Master's Thesis in Mathematical Statistics

Portfolio Optimization with Structured Products

A quantitative approach to rebalancing portfolios of index linked principle protected notes and non-principle protected certificates

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Abstract

Investors holding structured products are advised to rebalance their positions in order to lock in profits or to enhance their portfolios' return potential. This thesis analyzes rebalancing situations that arise when investing in principle protected notes and non-principle protected certificates, which are linked to a stock index. The rebalancing decision is determined in a two-stage procedure. In the first stage, return scenarios are generated by simulating the evolution of the structured products' risk factors under the physical measure. Structured products are traded over-the-counter and therefore the investor is exposed to credit risk. The presented approach incorporates this risk factor. The simulation procedure is based on a statistical factor model using Principle Component Analysis. In the second stage, the portfolio weight adjustments are determined by solving a scenario optimization program that takes into account trading constraints and proportional transaction costs. Experiments conclude that the representative quality of the scenarios is insufficient when the simulation procedure is solely based on historical data. Adding a subjective view to the simulation methodology can increase the representativeness of the scenarios, depending on the accuracy of the view. The conducted investigations conclude that rebalancing is necessary in order to meet the investor's risk requirements and to maximize the reward potential.

Keywords: Structured products, portfolio optimization under transaction costs, scenario optimization, statistical factor models

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Introduction

Structured products, such as principle protected notes (PPN), have been a popular asset class for both institutional and private investors. In marketing material they are often advertised as a safe investment that guarantees a minimum payoff at maturity and permits profiting from positive developments in the underlying markets. Most investors see a structured investment (here interchangeably used with structured product) as a typical buy-and-hold asset. In other words, a structured product is bought and held until its maturity. This practice is due to the belief that the exposure to market risk is restricted by the guarantee feature of the PPN at maturity. One forgets though that the risk and reward characteristics of a structured product evolve with time and changing market conditions. This report will formulate how the characteristics of a portfolio of structured investments can be analyzed, and how a rational investor with a given risk conception should react to these changing attributes of his portfolio (referring to the investor as a male does not imply anything on the gender of an investor in general).

1.1 Definition of structured products

A structured product is an investment vehicle that is available in numerous variations, therefore it is important to clarify which type is considered in this investigation. A PPN is a structured investment that is a prepackaged combination of a zero coupon bond and an at-the-money (ATM) plain vanilla call option on an underlying. The bond and the option are denoted in the same currency and have matching maturity. The price of a PPN on the date of issue is equal to the face value of the bond, which is set to be the current price of the underlying. The amount of options included in the structure is the difference between the face value and the current price of the bond, divided by the price of the option. This is known as the participation rate[1]. At maturity the owner of a PPN receives the face value of the bond plus the payoff of the call option times the participation rate. Some PPNs have a minimum payoff feature which is equal to just a fraction of the issue price. These products have a higher participation rate since the difference between the issue price and the guaranteed payoff, which is equal to the face value of the bond, is invested in an ATM call option. In this report a PPN always guarantees a minimum payoff equal to the issue price. If an investor wishes to obtain a higher participation rate, he can purchase another structured investment called non-principle protected certificate (NPPC) which is identical to one option included in the

[1] It is common practice to refer to the participation rate as the percentage of the difference between the face value and the current price of the zero coupon bond times the ratio of the current value of the underlying and the price of the option

PPN. By adding NPPCs to a fixed amount of PPNs, any participation rate greater or equal to the one of the PPN can be reached. Figure 1.1.1 illustrates how a structured product is valued in terms of its components.

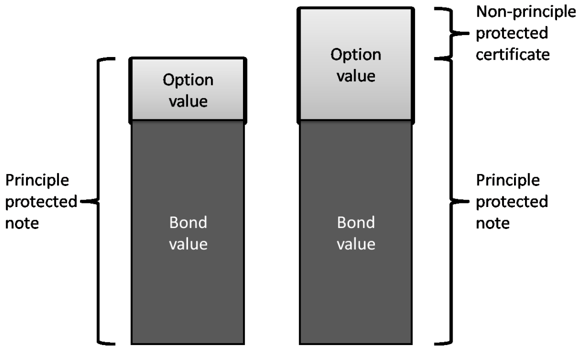


Figure 1.1.1: *Illustration of the value of structured product in term of its components. The left bar shows a PPN and the right bar shows a PPN plus a NPPC.*

1.2 Motivating example

The following example, which is closely related to an illustration in Nyman (2009), shows investment situations that arise when holding structured products. The goal is to motivate that a portfolio of structured investments should be evaluated on a regular basis in terms of its risk and reward characteristics. Repositioning has two main reasons. One is that the investor holds a position which becomes more risky than his risk tolerance allows. The second is that the portfolio's reward potential is below the reward requirement of the investor.

In this motivating example, the investor is assumed to have a time horizon of ten years and exclusively hold PPNs with three years time to maturity. To simplify the situation the interest rate and the volatility of the underlying are assumed to be constant. Figure 1.2.1 presents the development of the underlying index and the evolution of the investor's portfolio value.

The time span is divided into four different investment periods. At the beginning of each period the investor's total wealth is put in the PPN issued at that time.

