related instruments changes due to changes in commodity prices.

2.1.4. Business Risk

Business risk is defined as the risk associated with uncertainty in the business conditions such as market environment, client behaviour and technological progress. This uncertainty is for example materialised through earnings uncertainty in margins, fees, volumes and costs) and the size of the fixed cost base.

2.1.5. Operational Risk

Operational risk is defined as the risk of direct or indirect loss, or changed reputation resulting from inadequate or failed internal processes, people and systems or from external events.

Within Economic Capital, operational risk is defined as the capital needed to cover unexpected losses in accordance with the above definition of the operational risk.

2.1.6. Life Risk

Life risk is defined as the risk of unexpected losses due to changes in mortality rates, longevity rates, disability rates and selection effects.

Life risk has not been further quantified as correlation vis-à-vis the other risk types is assumed to be zero.

2.2. Mathematical Definitions

2.2.1. Correlation

Correlation within Economic Capital is often mentioned in terms of Inter-Risk (between risk types) and Intra-Risk (between components within a specific risk type). This thesis will focus on the Inter-Risk correlation, which refers to the correlation between for example market and credit risk, and is the major factor that determines the diversification effects. This correlation is a linear correlation and is defined as:

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)} \times \sqrt{Var(Y)}} \qquad \textit{where } X,Y = \textit{risk type}$$

2.2.2. Correlation matrix

The correlation matrix is defined as the complete linear Pearson correlation matrix that expresses the above given defined correlation. The correlation matrix is a symmetric matrix as correlation between X and Y is the same as the correlation between Y and X. With the specified risks within the Economic Capital, the correlation matrix described as:

$$Correlation\ Matrix = \begin{pmatrix} 1 & \rho_{\text{Credit,Market}} & \cdots & \cdots & \cdots & \rho_{Credit,Life\ insurance} \\ \rho_{Market,Credit} & 1 & \cdots & \cdots & \cdots & \rho_{Market,Life\ insurance} \\ \rho_{Business,Credit} & \cdots & 1 & \cdots & \cdots & \rho_{Business,Life\ insurane} \\ \rho_{Operational,Credit} & \cdots & \cdots & 1 & \cdots & \rho_{Operationl,Life\ insurane} \\ \rho_{Life\ insurance,Credit} & \cdots & \cdots & \cdots & \cdots & 1 \end{pmatrix}$$

2.2.3. Diversification

In the Bank, diversification benefits that accrue from aggregation are driven by three main factors, correlation being one of them. The other two main drivers are the number of risk positions and the concentration of those risk positions (i.e. the relative weights in a portfolio). The formula for calculating the diversified Economic Capital is defined as:

$$EC_{total\ div} = \sqrt{\begin{pmatrix} EC_{Credit} \\ EC_{Market} \\ EC_{Business} \\ EC_{Operational} \\ EC_{Life\ insurance} \end{pmatrix}' \begin{pmatrix} 1 & \rho_{\text{Credit,Market}} & \cdots & \cdots & \cdots & \rho_{Credit,Life\ insurance} \\ \rho_{Market,Credit} & 1 & \cdots & \cdots & \cdots & \rho_{Market,Life\ insurance} \\ \rho_{Business,Credit} & \cdots & 1 & \cdots & \cdots & \rho_{Business,Life\ insurane} \\ \rho_{Operational,Credit} & \cdots & \cdots & 1 & \cdots & \rho_{Operationl,Life\ insurane} \\ \rho_{Life\ insurance,Credit} & \cdots & \cdots & \cdots & \cdots & 1 \end{pmatrix} \begin{pmatrix} EC_{Credit} \\ EC_{Market} \\ EC_{Business} \\ EC_{Operational} \\ EC_{Life\ insurance} \end{pmatrix}}$$

Where, $EC_{total\ div}$ is the aggregate Economic Capital taking diversification effects into account. At a given level of aggregation, the diversification benefit (DB) is simply one minus the ratio of the aggregated Economic Capital, i.e.,

$$DB = 1 - \frac{EC_{total\ \ div}}{\sum_{i=1}^{n} EC_{total}}$$

If all risks where perfectly correlated, i.e., all losses would occur at the same time, no diversification effects would exist. Hence, the diversification effects determine the level of Economic Capital. High diversification effects, i.e., low correlation, decrease Economic Capital which implies that a bank can generate higher returns on its invested capital.

3. Analysis of loss data and macroeconomic variables

In order to establish a correlation matrix for credit, market, business, operational and life risk it is important that calculation of correlation is based on a sufficient number of data points. Moreover, it is important that the time period studied reflects a sufficient number of economic cycles. The lowest common denominator in time, for which actual loss data exists, is the second quarter 2004, which implies that the actual number of loss data points is 16. Hence, the available historical loss data is neither enough in terms of number of observations, nor does it reflect a sufficient number of economic cycles. The time period from 2004 to 2008 can be considered as a period of economic growth in the Nordic countries and hence, a correlation matrix based on the data form this period of time would not be reliable due to the lack of observations during times of declining economy. Moreover, as the Bank experienced several organisational changes before 2000, a calculation of correlation based on historical losses would not reflect the Bank's current risk exposure.

In order to capture the organisational structure of today, several economical cycles, and to have a sufficient number of data points (i.e. quarterly risk figures), This thesis will simulate credit, market and business risk back in time. The simulation is based on relationships between risk types and macroeconomic variables and the specific parameters, i.e., the impact of each macroeconomic variable on the specific risk. This relationship is determined by a linear regression. Operational and life risks have not been historical simulated due to the lack of correlation with macroeconomic variables.

Before calculation of correlation can be done, loss data and macroeconomic variables need to be gathered and analysed. The process will be presented in this chapter. The initial conditions of calculating correlations are followed by a section where credit, market and business risks are quantified. Further on, macroeconomic variables are identified and the optimal time lag between each risk type and their corresponding set of macroeconomic variables is studied.

The preparation processes can be described by the following steps.

1. Define and quantify each risk that shall be modelled
2. Identify and gather data of macroeconomic variables
3. Study the optimal time lag between each risk type and their corresponding set of macroeconomic variables, i.e., find the time where the impact of each macroeconomic variable has the highest correlation with each risk type

3.1. Credit Risk

3.1.1. Definition and calculation

As described in chapter 2, the Bank's credit risk is defined as the risk that counterparties fail to fulfil their agreed obligations and that the pledged collateral does not cover the Bank's claims.

The Bank is mainly a Nordic bank why this analysis is confined to contain credit losses associated with the Nordic countries.

To further specify credit risk, this thesis has divided credit risk into eight subgroups, through two different levels of distinction. Firstly, the four Nordic countries will be assessed separately and secondly, a separation has been made between households- and corporate segments. The first distinction, i.e., between countries is important since loan losses are likely to differ mainly due to changes in exposure between countries but also due to changes in economic cycles, governmental laws and the definition of default. During the 1980s, the Nordic countries liberalised their financial markets and experienced a consecutive period of large swings in credit, asset prices and economic activity. In Denmark, this financial cycle started earlier than in the rest of the Nordic countries since the financial deregulation started earlier and was more prolonged. Hansen (2003) studies, in his report for the Swedish Riksbank, the financial cycles and bankruptcies in the Nordic countries and the predicting power of macroeconomic factors for bankruptcies and finds that several empirical results appear to be similar for Sweden, Norway and Finland, while patterns for Denmark often differ.

The second distinction of loans to the public, i.e., into loans to the household and corporate sector is made due to their specific risk characteristics and the fact that they are treated as different segments by financial institutions. This separation, into households and corporate, is also made by Evjen (2004) in a survey conveyed from Norway's Bank.

To only use macroeconomic variables to explain the dynamics in loan losses, i.e., credit risk, is likely a restriction since a lot of information is not captured by macroeconomic variables. Evjen (2004) concludes that loan losses for households and corporate depend on both macroeconomic development and microeconomic conditions associated to the individual borrower, and to properly understand the dynamics of loan losses, both macro and micro economic variables should be modelled. A contrary conclusion is made by Jacobsen et al. (2003) in a report for the Swedish Riksbank, concluding that macroeconomic variables are of crucial importance for explaining changes in absolute default risk, while firm specific variables are very useful in ranking the riskiness of firms. The risk specific variables of a firm are often of idiosyncratic origin, and hence difficult to enter as explanatory variables. The microeconomic variables they use as explanatory variables are earnings ratio, interest coverage ratio, debt ratios, cash ratios and turnover ratio. Hansen's result, that firm specific variables often are of idiosyncratic origin, is of importance for this thesis when analysing the credit risk for the Nordics since only macroeconomic variables will be modelled.

This thesis will quantify credit risk for the Nordic countries as the actual gross loss divided by the outstanding amount in order to reflect the risk exposure of each division and country, i.e.,

$$Credit\ Risk_t^{k,i} = \frac{Actual\ Gross\ Loan\ Loss_t^{k,i}}{Lending\ to\ the\ Public_t^{k,i}} \qquad \begin{aligned} &t = 1,2 \ldots quarters \\ &i = SE, DK, NO\ and\ FI \\ &k = HH, C \end{aligned}$$

This approach of modelling loan losses is also used by Schuermann (2004) and Carling (2006).

3.1.2. Macroeconomic variables

This thesis will model loan losses with GDP, inflation, unemployment rate, interest rate, output gap (i.e. deviation of GDP around its trend value), households' savings and debts and the households' consumption. Among those, one could assume the evolution of GDP, inflation and interest rate to have a high impact on the prediction of loan losses for the household and corporate sector. Moreover, a strong relationship between unemployment rate and loan losses associated with households would likely exist.

A low and stable inflation provides households and undertakings with a clear indication of changes in relative prices, thereby making it easier for economic agents to make the correct decisions. Low and stable consumer price inflation also contributes to price stability in financial and property markets. An unexpected decline in inflation increases the real value outstanding debt, making defaults more likely. Furthermore, the vulnerability of the financial system tends to rise when inflation is high. Hence, the traditional view has been that low and stable inflation provides an indication of financial stability.

GDP is a main indicator of the state in an economy and is likely to have a predicting power on the loan losses. Laeven and Majnoni (2002) find a negative relationship between GDP growth and loan loss provisions and conclude that one standard deviation increase in GDP growth is associated with a decrease in loan loss provisions of 0.20 percent. Loan loss provisions can be seen as a proxy for loan losses as a provision needs to be charged when a bank recognises a loan as impaired[4].

When modelling loan losses for Norway's Bank, Evjen (2004) uses the unemployment rate, real value of private houses and the debt burden, measured as debt in disposable income, to model loan losses for the household sector. A significant relationship is found between households' loan losses and debt in per cent of disposable income, real value of private houses, the interest rate and unemployment rate. To model loan losses associated to the corporate sector, they use the risk-weighted debt for all enterprises and the real price of existing dwellings.

When modelling bankruptcies in the Nordic countries Hansen (2003) uses lending to the private sector, share prices and house prices, investment in construction, GDP, inflation and short-term interest rates. Following Jacobsen et al. (2005), they use output gap, the annual inflation rate, the REPO nominal interest rate and the real exchange rate. Åsberg and Shahnazarian use industrial production, consumer price index, three month treasury bills, when modelling expected default frequency for the corporate sector in their report for the Swedish Riksbank.

A complete list of macroeconomic variables that will be used in the regression is demonstrated below.

