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Risk analysis of structured products

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June 2009

Abstract

During the last decade investors' interest in structured products, especially Equity-Linked Notes(ELN), has increased dramatically. An ELN is a debt instrument which differs from a typical fixed income security in that the final payout is based partly on the return of an underlying equity, in this case the Swedish equity index OMXS30$^{\mathrm{TM}}$. The ELN is specified as a portfolio of a bond and a call option on the index.

This thesis investigates the risks with investing in an ELN on the Swedish market, and also compares the ELN to investing in portfolios of different combinations of the bond and the index. The risks are measured using Value-at-Risk and Expected Shortfall with three different approaches; historical simulation, analytical solution and Monte Carlo analysis.

The ELN is found to have a risk profile that varies significantly with changing market conditions. Though, the major setbacks of the ELN seem to be the risk of losing the interest rate normally paid by a bond, the high upfront fee charged and for some investors the difficulty to easily adjust the portfolio composition.

Acknowledgements

I would like to thank my tutor Filip Lindskog at the Royal Institute of Technology for interesting discussions and valuable comments, and Viktor Östebo at Derivatinfo.com for helping me with data.

I would also like to thank my family for their tireless support, and my friends for making my years at college unforgettable.

Stockholm, June 2009

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Contents

Chapter 1

Introduction

1.1 Background

Despite subject to sharp criticism from media and the Swedish Financial Supervisory Authority (FI), investors' interest in structured products, especially Equity-Linked Notes (ELN), continues to increase. Between 2001 and 2007, investments in ELNs on the Swedish market increased from just over 10 MM SEK to 95 MM SEK [4, 11]. By January 1, 2009 the total issued volume was just short of 170 MM SEK [2].

1.2 The Market

The most popular ELNs on the Swedish market have a return linked to the OMXS30™. Handelsbanken, Nordea, SEB and Swedbank, who together issued more than 60 percent of all ELNs in 2008 [3], all market the ELNs in similar ways. They are said to be products that provide both safety and opportunity. SEB in particular writes: "An ELN combines the opportunity to a good return with the safety of the bond. You participate in possible increases in the market and at the same time you have a protection against decreases". Usually the various issuers offer the same kind of products, for instance an ELN with a return linked to the OMXS30™. But often the conditions differ, and special features are applied differently by the issuers making it hard to compare similar products. An example of a special feature is that the return of each individual stock in an index is "caped", i.e. a maximum return is set to a specific level.

1.3 Criticism

There are two main areas of criticism directed towards ELNs. The first is the high fees associated with ELNs. According to an article from E24 [4], the brokerage fee is between 1-2 percent of the invested capital. Then there are annual fees of between 0.5 and 1 percent. According to Handelsbanken, the total fee is circa 1 percent annually [4]. By investing in an ELN, the investor also takes on the risk of losing the interest rate normally paid by a bond.

The second area of criticism concerns how ELNs are presented to investors. In a report from FI dated December 22, 2006 it is stated that they have "found shortcomings in the way that information is presented to clients. This applies foremost how risks are described in the marketing material...", see [10]. According to FI, the risk on an ELN can be "divided into an interest portion and an equity portion. The risk in the equity portion is that this portion can be positive and then later weaken in a stress scenario".

1.4 The purpose of this thesis

I have decided to focus on the risks associated with investing in ELNs. I believe many investors do not know what they have actually invested in, and I think it is fair to assume that most of them would never buy an option.

"The opportunity to a good return with the safety of the bond" almost sounds too good to be true, and in this thesis I will investigate the risks with investing in an ELN on the Swedish market, and if there are any interesting alternative investments.

Chapter 2

Methods

2.1 Creating the structured product

The structured product examined in this thesis will be of the type Equity-Linked Notes (ELN). An ELN is a debt instrument which differs from a typical fixed income security in that the final payout is based partly on the return of an underlying equity, in this case the Swedish equity index OMXS30$^{\text{TM}}$. A common feature of an ELN is that it has a guaranteed payout, usually the same amount as the initial price.

I will define the ELN as a portfolio $\mathbf{P}_t$ composed of a bond B_t and an at-the-money call option C_t on OMXS30$^{\text{TM}}$, both maturing two years after issuance. On the Swedish market it is common that the bond is issued by the seller of the ELN. For instance SEB write in their prospectus that even though there is a guaranteed payout, the owner has a credit risk on SEB [9]. I will assume that the ELN, and hence the bond, is issued by an average Swedish bank, and that the option is bought on the market.

I have specified two requirements on the portfolio. The first is that the initial price of the portfolio is the same as the face value of the bond, which ensures the guaranteed payout. The second requirement is that the return of the portfolio at maturity is zero or equal to the return of the OMXS30$^{\text{TM}}$multiplied by a participation rate w_o, whichever is highest. The participation rate varies with the market conditions, essentially the bond yield and the implied volatility of OMXS30$^{\text{TM}}$, and is set just before issuance. It specifies how many at-the-money (at time zero) options the portfolio contains. In this thesis the participation rate is approximately 0.5. An indicative payoff diagram of the ELN can be found in figure 2.1. The construction of the ELN portfolio is described in three steps below.

1. The initial capital is IC.
2. Buy 1 bond B_0 with face value equal to IC.
3. Spend the remaining capital IC - B_0 on w_o at-the-money options C_0.

The value of the portfolio at time t is

$$V_t = w_t^T \mathbf{P}_t = 1 \cdot B_t + w_o \cdot C_t. \tag{2.1}$$

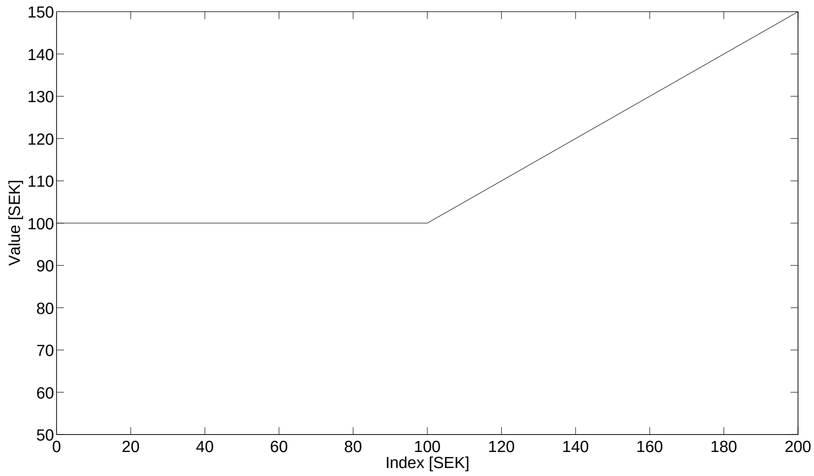

Figure 2.1: The indicative pay off against index level at maturity.

2.2 Maximizing expected utility

To put the ELN portfolio into a bigger perspective, one can consider a market with three assets; the bond, the index and the at-the-money call option on the index. On this market short selling is allowed. Given an investors preferences, in terms of a utility function, one can calculate an optimal portfolio allocation for the investor. To maximize expected utility can be thought of as maximizing the investors satisfaction or happiness.

A utility function is a function on the real numbers that is typically increasing and concave meaning that the investor always wants to have more money, but the additional utility of one extra SEK decreases the wealthier the investor gets [8]. The investors problem is

$$\begin{array}{ll} \text{maximize} & \mathrm{E}[U(W_1)] \\ \text{subject to} & \mathbf{w}^T \mathbf{1} = 1 \end{array}$$

where $U(W_1)$ is the utility function, $W_1 = W_0(1 + \mathbf{w}^T\mathbf{r})$ is the final wealth, W_0 is the initial wealth, $\mathbf{r} = (r_1, r_2, r_3)$ are the returns of the assets and $\mathbf{w} = (w_1, w_2, w_3)$ are the portfolio weights. This is a typical nonlinear optimization problem which, using Lagrange relaxation, can be rewritten into a nonlinear equation system

$$\begin{aligned}
&\mathrm{E}\left[\frac{dU(W_1)}{dw_i}\right] - \lambda = 0, \quad i = 1, 2, 3 \\
&\mathbf{w}^T\mathbf{1} - 1 = 0.
\end{aligned}$$

From a utility maximization perspective the ELN is just a standardized portfolio choice made by the issuer. This choice probably corresponds to few investors' optimal portfolios, just the ones with the exactly matching utility function. An example of a portfolio optimization using a specific utility function, not necessarily the one leading to the ELN, can be found in section 3.2.

2.3 Bond-Stock portfolios

Reasonable substitutes to the ELN are portfolios consisting of the bond and the index. I will call these Bond-Stock portfolios. Below, I have defined two Bond-Stock portfolios, both employing different properties of the ELN. These portfolios can also be thought of as standardized portfolio choices from the utility maximization. They will later be compared to the ELN with regards to risk and return.

2.3.1 Bond-Stock portfolio 1

The ELN has a return at maturity that is zero or equal to the return of the OMXS30™multiplied by a participation rate w_o, whichever is highest. It is natural to compare the ELN to a Bond-Stock portfolio that has the same return in a positive market environment. Hence, this portfolio will consist of approximately 50 percent bond and 50 percent index.

2.3.2 Bond-Stock portfolio 2

The second portfolio is created using a different approach. The ELN contains just over 91 percent bond, the rest is used to by options. This is to ensure the requirement of a guaranteed payout. It is then equally natural to compare the ELN to a Bond-Stock portfolio that has the same exposure to the bond. Hence, this portfolio will consist of approximately 91 percent bond and 9 percent index.

2.4 Pricing

2.4.1 Bond pricing

The bond pricing will be based on continuously compounded interest rate.

$$B_t = e^{-(r_t+p_t)(T-t)}$$

where r_t is the risk-free interest rate at time t and p_t is the credit risk premium of the bond at time t. Hence, the bond price can be modelled by

$$B_t = f_1(t, r_t, p_t).$$

2.4.2 Option pricing

For the option pricing I will use the Black-Scholes formula [6]. The reason I am choosing this pricing model is that it is the most well known method for pricing options, and the fact that the formula itself as well as its derivatives have closed form solutions. The price of the option at time t is given by

$$C_t = I_t N(d_1) - Ke^{-r(T-t)} N(d_2)$$

where

$$\begin{aligned} d_1 &= \frac{\ln I_t/K + (r_t + \sigma_t^2/2)(T - t}{\sigma_t \sqrt{T-t}} \\ d_2 &= \frac{\ln I_t/K + (r_t - \sigma_t^2/2)(T - t)}{\sigma_t \sqrt{T-t}} = d_1 - \sigma_t \sqrt{T-t} \end{aligned}$$

and

$$\begin{aligned} r_t &= \text{the risk-free interest rate at time t} \\ I_t &= \text{the price of the underlying index} \\ K &= \text{the strike price of the index} \\ \sigma_t &= \text{the volatility of } I_t. \end{aligned}$$

Hence, the option price can be modelled by

$$C_t = f_2(t, r_t, \ln I_t, \sigma_t).$$

2.5 Loss distribution of a portfolio

2.5.1 Modelling the value

Generally one can model the value of a portfolio with a function of a d-dimensional random vector $\mathbf{Z}_t = (Z_1, ..., Z_d)$ of risk-factors,

$$V_t = f(t, \mathbf{Z}_t).$$

If we introduce the random vector $\mathbf{X}_{t+1} = \mathbf{Z}_{t+1} - \mathbf{Z}_t$ of risk-factor changes, the loss of the portfolio can be expressed as

$$\begin{aligned} L_{t+1} &= -\left(V_{t+1} - V_t\right) \\ &= -\big(f(t+1, \mathbf{Z}_t + \mathbf{X}_{t+1}) - f(t, \mathbf{Z}_t)\big). \end{aligned} \tag{2.2}$$

For a more detailed description, see [7].

2.5.2 Choice of risk-factors

The loss of the ELN portfolio is dependent on the simultaneous loss of the bond and the option. The values of the bond and the option can be modelled as two separate functions as seen in sections 2.4.1 and 2.4.2,

$$\begin{aligned} B_t &= f_1(t, r_t, p_t) \\ C_t &= f_2(t, r_t, \ln I_t, \sigma_t). \end{aligned}$$

To simplify the simulations that are the main part of this thesis I will use logarithmic implied volatility instead of the plain implied volatility used in Black-Scholes formula. This eliminates major complications in the calculations. With this adjustment, we have the risk-factors

$$\mathbf{Z}_t = (r_t, p_t, \ln I_t, \ln \sigma_t),$$

and introducing the notation $\Delta x_t = x_{t+1} - x_t$, I define the risk-factor changes as

$$\mathbf{X}_{t+1} = (\Delta r_t, \Delta p_t, \Delta \ln I_t, \Delta \ln \sigma_t).$$

Finally, the loss can be written as

$$\begin{aligned} L_{t+1} &= -\big(f(t+1, \mathbf{Z}_t + \mathbf{X}_{t+1}) - f(t, \mathbf{Z}_t)\big) \\ &= -\big(f(t+1, r_t + \Delta r_t, p_t + \Delta p_t, \ln I_t + \Delta \ln I_t, \ln \sigma_t + \Delta \ln \sigma_t) \\ &\quad - f(t, r_t, p_t, \ln I_t, \ln \sigma_t)\big). \end{aligned}$$

In the same way as for the ELN above, the loss distributions of the Bond-Stock portfolios become functions of the risk-factors r_t, p_t and $\ln I_t$.