During the first period the underlying asset experiences a bull market. The option included in the structure goes deep in-the-money, and the market value of the PPN increases significantly. At the end of the first period the PPN with one year time to maturity is sold and the received cash flow is reinvested in a newly issued PPN. The portfolio owner secures accomplished gains by selling the structured investment prior to its maturity. Why is this a rational decision? At the beginning of period one, the largest loss the investor can realize over the next three years is equal to zero. Two years later, the structured product has a current market value above its issue price. With one year time to maturity the largest loss the investor can suffer over the next year is no longer equal to zero. The owner risks to lose the difference between the PPN's current price and its issue price over the next year. The current value of the PPN is 90% above the issue price and therefore the investor risks to lose up to 47% of his current wealth over the next year. An alternative investment opportunity is the currently issued PPN. The maximum loss which can be realized over the next year when holding this

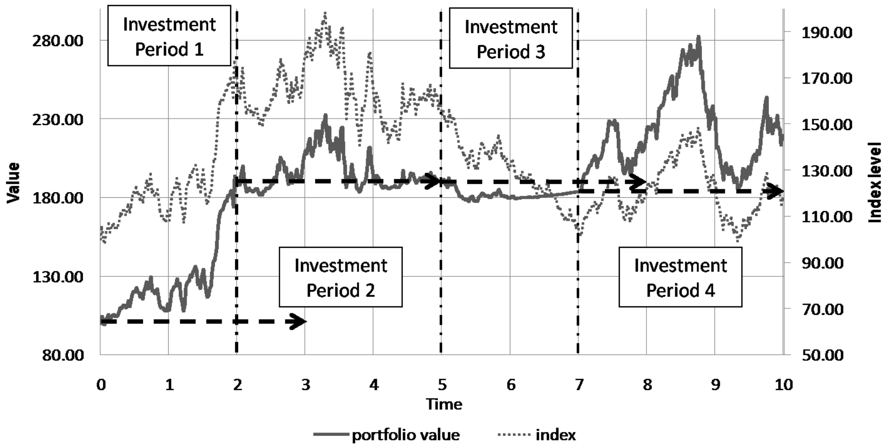


Figure 1.2.1: *Portfolio value development of an investor who has a time horizon of 10 years. The solid line represents the portfolio value measured on the left-hand scale. The dotted line corresponds to the underlying index level measured on the right-hand scale. The dashed-dotted lines indicate the portfolio rebalancing times. The guarantee feature of the PPNs is illustrated by the dashed arrows. The length of the arrow symbolizes the time to maturity. The rebalancing situation at the end of period 1 illustrates early selling to secure realized gains and the one at the end of period 3 shows early selling to increase the reward potential of the portfolio.*

structure is around 7%. This corresponds to the scenario where the option becomes nearly worthless and the value of the structured product is equal to the price of a zero coupon bond with two years time to maturity.

In period two the investor holds the structured product until its maturity. In this case the terminal value of the underlying is below the strike and therefore the received payoff is equal to the initial investment, which is then reinvested in a newly issued PPN.

During period three the underlying asset experiences a bear market. At the end of the period the structured investment has one year left until maturity and the option is deep out-of-the-money. The potential reward from holding the option over the next period is very small, since the event of the underlying's value being above the strike is unlikely. The worst case scenario loss the investor can carry out over the next year is a negative value, since the current value of the PPN is below the issue price. From a risk perspective this situation is favorable, because the PPN will give a positive return over the next year regardless of the price of the underlying at maturity. Examining the situation from a return perspective, it is almost equivalent to just holding a bond with one year time to maturity. Since this is below the return requirements of the investor, he sells his position and receives the market value of the PPN which is equal to the current price of the bond plus current price of the option. With the received cash flow the investor purchases the currently issued PPN that has a higher potential reward than holding his previous position over the next period. The new position is risky and highly dependent on the performance of the underlying.

In the fourth investment period the structured investment is held until its maturity. The investor achieves a profit due to a high performance of the underlying at the end of the period.

To keep the example simple some effects were ignored. First of all, interest rates do change through time and future interest rates are stochastic as see from today. This means that the future price of a bond prior to its maturity is unknown. Also the volatility of the market changes through time which will have a significant impact on the option price. Changes in the interest rates and in the underlying's volatility will lead to variation in the participation rate of PPNs issued at different points in time, since this rate is dependent on those factors. Structured products are traded over-the-counter (OTC), which introduces credit risk as an additional factor influencing their price prior to their maturity. Moreover, buying and selling financial assets also involves costs which influence the rebalancing decision. In this report, the investor's portfolio is analyzed in terms of expected return and Conditional Value-at-Risk[2] (CVaR), and rebalancing decisions are made to optimize his position according to his risk and reward requirements, in the presence of transaction costs.

1.3 Report outline

The thesis is structured in the following way: Readers who are unfamiliar with the following concepts: CVaR as a risk measure, scenario optimization, portfolio optimization with transaction costs, statistical factor models and credit default swaps should first read the chapter called *Theoretical Background*, which explains these concepts that are necessary to follow later investigations. The chapter *Analysis of the rebalancing decision* explains how a model of the underlying risk factors influencing the market value of the portfolio can be built and how the portfolio choice problem is formulated. The main part of the report is presented in the chapter called *Investigations*. This chapter consists of four sections with different topics. Each section opens with its purpose and closes with conclusions from the conducted experiments. The first section investigates the effects of the risk specification on the portfolio weights and on the performance of the rebalancing strategy. The second section incorporates the investor's subjective view in order to reduce the return scenarios' dependence on historical trends in the risk factors. Experiments are carried out showing that a subjective view of good quality can enhance the performance of the rebalancing strategy. The third part studies the impact of transaction costs on the portfolio weights and on the performance of the rebalancing strategy. In the fourth part, the model of the investment decision is extended in order to take into account credit risk. The effects of credit risk on the portfolio weights are examined. The report closes with the final chapter called *Conclusions*.