[4] A loan is impaired if there is objective evidence that full repayment is unlikely

Table 1: Macroeconomic variables used in the regression for credit risk

Type of regressor	Sweden	Denmark	Norway	Finland
GDP				
constant prices	X*	X*	X *	X*
current prices				
Output gap	X			X
Unemployment rate	X	X	X	X*
Inflation (CPI)	X	X	X*	X
3 M T-bill	X	X	X	
Government bond				
2 Y		X		
5 Y	X	X	X	
10 Y	X	X		X
Households' assets			X	
Households' liabilities			X	
Industry production	X*			
Households' consumption	X*			
Investment in construction	X		X	
Number of bankruptcies	X (1994)	X (1984)	X (1986)	

* Seasonal adjusted data is available

3.1.3. Optimal lag coefficient

This section presents and evaluates leading and lagged correlation coefficients between credit risk and the set of macroeconomic variables. The sample ranges from the first quarter 2002 to the second quarter 2008. Figure 1-8 shows leading and lagged coefficients for macroeconomic variables and credit risk

Figure 1: The Swedish corporate sector

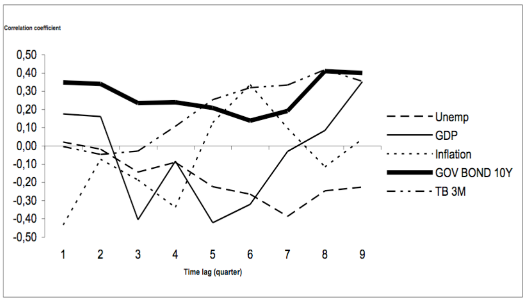


Figure 2: The Swedish household sector

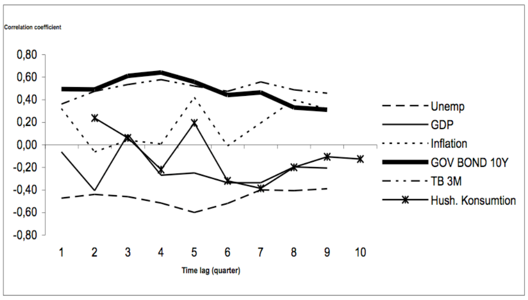


Figure 3: The Danish corporate sector

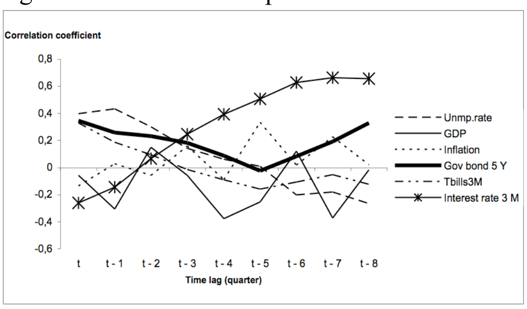


Figure 4: The Danish household sector

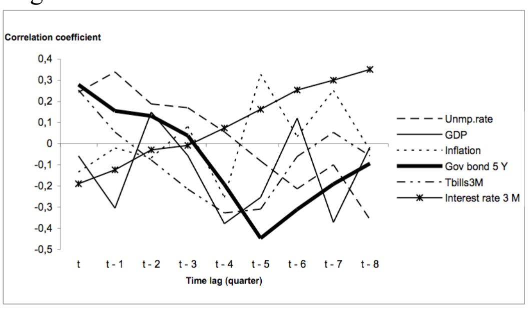


Figure 5: The Finnish corporate sector

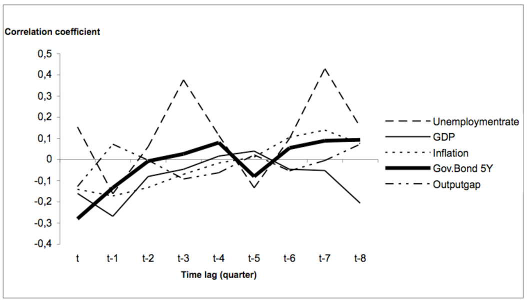


Figure 6: The Finnish household sector

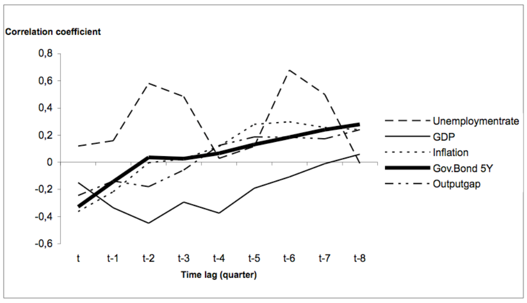


Figure 7: The Norwegian corporate sector

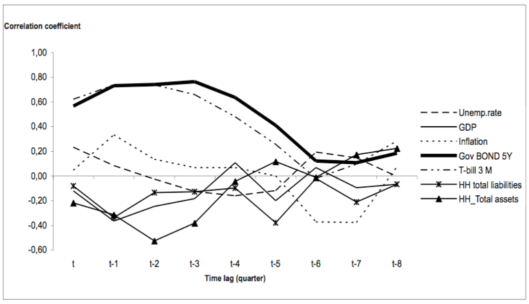
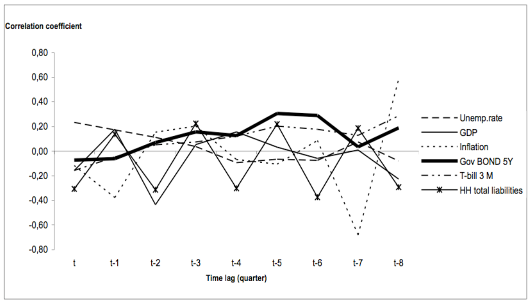


Figure 8: The Norwegian household sector



As can be seen in figure 1-8 above, some macroeconomic variables show a very irregular behaviour, thus, does not allow for any conclusions to be drawn as to the optimal time lag. This is especially true for the Danish and Finnish unemployment rate and inflation in Sweden and Norway. Nevertheless some variables can be analysed and "stylised facts" emerge and are as follows:

- Lagged unemployment rate correlates positively with credit risk and the strongest correlation is found between four to six years ahead for all countries but Sweden and Norway
- Lagged inflation correlates in general positively with credit risk four to seven quarters ahead for all countries. Correlations are in general higher for households than for the corporate sector.
- Lagged GDP correlates in general negatively with credit risk one to five quarters ahead for all the Nordic countries except for Norway
- Interest rates correlate positively with credit risk for all countries with a time lag between 1 to 8 quarters.

The preceding "stylized facts" corresponds well to Hansen's results from investigating the relationship between macroeconomic imbalances and bankruptcies in the Nordic countries. The most relevant findings are listed below;

- Lagged inflation correlates positively with bankruptcies one to two years ahead
- Correlation between lagged nominal interest rates and bankruptcies are positive
- Several key relationships appear to be similar for Sweden, Norway and Finland while patterns for Denmark differ and the share of insignificant correlations is higher.

3.2. Market Risk

3.2.1. Definition and calculation

As mentioned earlier the market risk is defined as the risk of loss in market value as a result of movements in financial market variables such as interest rates, foreign exchange rates, equity prices and commodity prices.

This thesis will consider market risk as the risk associated with interest rates, foreign exchange rates and equity prices due lack in loss observations in commodity prices. To further analyse market risk and the associated risk drivers in order to model market loss back in time, two things are important: firstly, the distribution of risk between countries and secondly, the distribution between risk types. In order to properly understand the risk associated with each country and the distribution between interest rate, foreign exchange and equity risk, each country and risk have been investigated for the total group between 2002-2008. Though the distribution has changed somewhat during the examined period, the general distribution between countries and risks has been the same. The third quarter's VaR-figures[5] give a relevant indication of the distribution between countries and risks as this value is believed to reflect turnover within divisions, net positions in the market and their risk appetite. The distributions of the VaR-figutes between the countries give an important indication: interest rate risk is the major risk and Denmark contributes to a high amount of the total market risk. This observation is important as it gives an indication of the macroeconomic variables to focus on when performing regression.

The definition of market risk given above, used in this thesis, has been narrowed down to contain market risk for the Bank's group A and B[6] due to the difficulty in data gathering of loan losses for all divisions. This limitation is reasonable to make as the two groups are the main contributors to market risk as they corresponds to 87%[7] of the total market risk within the Economic Capital. Actual profits and losses (APL) have been registered at group A and B and used in this thesis to reflect quarterly market losses. These data are daily actual profit and losses from the second quarter of 2004 to the third quarter 2008. Since only the losses are of interest, positive values are excluded, and the quarterly losses are computed as the sum of all negative observations separately for each division.

As this thesis aims to measure the correlation between risks, it is of importance that calculations are based upon market risk and not actual market loss. The above quarterly losses are computed from actual loss observations and cannot be translated into market risk. Hence, to better reflect the market risk, a quarterly 99% Value-at-Risk measure (VaR) is used to reflect market risk by dividing the actual loss with the VaR measure. Daily VaR measures are collected for group A and B during the same period as daily APL and the values are believed to reflect turnover within divisions, net positions in the market and their risk appetite. In order to arrive at a quarterly VaR-figure, the average daily VaR-figures have been summarised.

[5] The VaR-figures cannot be published in this paper as they are classified as secret information.

[6] The real names of the groups cannot be published in this paper as they are classified as secret information.

[7] Internal information from the Bank and is the total exposure if the Bank life & pensions, risk in pension funds and real estate risk are excluded.

$$VaR(group\ A_q + group\ B_q)_{0,99} = \sqrt{n}\overline{\sigma}_{group\ A+group\ B}\Phi^{-1}(0,99)$$

$$\overline{\sigma}_{group\ A+group\ B} = \sqrt{\overline{\sigma}^2_{group\ A} + \overline{\sigma}^2_{group\ B}}$$

The last summation assumes that VaR_{groupA} and VaR_{groupB} are independent. This assumption is made based on the measured correlation that is 0,72%.

3.2.2. Macroeconomic variables

Market risk captures movements in interest rate, equity prices and foreign exchange rate. Hence, to capture those movements one has to analyse the characteristics of the Bank's market positions in each of those risk factors since the risk factors depend on the nature of the underlying asset or position. For example, equity risk will depend on the underlying asset of the Bank's portfolio. Evidently, the exposure of all these positions is impossible to model but one can improve the significance of the regression model through for example, using a relevant stock index.

Dimakos et al (2005) use 14 different risk drivers to model market risk for DnB NOR, and like the approach of this thesis they focus on equity risk, exchange rate risk and interest rate risk. For equity risk, they found the most important risk drivers to be the Norwegian financial index and the Norwegian stock index. For exchange rate risk, they used USD/NOK, EUR/NOK, JPY/NOK exchange rates. For interest rate risk they used 3-month and 5-year NIBOR, EURIBOR and LIBOR[8].

According to the Basel II regulations, under which Economic Capital is regulated, financial institutions are obliged to stress test their calculations of market risk in order to assure that their capital adequacy corresponds to financial and macroeconomic conditions. The risk drivers used for stress testing are likely to give an indication of the main risk drivers behind market risk. Equity risk is stressed by using different indices, such as different OMX:s indices and the Standard & Poor's 500. Interest rate risk is stressed with government curves in the Nordic currencies and USD. Foreign exchange risk is stressed with the Nordic currencies vis-à-vis EUR and USD.

Following Dimakos and the Bank's guidelines for market stress testing, this thesis will model equity risk by using European and US stock indices in order to capture movements in equity markets. The Bank is a Nordic bank and intuitionally, Nordic stock indices are likely to reflect equity risk. Since the Nordic is a relevant small district of finance, we are likely to be influenced by global movements in for example Europe, Asia and the US. Hence, stock indices for those areas will be used in the regression in order to find indices that capture most of the explanation in equity movements for the Bank's portfolio and can be found in Table 2.

Similar to the risk factors used in stress testing interest rate risk, both short and long term Nordic's interest rate will enter the regression as explanatory variables. The interest rates of small monetary unions, such as the Nordic countries, are likely to be strongly influenced by stronger monetary unions such as the Euro area and the US. Hence, except the Nordic interest rates, the US interest rate will enter the model as an explanatory variable in order to investigate its impact on the interest rate risk.

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[8] These three interest rates are the inter bank offered rate, i.e the interest rate offered between banks, in Norway (NIBOR), Europe (EURIBOR) and Great Britain (LIBOR).

To model foreign exchange risk it is important to use exchange rates that represent maximum exposure in foreign exchange positions. Those positions have changed over time but the main positions are likely to have remained relatively constant. Following the procedure in market stress testing of foreign exchange risk, this thesis will use the Nordic currencies vis-à-vis USD and Euro in order to find the risk factors capturing most of the explanation in market risk movements.

The risk factors given above are related to movements in the market, since they are directly related to equity, interest rate and foreign exchange risk. In addition to those factors economic indicators such as GDP are likely to have an impact on the market in general. Those macroeconomic variables will therefore enter the model as explanatory variables in order to investigate if they will increase the significance. A complete table for the macroeconomic variables used as explanatory variables in the model for market risk is demonstrated in Table 2.