2.5.3 Linearized loss distribution

To simplify calculations it is convenient to have a linearized relation between L_{t+1} and $\mathbf{X}_{t+1}$. This is done by differentiating f with respect to t and Z_i. The linearized loss becomes

$$L_{t+1}^{\Delta} = -\big(f_t(t, \mathbf{Z}_t)\Delta t + \sum_{i=1}^{d} f_{z_i}(t, \mathbf{Z}_t) X_{t+1,i}\big). \tag{2.3}$$

With risk-factors $\mathbf{Z}_t$, risk-factor changes $\mathbf{X}_{t+1}$ and the value of the portfolio $\mathbf{P}_t$, see (2.1), the linearized loss is written

$$L_{t+1}^{\Delta} = -\left(\frac{\partial B_t}{\partial t}\Delta t + \frac{\partial B_t}{\partial r_t}\Delta r_t + \frac{\partial B_t}{\partial p_t}\Delta p_t \right.$$
$$\left. + w_o \cdot \left(\frac{\partial C_t}{\partial t}\Delta t + \frac{\partial C_t}{\partial r_t}\Delta r_t + \frac{\partial C_t}{\partial I_t} I_t \Delta \ln I_t + \frac{\partial C_t}{\partial \sigma_t}\sigma_t \Delta \ln \sigma_t\right)\right). \tag{2.4}$$

To know how reliable the above expression is, I will test the robustness of the linearization by comparing it to the non-linear loss. This is done by stressing two risk-factors at the time which results in an error surface where the linearization's deviation from the non-linear formula can be seen.

2.6 Data

2.6.1 Dependence structure of the risk-factor changes

To analyse the risk in a realistic way it is important to examine whether the different risk-factors are dependent or not. Also, even though a pair of risk-factors might seem uncorrelated when data is aggregated and measured over a large sample, it is possible that they behave as if correlated during one or more shorter time periods. The dependence structure can be examined using e.g. scatter plots and correlation calculations.

2.6.2 Fitting data to distributions

In many of the models used in financial theory data is assumed to be normally distributed. A quick examination of almost any financial time series shows that this assumption does not provide the best fit possible. According to Broadie and Detemple [1] the probability of a crash equal to, or worse than the Black Monday crash on October 19, 1987 is approximately 10^{-97} under the assumption of normal distribution. Statistical tests show that in general financial data has a kurtosis far larger (i.e. fatter tails) than implied by the normal distribution.

A useful tool when studying the extremal properties of a sample is quantile-quantile plots (qq-plots). A sample $X_1, ..., X_n$ is compared to a reference distribution F by plotting

$$\left\{\left(X_{k,n}, F^{\leftarrow}\left(\frac{n-k+1}{n+1}\right)\right) : k = 1, ...n\right\}$$

with the sample sorted according to $X_{n,n} \leq X_{n-1,n} \leq ... \leq X_{1,n}$. If F is a more heavy tailed distribution than the sample the plot will curve down at the left and/or up at the right, and the other way around if F has lighter tails. If the sample comes from the same distribution as F, the plot appears linear. More info on qq-plots can be found in Hult and Lindskog [7].

2.7 Risk measurement

The two risk measures that will be used in this thesis are Value-at-Risk and Expected Shortfall. They are defined as follows:

$$\begin{aligned}\mathrm{VaR}_\alpha(L) &= \inf\{l \in \mathbb{R} : P(L > l) \leq 1-\alpha\} \\ &= \inf\{l \in \mathbb{R} : 1 - F_L(l) \leq 1-\alpha\} \\ &= \inf\{l \in \mathbb{R} : F_L(l) \geq \alpha\} \\ &= F_L^{-1}(\alpha).\end{aligned}$$

$$\begin{aligned}\mathrm{ES}_\alpha(L) &= \mathrm{E}(L | L \geq \mathrm{VaR}_\alpha(L)) \\ &= \frac{\mathrm{E}(L\mathbb{I}_{[q_\alpha(L),\infty)}(L))}{\mathrm{P}(L \geq q_\alpha(L))} \\ &= \frac{1}{1-\alpha}\mathrm{E}(L\mathbb{I}_{[q_\alpha(L),\infty)}(L)) \\ &= \frac{1}{1-\alpha}\int_{q_\alpha(L)}^{\infty} l \mathrm{d}F_L(l).\end{aligned}$$

As stated in the introduction, according to FI the risk on an ELN can be "divided into an interest portion and an equity portion. The risk in the equity portion is that this portion can be positive and then later weaken in a stress scenario". What this basically means is that if the underlying index has increased drastically, a "crash" in the index will cause a lot more damage than if it occurs with the index at approximately the same level as at issuance. With the intention to give a comprehensive risk profile of the ELN, two risk scenarios are presented below.

2.7.1 Risk scenarios

The ELN portfolio has an initial term to maturity of two years. During this period, two risk scenarios will be considered. One will take place one month after issuance, the other one year after issuance. The two scenarios are based on stressing the two risk-factor pairs (r_t, p_t) and $(\ln I_t, \ln \sigma_t)$ one at the time with the other held constant. The reason for the choice of these pairs is the fact that they are the only two found correlated, see section 3.3.2. Note that the scenarios are constructed to show how the risks of the portfolio change due to changes in the risk-factors, i.e. market conditions, during the period from issuance until just before risk measurement.

Risks will be measured for 1-day losses and also 20-day losses where applicable. The 1-day losses are chosen to give a sense of the magnitude of the day-to-day losses, while the 20-day losses are supposed to represent the frequency at which a typical investor reviews an investment.

2.7.1.1 Scenario 1 - Stressing (r_t, p_t)

One month after issuance, the probability that the underlying index has increased heavily is low. Therefore, I will consider a scenario where the pair $(\ln I_t, \ln \sigma_t)$ is held constant (i.e. the same as at issuance) while I allow r_t and p_t to vary. This leads to a risk function that, instead of being one single number, becomes a surface. This risk surface is dependent on how r_t and p_t move during the first month. An example of a situation that this scenario covers is if the Swedish central bank, Riksbanken, decides to change the repo rate during this first month, and how this changes the risk of the ELN.

2.7.1.2 Scenario 2 - Stressing $(\ln I_t, \ln \sigma_t)$

After one year, with one year to maturity, the other scenario takes place. Holding the pair (r_t, p_t) constant, $\ln I_t$ and $\ln \sigma_t$ are allowed to vary. This gives a risk surface dependent on how $\ln I_t$ and $\ln \sigma_t$ have moved during the first year. For instance, this scenario will show whether the risk increases a lot after a period of bullish stock market behaviour.

2.7.2 Historical simulation

Calculating the risk measures with historical simulation is a rather straight forward exercise. Historical data of the risk-factor changes has to be collected. Then, the data is simply plugged in to (2.2) which gives the empirical loss distribution $\mathbf{L}_n$. The empirical VaR and ES can be written

$$\widehat{\mathrm{VaR}}_\alpha(\mathbf{L}_n) = L_{[n(1-\alpha)]+1,n}$$

$$\widehat{\mathrm{ES}}_\alpha(\mathbf{L}_n) = \frac{\sum_{k=1}^{[n(1-\alpha)]+1} L_{k,n}}{[n(1-\alpha)]+1}$$

where $[x]$ is the integer part of x, and with the empirical loss distribution ordered such that $L_{1,n} \geq ... \geq L_{n,n}$.

2.7.3 Analytical solution

For the analytical solution the linearized loss in (2.4) can be used. The partial derivatives of B_t are easily calculated using the bond pricing expression in section 2.4.1. For C_t we need the partial derivatives of the Black-Scholes formula who are the well known Greeks,

$$\frac{\partial C_t}{\partial t} \text{ is called } \textit{theta} \qquad \frac{\partial C_t}{\partial r_t} \text{ is called } \textit{rho}$$
$$\frac{\partial C_t}{\partial I_t} \text{ is called } \textit{delta} \qquad \frac{\partial C_t}{\partial \sigma_t} \text{ is called } \textit{vega}.$$

The partial derivatives become constants used as weights in the equation. To calculate the risk analytically using one of our preferred risk measures we need to find a multidimensional distribution $F_{\mathbf{X}_{t+1}}$ of the risk-factor changes. This can be done with help of qq-plots, described in section 2.6.2. (2.4) can now be written

$$\begin{aligned} L_{t+1}^{\Delta} &= -\left(\frac{\partial B_t}{\partial t} + w_o \frac{\partial C_t}{\partial t}\right)\Delta t - \left(\frac{\partial B_t}{\partial r_t} + w_o \frac{\partial C_t}{\partial r_t}\right)\Delta r_t \\ &\quad - \frac{\partial B_t}{\partial p_t}\Delta p_t - w_o \frac{\partial C_t}{\partial I_t} I_t \Delta \ln I_t - w_o \frac{\partial C_t}{\partial \sigma_t}\sigma_t \Delta \ln \sigma_t \\ &= -\left(\frac{\partial B_t}{\partial t} + w_o \frac{\partial C_t}{\partial t}\right)\Delta t + \mathbf{w}^T \mathbf{X}_{t+1}. \end{aligned}$$

If $F_{\mathbf{X}_{t+1}}$ is multivariate elliptically distributed with mean vector μ and covariance matrix Σ, VaR and ES can be written

$$\begin{aligned} \mathrm{VaR}_{\alpha}(L_{t+1}^{\Delta}) &= -\left(\frac{\partial B_t}{\partial t} + w_o \frac{\partial C_t}{\partial t}\right)\Delta t + \mathbf{w}^T \mu \\ &\quad + \sqrt{\mathbf{w}^T \Sigma \mathbf{w}}\, \mathrm{VaR}_{\alpha}(X_{t+1}) \\ &= -\left(\frac{\partial B_t}{\partial t} + w_o \frac{\partial C_t}{\partial t}\right)\Delta t + \mathbf{w}^T \mu \\ &\quad + \sqrt{\mathbf{w}^T \Sigma \mathbf{w}}\, F_{X_{t+1}}^{-1}(\alpha) \end{aligned}$$

and

$$\begin{aligned} \mathrm{ES}_{\alpha}(L_{t+1}^{\Delta}) &= -\left(\frac{\partial B_t}{\partial t} + w_o \frac{\partial C_t}{\partial t}\right)\Delta t + \mathbf{w}^T \mu \\ &\quad + \sqrt{\mathbf{w}^T \Sigma \mathbf{w}}\, \mathrm{ES}_{\alpha}(X_{t+1}) \\ &= -\left(\frac{\partial B_t}{\partial t} + w_o \frac{\partial C_t}{\partial t}\right)\Delta t + \mathbf{w}^T \mu \\ &\quad + \sqrt{\mathbf{w}^T \Sigma \mathbf{w}}\, \frac{1}{1-\alpha} \int_{F_{X_{t+1}}^{-1}(\alpha)}^{\infty} x \mathrm{d} F_{X_{t+1}}(x) \end{aligned}$$

where $F_{X_{t+1}}$ is a one-dimensional standardized elliptical distribution of the same kind as $F_{\mathbf{X}_{t+1}}$. For instance, if $F_{\mathbf{X}_{t+1}}$ is multivariate normally distributed then $F_{X_{t+1}}$ is the standard normal distribution.

2.7.4 Monte Carlo simulation

In the Monte Carlo simulation, the value change of the ELN is modelled with the help of a copula, C_R. This is done to achieve a certain dependence structure of the risk-factor changes. Thereafter, using the best fitted distribution for each of the risk-factor changes one can simulate values of the risk-factor changes from the copula and get simulated "historical data". The risks are then calculated in the same way as in the historical simulation, see section 2.7.2. A more detailed description of copulas can be found in Hult and Lindskog [7].

Chapter 3

Results

3.1 Creating the structured product

The ELN is created as a portfolio $\mathbf{P}_t$ of a bond and w_o at-the-money call options, see section 2.1. The initial value of the portfolio is set to 100 SEK. The risk-factors were chosen as $\mathbf{Z}_t = (r_t, p_t, \ln I_t, \ln \sigma_t)$. r_t is represented by Swedish Treasury bills, SSVX, with 12 months maturity. For the credit risk premium, p_t, I use the so called TED-spread as a proxy. The TED-spread is used as a measure of credit quality and is defined as the difference between the interest rate on interbank loans, in the Swedish case STIBOR, and Treasury bills for a given time to maturity. Applied to the Swedish market the equation becomes

$$\text{TED-spread} = \text{STIBOR} - \text{SSVX}.$$

I_t is as previously stated the Swedish equity-index OMXS30$^{\text{TM}}$. The implied volatility σ_t of the index is represented by DVIS, which is an indicator of the expected market volatility the following 30 calendar days, calculated from the price of OMXS30$^{\text{TM}}$options.

What remains is to set the initial values of the risk-factors in agreement with the requirements specified in section 2.1.

Risk-factor	Initial value
r_t	4.4 %
p_t	0.2 %
I_t	100 SEK
σ_t	24 %

r_t and p_t are set to their market averages during the examined period. I_t is chosen out of simplicity, and finally the implied volatility is set to obtain a participation rate, w_o, of approximately 0.5.

3.2 Maximizing expected utility - An example

Assume that, on a market with three assets, an investor has the utility function $U(W_1) = \ln(W_1)$. Recall that $W_1 = W_0(1 + \mathbf{w}^T\mathbf{r})$ is the final wealth where W_0 is the initial wealth, $\mathbf{r} = (r_1, r_2, r_3)$ are the returns of the assets and $w = (w_1, w_2, w_3)$ are the portfolio weights. The investor faces the nonlinear equation system

$$\begin{aligned} &\mathrm{E}\left[\frac{W_0(1+r_i)}{W_0(1+\mathbf{w}^T\mathbf{r})}\right] - \lambda = 0, \quad i = 1,2,3 \\ &\mathbf{w}^T\mathbf{1} - 1 = 0. \end{aligned} \tag{3.1}$$

This can be solved using e.g. MATLAB®and Newtons Method [5]. 20-day returns of the bond, the index and the call option calculated from the historical data, see section 3.3.1, are used to compute the expectation values of (3.1). Finally, this yields the portfolio

$$w_1 = 1.4 \quad w_2 = -2.1 \quad w_3 = 1.7.$$

The interpretation of this portfolio is that the investor with utility function $\ln(W_1)$ should short sell 2.1 units of the index, buy 1.4 units of the bond and buy 1.7 units of the call option on the index. Note that in this example it is assumed that the assets all have the same price.

3.3 Data

3.3.1 Data collection

Data has been collected from a 15 year period, January 1, 1994 to December 31, 2008. 616 days has been left out due to missing data, leaving a total of 3152 days of observations to use in the analysis. r_t, p_t and I_t can all easily be collected in the market from e.g. `www.riksbank.se` and `finance.yahoo.com`. Regarding σ_t, the implied volatility-indicator DVIS is published on a daily basis by `www.derivatinfo.com`.