[2] also known as Expected Shortfall

2

Theoretical Background

2.1 Risk measures

In traditional portfolio theory the risk of a portfolio is measured in terms of its variance. This approach is based on the assumption that the portfolio's return distribution is symmetric. Since the return distribution of a portfolio containing derivatives becomes non-symmetric, there is a need for new risk measures. Two risk measures used for portfolios with non-symmetric return distributions are Value-at-Risk (VaR) and CVaR.

VaR describes the predicted maximum loss with a specified confidence level α over a period of time. If $x \in X$ denotes a portfolio from the set of available portfolios $X = \{(x_1, x_2, \ldots, x_n) \mid x_i \geq 0\ \forall i,\ \sum_{i=1}^{n} x_i = 1\}$ and ω a random vector of future asset prices, then the loss of portfolio x can be described as $L(x,\omega) = V_0(x) - V_1(x,\omega)$ where $V_0(x)$ is the current portfolio value and $V_1(x,\omega)$ is the random future portfolio value. The probability of the loss not exceeding a fixed threshold γ is $P(L(x,\omega) \leq \gamma)$. VaR can be defined as

$$\mathrm{VaR}_\alpha(x) = \min_{\gamma}\{P(L(x,\omega) \leq \gamma) \geq 1-\alpha\}$$

In the second Basel Accord, internal models method for measuring market risk in order to determine minimum capital requirements are based on VaR (Basel Committee on Banking Supervision, 2004). Despite the fact of its popularity in the financial world, VaR is difficult to implement in portfolio optimization. First of all, it lacks subadditivity, which implies that it is not in line with the general idea that diversification leads to risk reduction. Moreover, it does not take into account the size of losses beyond the VaR threshold, which can result in promoting portfolios with enormous losses far out in the tail of the loss distribution. Furthermore, VaR is a non-convex and non-smooth function, that exhibits multiple local extrema, which makes global optimization computationally intensive.

An alternative measure of risk is CVaR, which has been developed taking into account the weak points of VaR. CVaR is the expected loss under the condition that the loss exceeds the VaR threshold. In general, it can be defined as

$$\mathrm{CVaR}_\alpha(x) = \frac{1}{\alpha}\int_0^\alpha \mathrm{VaR}_p(x)\,dp$$

Assuming that ω is a continuous random variable with probability density function

$p(\omega)$, CVaR can also be defined as

$$\mathrm{CVaR}_\alpha(x) = \frac{1}{\alpha} \int\limits_{L(x,\omega)\geq \mathrm{VaR}_\alpha(x)} L(x,\omega)p(\omega)d\omega$$

CVaR is not only a coherent measure of risk, but also an upper bound to VaR, as shown by the following argument.

$$\begin{aligned}
\mathrm{CVaR}_\alpha(x) &= \frac{1}{\alpha} \int\limits_{L(x,\omega)\geq \mathrm{VaR}_\alpha(x)} L(x,\omega)p(\omega)d\omega \\
&\geq \frac{1}{\alpha} \int\limits_{L(x,\omega)\geq \mathrm{VaR}_\alpha(x)} \mathrm{VaR}_\alpha(x)p(\omega)d\omega \\
&= \frac{\mathrm{VaR}_\alpha(x)}{\alpha} \int\limits_{L(x,\omega)\geq \mathrm{VaR}_\alpha(x)} p(\omega)d\omega \\
&\geq \mathrm{VaR}_\alpha(x)
\end{aligned}$$

However, CVaR cannot be implemented in portfolio optimization using its general form, since its definition is still based on VaR. It is though possible to express CVaR in a different way without first computing VaR. To do this, the function $G_\alpha(x,\gamma)$ is introduced as

$$G_\alpha(x,\gamma) = \gamma + \frac{1}{\alpha} \int\limits_{\omega\in\mathbb{R}} \max(L(x,\omega)-\gamma,0)p(\omega)d\omega \tag{2.1.1}$$

This is a convex function in γ and the minimum value of $G_\alpha(x,\gamma)$ with respect to γ and given x corresponds to $\mathrm{CVaR}_\alpha(x)$. An additional advantageous feature of $G_\alpha(x,\gamma)$ is that the minimizer over γ is equal to $\mathrm{VaR}_\alpha(x)$. VaR and CVaR in terms of G can be summarized as

$$\begin{aligned}
\mathrm{VaR}_\alpha(x) &= \arg\min_\gamma G_\alpha(x,\gamma) \\
\mathrm{CVaR}_\alpha(x) &= \min_\gamma G_\alpha(x,\gamma)
\end{aligned}$$

To find the minimum CVaR portfolio in the set of available portfolios X, one simply minimizes the function $G_\alpha(x,\gamma)$ simultaneously with respect to x and γ.