Table 2: Macroeconomic variables for market risk

Interest rate risk	**Stock indices**
US Government bond 10Y, 5Y SE Government bond 10Y, 5Y DK Government bond 10Y, 5Y	MSCI Swedish index (SEK) OEAX (NOK) S&P 500 (USD) KAX(EURO)
Foreign Exchange Risk	**General Macro variables**
USD/SEK USD/NOK EUR/SEK EUR/DKK	GDP (SE, DK, NO, FI)* Output gap (SE, FI)

3.2.3. Optimal lag coefficients

This section presents and evaluates leading and lagged correlation coefficients between the market risk and the set of macroeconomic variables. The sample ranges from the first quarter 2004 to the second quarter 2008.

Figure 9: Equity indices

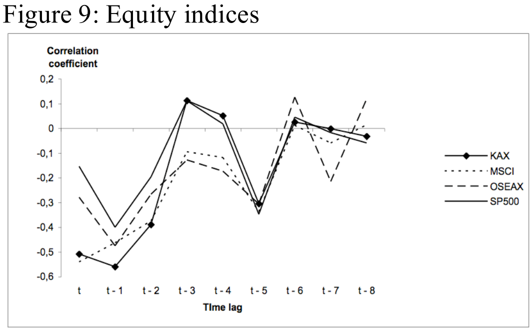


Figure 10: FX-rates

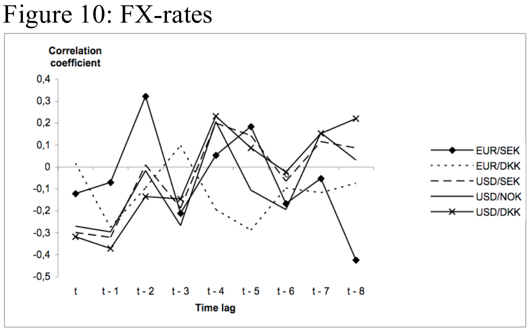
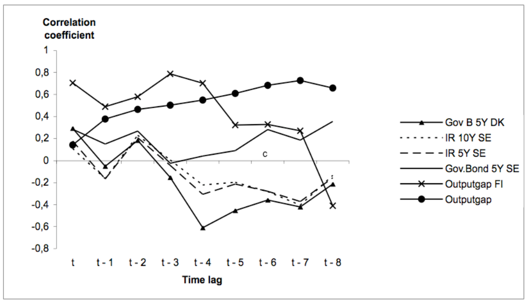


Figure 11: Interest rates and output gap



When time lag between equity indices and market risk, which is demonstrated in Figure 9, is larger than 2 quarters, an irregular behaviour can be observed. However, the correlation between lagged equity indices and market risk is negative when the time lag is between 0 and 2 quarters. Lagged FX-rates and credit risk show a relationship that is too irregular to warrant any conclusions.

Figure 11, shows leading and lagged correlation coefficients for interest rates and output gap where it can bee seen that a positive relationship between output gap and market risk is likely to exist.

3.3. Business Risk

3.3.1. Definition and calculation

In the Bank, business risk is defined as the risk associated with uncertainty in the business conditions such as market environment, client behaviour and technological progress. This uncertainty is materialised through earnings uncertainty (uncertainty in margins, fees, volumes and costs) and the size of the fixed cost base. The main drivers for business risk are the size of fixed costs, volatility in volumes, volatility in margins and cost volatility.

This thesis will model business risk as the change in cleansed operating profits, i.e., the change in profits cleansed from the influence of market and credit risk.

$$Business\ Risk_t = \frac{-(oper\ profit_t - oper\ profit_{t-1})}{oper\ profit_{t-1}} \qquad t = 1,2..quarters$$

To be consistent with the definitions of credit and market risk, a positive business risk means a decrease in operating cleansed profit which explains the minus sign in front of the formula.

3.3.2. Macroeconomic variables

Since business risk is the fluctuations in cleansed operating profit the risk drivers are intuitively factors which contribute to change in profits, i.e., margin and volume volatility. Böcker (2008) concludes that such losses can result above all from a serious deterioration of the market environment, customers' shift, changes in competitive situation, or internal restructuring. This thesis will model risk as a function of macroeconomic variables due to the lack of internal data, and hence the three last mentioned risk drivers, customer shift, changes in competitive situation and internal restructuring, will not be captured in the model. The only risk driver that will be modelled is the changes in the market environment that are captured with macroeconomic variables such as GDP, inflation, unemployment rate, interest rates. Moreover, other market variables that are likely to reflect the market environment, and hence, changes in operating profits, are foreign exchange rates and stock indices. Hence, they enter the regression as explanatory variables in order to improve the level of explanation. A complete list of variables tested for in regression is demonstrated in Table 3 below.

Table 3: Macroeconomic variables for business risk

Interest rate risk	**Stock indices**
US Government bond 10Y, 5Y SE Government bond 10Y, 5Y DK Government bond 10Y, 5Y	MSCI Swedish index (SEK) OEAX (NOK) S&P 500 (USD) KAX(EURO)
Foreign Exchange Risk	**General Macro variables**
USD/SEK USD/NOK EUR/SEK EUR/DKK	GDP (SE, DK, NO, FI)* Output gap (SE, FI) Inflation (SE, DK, NO, FI) Unemployment rate (SE, DK, NO, FI)

* A weighted GDP, measured in Euro for all countries

3.3.3. Optimal lag coefficients

This section presents and evaluates leading and lagged correlation coefficients between the business risk and the set of macroeconomic variables. The sample ranges from the third quarter 2001 to the second quarter 2008.

Figure 12: Equity indices

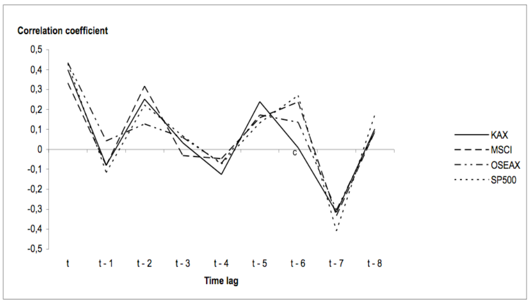


Figure 13: Foreign Exchange rates

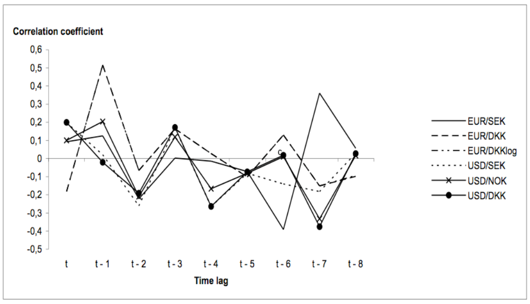


Figure 14: Interest rates

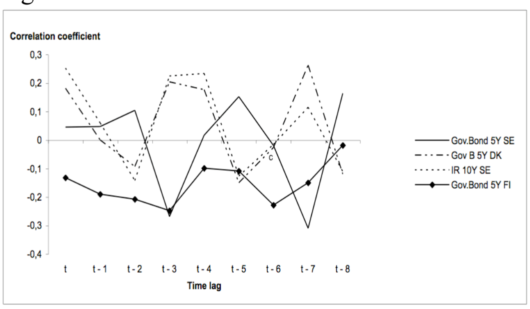
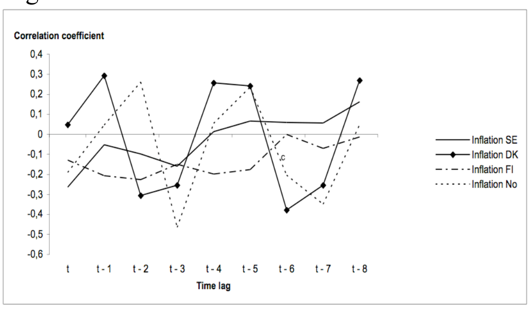


Figure 15: Inflation



As analysis of Figure 12-15 above, indicates that all variables show a behavior being too unstable to warrant any conclusions. One reasonable explanation is that business risk is a highly volatile risk (see Figure 18 for a graph) and thus long term relationship between the time lag in the variables and business risk are hard to find.

4. Mathematical methods

4.1. Regression

This section describes the regression process in detail. A short description of the chosen regression model is presented, followed by an account for the performed tests and the obtained models for each risk.

4.1.1. Regression model

Macroeconomic variables have now been identified for credit, market and business risk and furthermore the lag coefficients have been studied. In order to identify the impact from each variable on each risk, a regression will be made and two methods considered; Ordinary Least Square (OLS) and Principal Component Analysis (PCA). Linear regression is a suitable tool for finding the relationship between a set of parameters and most of all there is a closed form expression for the best-fitting parameters. Moreover, the estimates of the unknown parameters obtained form linear least squares regression are unbiased. Though there are types of data that are better described by nonlinear functions, many processes in science and engineering are well described by linear models. PCA analysis is a suitable tool when finding out which of the macro variables that contribute the most to the variance in the risk parameters. Since the extent of explanation is more important in order to find the best predicting model for each risk to deduce which macroeconomic variables contribute most to each risk, the most appropriate procedure is OLS.

The OLS regression has been made in SAS with a regression procedure that fits least-squares estimates to linear regression models, i.e.,

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \dots \quad \dots + \beta_p X_{ip} + \varepsilon_i \qquad\qquad i = 1 \dots. n$$

In this case, Y_i is the observed risk factor, $X_{ip}$is the p:th macroeconomic regression coefficient for the i:th quarter, β_p the coefficient for the p:th macroeconomic variable and ε_I the i:th error term.

The above linear regression is based on four assumptions;

- i linearity between dependent (risk) and independent (macroeconomic) variables
- ii independence of the errors (no serial correlation)
- iii homoscedasticity (constant variance) of the errors
  - a) versus time
  - b) versus the predictions, i.e., risk parameters

If any of the above assumptions is violated, the forecast of the risk parameters and economic insights yielded by a regression model may be inefficient or seriously biased.

4.1.2. Regression tests and parameters

In order to verify the conditions given above, the following set of tests have been performed and listed beneath.

- Linearity between the dependent variables and the independent variables have been tested through scatter plots
- Autocorrelation have been tested by a Durbin Watson test

Autocorrelation

When regression is performed on time series data, the errors may not be independent. Often errors are auto-correlated that is, each error is correlated with the error immediately before it. Autocorrelation is also a symptom of systematic lack of fit. The errors have been tested in each model in order to detect autocorrelation with the Durbin Watson *d* statistic to test that the autocorrelation is zero:

$$d = \frac{\sum_{i=1}^{n}(e_i - e_{i-1})^2}{\sum_{i=1}^{n} e_i^{\ 2}} \qquad \text{where } e_i = y_i - \hat{y}_i$$

If the value of *d* is close to 2, the errors are uncorrelated.

This autocorrelation of the residuals may not be a very good estimate of the autocorrelation of the true errors, especially if there is a limited amount of observations and the independent variables have certain patterns. In the case for this thesis, we are suffering from a lack of observations and hence, the d-statistic might not give the optimal estimate.

Positive autocorrelation of the errors generally tends to make the estimate of the error variance too small, so confidence intervals are too narrow and true null hypotheses are rejected with a higher probability than the stated significance level. Negative autocorrelation of the errors generally tends to make the estimate of the error variance too large, thus confidence intervals are too wide and the power of significance tests is reduced. With either positive or negative autocorrelation, least-squares parameter estimates are usually not as efficient as generalized least-squares parameter estimates.

Multi-collinearity

When a regressor is nearly a linear combination of other regressors in a model, the affected estimates are unstable and have high standard errors. This problem is called collinearity or multi-collinearity. Now, this would occur a problem if one would want to find the level of contribution of each variable, i.e., the variable that contributes most to the variance of the dependent variable. In this thesis, however, since it is only of concern to find the best predicting model and hence multi-collinearity does not constitute a problem. Nevertheless, Belsey, Kuh and Welsch (1980) suggest that when the conditional index, defined as the square roots of the ratio of the largest eigenvalue to each individual eigenvalue, is larger than 100, the estimates may have a fair amount of numerical error (although the statistical standard error almost always is much greater than the numerical error). To be rigorous, collinearity tests have been performed in order to identify variables with a strong dependency. The collinearity tests are demonstrated in Appendix B

For each variable, the proportion of the variance of the estimate accounted for by each principal component has been computed. A collinearity problem occurs when a component associated with a high condition index contributes strongly (variance proportion greater than about 0.5) to the variance of two or more variables.