3.3.2 Dependence structure of risk-factor changes

I have investigated the dependence structure of the following pairs of daily risk-factor changes.

$$\begin{array}{ccc} (\Delta \ln I_t, \Delta \ln \sigma_t) & (\Delta r_t, \Delta p_t) & (\Delta \ln I_t, \Delta r_t) \\ (\Delta \ln I_t, \Delta p_t) & (\Delta \ln \sigma_t, \Delta r_t) & (\Delta \ln \sigma_t, \Delta p_t) \end{array}$$

Scatter plots of all pairs can be found in figure 3.1, placed in the same order as they are listed above. 250 day rolling correlations for the whole data set of each pair above have been calculated and can be found in the

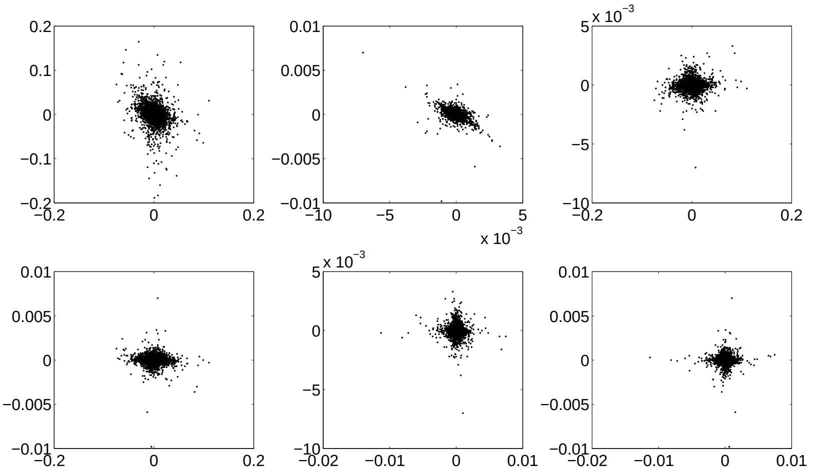

Figure 3.1: Scatter plots of all pairs of risk-factor changes.

appendix, section 5.2. The only pairs that are systematically correlated are $(\Delta \ln I_t, \Delta \ln \sigma_t)$ and $(\Delta r_t, \Delta p_t)$, these will instead be presented below and examined more thoroughly.

3.3.2.1 $(\Delta \ln I_t, \Delta \ln \sigma_t)$

Figure 3.2 shows a 250 day rolling correlation between $\Delta \ln I_t$ and $\Delta \ln \sigma_t$. The correlation is negative throughout the whole 15 year period, has an average of -0.27 and varies over time between [-0.61, 0].

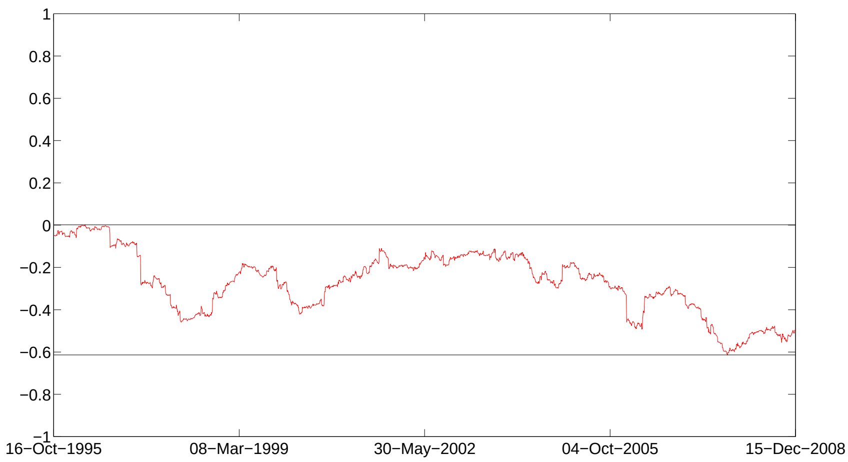

Figure 3.2: 250 day rolling correlation of $(\Delta \ln I_t, \Delta \ln \sigma_t)$.

3.3.2.2 $(\Delta r_t, \Delta p_t)$

The pair $(\Delta r_t, \Delta p_t)$ has a higher correlation with an average of -0.58, varying between [-0.92 -0.22]. The 250 day rolling correlation can be found in figure 3.3.

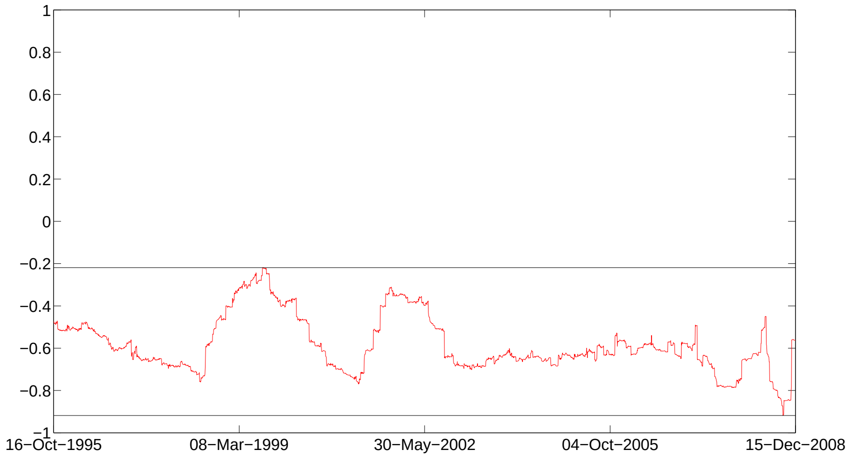

Figure 3.3: 250 day rolling correlation of $(\Delta r_t, \Delta p_t)$.

3.3.3 Fitting data to distributions

Histograms of all four risk-factor changes can be found in figure 3.4. All histograms show a significant difference from normal distribution in the way that they have heavier tails. In figure 3.5 the reader as a reference finds histograms from four different t_ν-distributions.

3.3.3.1 Distribution of $\Delta \ln I_t$

In figure 3.6 $\Delta \ln I_t$ is plotted in qq-plots against nine different t_ν-distributions. The best fit is provided by the $t_{5.0}$-distribution, found in the middle of figure 3.6.

3.3.3.2 Distribution of $\Delta \ln \sigma_t$

$\Delta \ln \sigma_t$ has heavier tails than $\Delta \ln I_t$, see figure 3.4. When qq-plotted against different t_ν-distributions in figure 3.7, the $t_{2.4}$-distribution is found to be the best fit.

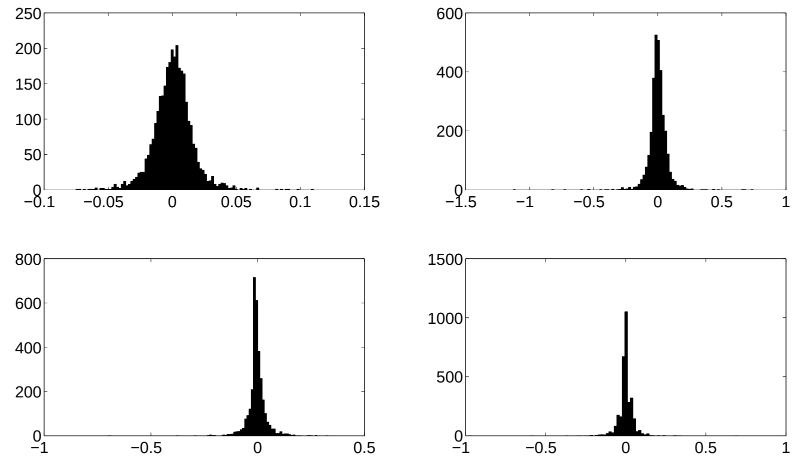

Figure 3.4: Histograms of risk-factor changes. From upper left corner: $\Delta \ln I_t$, $\Delta \ln \sigma_t$, Δr_t, Δp_t.

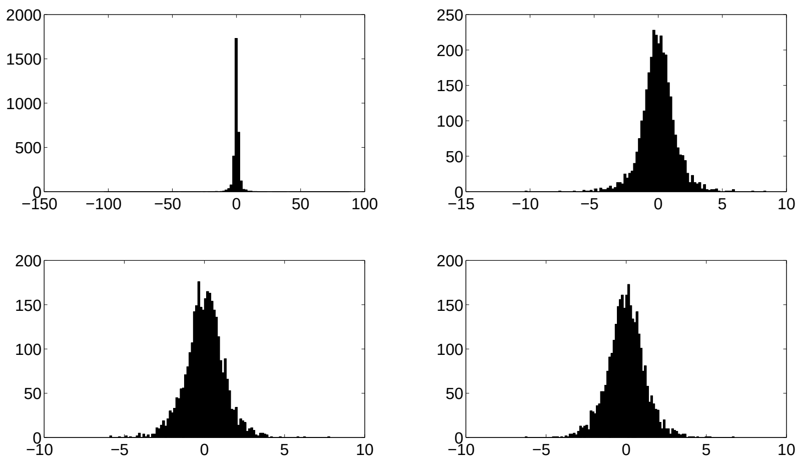

Figure 3.5: Histograms from four t_ν-distributions. From upper left corner: t_2, t_4, t_6, t_8

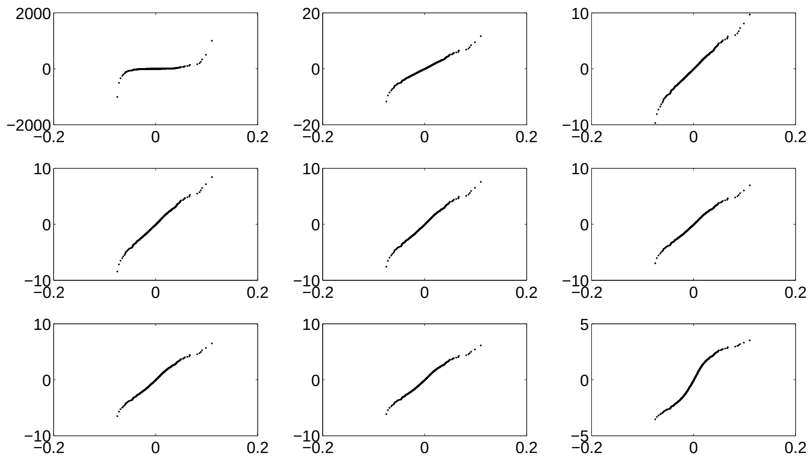

Figure 3.6: QQ-plots of $\Delta \ln I_t$ vs. t_ν-distributions. From upper left corner: $\nu = (1, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 100)$.

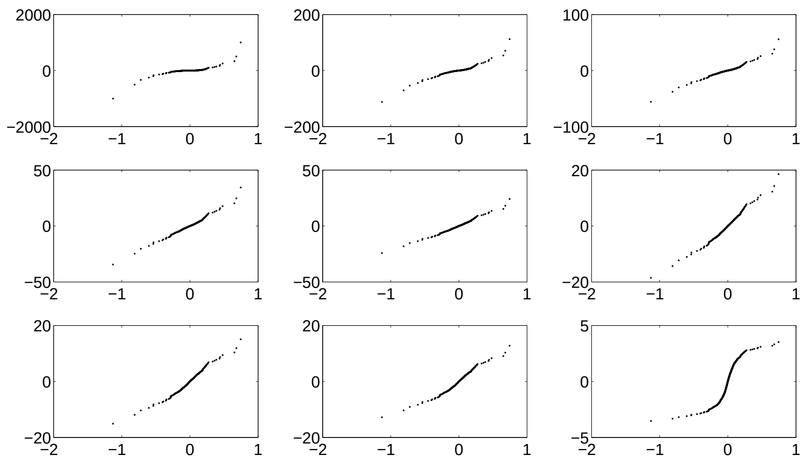

Figure 3.7: QQ-plots of $\Delta \ln \sigma_t$ vs. t_ν-distributions. From upper left corner: $\nu = (1, 1.5, 1.8, 2.1, 2.4, 2.7, 3.0, 3.3, 100)$.

3.3.3.3 Distribution of Δr_t

When making a qq-plot with Δr_t one notices that it is hard to determine which distribution provides the best fit, see figure 3.8, due to the most extreme points in the lower left of the figure. However, the $t_{3.5}$-distribution is found to be the best fit.

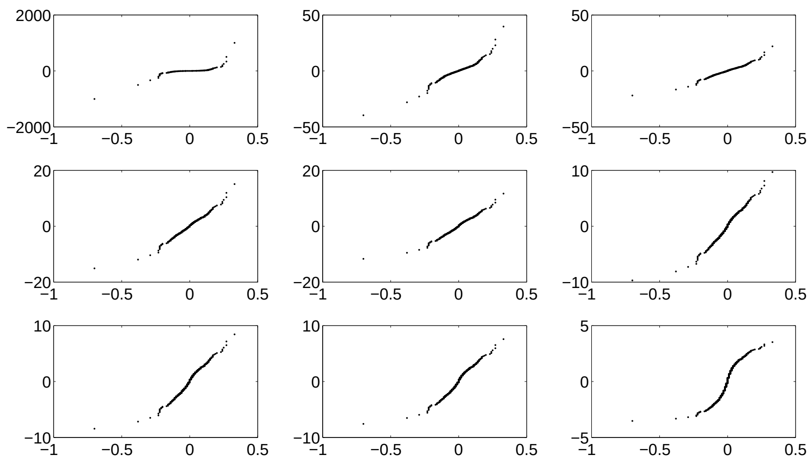

Figure 3.8: QQ-plots of Δr_t vs. t_ν-distributions. From upper left corner: $\nu = (1, 2, 2.5, 3, 3.5, 4, 4.5, 5, 100)$.

3.3.3.4 Distribution of Δp_t

In figure 3.4 we see that Δp_t is the most heavy tailed risk-factor. The qq-plots in figure 3.9 indicate that the $t_{2.5}$-distribution provides the best fit.