$$\min_{x\in X} \mathrm{CVaR}_\alpha(x) \Leftrightarrow \min_{x\in X,\gamma} G_\alpha(x,\gamma)$$

Since $\mathrm{CVaR}_\alpha(x) \leq G_\alpha(x,\gamma)$, the function G can also be used to formulate a CVaR constraint on a portfolio. Let $R(x,\omega)$ denote a given portfolio's return, then the maximum expected return portfolio with a CVaR below ξ is the solution to the following optimization problem

$$\begin{array}{llcll}
\min\limits_{x} & -E[R(x,\omega)] & \Leftrightarrow & \min\limits_{x,\gamma} & -E[R(x,\omega)] \\
\text{subject to:} & \mathrm{CVaR}_\alpha(x) \leq \xi & & \text{subject to:} & G_\alpha(x,\gamma) \leq \xi \\
& x \in X & & & x \in X \\
& & & & \gamma \in \mathbb{R}
\end{array}$$

Further information on CVaR as a risk measure in portfolio optimization can be found in Uryasev (2000).

2.2 Portfolio Construction

2.2.1 Stochastic Programming

The future value of a portfolio is unknown today, since it depends on the future prices of the assets, which are subject to randomness. However, the investment decision has to be faced today. Portfolio optimization belongs to the class of stochastic programming. There are several ways to handle the uncertainty in the parameters. The key idea is to transform the stochastic program into a deterministic equivalent. The simplest approach is to replace all random variables by their expected values. This is known in the optimization terminology as a mean value problem. This simple approach may fail to deliver a good solution. A more sophisticated way of incorporating randomness is the so called two-stage stochastic problem with recourse (Birge and Louveaux, 1997). The general formulation of a two-stage linear stochastic program with recourse reads:

$$\begin{aligned}
\min_{x} \quad & c^T x + E\left[\min_{y} a(\omega)^T y(\omega)\right] \\
\text{subject to:} \quad & Ax = b \\
& B(\omega)x + C(\omega)y(\omega) = d(\omega) \\
& x \geq 0, \qquad y(\omega) \geq 0
\end{aligned} \tag{2.2.1}$$

In equation (2.2.1) x are the first-stage variables, which represent decisions that have to be made before randomness occurs. $y(\omega)$ are the second-stage variables which can be interpreted as some kind of adjustments for each event ω so that the constraint $B(\omega)x + C(\omega)y(\omega) = d(\omega)$ is satisfied. Solving such a problem can be extremely difficult when the sample space has a large cardinality or is infinite. Instead the sample space can be approximated by a smaller finite set of scenarios $\{\omega_k | \, k = 1, \ldots, S\}$ where each scenario occurs with probability p_k. The deterministic equivalent using this approximation reads

$$\begin{aligned}
\min_{x,y} \quad & c^T x + \sum_{k=1}^{S} p_k a_k^T y_k \\
\text{subject to:} \quad & Ax = b \\
& B_k x + C_k y_k = d_k \qquad k = 1, \ldots, S \\
& x \geq 0 \\
& y_k \geq 0 \qquad k = 1, \ldots, S
\end{aligned} \tag{2.2.2}$$

The complexity of this optimization program depends on the number of scenarios S, since both the number of variables and the number of constraints is proportional to S. The solution to these kinds of problems can be determined very fast, due to efficient algorithms for solving linear programs, such as Simplex or Interior point method, and the steady increase of computational power in personal computers. However, it is important to keep the class of problems as simple as possible. Therefore the optimization problems for portfolio selection are restricted to linear programs in order to keep the computation time at an acceptable level.

2.2.2 Linearization of CVaR

The CVaR objective / constraint needs to be linearized in order to express the portfolio choice problem as a linear stochastic program of the form (2.2.2). Assume that the loss function $L(x, \omega)$ is linear in x and that S realizations of the random vector ω are given in form of scenarios with equal probability. The function $G_\alpha(x, \gamma)$ as defined in (2.1.1) can be approximated by

$$\tilde{G}_\alpha(x, \gamma) = \gamma + \frac{1}{\alpha S} \sum_{k=1}^{S} \max(L(x, \omega_k) - \gamma, 0) \tag{2.2.3}$$

With the help of artificial variables z_k, equation (2.2.3) can be written as a linear objective function or a linear constraint. In the case of a portfolio choice problem with CVaR objective the linear approximation with scenarios reads

$$\begin{array}{lll} \min\limits_{x,\gamma,z} & \gamma + \frac{1}{\alpha S} \sum\limits_{k=1}^{S} z_k & \\ \text{subject to:} & z_k \geq 0 & k = 1, \ldots, S \\ & z_k \geq L(x, \omega_k) - \gamma & k = 1, \ldots, S \\ & x \in X & \\ & \gamma \in \mathbb{R} & \end{array}$$

In the situation where a linear reward function, such as expected return, is maximized subject to a CVaR constraint, the linear optimization problem is defined as follows: where μ is the vector of expected asset returns and ξ is the upper bound on CVaR

$$\begin{array}{lll} \min\limits_{x,\gamma,z} & -\mu^T x & \\ \text{subject to:} & \gamma + \frac{1}{\alpha S} \sum\limits_{k=1}^{S} z_k \leq \xi & \\ & z_k \geq 0 & k = 1, \ldots, S \\ & z_k \geq L(x, \omega_k) - \gamma & k = 1, \ldots, S \\ & x \in X & \\ & \gamma \in \mathbb{R} & \end{array}$$

For a more detailed discussion on CVaR as an objective and a constraint in portfolio optimization see Krokhmal et al. (2002)

2.2.3 Transaction costs

Transaction costs have become increasingly important in portfolio optimization. Their impact varies depending on the asset class. While large-cap stocks can be bought and sold in moderate sizes without much trading friction, the trading costs can amount to a large sum in small and illiquid markets. Transaction costs arise from three different sources; namely commissions such as brokerage fees, bid-ask spread that is the cost of buying an asset and immediately selling it, and finally market impact which is the cost due to unloading large positions compared to average traded volume (Scherer, 2007).