4.1.3. Regression models for Credit Risks

Credit risk regression will be performed separately for each country in order to capture special characteristics (e.g. governmental changes, financial and economic cycles) associated to each country. Each linear model will be presented, per country, in the following sections. Lower case letters indicates that the change in macroeconomic variables has been modelled. The use of upper case letters indicates that the actual percentage value has been modelled. Each model with a further description of the regression statistics and explanations of macroeconomic variables can be found in appendix B and D.

Starting with Sweden, the evolution of credit risk is strongly related to the Swedish government bond and unemployment rate for the household as well as the corporate sector. The established relationships are in detail:

$$Credit\ Risk_t^{HH\ SE} = -0{,}002 + 0{,}0076 GOVB10Y_{t-3}^{SE} - 0{,}0016 UMPR_{t-2}^{SE} - 0{,}0037 hhcons_{t-5}^{SE}$$
$$Credit\ Risk_t^{C\ SE} = 0{,}036 + 0{,}01764 GOVB10Y_{t-1}^{SE} - 0{,}046 gdp_{t-2}^{SE} - 0{,}17 indprod_{t-4}^{SE}$$

Where, GOVB10Y is the 10 year government bond, UMPR is the quarterly unemployment rate, hhcons is the quarterly changes for the households' consumptions, gdp is the quarterly changes in gross domestic product and indprod is the quarterly changes in the industry production.

Some of the partial effects of the macroeconomic variables on credit risk are intuitive. Higher interest rates increase loan losses for the household and corporate sector meaning that an increase in the Swedish government bonds is followed by an increase in credit risk. A negative relationship between GDP and industry production and credit risk holds, i.e., when GDP and industry production increases are negative related to loan losses

For Finland, the regression indicates that the evolution of credit risk is related to GDP and unemployment rate for the household and corporate sector. The established relationships are in detail;

$$Credit\ Risk_t^{HH\ FI} = -0{,}065 + 0{,}012 UMPR_{t-6}^{FI} - 0{,}056 gdp_{t-2}^{FI}$$
$$Credit\ Risk_t^{C\ FI} = 0{,}010 + 0{,}020 UMPR_{t-7}^{FI} - 0{,}023 gdp_{t-2}^{FI} - 0{,}037 INFL_{t-1}^{FI} + 0{,}11 outputgap_t^{FI}$$

Where, UMPR is the quarterly unemployment rate, gdp is the quarterly change in the domestic product and INLF is the quarterly inflation.

The partial effects of the macroeconomic variables on credit risk for Finland are intuitive. Higher unemployment rate increases credit risk for the corporate and household sector and an increase in GDP decreases credit risk. A rise in output gap results in an increase in credit risk which is intuitive as it indicates that actual output is higher than the potential output. Economic theory suggests that positive output gap will lead to inflation as production and labour costs rise.

The corporate credit risk model further indicates that inflation and credit risk are negatively related with a time lag of 2 quarters, i.e., the higher inflation the lower credit risk. This relationship is not as intuitive since, according to economy theory, a high inflation is often followed by higher interest rates, which normally increase the number of defaults. The explanation here is probably related to the time lag. The time lag is obtained through optimising the, by the model, explained variance (R^2). A longer time lag would likely result in a positive relationship between inflation and credit risk, but with the drawback that the coefficient of explanation decreases.

Similar patterns can be seen for Denmark: higher unemployment rate increases credit risk for the corporate and household sector and an increase in GDP decreases credit risk.

$$Credit\ Risk_t^{HH\,DK} = -2{,}14 - 0{,}0014 gdp_{t-4}^{DK} + 0{,}015 UMPR_{t-1}^{DK} + 2{,}15 INFL_{t-5}^{DK}$$
$$Credit\ Risk_t^{C\,DK} = -5{,}71 - 0{,}027 gdp_{t-4}^{DK} + 0{,}023 UMPR_t^{DK} + 5{,}62 INFL_{t-5}^{DK} + 0{,}039 IR3M_{t-8}^{DK}$$

Where, gdp is the change in gross domestic product, UMPR is the unemployment rate, INFL is the quarterly inflation and IR3M is the 3 month interest rate.

The positive relationship between inflation and credit risk, discussed in the above section, is captured by the regression for the household and corporate sector. Credit risk associated to corporate sector also shows a positive relationship with interest rates and credit risk, an intuitive relationship as higher interest rates likely results in higher default rates. This result is also found by Jacobsen (2005).

Credit risk for Norway has the same positive relationship between unemployment rate and credit risk for the corporate and household sector. Moreover, the household sector shows the intuitive relationship between households' liabilities and credit risk; as households' liabilities increase, credit risk increases. For the corporate sector, credit risk increases when the Norwegian government bond increases.

$$Credit\ Risk_t^{HHNo} = -0{,}055 + 0{,}009 hhliab_{t-3}^{No} + 0{,}015 UMPR_t^{No} + 0{,}008 gdp_{t-6}^{No}$$
$$Credit\ Risk_t^{C\,No} = -0{,}54 + 0{,}11 GOVB5Y_{t-3}^{No} + 0{,}06 UMPR_{t-7}^{No} - 0{,}037 gdp_{t-3}^{No}$$

Where, hhliab is the quarterly change in households' liabilities, gdp is the quarterly change in gross domestic product and GOVB5Y is the 5 year government bond.

4.1.4. Regression model for Market Risk

Market risk regression has been made with the macro variables discussed in section 3.2.1 and the final model includes the macro variables that captures maximum of the variation in market risk:

$$Market\ Risk_t = 0,35 + 0,21 msci_t^{Se} + 0,85 usd/dkk_{t-4} + 0,65 GOVB_{t-8}^{5Y\ Dk} + outputgap_{t-5}^{Se}$$

The model indicates that an increase in MSCI index, Danish government bond, output gap and as the Danish crown declines against the dollar result in an increase in total market risk for the entire group. The process of interpreting these relationships is a challenge as positions in market risk portfolio are likely to have changed during the examined period. Market loss is narrowed to the groups A and B and analysing for example group A, the positions taken by equity and FX-traders change on a day-to-day basis. Hence, long term relationships between for example equity indices, FX-change and market loss cannot be established. The important thing here is that the model captures the risks affecting the Bank's "market risk portfolio", i.e., equity, interest and FX-risk.

The macro economic variables for market risk given above are the variables explaining maximum of the dynamics of market risk and hence having the highest t-ratios. Moreover, each of the risks, FX, interest rate and equity risk are captured which is desired, due to the intuitively explanation factor. Nevertheless, some further motivations are needed to explain why some variables will be included while other will not. For example, one might argue that a Nordic equity index would be used due to the concentration in positions to the Nordic countries. The reason for excluding certain macro variables is due to a high correlation between the macro variables within each risk category e.g. the correlation between equity indices is between 0,91-0,96. Hence, adding more equity indices would only enhance the model slightly while decreasing the adjusted R^2. A complete list of correlations between the macroeconomic variables finally used in the model for market risk and tested for in regression is found in Table 4.

Table 4: Correlation between macroeconomic variables chosen for the market risk model and used in regression

FX Risk		**Equity Risk**		**Interest rate Risk**		**Macro variables**	
	USD/SEK		**MSCI**		**DK G.B$_{5Y}$**		**Output gap SE**
USD/SEK	1	**KAX**	0,96	**DK G.B$_{5Y}$**	1	**Output gap FI**	0,64
EUR/DKK	-0,66	**MSCI**	1	**DK G.B$_{10Y}$**	0,91		
EUR/SEK	0,58	**OSEAX**	0,91	**SE G.B$_{5Y}$**	0,84		
USD/NOK	0,90	**S&P 500**	0,96	**SE G.B$_{10Y}$**	0,85		
USD/DKK	0,54			**NO G.B$_{5Y}$**	0,86		
				NO G.B$_{10Y}$	0,88		
				FI G.B$_{10Y}$	0,72		

4.1.5. Regression model for Business Risk

The same principle as the preceding regression models has been used in business risk, i.e., the variables that account for the largest portion of variance is included in the model. The chosen macroeconomic variables are presented below:

$$Business\ Risk_t = -0,071 - 0,48 mcsi_t^{SE} + 2,76 eur/sek_{t-6} - 0,18 GOVB_{t-2}^{5Y\ US} + Infl_{t-3}^{NO} - 0,37 gdp_{t-4}^{Nordic}$$

The model suggests a positive relationship between Norwegian inflation, EUR/SEK exchange rate and business risk (i.e. change in operating cleansed profit) meaning that as inflation increases and as the Swedish crown declines against the Euro, the business risk increases, i.e., operating profit falls. Moreover, the model indicates a negative relationship between the MSCI-index, American government bond and business risk, meaning that an increase in the MSCI-index and American government bond is followed by a lower business risk, i.e., operating profit increases.

The parameters above are the optimal parameters from a statistical point of view but the choice of parameters need to be further motivated in order to explain why some variables have been excluded. One might argue that the obtained parameters do not explain the Bank’s total market risk, since, e.g. equity risk is not to be captured by the MSCI-index and the general macroeconomic condition is not explained by the Norwegian inflation. As correlation between the variables within each category, i.e., all equity indices, foreign exchange rates, interest rates and inflation, are overall highly correlated (see Table 5), this implies that adding for example all equity indices would only add a negligible (if any) explanation in variation of business risk.

Table 5: Correlation between macroeconomic variables used in the model for business risk and tested for in regression.

FX Risk		**Equity Risk**		**Interest rate Risk**		**General Macro variables**	
	EUR/SEK		**MSCI**		**US G.B$_{5Y}$**		**Inflation$_{NO}$**
EUR/SEK	1	**KAX**	0,96	**US G.B$_{10Y}$**	0,99	**Inflation$_{SE}$**	0,50
EUR/DKK	-0,49	**MSCI**	1	**US G.B$_{5Y}$**	1	**Inflation$_{DK}$**	0,61
USD/SEK	0,58	**OSEAX**	0,91	**DK G.B$_{5Y}$**	0,80	**Inflation$_{FI}$**	0,59
USD/NOK	0,24	**S&P 500**	0,96	**SE G.B$_{5Y}$**	0,73	**Inflation$_{NO}$**	1
USD/DKK	-0,34						

4.2. Historical Simulation of risks

In the preceding sections, it was described how appropriate models for credit, market and business risk were identified through linear regression. This section presents the historical simulations made by these established relationships. Simulations will be made with macro economic variables from 1983 since this period captures at least two big economic downturns; the dot.com crisis 2000 and the bank crisis around 1990.

This thesis will make a simulation, back in time, with the gathered macroeconomic variables. A historical simulation is superior to a Monte Carlo simulation due to difficulties making assumptions of the joint multivariate distribution including all macroeconomic risk factors. Historical simulation is easy to perform but includes some limitations. The largest limitation is the assumption that “what has happened in the past will happen in the future” (here this should be interpreted inversely as the relationships that holds between present macroeconomic variables and risk factors is assumed to have existed in the past). Recall the reason for performing this historical simulation being the lack of loss data and the small amount of loss data that existed could not represent the Bank’s current risk exposure, due to the organisational changes that existed before 2000.

4.2.1. Simulation of Credit Risk

A simulation back in time for each country’s sectors, i.e., household and corporate sector, have been made using the gathered macroeconomic variables. In order to verify that the performed simulations are reliable, the simulated risks have been compared with each country’s actual number of bankruptcies (see Appendix F)

Historical simulations of the household and corporate credit risk have been performed for each country, i.e., eight simulated credit risks. Actual loss observations for these eight segments are available from 2002 and are used to calculate the percentage loss of the total credit loss of each country's segment. These percentage contributions (α in the formula below) are demonstrated in Table 6.