3.4 Risk measurement - ELN

In this section the results of the different simulations are presented. Throughout the analysis, the confidence level $\alpha = 0.99$ will be used.

Recall that the risks will be measured using the two scenarios defined in section 2.7.1 where the risk-factor pairs (r_t, p_t) and $(\ln I_t, \ln \sigma_t)$ are stressed one at the time.

3.4.1 Historical simulation

A table with the underlying data for the VaR plots of this section can be found in section 5.3 in the appendix.

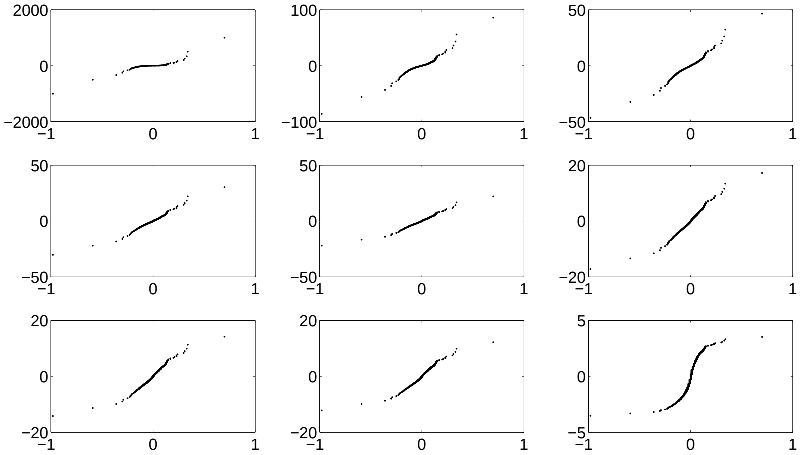

Figure 3.9: QQ-plots of Δp_t vs. t_ν-distributions. From upper left corner: $\nu = (1, 1.6, 1.9.2.2, 2.5, 2.8, 3.1, 3.4, 100)$.

3.4.1.1 Scenario 1 - Stressing (r_t, p_t)

The results can be found in figures 3.10 and 3.11.

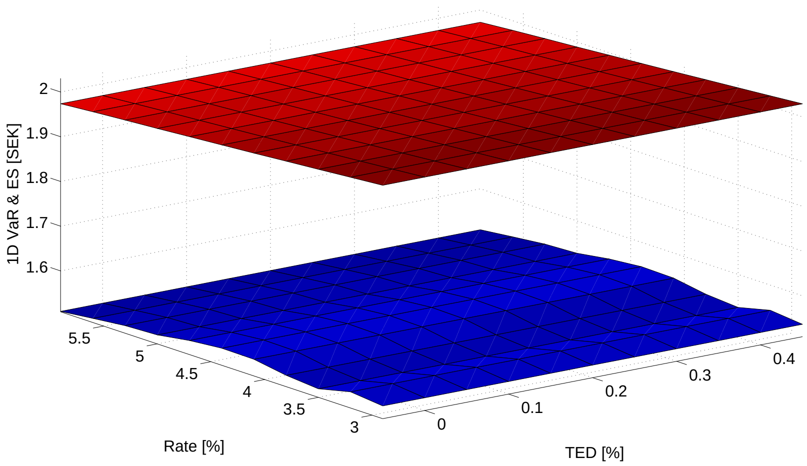

Figure 3.10: Scenario 1 with historical simulation, 1-day VaR and ES. The risk levels are fairly constant throughout the whole surfaces.

3.4.1.2 Scenario 2 - Stressing $(\ln I_t, \ln \sigma_t)$

The results are found in figures 3.12 and 3.13.

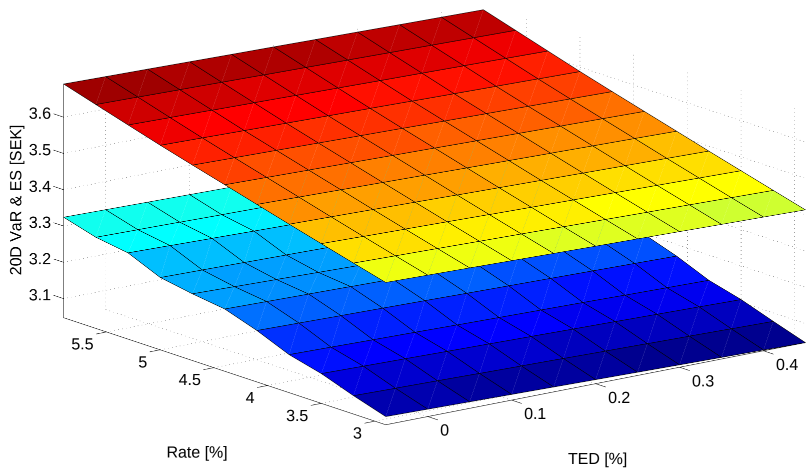

Figure 3.11: Scenario 1 with historical simulation, 20-day VaR and ES. As in the case for the 1-day risks, the changes in the risks over the surfaces are small.

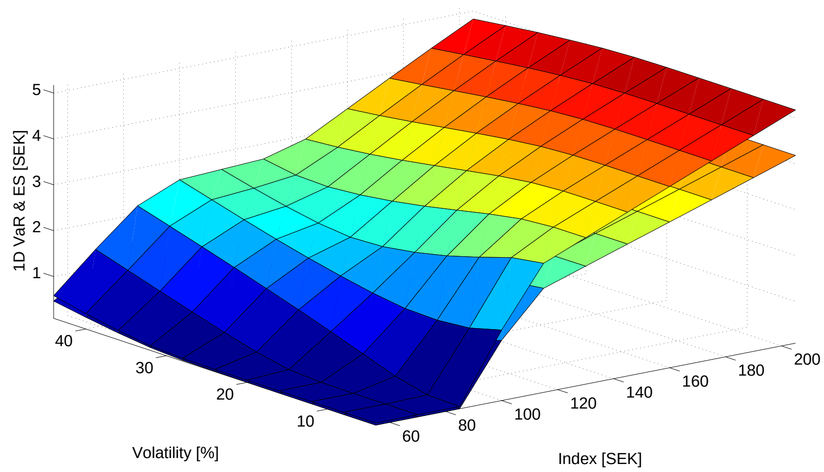

Figure 3.12: Scenario 2 with historical simulation, 1-day VaR and ES. Note that the changes in the index have a large effect on the risk level. The level of the implied volatility has different effects on the risk depending on the index level.

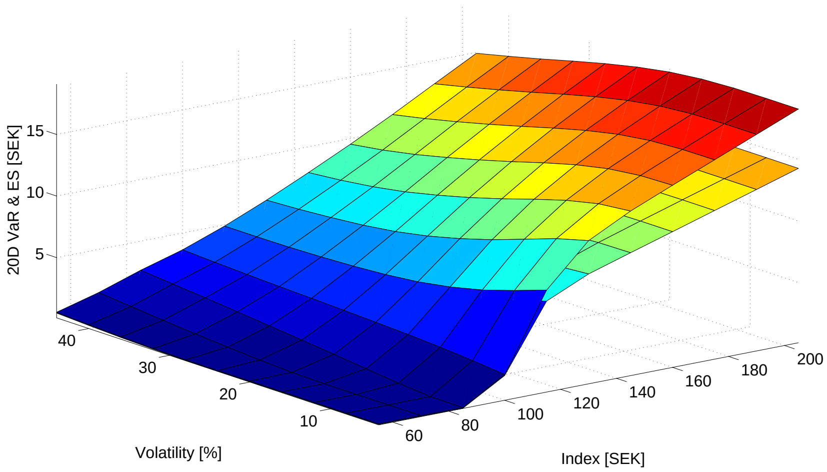

Figure 3.13: Scenario 2 with historical simulation, 20-day VaR and ES. As in the case of the 1-day risks, the index has a huge impact on the risk level. Note that the risks dependence on the implied volatility is weaker than in the 1-day case.

3.4.2 Robustness of the linearization

The error of the linearization is calculated as the relative difference between the linearized loss (2.3) and the non-linear loss (2.2), $(L^{\Delta}_{t+1} - L_{t+1})/(L_{t+1})$. Hence, the error is a function of the risk-factors. The linearization is tested by stressing the risk-factor pairs (r_t, p_t) and $(\ln I_t, \ln \sigma_t)$ one at the time. The results can be found in figures 3.14 and 3.15. Note that the centre points of the x-axis and y-axis corresponds to a zero loss where naturally the error is zero. Due to the characteristics of these figures the 20-day risks calculated with the linearized analytical solution will not be used.

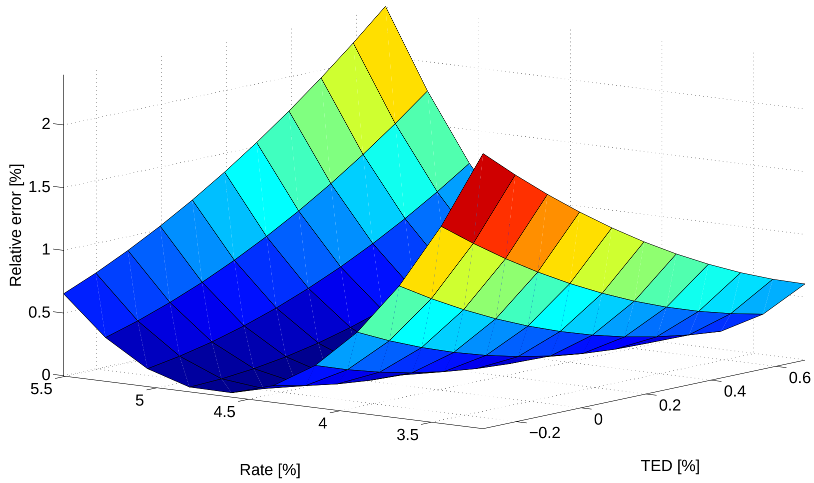

Figure 3.14: Robustness of the linearization. The relative error caused by stressing the risk-factor pair (r_t, p_t).

3.4.3 Analytical solution

Using the distributions found in section 3.3.3 one can conclude that the multivariate elliptical distribution $F_{\mathbf{X}_{t+1}}$, and hence the marginal distribution F_{X_1}, can be approximated by a t_3 distribution. The analytical VaR and ES is then written

$$
\begin{aligned}
\mathrm{VaR}_{0.99}(L^{\Delta}_{t+1}) &= -\left(\frac{\partial B_t}{\partial t} + w_o \frac{\partial C_t}{\partial t}\right)\Delta t + \mathbf{w}^{\mathrm{T}}\mu \\
&\quad + \sqrt{\mathbf{w}^T \Sigma \mathbf{w}}\,\mathrm{VaR}_{0.99}(X_{t+1}) \\
&= -\left(\frac{\partial B_t}{\partial t} + w_o \frac{\partial C_t}{\partial t}\right)\Delta t + \mathbf{w}^{\mathrm{T}}\mu \\
&\quad + \sqrt{\mathbf{w}^T \Sigma \mathbf{w}}\,F^{-1}_{t_3}(0.99)
\end{aligned}
$$

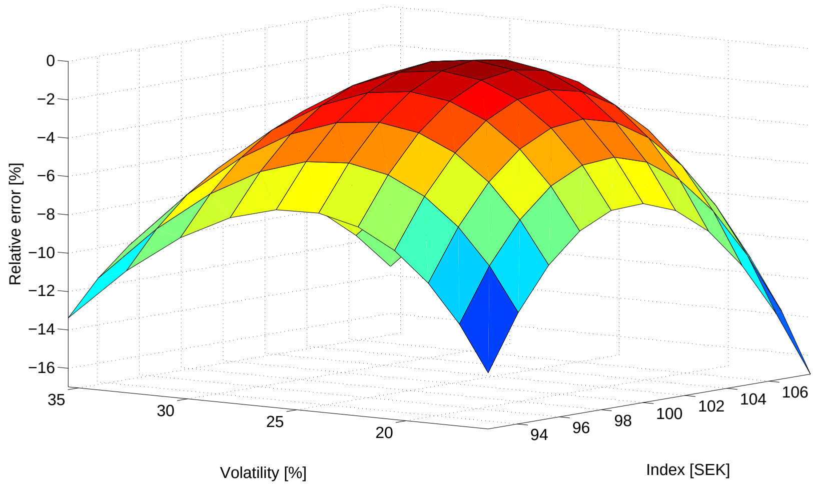

Figure 3.15: Robustness of the linearization. The relative error caused by stressing the risk-factor pair $(\ln I_t, \ln \sigma_t)$.

and

$$\begin{aligned}
\mathrm{ES}_{0.99}(L^{\Delta}_{t+1}) &= -\left(\frac{\partial B_t}{\partial t} + w_o \frac{\partial C_t}{\partial t}\right)\Delta t + \mathbf{w}^{\mathrm{T}}\mu \\
&\quad + \sqrt{\mathbf{w}^T \Sigma \mathbf{w}}\mathrm{ES}_{0.99}(X_{t+1}) \\
&= -\left(\frac{\partial B_t}{\partial t} + w_o \frac{\partial C_t}{\partial t}\right)\Delta t + \mathbf{w}^{\mathrm{T}}\mu \\
&\quad + \sqrt{\mathbf{w}^T \Sigma \mathbf{w}} \frac{1}{1-0.99} \int_{F_{t_3}^{-1}(0.99)}^{\infty} x \mathrm{d}F_{t_3}(x).
\end{aligned}$$

3.4.3.1 Scenario 1 - Stressing (r_t, p_t)

The result is found in figure 3.16.

3.4.3.2 Scenario 2 - Stressing $(\ln I_t, \ln \sigma_t)$

The result is found in figure 3.17.