In portfolio construction trading costs can be handled in two different ways. The indirect approach tries to restrict actions which cause transaction costs to increase. Following this line of thought, turnover constraints have been introduced to portfolio management. If the current holdings are described by the vector x^{initial}, then the assets bought are identified by x^+ and the ones sold by x^-, so that the new holdings satisfy

$x = x^{\text{initial}} + x^{+} - x^{-}$. The turnover of the portfolio is defined as $\sum_{i=1}^{n} x_i^{+} + x_i^{-}$. The following linear program maximizes the expected return of the portfolio subject to constraints that restrict the portfolio to a maximum turnover τ, and to x being a feasible portfolio.

$$\begin{aligned} \min_{x,x^{+},x^{-}} \quad & -\mu^T x \\ \text{subject to:} \quad & x = x^{\text{initial}} + x^{+} - x^{-} \\ & \sum_{i=1}^{n} x_i^{+} + x_i^{-} \leq \tau \\ & x^{+} \geq 0 \\ & x^{-} \geq 0 \\ & x \in X \end{aligned}$$

Another idea to indirectly control transaction costs uses trading constraints. An asset may for example only enter the portfolio if the invested proportion in the asset lies above a certain minimum level. In the same fashion one can restrict the asset to be included in the portfolio if the invested proportion lies below a fixed maximum level, to reduce market impact. Not only weights can be limited, but also the number of assets in a portfolio can be subject to a constraint, which will result in lowering transaction costs. For modeling this aforementioned type of trading constraint, it is necessary to introduce binary variables δ_i which indicate if an asset i is included in the portfolio or not.

$$\delta_i = \begin{cases} 1 & \text{if asset } i \text{ is included in the portfolio} \\ 0 & \text{otherwise} \end{cases}$$

The weight constraints can be formulated as

$$\delta_i x_i^{\min} \leq x_i \leq \delta_i x_i^{\max}, \qquad \delta_i \in \{0,1\}$$

Different choices for the parameters $x_i^{\min}$ and $x_i^{\max}$ will then result in different types of buy-in thresholds, which are summarized in the following table.

Type	$x_i^{\min}$	$x_i^{\max}$
Either above or out	smallest proportion of wealth invested in asset i	large number
Either below or out	0	largest proportion of wealth invested in asset i
Either in between or out	smallest proportion of wealth invested in asset i	largest proportion of wealth invested in asset i

Table 2.1: *Parameter settings for weight constraints to control transaction costs*

The so called cardinality constraint, which limits the number of assets included in the portfolio, can be formulated as

$$\begin{aligned} x_i &\leq \delta_i x_i^{\max}, \qquad i = 1,\ldots,n \\ \sum_{i=1}^{n} \delta_i &\leq \psi \\ \delta_i &\in \{0,1\} \end{aligned} \tag{2.2.4}$$

where $x_i^{\max}$ is set to be a large number and ψ is the maximum number of different assets included in the portfolio. The cardinality constraint can easily be combined with one of the weight constraints by simply replacing (2.2.4) with the appropriate formulation.

The disadvantage of the trading constraints is that their formulation relies on binary variables [1], which change the class of optimization problems from linear program to mixed integer linear program. Optimization problems with integer restriction on some of the decision variables are very complex and therefore require a lot of computation time.

A direct approach for handling transaction costs models the cost of buying or selling an asset proportional to the capital invested in the asset. Let TC_i^+ denote the proportional cost associated with buying asset i and TC_i^- the proportional cost of selling asset i. To include this transaction cost model in the usual formulation of a portfolio choice problem, the budget constraint $\sum_{i=1}^n x_i = 1$ has to be modified. In the new setting wealth is not only invested in assets but also used to pay for the cost that originates from making investment decisions. The new budget equation reads

$$\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \left(TC_i^+ x_i^+ + TC_i^- x_i^- \right) = 1$$

Since the sum of weights no longer adds up to one, the reward function $\sum_{i=1}^n \mu_i x_i$ needs to be modified to $\sum_{i=1}^n (1 + \mu_i x_i)$. A portfolio that minimizes CVaR subject to attaining a certain level of expected return r_{target}, and taking into account proportional transaction costs can be found by solving the following linear program.

$$\begin{array}{lll} \min\limits_{x,x^+,x^-,z,\gamma} & \gamma + \dfrac{1}{\alpha S} \sum\limits_{k=1}^{S} z_k & \\ \text{subject to:} & z_k \geq 0 & k = 1,\ldots,S \\ & z_k \geq L(x,\omega_k) - \gamma & k = 1,\ldots,S \\ & \sum\limits_{i=1}^{n} (1+\mu) x_i \geq 1 + r_{\text{target}} & \\ & \sum\limits_{i=1}^{n} x_i + \sum\limits_{i=1}^{n} \left(TC_i^+ x_i^+ + TC_i^- x_i^- \right) = 1 & \\ & x_i = x_i^{\text{initial}} + x_i^+ - x_i^-, & i = 1,\ldots,n \\ & x_i \geq 0, & i = 1,\ldots,n \\ & x_i^+ \geq 0, & i = 1,\ldots,n \\ & x_i^- \geq 0, & i = 1,\ldots,n \end{array}$$

[1] The either below or out type of trading constraint can be formulated without using binary variables.