Table 6: Each country's segment contribution (α_{Iik}) to the total credit gross loan loss

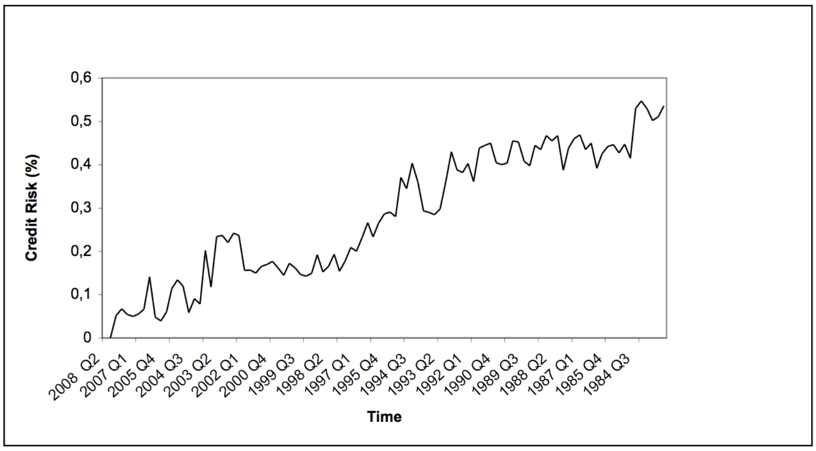
Segment\Country	Sweden	Denmark	Norway	Finland	Total
Corporate	14%	29%	20%	16%	79%
Household	3%	12%	2%	4%	21 %
Total	17%	41%	22%	20%	100%

To finally aggregate the total credit risk for all the Nordic countries, each country's amount ($\alpha_{i,k}$) of the total credit loss are used to aggregate the Bank's total credit risk. The formula for aggregation is defined as:

$$Credit\ Risk_t = \sum_{t,i,k} \alpha_{i,k} Credit\ Risk_t^{i,k} \qquad \begin{array}{l} t = 1,2...quarters \\ i = SE, DK, NO \text{ and } FI \\ k = HH \text{ and } C \end{array}$$

where i is the index for each country, k is the index for households or corporate and $\alpha_{i,k}$ is the part of loan loss for the i:th country's k:th segment. The aggregated total simulated credit risk is demonstrated in Figure 16.

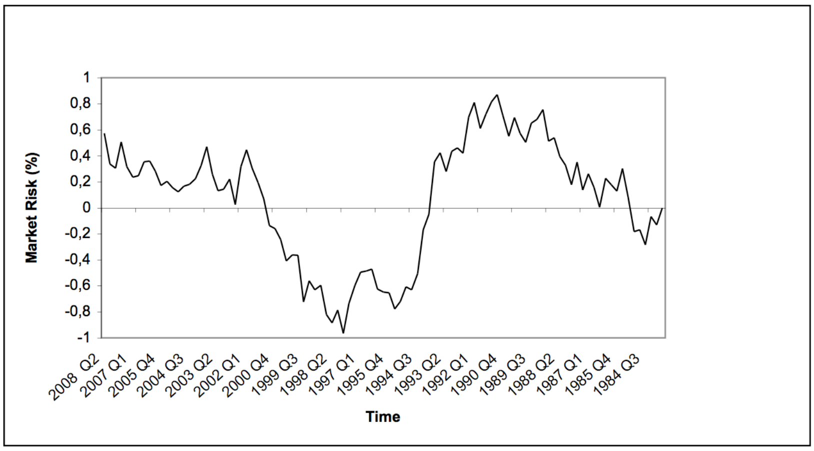
Figure 16: Simulated credit risk for all the Nordic countries



4.2.2. Simulation of Market Risk

As the optimal model, due to explanation in variance, has been found, a historical simulation of market risk can be performed with the macroeconomic time series. Figure 17 shows the result of the historical regression with the macroeconomic variables.

Figure 17: Simulated market risk between 1983-2000



As can be seen, market risk displays two intervals of negative market risk, which needs to be further analysed. Recalling the definition of market risk used in the model, i.e.,

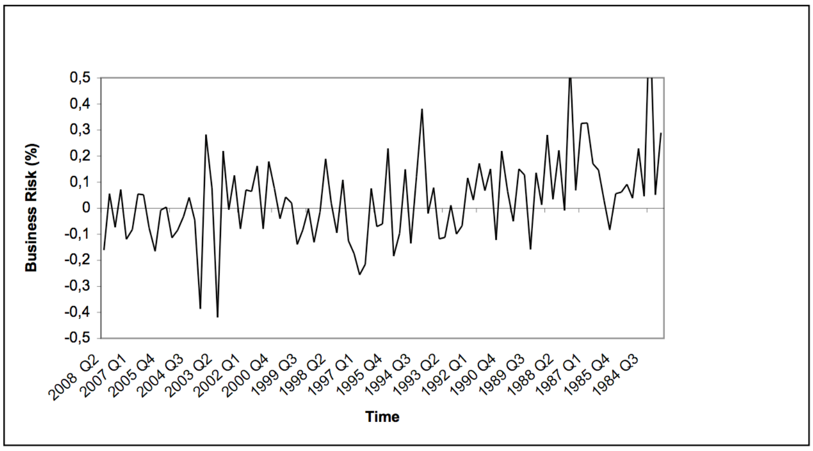
$$Market\ risk_t = \frac{Actual\ Market\ Loss_t}{VaR_t^{99\%}} \qquad t = 1,2\ldots quarters$$

where n is the number of quarters. A negative market risk simply means that the simulated market risk is a negative loss, i.e., a profit. Hence, these periods are not of our concern, and will be excluded when computing correlation in stressed events. Another remark to make, is that the simulated as well as observed market risk never hits a ratio larger than one, indicating that the actual loss is never larger than the simulated VaR, which is a good indication of that the simulated market risk is reliable.

4.2.3. Simulation of Business Risk

As the relationship between macroeconomic variables and business risk has been identified through regression, a historical simulation is made back to 1983 and is demonstrated in Figure 18.

Figure 18: Simulated Business Risk between 1983-2008



Recall the definition of business risk, i.e.,

$$Business\ Risk_t = \frac{-(oper\ profit_t - oper\ profit_{t-1})}{oper\ profit_{t-1}}$$

$$t = 1,2..quarters$$

Risks are positive, i.e., a decline in operating cleansed profit indicates a risk. Hence, in the above figure, a negative business risk is an increase in profit, i.e., not a risk. As can be seen, business risk has a high volatility, which is expected, as business risk is the change in operating profit that changes from each quarter.

4.3. Correlation

The preceding sections have described how historical losses have been recreated and a sufficient amount of loss data have been created which capture at least two economic cycles. This section will describe the performance when calculating correlation between risk types starting with the assumptions being made, how extreme events have been identified, and finally discussing potential implications of transfer correlation in terms of diversification.

4.3.1. Time lag for losses

Losses usually occur with a time lag between the actual default and the realisation of the loss, i.e., there is a time lag between default and realised loss for the Bank. Internal analysis performed by the Bank shows that the major part of credit losses are realised after X years[9], and hence, credit risk is provided with a time lag of X years when analysed against market and business risk in order to capture the actual maximum correlation.

4.3.2. Pearson Correlation Matrix

Pearson correlation matrix was defined in chapter 2 and linear correlations will be calculated with the formula that can be seen in Appendix E. As can be seen in the formula, the covariance of the error term must be taken into consideration. The error terms are true observations between the years when actual risk observations exist and in order to capture the covariance related to the error terms, covariance of the error terms will be estimated by the true covariance when predictions of each risk are made back in time.

4.3.3. Identifying extreme events

This thesis strive to model the maximum correlation between all risks, i.e., the correlation measured when credit, market and business risk are at their highest all at once. The desired correlation is the highest correlation between all risks, i.e., the correlation measured in stressed periods. It is of importance that the Economic Capital covers unexpected losses, but at the same time, the Bank should strive to maintain as little Economic Capital as needed, so as to invest capital at a higher rate of return

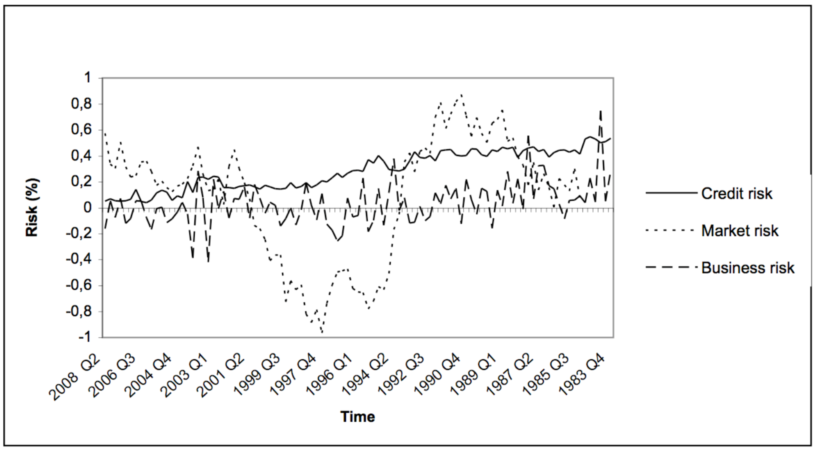
[9] The actual time lag cannot be demonstrated in this thesis as it is classified as secret information.

In order to capture the highest correlation, to insure that Economic Capital cover extreme losses, i.e., when all risks are high at once, this thesis will calculate the correlations in stressed scenarios. The stressed intervals will be visually selected and shall represent the intervals in which credit, market and business risk peak all at once. It is convenient to select those intervals visually, as it is a historical simulation and hence the extreme events are known e.g. the bank crisis during 1990 and can easily be captured. Moreover, a statistical evaluation of the data by Extreme Value Theory is not appropriate due to the lack of observations. In case of longer time series, extreme periods would be chosen by for example a mean-excess plot.

Table 7: Visually selected stressed intervals for all risk types

Visually selected intervals for all risk types
1991Q4-1993 Q3
2002Q2-2005 Q2

Figure 19: Simulated credit, market and business risk between 1983-2008



As credit and market risk are the main risks within the Economic Capital, the correlation between these risks are of high importance and hence focusing on when to choose stressed scenarios. Business risk fluctuates highly over time and hence extreme peaks of losses solely are in general hard to identify and in particular, when considering peaks of credit and market risk. As can be seen in Figure 19 above, business risk is negative at certain quarters and will be set to zero as only losses are of concern at those stressed intervals.

4.4. Correlation matrices

In order to give a complete view of correlations, three correlation matrices will be calculated; one during the entire interval, one during losses only and one during extreme events.

The first calculation is made during the whole period, i.e., between the first quarter 1987 and the second quarter 2008, i.e., 87 quarters. This interval contains both positive and negative risks, i.e., both losses and gains in order to provide a complete description of the dependency structure. It can be argued that it is of importance to incorporate negative risks, i.e., profits in order to illustrate how risks correlate during a constant period of time.

Table 8: Established Correlation between 1987 and 2008

	Credit risk	Market risk	Business risk
Credit risk	1	0,48	0,34
Market risk	0,48	1	0,16
Business risk	0,34	0,16	1

The highest correlation is found between lagged credit and market risk. These risks can be considered as asset based risks e.g. the value of the corporate and consumer assets being financed. The high correlation is intuitive as market and credit risk are highly correlated with the general economy; economic downturns generally result in drops in all asset values. One might also argue this being specifically true for the Bank, given the geographically concentrated nature of its portfolio. When analysing correlation between credit and market risk it is important to consider the Bank's market risk portfolio, i.e., the different market risk sub-types. The Bank's market risk portfolio consists mainly of interest rate risk, which can be assumed to have a high correlation with lagged credit losses. Higher interest rates create a greater burden for a borrower attempting to service their debt position, resulting in a higher probability of default.

Business and credit risks are not as highly correlated as credit and market risks. Business risk is the most volatile of the three risks and is driven by fluctuations in operating profit. Hence, the general state of the economy is one major driver of business risk, which explains the positive correlation with the lagged credit risk. Though the general state of the economy is a major contributor to business risk, other contributors exists such as fluctuations in market share, which have not been captured in the model for business risk.

Business and market risks have a relatively low correlation, which by a first view might seem abnormal. Market risk consists of equity, interest rate and FX-risk, which also contributes to changes in operating profits, i.e., business risk. However, even if both risks carry a common set of risk drivers, this does not necessary implies a high positive correlation. If the macroeconomic variables contribute to the risk in opposite ways e.g. if one variable increases the risk and decreases the other risk they counteract each other. This is the case for two of the variables included in the models for business risk and market risk. MCSI- index represents interest rate risk for market and business risk but contributes oppositely to each risk; positively on market risk while negatively on business risk. Danish Government bond and US government bond represent interest rate risk that also contributes in opposite ways; positively on market risk while negatively on business risk. Hence, even if both models have a common set of macro variables; a lower correlation is to be expected.