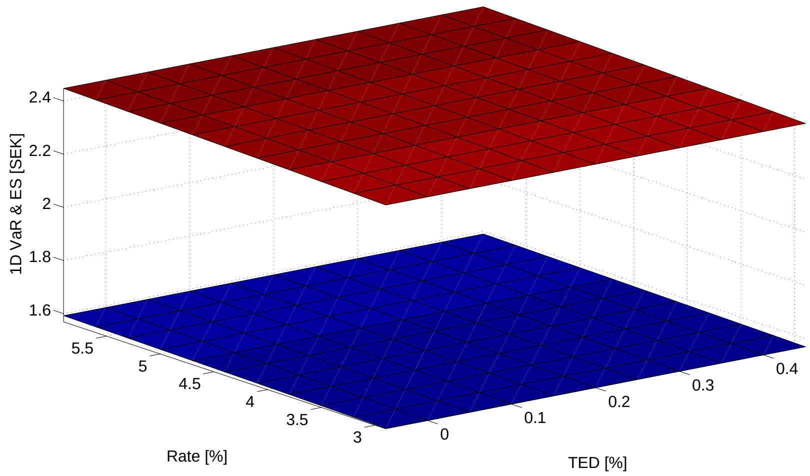

Figure 3.16: Scenario 1 with analytical solution, 1-day VaR and ES. As in the historical simulation, the risk surfaces are quite flat.

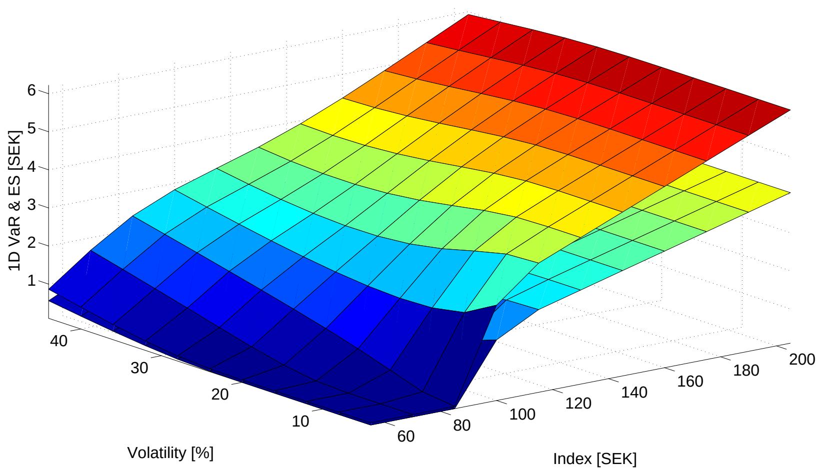

Figure 3.17: Scenario 2 with analytical solution, 1-day VaR and ES. The behaviour is quite similar to the historical simulation, although the surfaces are smoother.

3.4.4 Monte Carlo simulation

As in section 3.4.3 we approximate the multivariate distribution as t_3. The generation of the 4-dimensional t-copula $C^t_{3,R}$ can be summarized in the following eight steps:

1. Measure pairwise dependence between risk-factor changes.
2. Collect in a 4-by-4 dependence matrix R.
3. Find the Cholesky decomposition A of R where $\mathrm{R} = \mathrm{AA}^{\mathrm{T}}$.
4. Simulate 4 independent random variates Z_1,...,Z_4 from N(0,1).
5. Simulate a random variate S from χ^2_3.
6. Set $\mathbf{Y} = A\mathbf{Z}$ and $\mathbf{X} = \frac{\sqrt{3}}{\sqrt{S}}\mathbf{Y}$.
7. Set $U_k = t_3(X_k)$ for $k = 1, ..., 4$.
8. Start over from 4.

This is iterated a desirable number of times to yield a sample from the copula $C^t_{3,R}$. In figure 3.18 pairwise scatter plots of the risk-factor changes simulated from the copula can be found. If these are compared with the pairwise scatter plots of the original data in figure 3.1 one can see that they are quite similar.

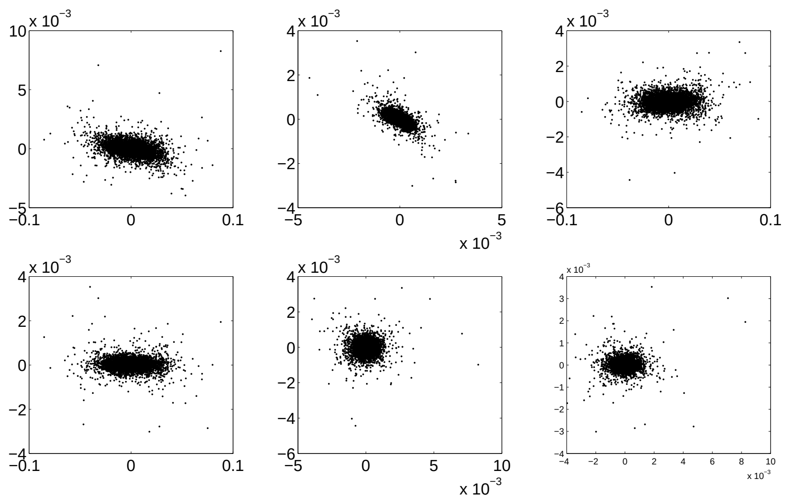

Figure 3.18: Pairwise scatter plots of copula parameters, ordered in the same way as originally listed in page 14.

3.4.4.1 Scenario 1 - Stressing (r_t, p_t)

Value-at-Risk and Expected Shortfall calculated with a Monte Carlo simulation can be found in figures 3.19 and 3.20.

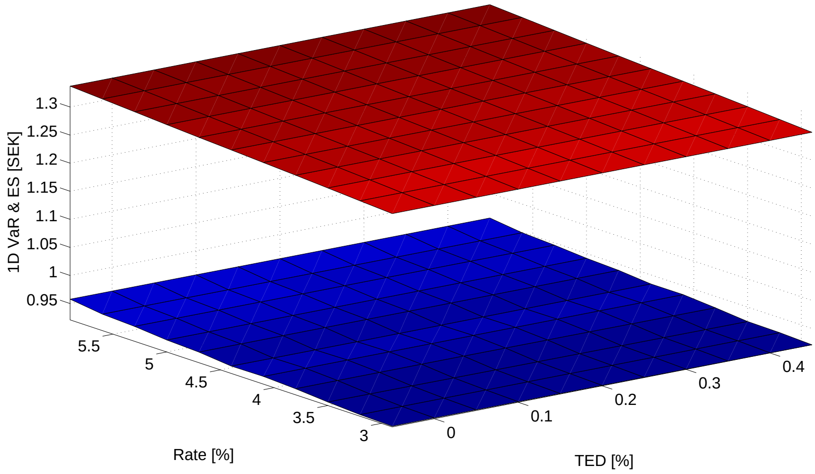

Figure 3.19: Scenario 1 with Monte Carlo simulation, 1-day VaR and ES. The Monte Carlo analysis confirms the results provided by the previous methods.

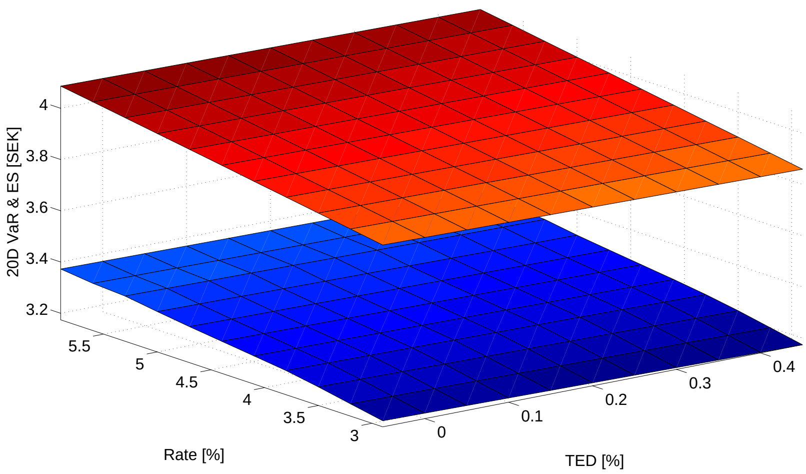

Figure 3.20: Scenario 1 with Monte Carlo simulation, 20-day VaR and ES.

3.4.4.2 Scenario 2 - Stressing $(\ln I_t, \ln \sigma_t)$

Value-at-Risk and Expected Shortfall calculated with a Monte Carlo simulation can be found in figures 3.21 and 3.22.

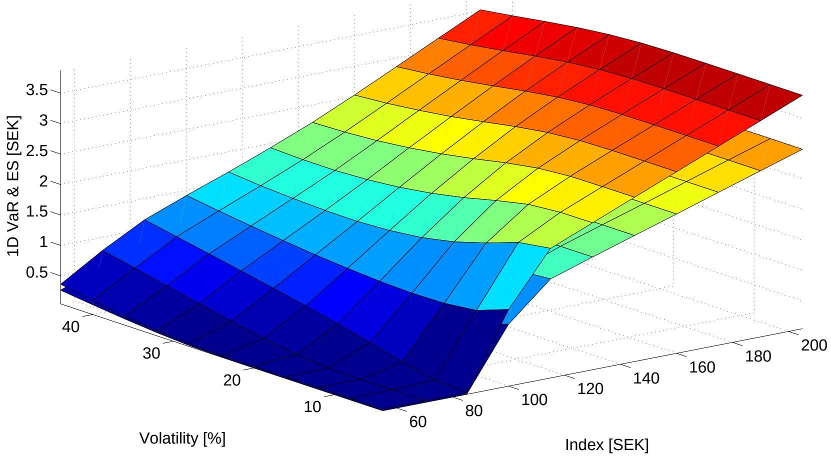

Figure 3.21: Scenario 2 with Monte Carlo simulation, 1-day VaR and ES. Also in this second scenario, the results of the previous sections are confirmed. The risk surfaces are affected drastically by changes in the index.

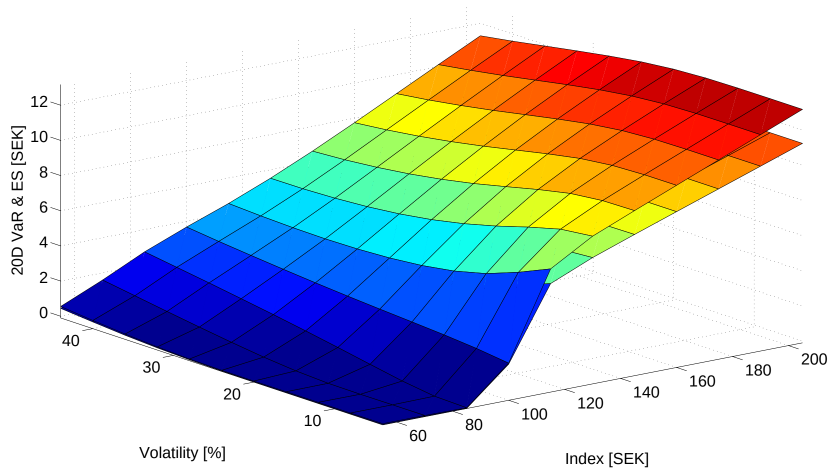

Figure 3.22: Scenario 2 with Monte Carlo simulation, 20-day VaR and ES.

3.4.5 A recapitulation of the ELN results

The stress tests are divided into two different scenarios where the risk-factor pairs $(\Delta r_t, \Delta p_t)$ and $(\Delta \ln I_t, \Delta \ln \sigma_t)$ are stressed one at the time. The stress tests are performed with historical simulation, computed analytically and with a Monte Carlo simulation. All three methods give approximately the same results throughout the analysis for both VaR and ES. Therefore, only the 1-day VaR from the historical simulation will be referred to below.

3.4.5.1 Analysis of Scenario 1 - Stressing $(\Delta r_t, \Delta p_t)$

Figure 3.10 shows that the risks are fairly constant over the whole surface, which is almost a plane. The lowest risk on the surface is in the corner with high interest rate r_t and high credit risk premium p_t. The maximum difference over the whole VaR surface is 2.5 percent and hence, the changes in the risks are very much controllable.

3.4.5.2 Analysis of Scenario 2 - Stressing $(\Delta \ln I_t, \Delta \ln \sigma_t)$

Changes in the index I_t has large implications to the risk level. The 1-day risk surface, see figure 3.12, shows that from the starting point at $I_t = 100$, the risk increases with 219 percent when I_t is doubled to 200. And if I_t is halved to 50, the risk decreases with 93 percent. The implied volatility σ_t does also have an effect on the risk, but it varies with I_t. With index between 50 and 100, a high implied volatility causes higher risk. Then with index between 120 and 150, the exact opposite occurs. Hence, this scenario causes major changes to the risk level of the ELN.

3.5 Risk measurement - Bond-Stock portfolios

Risks are, as in the case with the ELN, measured using the two scenarios defined in section 2.7.1. Since the results of the ELN were consistent throughout the analysis above, the risks of the Bond-Stock portfolios will only be measured with historical simulation.

As for the ELN, a table with the underlying data for the 1-day VaR plots of this section can be found in section 5.3 in the appendix.

3.5.1 Bond-Stock portfolio 1

3.5.1.1 Scenario 1 - Stressing (r_t, p_t)

The results of the scenario 1 simulations of Bond-Stock portfolio 1 can be found in figures 3.23 and 3.24.

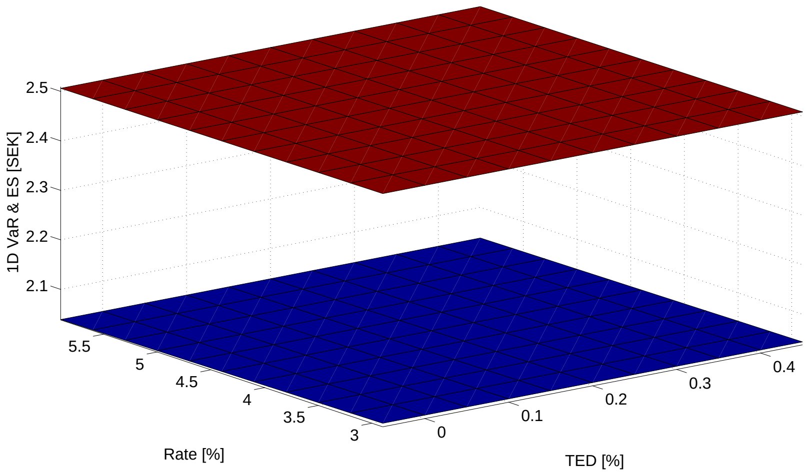

Figure 3.23: Scenario 1 with Bond-Stock portfolio 1, 1-day VaR and ES. The risk surfaces look similar to the ones of the ELN.