The advantage of using proportional transaction costs is that the complexity of the optimization problem hardly increases while taking into account a main part of costs related to trading financial assets.

However, not all costs are related to the size of the trade. Some fees and commissions have to be met when entering a certain market. Such costs are known as fixed transaction costs and are denoted FC_i^+ and FC_i^- for the i asset. Their formulation requires two binary variables $\delta_i^+ \in \{0,1\}$ and $\delta_i^- \in \{0,1\}$ and these two constraints for each asset

$$\begin{aligned} x_i^+ &\leq \delta_i^+ x^{\max} \\ x_i^- &\leq \delta_i^- x^{\max} \end{aligned}$$

where $x^{\max}$ is an arbitrary large number. The new budget constraint with fixed and proportional transaction costs reads :

$$\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \left(\delta_i^+ FC_i^+ + \delta_i^- FC_i^-\right) + \sum_{i=1}^{n} \left(TC_i^+ x_i^+ + TC_i^- x_i^-\right) = 1$$

Taking into account both fixed and proportional transaction costs is a closer description of the real world but comes at the expense of a dramatical increase in the complexity of the optimization problem. As already mentioned earlier, the use of binary variables changes the problem class to a mixed integer linear program.

2.2.4 Scenario generation

Scenario based optimization uses a finite set of outcomes of the random parameters to formulate the deterministic equivalent of the stochastic problem. The generation of scenarios is a key issue since the quality of the scenarios has substantial influence on the quality of the solution. In general, scenarios can be generated by any model which describes the random parameters of the optimization problem. It is assumed that the random parameters are asset returns. Scenario asset returns could be produced by simulating from an autoregressive (AR) model or by bootstrapping historical return observation. There is no general way of generating good scenarios, but there are some ideas that can give guidance to scenario generation (Scherer, 2007).

- **parsimonious:** the size of scenarios should be as small as possible to reduce the optimization program's computational complexity
- **representative:** the scenarios should reflect the random parameters in a realistic fashion
- **free of modeling errors:** situations such as arbitrage should be removed since they are uncharacteristic and usually dissolve very quickly in the market

To create scenarios that are representative and parsimonious, variance reduction methods can be used. Two of such methods are adjusted random sampling and tree-fitting. The first approach creates random samples in pairs which have perfect negative correlation. Assume a simple AR model of a return series

$$r_t = a + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} r_{t-1} + \varepsilon_t$$

where ε_t is simulated from some symmetric distribution. A simulated pair of residuals using the adjusted sampling approach is $(\varepsilon_k, -\varepsilon_k)$.

The tree-fitting approach has the goal of matching some prespecified moments of the sample and the assumed distribution. These kinds of matching problems can be solved as a nonlinear optimization problem. Assume that the residual term of the AR model introduced earlier is normally distributed with zero mean and covariance matrix Σ. S scenarios are generated where each one has equal probability of occurrence. A sample of ε_k having equal mean and a covariance matrix as close as possible to the population is the solution to the following optimization problem

$$\begin{aligned} \min_{\varepsilon} \quad & \sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{S}\left(\frac{\varepsilon_{ki}\varepsilon_{kj}}{S} - \Sigma_{ij}\right)^2 \\ \text{subject to:} \quad & \sum_{k=1}^{S}\varepsilon_k = 0 \end{aligned}$$

where ε_{ki} is the i element of the k scenario residual.

The third desirable scenario property can also be imposed by optimization program. If scenarios are to be free of arbitrage the following situation should not appear (Cornuejols and Tütüncü, 2007).

- Arbitrage I: A strategy which has a negative initial cost and a non-negative value for all scenarios
- Arbitrage II: A strategy which has zero initial cost and at least one scenario outcome with a positive value, while all other scenarios will result in a non-negative value

Both of these arbitrage situations can be formulated as linear programs. Let r_i^k denote the return of the i asset in the k scenario, then the solution to the following linear program is unbounded if arbitrage I is present in the scenarios.

$$\begin{aligned} \min_{x} \quad & \sum_{i=1}^{n} x_i \\ \text{subject to:} \quad & \sum_{i=1}^{n} r_i^k x_i \geq 0, \qquad k = 1,\ldots,S \end{aligned}$$

To check if arbitrage II situations exist, the next optimization problem can be used. In the case of arbitrage, the solution will be unbounded.

$$\begin{aligned} \min_{x} \quad & -\sum_{k=1}^{S}\sum_{i=1}^{n} r_i^k x_i \\ \text{subject to:} \quad & \sum_{i=1}^{n} x_i = 0 \\ & \sum_{i=1}^{n} r_i^k x_i \geq 0, \qquad k = 1,\ldots,S \end{aligned}$$

By examining the scenarios with the two aforementioned linear programs, a set of scenarios containing arbitrage can be abandoned or extended with more random samples until it becomes arbitrage free. Even in situations where arbitrage cannot exist due to restrictions in the portfolio weights, like disallowing short selling, it is important to check for these kinds of modeling errors, since certain assets will become extremely desirable and therefore will be overrepresented in the constructed portfolio.