Next step is to calculate correlation when excluding profits. This will be done by only calculating correlation when credit, market and business risk are positive all at once which corresponds to 34 quarters (i.e.34 data points).

Table 9: Correlation when excluding profits

	Credit risk	Market risk	Business risk
Credit risk	1	0,43	0,20
Market risk	0,43	1	-0,23
Business risk	0,20	-0,23	1

Table 9 shows that the correlation between credit, market and business risk is lower when considering losses only. This can be interpreted, as risks are having a strong positive correlation in case of a decreasing economy or even a recession as during times of a strong economy. A reasonable explanation for this is that losses are not perfectly correlated, i.e., all losses do not occur and peak at once. A time lag between losses is likely to exist and as market risk is provided with a time lag of five quarters, the correlation increased from -0,23 to 0,49. As the Bank strive to hold the Economic Capital needed to cover scenarios in which risks have the highest correlation, it cannot apply the correlation matrix based on losses solely, when calculating the diversified Economic Capital. Therefore, the correlation needs to be calculated in stressed scenarios in order to investigate if the correlation between risks is higher.

Continuing with correlation calculated in stressed scenarios, these results are higher for all risks. During these stressed scenarios, business risk was negative (i.e., profit) at certain quarters and those were set to zero. As can be seen in
Table 10 below, the correlation between credit and market risk is the strongest correlation. As arguing in the beginning of this section, a strong correlation between credit and market risk is likely to exist due to the fact that both risks are strongly correlated with the general economy.

Table 10: Established correlation in stressed scenarios

	Credit risk	Market risk	Business risk
Credit risk	1	0,73	0,41
Market risk	0,73	1	0,51
Business risk	0,41	0,51	1

Correlation between business risk vis-à-vis all other risks are significantly higher in stressed scenarios than in the two cases studied above (i.e., during the whole interval and the period of losses solely). As argued in the section when stressed intervals was chosen, a negative business risk is set to zero which is a reasonable explanation to the higher correlation with other losses.

4.4.1. Including Operational Risk and Life Risk

Operational losses are collected by the Group Operational Risk Management and are used to reflect operational risk. Loss observations are available from 2004, i.e., 18 quarters.

$$operational\ risk_t = \frac{operational\ loss_t - operational\ loss_{t-1}}{operational\ loss_{t-1}}$$

$$t = 1,2\ldots\ldots quarters$$

Due to the difficulty in finding risk drivers for operational risk, historical simulations will not be made and correlation will be measured on actual risk figures. The established correlation is demonstrated in
Table 11.

Life risk is assumed to be uncorrelated to all other risk types as fluctuations in mortality rates are assumed to be uncorrelated to other risk drivers.

Table 11: Correlation between operational risk and life risk

	Operational risk	Life risk
Credit risk	0,49	0
Market risk	-0,07	0
Business risk	0,21	0

5. Conclusion

The recommended correlation matrix is the one established in this thesis, based on correlations calculated during extreme events, and is shown
Table 12.

Table 12: Established and recommended correlation matrix

	Credit risk	Market risk	Business risk	Operational risk	Life risk
Credit risk	1	0,73	0,41	0,49	0
Market risk	0,73	1	0,51	0	0
Business risk	0,41	0,51	1	0,21	0
Operational risk	0,49	0	0,21	1	0
Life risk	0	0	0	0	1

The importance of considering the highest correlation possible, which has already been stressed in this thesis, is that a bank needs to hold the amount of Economic Capital that covers the maximum scenario of losses, i.e., when all losses peak at once. Over a cycle, the correlation between risks changes but a bank always needs to be prepared for the worst scenario (i.e., the highest correlation between risks) and the maximum correlation matrix should be used when calculating the diversified Economic Capital.

The correlation between operational risk vis-à-vis credit and business risk is based on calculations of the actual observations between 2004 and 2008. Correlation between operational and market risk is set to zero, as negative correlation is not allowed.

Correlations between life risk vis-à-vis all other risks are assumed to be zero, since mortality rates are assumed to be uncorrelated with all other risk drivers.

For credit, market and business risks, historical simulations have been performed with macroeconomic variables that represent the risk characteristics of the Bank's risk exposure. For example credit risk has been analysed on country level and separated into households and corporations and regressions have been made against macroeconomic variables that represent each country and segment.

6. Improvements

Though the aim of this thesis, to calculate correlations between risks based on risk data that correspond to the specific risk characteristics of the Bank's portfolio, can be considered fulfilled, some needs for improvements have been identified to improve the correlation matrix further.

The models for credit, market and business risks, are linear, and even though most of the relationships between the variables showed such a relationship some information is likely lost. Other models, such as non-linear, logit-models and AR(p) processes could be developed in order to explore their significance in predicting each risk.

Another improvement to be considered is the choice of simulation with macroeconomic variables back in time as it presupposes that the relationship between the actual number of observations also holds back in time (in this case back to 1983). This is the inverse limitation to the general limitation to historical simulation; "what we have seen in the past will happen in the future". Hence, an improvement would be to allow for changes in dependency structure, i.e., to allow for modifications of the contribution of each macroeconomic variable in the model. Instead of simulating losses back in time one could consider simulating losses in the future in order to calculate a correlation matrix built on future scenarios. Such an implementation could be made by Monte Carlo simulation that allows for possible scenarios that have not yet been mirrored in history.

Moreover, the choice of extreme events should, as new losses occur, be selected by statistical methods and not only visually.

Finally, rank correlations are defined at a copula level and would allow for dependency structures that are non-elliptical. The use of rank correlation would be necessary if the currently used aggregation method, variance-covariance, should be replaced by a copula approach.

Appendix A: Data sources

Variable	Sweden	Denmark	Norway	Finland
GDP (current prices)	EW: (1980)	EW:	EW: (1978)	EW: (1975)
GDP (current prices, SA)	EW: (1980)	EW:(1977)	EW: (1978)	EW: (1975)
GDP (constant prices, SA)	EW: SEK, Index	EW: (1993) DKK Eurostat (1977) EURO	EW: (1978) Eurostat (1978) EUR, Index,	EW: (1975)
GDP (constant prices)	Eurostat (1980) SEK, EUR, Index	EW: (1993) DKK Eurostat (1977) EURO	EW: (1978) NOK Eurostat(1978) EUR, Index	EW: (1975)
CPI	EW: M; 1920-	EW: M; 1967-	EW: M; 1920 SA: M;1979-	SF:Q;1980-
CPI SA				
Unemployment rate 1960- M	EW: M; 1970-		OECD**; Q;1980-	EW: M; SA (1988-)
Output gap	SCB: M;1980-			IMF: Q 1980-
Households total assets			EW: Q; 1980- NOK	
Households total liabilities			EW: Q;1980- NOK	
Households total consumption, constant prices	IH: Q; 1980-*			
Households total assets				
Industry production constant prices	IH: Q:1980-*			
Households Investments current prices	IH: Q; 1980-*			
Stock Index	EW: M; 1969- MSCI	EW: M; 1957- KAX	EW: M; 1957- OSEAX	

# Bankruptcies	EW: (1994-)M	EW: (1984)*M		EW:,M; 1986-
Government bond				
- 10Y	EW:,M 1918-	EW:;M;1983	EW:,M;1921	
- 5Y	EW:,M; 1960-	EW:,M; 1960-	EW:,M;1961-	EW: M; 1948
- 2Y		EW: M; 1982		
T-bills 3M	EW: M; 1960-	EW: M;1972-	EW:1971-	
FX- rates	EW:, M; USD/SEK; 1913- EUR/SEK:1978-	EW:, M; USD/DKK;1957- EUR/DKK:1978-	EW:, M; USD/NOK; 1957	EW:, M;
US Government Bond	EW:			
- 10Y	(1900-)			
- 5Y	(1962-)			
S&P 500	EW: (1928-)			

** q1, q2 2008 från ssb.no
EW: Ecowin

Appendix B: Collinearity tests

Table 13: Collinearity analyse business risk

Number	Eigenvalue	Condition Index	Intercept	MSCI	EUR/SEK	$GovBond_{5Y\ US}$	Inflation	GDP
1	2,00	1,00	0,074	0,079	0,13	0,31	0,03	0,03
2	1,26	1,26	0,073	0,078	0,01	0,01	0,09	0,09
3	1,02	1,40	0,024	0,010	0,60	0,20	0,02	0,04
4	0,73	1,66	0,461	0,172	0,20	0,17	0,01	0,08
5	0,66	1,75	0,039	0,058	0,01	0,01	0,54	0,54
6	0,33	2,45	0,330	0,141	0,32	0,32	0,86	0,23

Table 14: Collinearity analysis market risk

Number	Eigenvalueue	ConditionIndex	Intercept	MSCI	USD/DKK	$GovBond_{5YDK}$	Outputgap
1	2,10	1,00	0,08	0,06	0,02	0,01	0,05
2	1,17	1,34	0,00	0,02	0,25	0,32	0,01
3	0,93	1,51	0,09	0,03	0,34	0,17	0,07
4	0,63	1,82	0,58	0,11	0,33	0,02	0,00
5	0,17	3,53	0,25	0,79	0,05	0,48	0,87

Table 15: Collinearity analysis credit risk; Swedish corporations

Number	Eigenvalue	Condition Index	Intercept	$Gov\ Bond_{10Y}$	Industry production	GDP
1	2.84	1.00	0.00	0.00	0.00	0.02
2	0.97	1.71	0.00	0.00	0.97	0.00
3	0.18	4.03	0.01	0.03	0.00	0.84
4	0.01	16.83	0.99	0.97	0.02	0.14

Table 16: Collinearity analysis credit risk; Swedish households'

Number	Eigenvalue	Condition Index	Intercept	$GovBond_{10Y}$	Unemploymentrate	HHconsumpt
1	3.51	1.00	0.00	0.00	0.00	0.02
2	0.44	2.82	0.00	0.00	0.00	0.82
3	0.04	9.26	0.00	0.17	0.28	0.11
4	0.00	30.49	1.00	0.83	0.72	0.05

Table 17: Collinearity analysis credit risk for the Norwegian corporate sector

Number	Eigenvalue	Condition Index	Intercept	$Gov\ Bond_{5Y}$	Unemployment rate	GDP
1,00	3,28	1,00	0,00	0,00	0,00	0,03
2,00	0,65	2,25	0,00	0,00	0,00	0,94
3,00	0,07	7,02	0,00	0,17	0,02	0,04
4,00	0,00	49,52	1,00	0,83	0,98	0,00
1,00	3,28	1,00	0,00	0,00	0,00	0,03

Table 18: Collinearity analysis credit risk for the Norwegian household sector

Number	Eigenvalue	Condition Index	Intercept	HH Total Liab	GDP	Unemployment rate
1	3,19	1,00	0,00	0,01	0,03	0,00
2	0,71	2,11	0,00	0,01	0,84	0,00
3	0,08	6,13	0,01	0,65	0,08	0,17
4	0,02	14,47	0,99	0,33	0,05	0,82

Table 19: Collinearity analysis credit risk for the Finnish's household sector

Number	Eigenvalue	Condition Index	Intercept	Unemployment rate	GDP	GovBond $_{5Y}$
1	3,62	1,00	0,00	0,00	0,02	0,00
2	0,36	3,18	0,00	0,00	0,67	0,01
3	0,02	12,71	0,06	0,11	0,19	0,97
4	0,01	26,66	0,94	0,89	0,12	0,01

Table 20: Collinearity analysis Credit risk, Finland's corporate sector

Number	Eigenvalue	Condition Index	Intercept	Unemployment rate	GDP	Inflation	Output gap
1	3,66	1,00	0,00	0,00	0,02	0,01	0,01
2	0,97	1,95	0,00	0,00	0,00	0,03	0,23
3	0,30	3,47	0,00	0,00	0,79	0,02	0,09
4	0,06	7,74	0,02	0,04	0,05	0,85	0,65
5	0,00	29,19	0,98	0,96	0,14	0,09	0,03