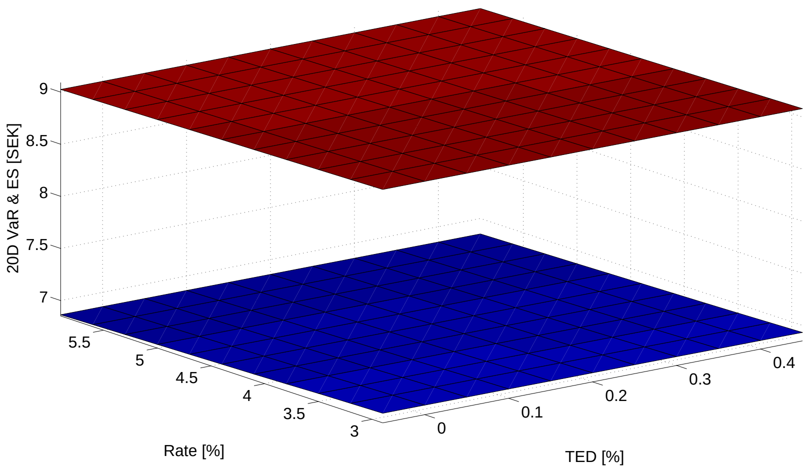

Figure 3.24: Scenario 1 with Bond-Stock portfolio 1, 20-day VaR and ES.

3.5.1.2 Scenario 2 - Stressing $(\ln I_t, \ln \sigma_t)$

Results of the historical simulation of Bond-Stock portfolio 1 done according to the specifications of scenario 2 can be found in figures 3.25 and 3.26.

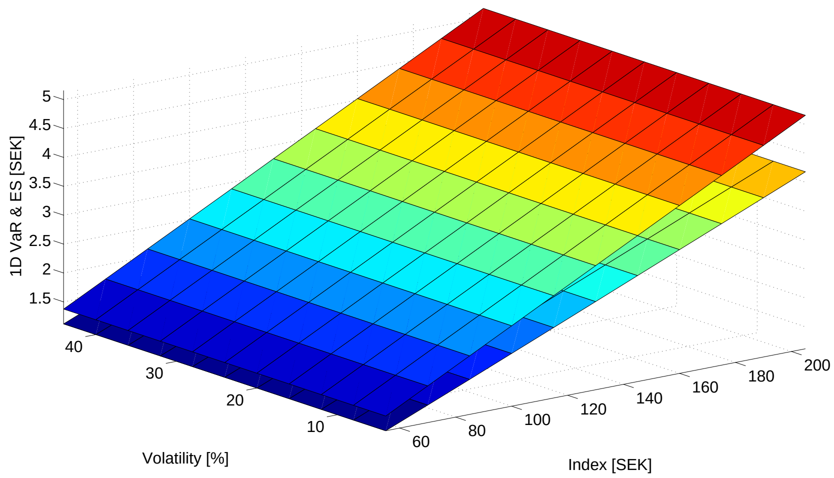

Figure 3.25: Scenario 2 with Bond-Stock portfolio 1, 1-day VaR and ES. The dependence of the index is the same as in the case of the ELN.

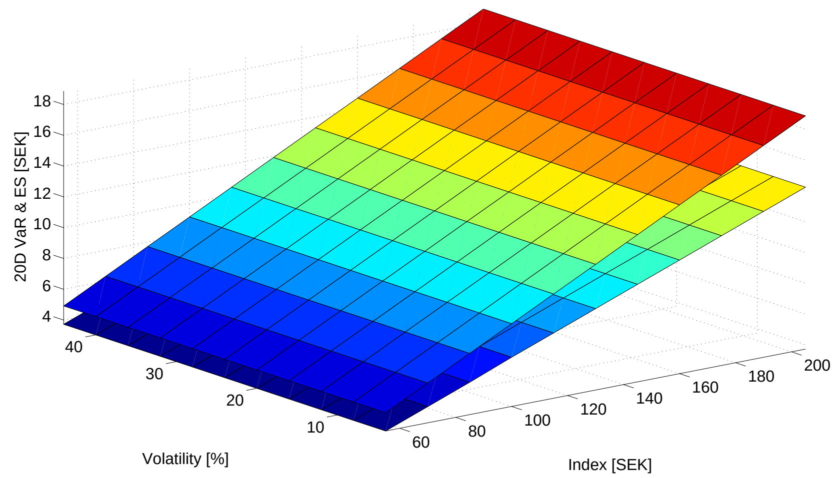

Figure 3.26: Scenario 2 with Bond-Stock portfolio 1, 20-day VaR and ES.

3.5.2 Bond-Stock portfolio 2

3.5.2.1 Scenario 1 - Stressing (r_t, p_t)

Scenario 1 results of the historical simulation of Bond-Stock portfolio 2 can be found in figures 3.27 and 3.28.

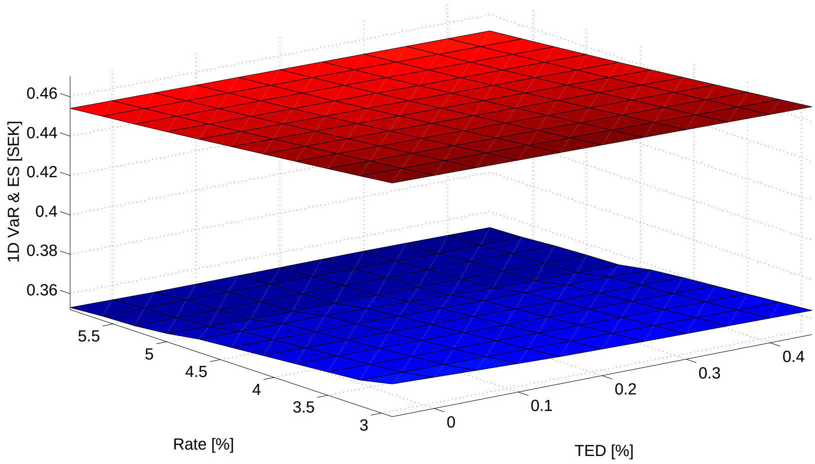

Figure 3.27: Scenario 1 with Bond-Stock portfolio 2, 1-day VaR and ES.

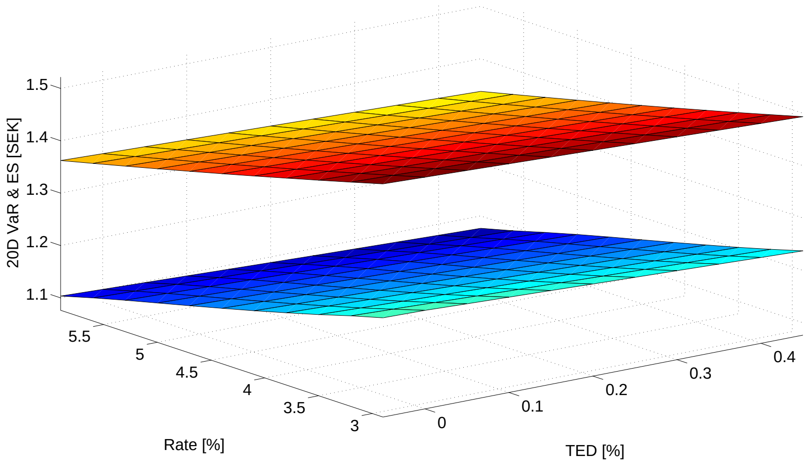

Figure 3.28: Scenario 1 with Bond-Stock portfolio 2, 20-day VaR and ES.

3.5.2.2 Scenario 2 - Stressing $(\ln I_t, \ln \sigma_t)$

Results of scenario 2 simulations of Bond-Stock portfolio 2 can be found in figures 3.29 and 3.30.

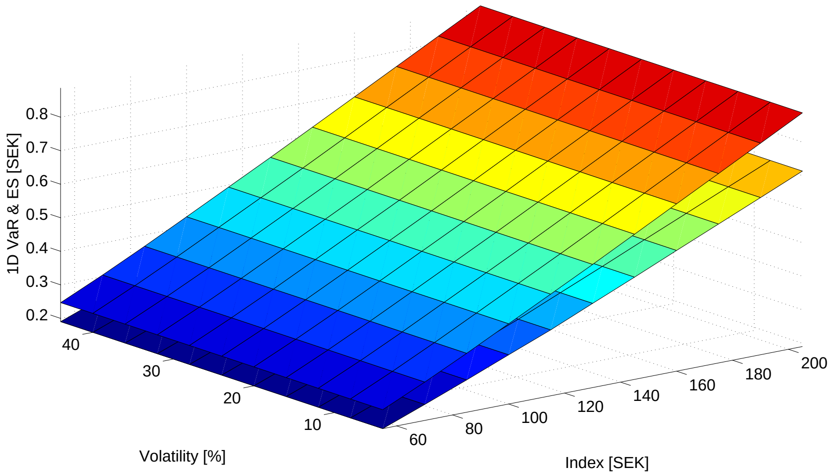

Figure 3.29: Scenario 2 with Bond-Stock portfolio 2, 1-day VaR and ES. The risk surfaces shows the same behaviour as Bond-Stock portfolio 1.

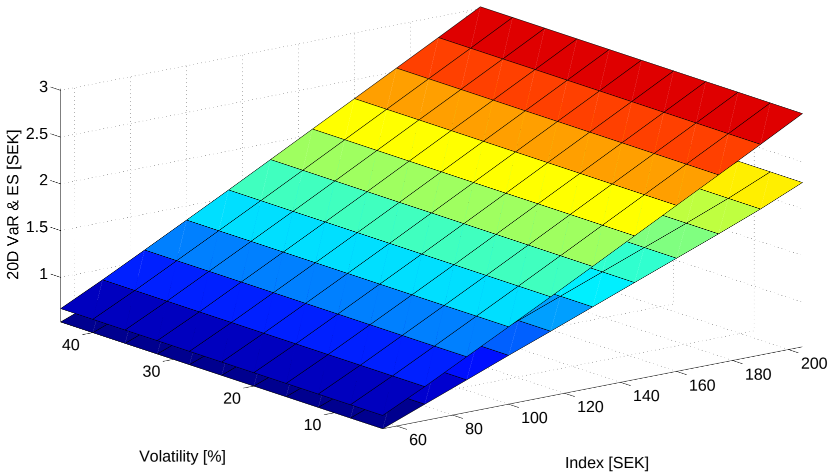

Figure 3.30: Scenario 2 with Bond-Stock portfolio 2, 20-day VaR and ES.

3.5.3 A recapitulation of the B-S portfolio results

The risk profiles of the two Bond-Stock portfolios, below called B-S 1 and B-S 2, are examined in the same way as the ELN. Recall that B-S 1 has the same participation rate as the ELN, and that B-S 2 has the same guaranteed payout as the ELN. In this recapitulation only 1-day VaR from the historical simulation is considered.

3.5.3.1 Analysis of Scenario 1 - Stressing $(\Delta r_t, \Delta p_t)$

Just like in the case of the ELN, the risk surfaces on both B-S 1 and B-S 2 are very close to being planes. The maximum difference between any two points on the two portfolios' risk surfaces are 0.4 percent for B-S 1 and 4.7 percent for B-S 2. As in the case for the ELN, the lowest risk on the surfaces is found at high interest rate r_t and high credit risk premium p_t. Hence, this scenario only causes small changes in the risk level.

3.5.3.2 Analysis of Scenario 2 - Stressing $(\Delta \ln I_t, \Delta \ln \sigma_t)$

In contrary to the ELN, the implied volatility does not have an effect on the risk surfaces. Changes in the index has a big impact on both portfolios. B-S 1 shows an increase of risk with 106 percent when the index is doubled, and a decrease of 45 percent when the index is halved. B-S 2 shows the same pattern but the numbers are a 112 percent increase and a 44 percent decrease respectively. This scenario causes significant changes in the risk level of the portfolios, but not to the extent of the ELN case.

3.6 Comparing the ELN to the B-S portfolios

3.6.1 The risk surfaces

The most extreme changes in the risks, both for the ELN as well as the Bond-Stock portfolios has occurred when stressing according to Scenario 2. Therefore, a graphical comparison of the 1-day VaR of the ELN and each of the Bond-Stock portfolios can be found in figures 3.31 and 3.32. The comparisons are presented as differences between the ELN and each of the Bond-Stock portfolios and calculated as $\text{VaR}_{ELN} - \text{VaR}_{B\text{-}S\,i}$, where $i = 1, 2$.

3.6.2 The portfolios' values

To put the risk measurement of all three portfolios, ELN, B-S 1 and B-S 2, into a bigger picture it is valuable to have an idea about how the values of the portfolios change with the index. This is done both with one year to maturity as well as at maturity and can be found in figures 3.33 and 3.34.

No transaction fees has been taken into account for the two Bond-Stock portfolios, but for the ELN a 2 percent upfront fee has been applied to reflect the difference in brokerage fees between the ELN and the B-S portfolios. To find a reasonable number to use, this has been discussed with a previous employee at one of the larger issuers of ELNs in Sweden.