2.2.5 Scenario optimization with CVaR objective and constraint

To summarize the presented modeling approaches of CVaR and transaction costs, two mixed integer linear programs are presented. Before the formulation can be written out the loss function $L(x,\omega)$ is revised. The loss function is defined as the difference of the portfolio value at time 0 and the portfolio value at time 1. If r_i^k denotes the return of the i asset in the k scenario, the loss occurring in the k scenario can be described as

$$\begin{aligned} L(x,\omega_k) &= V_0 - V_1(x,\omega_k) \\ &= \sum_{i=1}^{n} x_i^{\text{initial}} - \sum_{i=1}^{n} \left(1+r_i^k\right) x_i \\ &= 1 - \sum_{i=1}^{n} \left(1+r_i^k\right) x_i \end{aligned}$$

Furthermore, the expected return of an asset is defined by the mean return of the scenarios $\mu_i = \frac{1}{S}\sum_{k=1}^{S} r_i^k$. The following two portfolio choice problems both incorporate fixed and proportional transaction costs. The first minimizes the portfolio's risk measured in CVaR subject to attaining a certain expected return level of the portfolio. The second maximizes the portfolio's reward potential measured in expected return subject to an upper bound on the portfolio's CVaR.

CVaR objective and return constraint

$$\begin{aligned} \min_{x,x^-,x^+,\delta^-,\delta^+,z,\gamma} \quad & \gamma + \frac{1}{\alpha S}\sum_{k=1}^{S} z_k \\ \text{subject to:} \quad & z_k \geq 0 && k=1,\ldots,S \\ & z_k \geq 1 - \sum_{i=1}^{n}\left(1+r_i^k\right)x_i - \gamma && k=1,\ldots,S \\ & \sum_{i=1}^{n}(1+\mu_i)x_i \geq 1 + r_{\text{target}} \\ & \sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\left(FC_i^+\delta_i^+ + FC_i^-\delta_i^-\right) + \\ & + \sum_{i=1}^{n}\left(TC_i^+x_i^+ + TC_i^-x_i^-\right) = 1 \\ & x_i = x_i^{\text{initial}} + x_i^+ - x_i^- && i=1,\ldots,n \\ & x_i \geq 0 && i=1,\ldots,n \\ & 0 \leq x_i^- \leq \delta_i^- x^{\max} && i=1,\ldots,n \\ & 0 \leq x_i^+ \leq \delta_i^+ x^{\max} && i=1,\ldots,n \\ & \delta_i^- \in \{0,1\}, \quad \delta_i^+ \in \{0,1\} && i=1,\ldots,n \end{aligned}$$

Return objective and CVaR constraint

$$
\begin{aligned}
\min_{x,x^-,x^+,\delta^-,\delta^+,z,\gamma} \quad & -\sum_{i=1}^{n}(1+\mu_i)\,x_i \\
\text{subject to:} \quad & \gamma + \frac{1}{\alpha S}\sum_{k=1}^{S} z_k \le \xi \\
& z_k \ge 0 && k = 1,\dots,S \\
& z_k \ge 1 - \sum_{i=1}^{n}\left(1+r_i^k\right)x_i - \gamma && k = 1,\dots,S \\
& \sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\left(FC_i^+\delta_i^+ + FC_i^-\delta_i^-\right) + \\
& + \sum_{i=1}^{n}\left(TC_i^+ x_i^+ + TC_i^- x_i^-\right) = 1 \\
& x_i = x_i^{\text{initial}} + x_i^+ - x_i^- && i = 1,\dots,n \\
& x_i \ge 0 && i = 1,\dots,n \\
& 0 \le x_i^- \le \delta_i^- x^{\max} && i = 1,\dots,n \\
& 0 \le x_i^+ \le \delta_i^+ x^{\max} && i = 1,\dots,n \\
& \delta_i^- \in \{0,1\}, \quad \delta_i^+ \in \{0,1\} && i = 1,\dots,n
\end{aligned}
$$

Both of these formulations can be used to trace out the efficient frontier. When minimizing risk, varying the target return r_{target} will result in different frontier portfolios. When instead maximizing return, changing the upper bound on CVaR ξ will give the various portfolios on the efficient frontier.

2.3 Factor models

A factor model represents some variable r, for example an asset return, in terms of some constant a, a finite amount of factors $f_1,\dots,f_m$ and a residual ε. In general, the model can be formulated as:

$$r = a + \sum_{k=1}^{m} b_k f_k + \varepsilon$$

Depending on the choice of factors, models for asset returns are classified as macroeconomic, fundamental, or statistical factor models (Tsay, 2005). Macroeconomic factor models use observable economic time series, like gross domestic product or inflation, as common sources of variation in returns, while fundamental factor models focus on asset specific properties as for example industry classification. Statistical factor models explain returns by unobservable variables which are derived by statistical techniques such as Principle Components Analysis (PCA).