Table 21: Collinearity analysis Credit risk, Denmarks's corporate sector

Number	Eigenvalue	Condition Index	Intercept	GDP	Unemployment rate	IR 3M	Inflation
1	3,52	1,00	0,002	0,0098	0,0018	0,0063	0,025
2	0,90	1,98	0,00015	0,93	0,00020	0,0025	0,0038
3	0,50	2,66	0,0030	0,0000022	0,0038	0,0076	0,896
4	0,066	7,32	0,12	0,063	0,028	0,85	0,0074
5	0,014	16,07	0,88	0,00041	0,97	0,130	0,0677

Table 22: Collinearity analysis Credit risk, Denmarks's housholds sector

Number	Eigenvalue	Condition Index	Intercept	GDP	Unemployment rate	Inflation
1	2.68	1.00	0.0033	0.0249	0.0033	0.050
2	0.87	1.75	0.00052	0.962	0.00058	0.027
3	0.44	2.47	0.00921	0.0116	0.00729	0.899
4	0.013	14.30	0.98691	0.0011	0.98887	0.0244

Appendix C: Regressions Statistics

Table 23: Variables and regressions statistics for the Swedish household sector

Variable	Time lag	Parameter Estimate	Standard error	t-value	p-value
Intercept		-0.00203	0.02312	-0.09	0.9307
Government bond 10Y	3	0.00758	0.00311	2.44	0.0234
Unemployment rate	2	-0.00157	0.00239	-0.65	0.5195
Households consumption	5	-0.00368	0.00344	-1.07	0.2962
Statistics Results					
R^2	0,449				
Adjusted R^2	0,374				
Durbin Watson	2,712				
1:st order of autocorrelation	-0,359				
Number of observations	26				

Table 24: Variables and regressions statistics for the Swedish corporate sector

Variable	Time lag	Parameter Estimate	Standard error	t-value	p-value
Intercept		0.03568	0.06895	0.52	0.61
Government bond 10Y	1	0.01764	0.0145	1.22	0.2365
GDP	2	-0.04601	0.02327	-1.98	0.0607
Industry production	4	-0.1752	0.10881	-1.61	0.1216
Statistics Results					
R^2	0,310				
Adjusted R^2	0,216				
Durbin Watson	1,115				
1:st order of autocorrelation	0,415				
Number of observations	26				

Figure 20: The Swedish household sector [10]

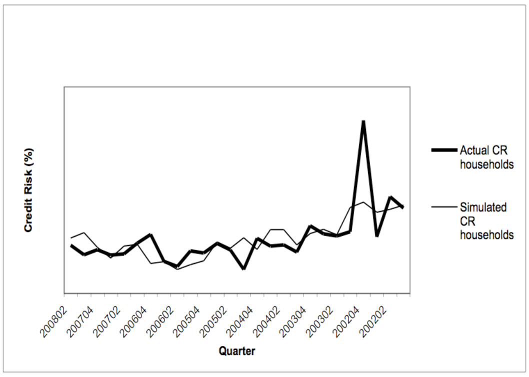
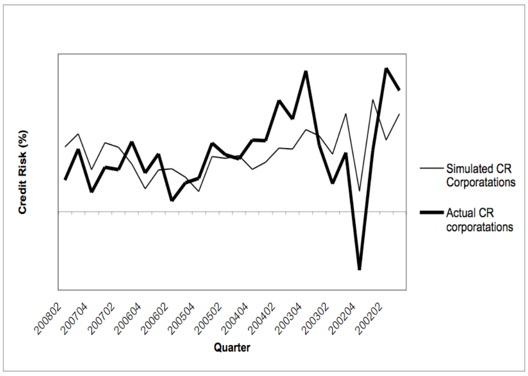


Figure 21: The Swedish corporate sector



[10] The credit risk values (the values of the y-axis) will not be demonstrated in the following graphs in this Appendix as it is classified as secret.

Table 25: Variables and regressions statistics for the Finnish household sector

Variable	Time lag	Parameter Estimate	Standard error	t-value	p-value
Intercept		-0,06538	0,02651	-2,47	0,0215
Unemployment rate	6	0,0122	0,00315	3,9	0,0008
GDP	2	-0,05595	0,01881	-1,33	0,1983
Statistics Results					
R^2	0,504				
Adjusted R^2	0,437				
Durbin Watson	2,375				
1:st order of autocorrelation	-0,196				
Number of observations	26				

Table 26:Variables and regressions statistics for the Finnish corporate sector

Variable	Time lag	Parameter Estimate	Standard error	t-value	p-value
Intercept		0,01027	0,10496	0,1	0,923
Unemployment rate	7	0,01988	0,0108	1,85	0,0784
GDP	2	-0,02252	0,01599	-1,41	0,1736
Inflation	1	-0,03673	0,01748	-2,1	0,0478
Output gap	0	0,11448	0,06201	1,85	0,079
Statistics Results					
R^2	0,3608				
Adjusted R^2	0,239				
Durbin Watson	2,322				
1:st order of autocorrelation	-0,191				
Number of observations	26				

Figure 22: The Finnish household sector

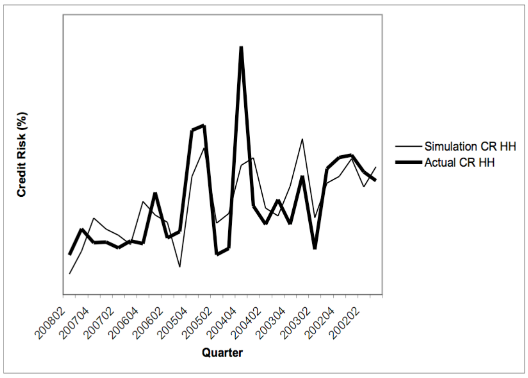


Figure 23: The Finnish corporate sector

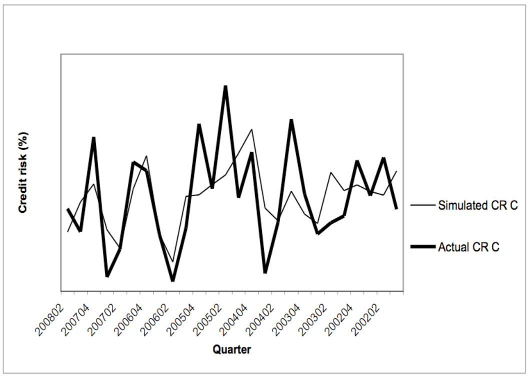


Table 27: Variables and regressions statistics for the Danish household sector

Variable	Time lag	Parameter Estimate	Standard error	t-value	p-value
Intercept		0,01327	0,04459	0,30	0,7689
GDP	6	-0,01199	0,00599	-2,0	0,0577
Unemployment rate	1	0,01436	0,00943	1,52	0,1418
Inflation	2	0,01978	0,01296	1,53	0,1413
Statistics Results					
R^2	0,3088				
Adjusted R^2	0,2146				
Durbin Watson	1,959				
1:st order autocorrelation	0,0115				
Number of observations	26				

Table 28: Variables and regressions statistics for the Danish corporate sector

Variable	Time lag	Parameter Estimate	Standard error	t-value	p-value
Intercept		-0,08873	0,10496	-1,06	0,2990
GDP	4	-0,02376	0,01198	-1,98	0,0605
Unemployment rate	0	0,02247	0,01951	1,15	0,2623
Interest rate 3M	8	0,04026	0,01430	2,81	0,0104
Inflation	5	0,05232	0,02573	2,03	0,0548
Statistics Results					
R^2	0,5885				
Adjusted R^2	0,5101				
Durbin Watson	2,282				
1:st order of autocorrelation	-0,289				
Number of observations	26				

Figure 24: The Danish household sector

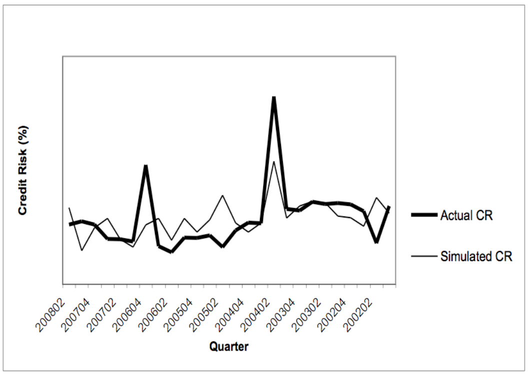


Figure 25: The Danish corporate sector

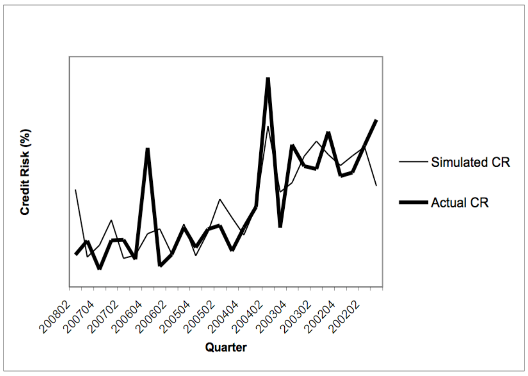


Table 29: Variables and regressions statistics for the Norwegian household sector

Variable	Time lag	Parameter Estimate	Standard error	t-value	p-value
Intercept		-0.05544	0.04060	-1.37	0.1859
HH liabilities	3	0.00870	0.00715	1.22	0.2369
GDP	6	0.00804	0.00717	1.12	0.2743
Unemployment rate	0	0.01482	0.00803	1.85	0.0784
Statistics Results					
R^2	0,180				
Adjusted R^2	0,07				
Durbin Watson	2,314				
1:st order of autocorrelation	-0,169				

Table 30: Variables and regressions statistics for the Norwegian corporate sector

Variable	Time lag	Parameter Estimate	Standard error	t-value	p-value
Intercept		-0.54292	0.86753	-0.63	0.5379
Gov Bond 5 Y	3	0.11467	0.06151	1.86	0.0757
Unemployment rate	7	0.05529	0.15251	0.36	0.7204
GDP	3	-0.03679	0.04262	-0.86	0.3974
Statistics Results					
R^2	0,310				
Adjusted R^2	0,217				
Durbin Watson	0,510				
1:st order of autocorrelation	0,652				
Number of observations	26				

Figure 26: The Norwegian household sector

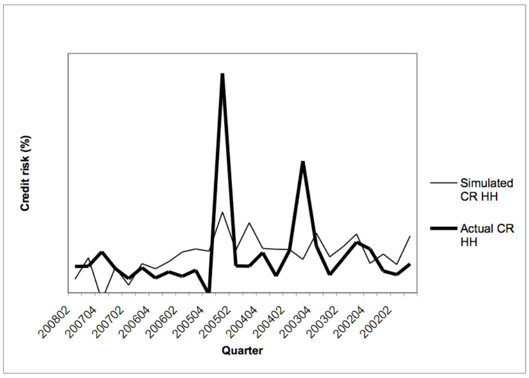


Figure 27: The Norwegian corporate sector

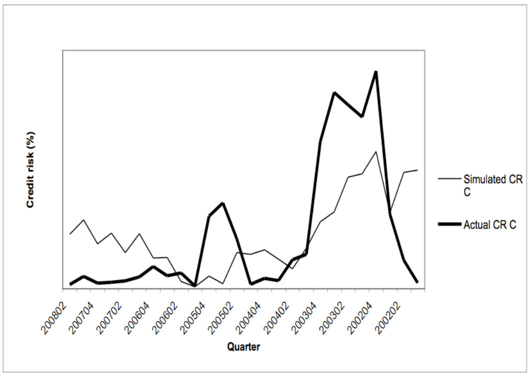


Table 31: Variables and regressions statistics for market risk

Variable	Time lag	Parameter Estimate	Standard error	t-value	p-value
Intercept		0,34503	0,02490	13,86	<0,0001
MSCI Sweden	0	0,21210	0,39390	0,54	0,6001
USD/DKK	4	0,84888	0,50387	1,68	0,1179
Government Bond 5Y DK	8	0,64763	0,31074	2,08	0,0592
Output gap SE	5	0,15082	0,04595	3,28	0,0066
Statistics Results					
R^2	0,654				
Adjusted R^2	0,539				
Durbin Watson	2,273				
1:st order of autocorrelation	-0,228				
Number of observations	17				