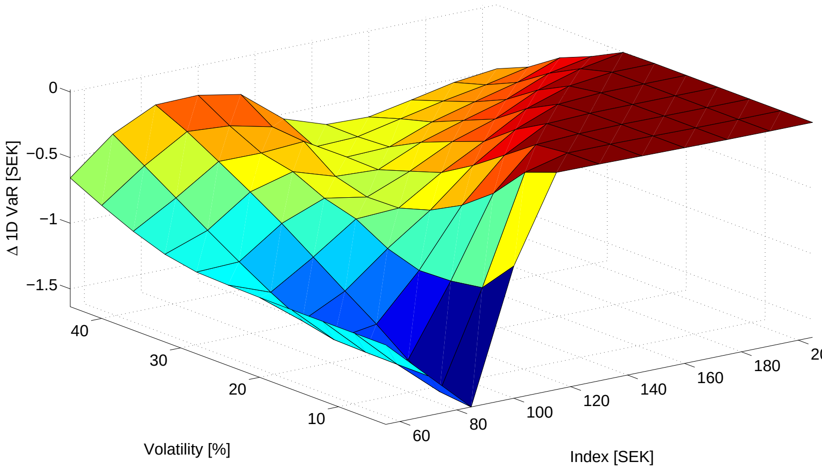

Figure 3.31: Comparing the risk surfaces of the ELN and B-S 1. This illuminates the difference in how the two portfolios depend on the implied volatility. The risk of the ELN is lower at almost the whole surface, although at low volatility and high index levels the difference is approximately zero.

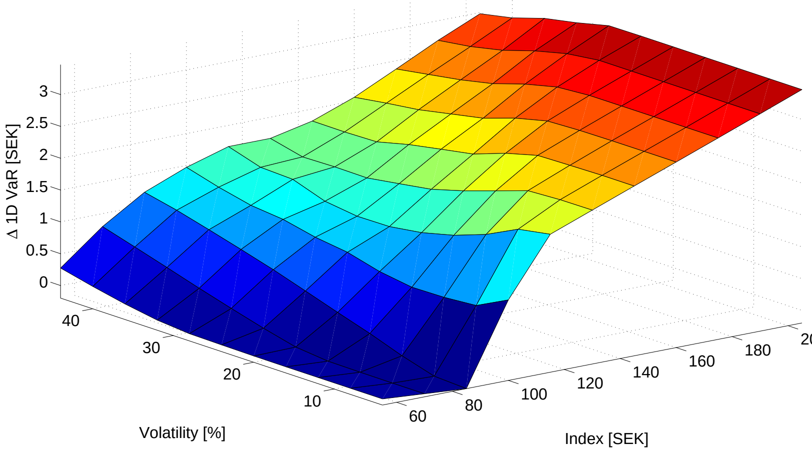

Figure 3.32: Comparing the risk surfaces of the ELN and B-S 2. Note that the risk of the ELN is higher at almost the whole surface, but at low volatility and low index levels the difference is approximately zero.

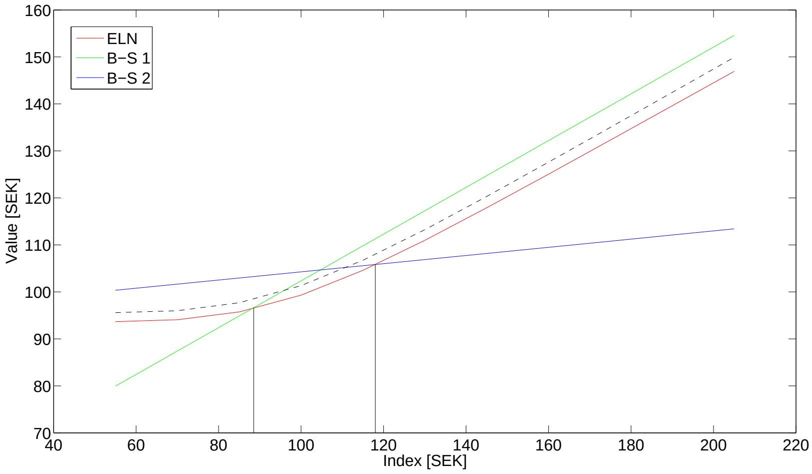

Figure 3.33: Comparing the values of the portfolios as the underlying index change with one year to maturity. Note that the ELN never has the highest value, and that between the index values of 89 SEK and 118 SEK it actually has the lowest value. The dashed line is the ELN without the upfront fee.

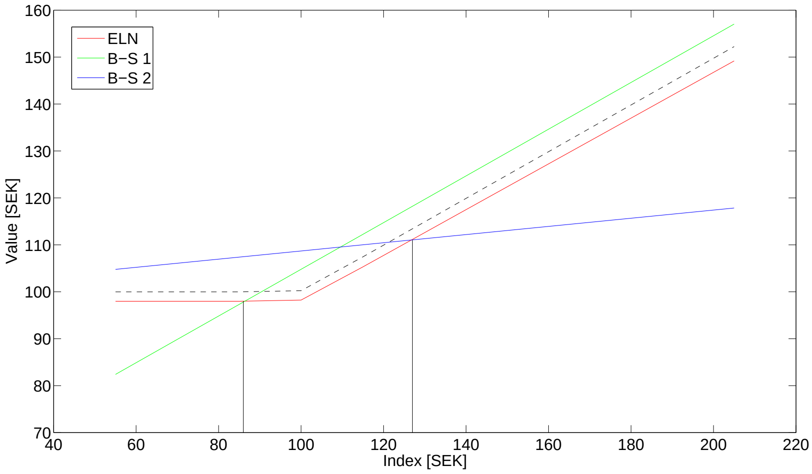

Figure 3.34: Comparing the values of the portfolios as the underlying index change at maturity. Note that the ELN never has the highest value, and that between the index values of 86 SEK and 127 SEK it actually has the lowest value. The dashed line is the ELN without the upfront fee.

Chapter 4

Conclusions and Discussion

4.1 ELN versus Bond-Stock portfolios

First of all, the goals with this thesis are to give a comprehensive risk profile of the ELN and to compare the ELN to alternative investments. To a large extent the risk profiles of the ELN as well as the two Bond-Stock portfolios are presented in the results chapter in terms of figures and the two recapitulations. Therefore, this section will most of all be focused on comparing the ELN to the Bond-Stock portfolios. This can be done using table 4.1 which provides a summary of the two recapitulations of the results chapter. From the results of the analysis it can be seen that the risk-factor with the largest influence on the risks of both the ELN and the Bond-Stock portfolios is the index. As we see in table 4.1 the risk of the ELN can change significantly

Portfolio	Scenario 1 Max Δ	Scenario 2 I_t: 100 $\rightarrow$ 200	Scenario 2 I_t: 100 $\rightarrow$ 50
ELN	2.5%	219%	-93%
B-S 1	0.4%	106%	-45%
B-S 2	4.7%	112%	-44%

Table 4.1: The table provides a summary of the two recapitulations in the previous chapter. *Max* Δ denotes the maximum difference in risk over a Scenario 1 risk surface. I_t*: 100* $\rightarrow$ *200* and I_t*: 100* $\rightarrow$ *50* shows how much the risk increases(decreases) when the index is doubled(halved) in Scenario 2.

with changing market conditions. When it is stressed according to scenario 2 and the index is doubled, the risk level increases by 219 percent. When the index is halved, the risk level decreases to 7 percent of the initial value. The same behaviour occurs for both of the Bond-Stock portfolios, although not to the same extent. As seen in section 3.6.1, at high index levels the ELN has a risk level approximately equal to B-S 1, and at low index levels the risk level is similar to B-S 2. This behavior is repeated if we look at figure

3.33 showing the values as functions of the index of the three portfolios. The characteristics of this figure shows that at low index levels the value functions of the ELN and B-S 2 are approximately parallel and at high index levels the value functions of the ELN and B-S 1 are parallel.

The ELN can be thought of as an "insurance" that gives the behavior of a B-S 1 portfolio in a bull market, and the behavior of a B-S 2 portfolio in a bear market. For this insurance the investor has to pay a premium. In figure 3.34 the ELN never has the highest value of the three portfolios, and between the index levels of 86 SEK and 127 SEK it has the lowest value. An analysis of the 15 years of OMXS30™ data used in this thesis shows that with a probability of 67 percent the index, starting at 100, gives a two year return within the interval [86 127], the interval in which the ELN gives the lowest return.

I believe that an ELN would be considered a safe investment by most investors, since with a two year time horizon the worst thing that can happen is that the investor gets the initial capital back. Two major setbacks of the ELN seem to be the risk of losing the interest rate normally paid by a bond and the high upfront fee charged.

4.2 Final reflections

Some investors might be looking for "The opportunity to a good return with the safety of the bond", meaning they will refrain the interest rate normally paid by a bond to instead bet on an eventual market increase. Other investors will set a risk level according to their financial goals, and others again will specify their utility function and optimize their portfolio accordingly.

For the second category of investors, an ELN will cause some problems. To keep the risk at a nearly constant level, the opportunity to easily rebalance the portfolio is important. The same problem occurs for the investor who has optimized a portfolio given a utility function. Once the values of the portfolio components starts to move, a rebalancing is needed. Easy rebalancing includes both low transaction costs as well as a liquid market for the assets, of which neither are typical features of the ELN.

Finally, for a very passive investor, i.e. one that wants to buy a portfolio and forget about it for two years, the risk of losing the interest rate and getting charged the upfront fee are premiums worth paying, but for investors' who keeps fairly good track of their portfolios it appears more rational to invest in a portfolio consisting of a combination of the bond and the index.

Chapter 5

Appendix

5.1 Plots of the Risk-factors

Figure 5.1: Daily quotes of I_t between January 1 1994 to December 31 2008 with bad data removed.

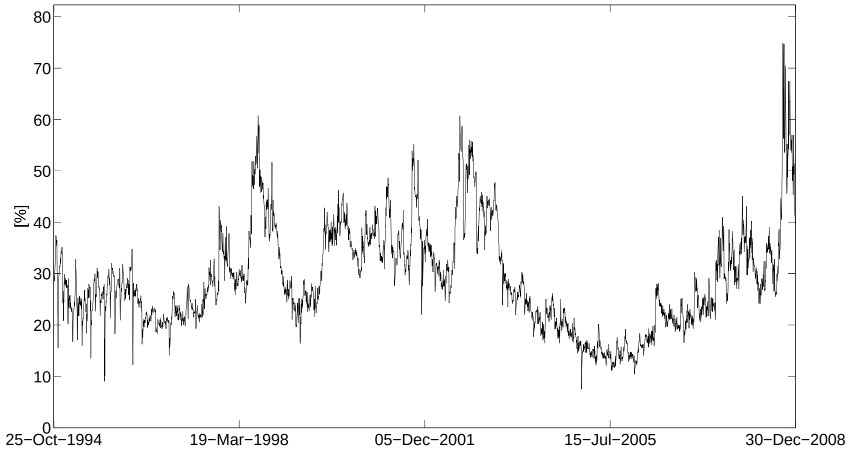

Figure 5.2: Daily quotes of σ_t between January 1 1994 to December 31 2008 with bad data removed.

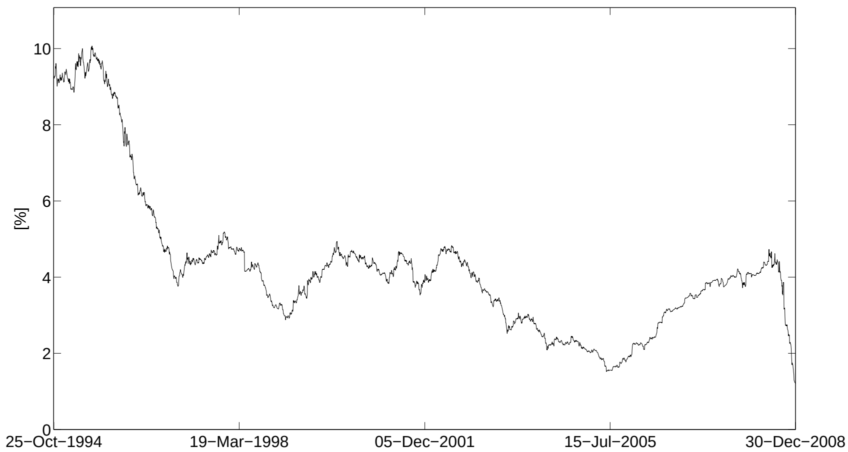

Figure 5.3: Daily quotes of r_t between January 1 1994 to December 31 2008 with bad data removed.

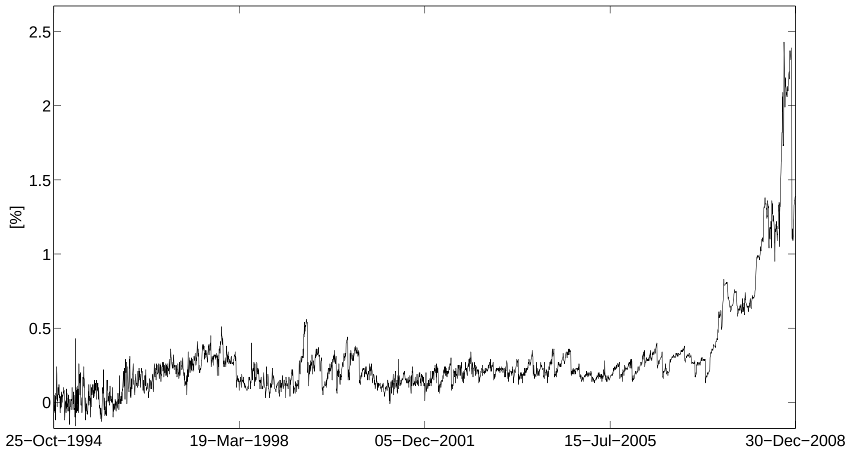

Figure 5.4: Daily quotes of p_t between January 1 1994 to December 31 2008 with bad data removed.