Figure 28: Simulated market risk

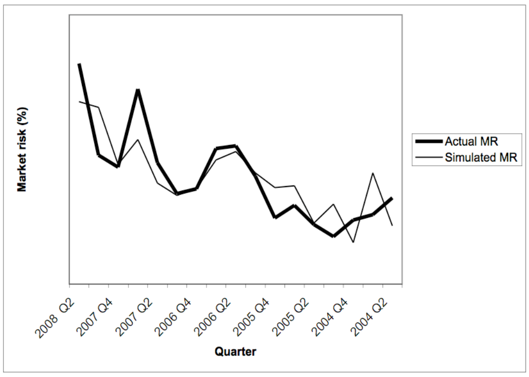
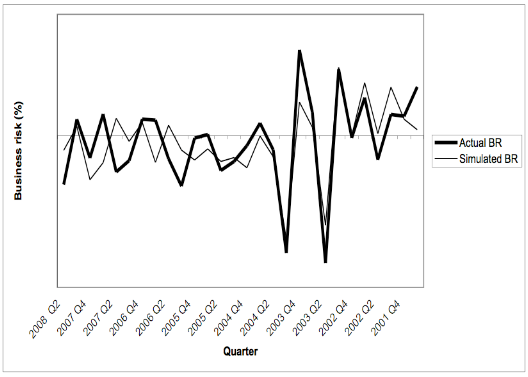


Table 32: Variables and regressions statistics for business risk

Variable	Time lag	Parameter Estimate	Standard error	t-value	p-value
Intercept		-0,07067	0,02244	-3,15	0,0047
MSCI Sweden	0	-0,48416	0,16899	-2,87	0,009
EUR/SEK	6	2,75984	0,84938	3,25	0,0037
Government Bond 5Y US	2	-0,18483	0,20629	-0,9	0,38
Inflation Norway	3	0,13674	0,03276	4,17	0,0004
GDP	4	-0,36999	0,66122	0,56	0,5814
Statistics Results					
R^2	0,637				
Adjusted R^2	0,5544				
Durbin Watson	1,895				
1:st order of autocorrelation	-0,019				
Number of observations	28				

Figure 29: Simulated business risk



Appendix D: Correlation between regressors

Table 33: Correlation matrix for inflation in the Nordic countries

	Inflation SE	Inflation DK	Inflation FI	Inflation NO
Inflation SE	1	-0,65	0,56	0,50
Inflation DK	-0,65	1	0,66	0,61
Inflation FI	0,56	0,66	1	0,59
Inflation NO	0,50	0,61	0,59	1

Table 34: Correlation matrix for Government bonds used in regression for market and business risk

	US G.B 10Y	USG.B 5Y	DK G.B. 5Y	SE G.B. 5Y
US G.B 10Y	1	0,99	0,83	0,75
US G.B 5Y	0,99	1	0,80	0,73
DK G.B 5Y	0,83	0,80	1	0,72
SE G.B 5Y	0,75	0,73	0,72	1

Table 35: Correlation between FX-rates used in regression for market and business risk

	EUR/SEK	EUR/DKK	USD/SEK	USD/NOK	USD/DKK
EUR/SEK	1	-0,49	0,58	0,24	-0,34
EUR/DKK	-0,49	1	-0,17	0,11	0,60
USD/SEK	0,58	-0,17	1	0,90	0,54
USD/NOK	0,24	0,11	0,90	1	0,76
USD/DKK	-0,34	0,60	0,54	0,76	1

Table 36: Correlation between equity indices used in regression for market and business risk

	KAX	MSCI	OSEX	S&P 500
KAX	1	0,96	0,97	0,91
MSCI	0,96	1	0,91	0,96
OSEAX	0,97	0,91	1	0,83
S&P 500	0,91	0,96	0,83	1

Appendix E: Correlation formula

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)} \times \sqrt{Var(Y)}}$$

where

$$COV(X,Y) = COV(\alpha + \sum_{i=1}^{n} \varphi_i x_i + \varepsilon_i, \beta + \sum_{k=1}^{m} \phi_i y_i + \eta_i) = COV(\sum_{i=1}^{n} \varphi_i x_i + \varepsilon_i, \sum_{k=1}^{m} \phi_i y_i + \eta_i) =$$

$$\varphi_1 \sum_{k=1}^{m} \phi_i COV(x_1, y_i) + \varphi_2 \sum_{k=1}^{m} \phi_i COV(x_2, y_i) + ... + \varphi_n \sum_{k=1}^{m} \phi_i COV(x_n, y_i) + \sum_{i=1}^{n} \sum_{k=1}^{m} COV(\varepsilon_i, \eta_k)$$

and the variances are specified as:

$$VAR(X) = VAR(\alpha + \sum_{i=1}^{n} \varphi_i x_i + \varepsilon_i) = VAR(\sum_{i=1}^{n} \varphi_i x_i) =$$

$$\sum_{i=1}^{n} \varphi_i^2 VAR(x_i) + 2\sum_{i=1}^{n-1} \varphi_i \varphi_{i+1} COV(x_i, x_{i+1})$$

$$VAR(Y) = VAR(\beta + \sum_{k=1}^{m} \phi_k x_k + \eta_k) = VAR(\sum_{k=1}^{m} \phi_k x_k) =$$

$$\sum_{k=1}^{m} \phi_k^2 VAR(y_i) + 2\sum_{k=1}^{k-1} \phi_i \phi_{i+1} COV(y_i, y_{i+1})$$

In the above formula, x and y are the being risks studied, ϕ, φ the coefficients identified by regression and ε, η the error terms from regression model.

The above formula is specified below for the correlation between credit and market risk:

$$Corr(CR,MR) = \frac{COV(CR,MR)}{\sqrt{VAR(CR,MR)}} = \frac{COV(\sum_{i,l,t} \alpha_i \beta_{i,l} X_{l,t}^i + \varepsilon_{i,t}, \sum_{i,l,t} \delta_m Y_{m.t} + \gamma_t)}{\sqrt{VAR(\sum_{i,l,t} \alpha_i \beta_{i,l} X_{l,t}^i + \varepsilon_{i,t}) VAR(\sum_{i,l,t} \delta_m Y_{m.t} + \gamma_t)}}$$

where

$$COV(\sum_{i,l,t} \alpha_i \beta_{i,l} X_{l,t}^i + \varepsilon_{i,t}, \sum_{i,l,t} \delta_m Y_{m.t} + \gamma_t) = \sum_{i,l,m,t} \alpha_i \beta_{i,l} \delta_m COV(X_{l,t}^i, Y_{m,t}) + \alpha_i COV(\varepsilon_{i,t}, \gamma_{m,t})$$

$$VAR(\sum_{i,l,t}\alpha_i\beta_{i,l}X^i_{l,t}+\varepsilon_{i,t})=\alpha_1^2\sum_{l,t}\beta_{1,l}VAR(X^1_{l,t}+\varepsilon_{1,t})+2\alpha_1\alpha_2\sum_{l,t}\beta_{1,l}\beta_{2,l}VAR(X^1_{l,t}+\varepsilon_{1,t},X^2_{l,t}+\varepsilon_{2t})$$

$$+...+2\alpha_1\alpha_8\sum_{l,t}\beta_{1,l}\beta_{8,l}VAR(X^1_{l,t}+\varepsilon_{1,t},X^8_{l,t}+\varepsilon_{8t})+...+2\alpha_8^2\sum_{l,t}\beta_{8,l}VAR(X^8_{l,t}+\varepsilon_{8t})$$

$t = 1,2..quarters$

$i = 1,2,...8$

$\alpha_i = percentage\ loan\ loss\ for\ \sec tor\ i$

$\beta_{i,l},\delta_m = coefficient\ for\ \sec tor\ i\ and\ macro\,\mathrm{var}\,iable$

$X_l^i = macro\,\mathrm{var}\,iable\ nr\ l\ for\ \sec tor\ i\ for\ credit\ risk$

$Y_{m.t} = macro\,\mathrm{var}\,iable\ nr\ m\ for\ market\ risk$

$\varepsilon_i,\gamma = error\ term\ for\ \sec tor\ i$

Appendix F: Simulated Credit compared to the actual number of bankruptcies

This appendix compares the simulated credit risks for each country compared to the actual number of bankruptcies. For Norway, there were no observations of bankruptcies available why they are not presented in this section. The times series of bankruptcies are the longest available but some of them does not capture the entire simulated interval.

Figure 30: The Swedish corporate sector

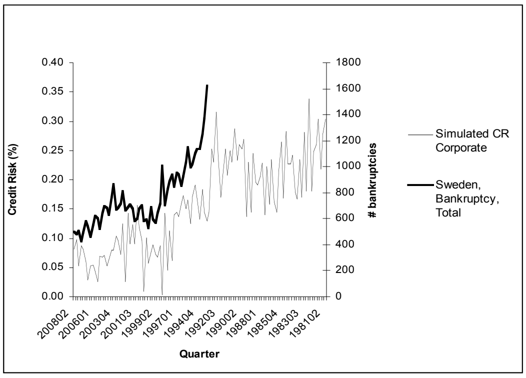


Figure 31: The Swedish household sector

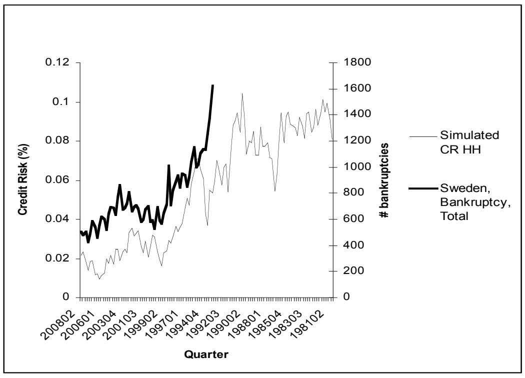


Figure 32: The Finnish corporate sector

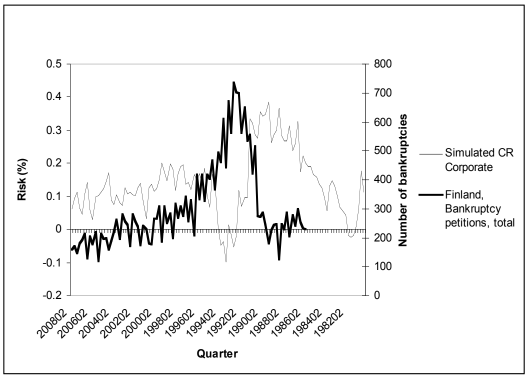


Figure 33: The Finnish household sector

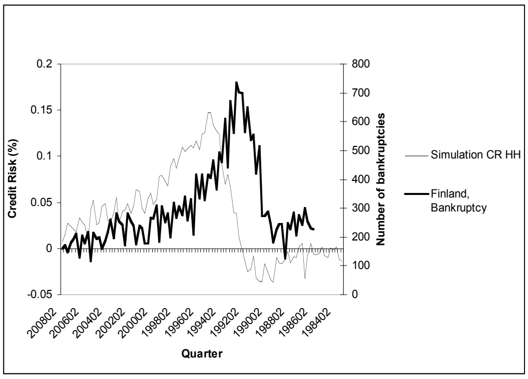


Figure 34: The Danish corporate sector

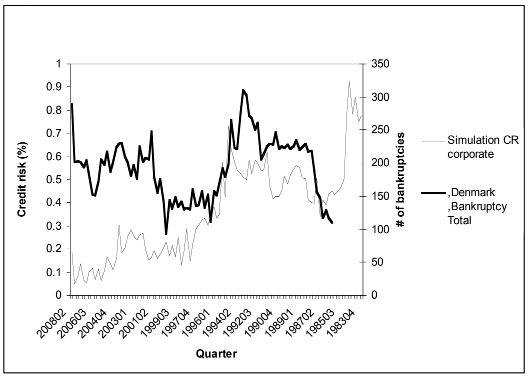
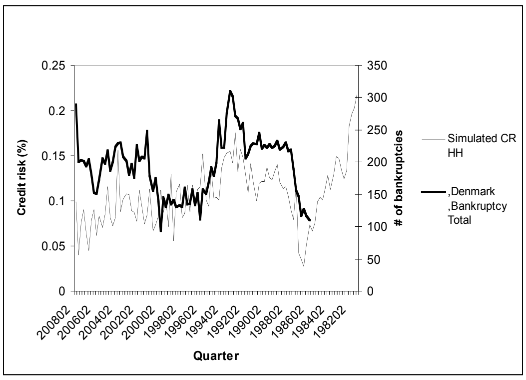


Figure 35: The Danish household sector



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