5.2 Additional rolling correlations

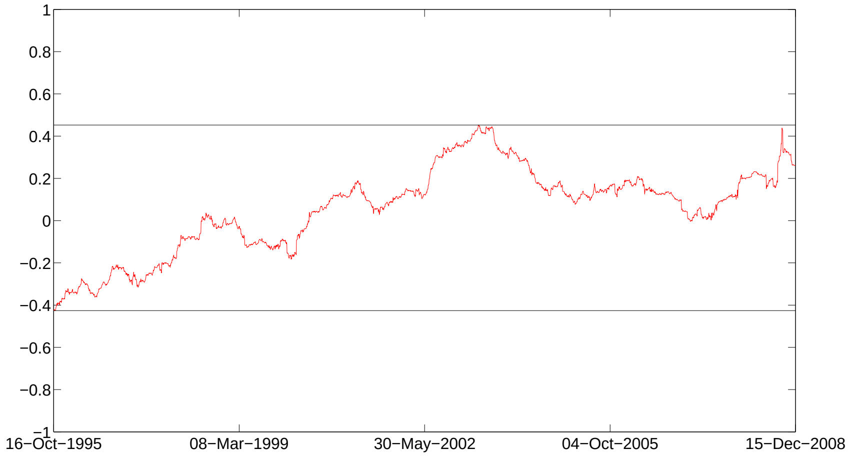

Figure 5.5: 250 day rolling correlation of $(\Delta \ln I_t, r_t)$.

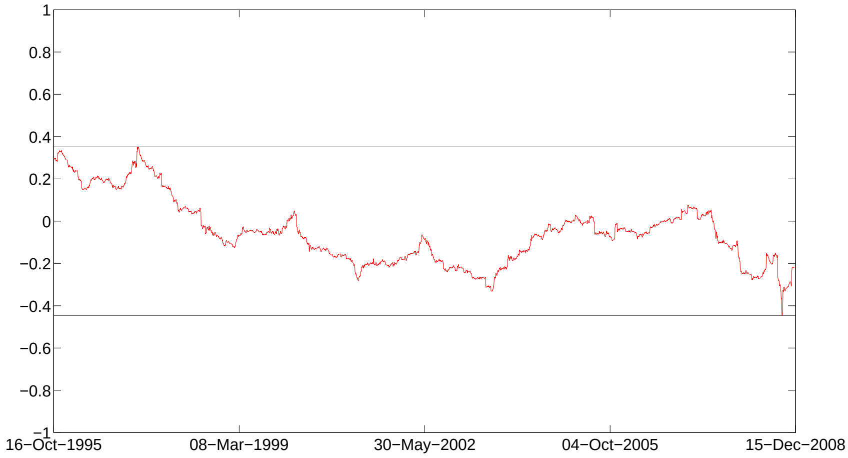

Figure 5.6: 250 day rolling correlation of $(\Delta \ln I_t, p_t)$.

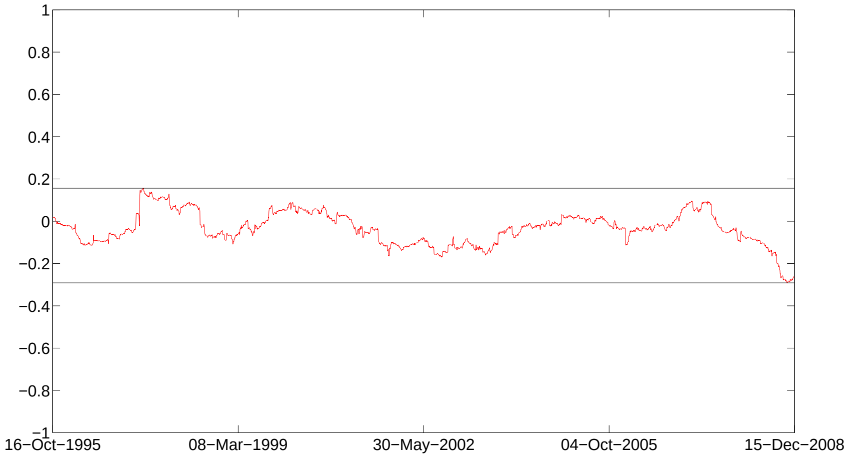

Figure 5.7: 250 day rolling correlation of $(\Delta \ln \sigma_{I_t}, r_t)$.

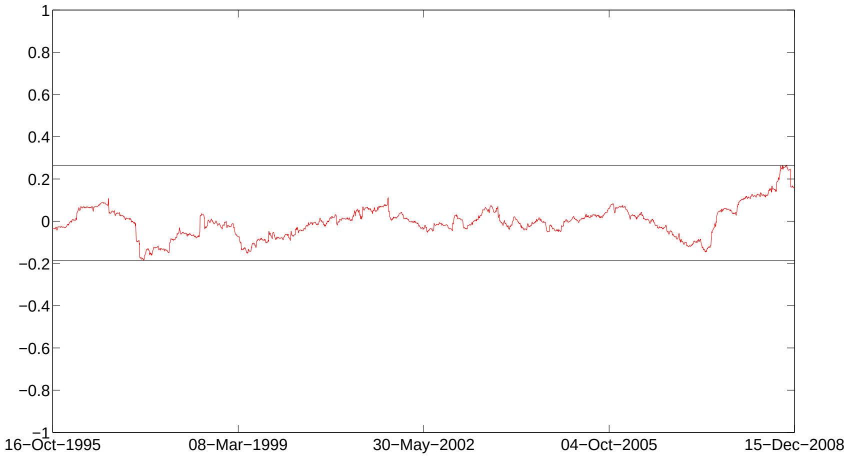

Figure 5.8: 250 day rolling correlation of $(\Delta \ln \sigma_{I_t}, p_t)$.

5.3 Risk measurement - Data from figures

In this section, underlying data of the VaR from the figures in the historical simulation section in the Results chapter are presented.

5.3.1 ELN - Scenario 1

r_t \ p_t	-0.05	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
2.90	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54
3.20	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54
3.50	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53
3.80	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53
4.10	1.55	1.55	1.55	1.55	1.55	1.54	1.54	1.54	1.54	1.54	1.54
4.40	1.55	1.55	1.55	1.55	1.55	1.55	1.55	1.55	1.55	1.55	1.55
4.70	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54	1.54
5.00	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53	1.53
5.30	1.53	1.53	1.53	1.53	1.53	1.52	1.52	1.52	1.52	1.52	1.52
5.60	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52	1.52
5.90	1.51	1.51	1.51	1.51	1.51	1.51	1.51	1.51	1.51	1.51	1.51

Table 5.1: Underlying data from Scenario 1, figure 3.10 showing 1-day losses from the historical simulation.

r_t \ p_t	-0.05	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
2.90	3.07	3.07	3.07	3.06	3.06	3.06	3.06	3.05	3.05	3.05	3.05
3.20	3.10	3.10	3.09	3.09	3.09	3.09	3.09	3.08	3.08	3.08	3.08
3.50	3.13	3.13	3.13	3.12	3.12	3.12	3.12	3.11	3.11	3.11	3.11
3.80	3.15	3.15	3.15	3.15	3.14	3.14	3.14	3.14	3.13	3.13	3.13
4.10	3.19	3.19	3.18	3.18	3.18	3.18	3.18	3.17	3.17	3.17	3.17
4.40	3.22	3.22	3.21	3.21	3.21	3.21	3.21	3.20	3.20	3.20	3.20
4.70	3.23	3.23	3.23	3.22	3.22	3.22	3.21	3.21	3.21	3.20	3.20
5.00	3.25	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.23	3.23	3.23
5.30	3.28	3.28	3.28	3.28	3.27	3.27	3.27	3.27	3.26	3.26	3.26
5.60	3.30	3.30	3.29	3.29	3.29	3.29	3.29	3.29	3.28	3.28	3.28
5.90	3.32	3.32	3.32	3.32	3.32	3.32	3.31	3.31	3.31	3.31	3.31

Table 5.2: Underlying data from Scenario 1, figure 3.11 showing 20-day losses from the historical simulation.

5.3.2 ELN - Scenario 2

σ_t \ I_t	55	70	85	100	115	130	145	160	175	190	205
4	0.09	0.09	0.09	1.40	2.36	2.66	2.96	3.27	3.57	3.88	4.18
8	0.09	0.09	0.12	1.15	2.27	2.66	2.96	3.27	3.57	3.88	4.18
12	0.09	0.09	0.27	1.11	2.02	2.63	2.95	3.27	3.57	3.88	4.18
16	0.09	0.10	0.45	1.10	1.84	2.46	2.94	3.26	3.57	3.88	4.18
20	0.09	0.15	0.61	1.18	1.71	2.29	2.81	3.24	3.57	3.88	4.18
24	0.10	0.28	0.78	1.31	1.64	2.15	2.63	3.12	3.53	3.88	4.18
28	0.11	0.42	0.95	1.38	1.63	2.07	2.53	2.95	3.41	3.81	4.18
32	0.15	0.57	1.10	1.49	1.70	1.99	2.44	2.85	3.25	3.69	4.06
36	0.24	0.73	1.24	1.55	1.88	2.00	2.32	2.75	3.14	3.53	3.96
40	0.34	0.88	1.38	1.66	1.90	1.99	2.30	2.68	3.06	3.42	3.80
44	0.46	1.04	1.49	1.81	2.05	2.11	2.31	2.61	2.98	3.35	3.69

Table 5.3: Underlying data from Scenario 2, figure 3.12 showing 1-day losses from the historical simulation.

σ_t \ I_t	55	70	85	100	115	130	145	160	175	190	205
4	0.15	0.15	0.11	2.11	7.50	9.03	10.08	11.13	12.18	13.22	14.27
8	0.15	0.15	0.15	2.36	6.65	8.83	10.04	11.13	12.18	13.22	14.27
12	0.15	0.13	0.37	2.54	5.85	8.42	9.89	11.08	12.15	13.22	14.27
16	0.15	0.12	0.65	2.66	5.44	7.71	9.58	10.90	12.12	13.18	14.23
20	0.13	0.17	0.94	2.74	5.03	7.15	9.02	10.62	11.87	13.07	14.22
24	0.12	0.30	1.22	2.78	4.84	6.81	8.54	10.11	11.62	12.81	14.06
28	0.12	0.46	1.47	2.92	4.62	6.38	8.09	9.73	11.13	12.60	13.72
32	0.17	0.65	1.70	2.99	4.52	6.16	7.75	9.18	10.78	12.11	13.43
36	0.25	0.88	1.91	3.08	4.41	6.04	7.34	8.96	10.18	11.79	13.08
40	0.36	1.12	2.12	3.19	4.42	5.84	7.21	8.62	9.99	11.14	12.76
44	0.50	1.35	2.31	3.28	4.42	5.68	7.11	8.33	9.65	10.97	12.07

Table 5.4: Underlying data from Scenario 2, figure 3.13 showing 20-day losses from the historical simulation.

5.3.3 Bond-Stock portfolios - Scenario 1

r_t \ p_t	-0.05	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
2.90	2.05	2.05	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04
3.20	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04
3.50	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04
3.80	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04
4.10	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04
4.40	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04
4.70	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04
5.00	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04
5.30	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04
5.60	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04
5.90	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04	2.04

Table 5.5: Underlying data from Scenario 1, figure 3.23 showing 1-day losses from the B-S 1 portfolio.

r_t \ p_t	-0.05	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
2.90	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.36	0.36	0.36	0.36
3.20	0.37	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
3.50	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
3.80	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
4.10	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
4.40	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36
4.70	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.35	0.35	0.35
5.00	0.36	0.36	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
5.30	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
5.60	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
5.90	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35

Table 5.6: Underlying data from Scenario 1, figure 3.27 showing 1-day losses from the B-S 2 portfolio.

5.3.4 Bond-Stock portfolios - Scenario 2

σ_t \ I_t	55	70	85	100	115	130	145	160	175	190	205
4	1.12	1.42	1.73	2.03	2.34	2.65	2.95	3.26	3.57	3.87	4.18
8	1.12	1.42	1.73	2.03	2.34	2.65	2.95	3.26	3.57	3.87	4.18
12	1.12	1.42	1.73	2.03	2.34	2.65	2.95	3.26	3.57	3.87	4.18
16	1.12	1.42	1.73	2.03	2.34	2.65	2.95	3.26	3.57	3.87	4.18
20	1.12	1.42	1.73	2.03	2.34	2.65	2.95	3.26	3.57	3.87	4.18
24	1.12	1.42	1.73	2.03	2.34	2.65	2.95	3.26	3.57	3.87	4.18
28	1.12	1.42	1.73	2.03	2.34	2.65	2.95	3.26	3.57	3.87	4.18
32	1.12	1.42	1.73	2.03	2.34	2.65	2.95	3.26	3.57	3.87	4.18
36	1.12	1.42	1.73	2.03	2.34	2.65	2.95	3.26	3.57	3.87	4.18
40	1.12	1.42	1.73	2.03	2.34	2.65	2.95	3.26	3.57	3.87	4.18
44	1.12	1.42	1.73	2.03	2.34	2.65	2.95	3.26	3.57	3.87	4.18

Table 5.7: Underlying data from Scenario 2, figure 3.25 showing 1-day losses from the B-S 1 portfolio.

σ_t \ I_t	55	70	85	100	115	130	145	160	175	190	205
4	0.19	0.24	0.29	0.34	0.39	0.44	0.50	0.55	0.60	0.66	0.71
8	0.19	0.24	0.29	0.34	0.39	0.44	0.50	0.55	0.60	0.66	0.71
12	0.19	0.24	0.29	0.34	0.39	0.44	0.50	0.55	0.60	0.66	0.71
16	0.19	0.24	0.29	0.34	0.39	0.44	0.50	0.55	0.60	0.66	0.71
20	0.19	0.24	0.29	0.34	0.39	0.44	0.50	0.55	0.60	0.66	0.71
24	0.19	0.24	0.29	0.34	0.39	0.44	0.50	0.55	0.60	0.66	0.71
28	0.19	0.24	0.29	0.34	0.39	0.44	0.50	0.55	0.60	0.66	0.71
32	0.19	0.24	0.29	0.34	0.39	0.44	0.50	0.55	0.60	0.66	0.71
36	0.19	0.24	0.29	0.34	0.39	0.44	0.50	0.55	0.60	0.66	0.71
40	0.19	0.24	0.29	0.34	0.39	0.44	0.50	0.55	0.60	0.66	0.71
44	0.19	0.24	0.29	0.34	0.39	0.44	0.50	0.55	0.60	0.66	0.71

Table 5.8: Underlying data from Scenario 2, figure 3.29 showing 1-day losses from the B-S 2 portfolio.

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