This paper discusses only the latter type of factor models. To formulate the model, both the factor loadings b_k and factors f_k need to be estimated. The goal is to choose the factor loadings in such a way that the residual term has zero expectation $E[\varepsilon] = 0$ and is uncorrelated with each of the factors $\mathrm{Cov}(f_k,\varepsilon) = 0$. Furthermore, one wants to explain most of the variation in the investigated variable by the factors, which is equivalent to minimizing the variance of the residual term.

2.3.1 Principle Component Analysis

PCA is a technique based on linear algebra that analyzes observations of several inter-correlated quantities and explains the variation in the data by a set of orthogonal variables. The m observations of n variables are described by the $m \times n$ matrix $\tilde{R}$. The sample means μ_j and the sample variances σ_{jj} of the columns of $\tilde{R}$ are estimated and summarized as follows.

$$
\begin{aligned}
\mu_j &= \frac{1}{m}\sum_{k=1}^{m} \tilde{R}_{kj} \\
M &= \mathbf{1}_m \mu^T \\
\sigma_{ij} &= \frac{1}{m-1}\sum_{k=1}^{m} \left(\tilde{R}_{ki} - \mu_i\right)\left(\tilde{R}_{kj} - \mu_j\right) \\
S &= \operatorname{diag}\{\sqrt{\sigma_{11}}, \sqrt{\sigma_{22}}, \ldots, \sqrt{\sigma_{nn}}\}
\end{aligned}
$$

Let R be the centered version of $\tilde{R}$ where every column is subtracted by its sample mean and divided by the square root of its sample variance. This can be expressed in vector notation as $R = \left(\tilde{R} - M\right) S^{-1}$ After this pre-processing, the matrix $R^T R$ can be referred to as the correlation matrix. It is assumed that R has the following singular value decomposition (SVD).

$$R = P \Delta Q^T$$

where P and Q are orthonormal $m \times m$ and $n \times n$ matrices and Δ is a $m \times n$ diagonal matrix of singular values. In fact, Δ^2 is equal to the diagonal matrix Λ which has the eigenvalue of $R^T R$ as diagonal elements.

$$R^T R = \left(P \Delta Q^T\right)^T P \Delta Q^T = Q \Delta^T P^T P \Delta Q^T = Q \Delta^T \Delta Q^T = Q \Delta^2 Q^T$$

It is assumed without loss of generality, that the column vectors of the matrix Q are ordered in such a way that $\delta_{11} \geq \delta_{22} \geq \cdots \geq \delta_{nn}$ holds for the diagonal elements of Δ. The PCA produces a set of new variables called principle components (PCs). The PCs are linear combinations of the original variables and computed in such a way, that the amount of variation they explain is in decreasing order, and that they are orthogonal to each other. The observations of PCs are known as factor scores F and can be computed by

$$F = RQ$$

where Q is referred to as the loading matrix with each column vector corresponding to one PC. So the ith observation of the original variables can be represented in terms of the principle components as

$$r_{i\bullet} = q_{\bullet 1}^T F_{i1} + q_{\bullet 2}^T F_{i2} + \cdots + q_{\bullet m}^T F_{im}$$

where $r_{i\bullet}$ corresponds to the ith row of the R matrix and $q_{\bullet j}$ represents the jth column of the Q matrix. The above representation replicates the original variables perfectly. In order to reduce the number of variables, one investigates the contribution of each PC to the total variation, which is given by $\frac{\delta_{ii}}{\sum_{j=1}^{n} \delta_{jj}}$. With highly inter-correlated data sets, one often experiences that most of the variation can be explained by just a few PCs. A model using the first K PCs is formulated in the following way.

$$r_{i\bullet} = \sum_{k=1}^{K} q_{\bullet k}^T F_{ik} + \varepsilon$$

The presented model for the centered observations satisfies all the wanted properties of a factor model and, in addition to that, has factors which are uncorrelated.

A more detailed presentation of PCA can be found in Abdi and Williams (in press 2010).

2.4 Reduced-form approach to modeling credit risk

This section introduces the reduced-form approach to modeling credit risk and how this model can be calibrated to market data. The discussed theoretical framework follows the outline as in O'Kane and Turnbull (2003).

2.4.1 Reduced-form models of credit risk

In reduced-form models the credit default event is described by the first jump of a Poisson process (Jarrow and Turnbull, 1995). The time of default τ of an entity is exponentially distributed. The probability of defaulting at time τ is defined by the conditional probability

$$P\{\tau < t+dt|\tau \geq t\} = \zeta(t)dt$$

It is the probability of defaulting in the time interval $[t, t+dt)$, conditioned on the event, that the entity has not defaulted up to time t. The probability is dependent on the length of the time interval dt and a deterministic function $\zeta(t)$, which is known as the hazard rate. The non-negative random variable τ describing the default time is assumed to be independent of all other economic quantities, such as interest rates or index level. The probability of surviving up to time T, given that the entity has not defaulted prior to time t, is

$$Q(t,T) = \exp\{-\int_t^T \zeta(s)ds\}$$

The hazard rate can be calibrated to market data using either credit default swaps (CDS) or corporate bonds. Before the calibration technique with CDS is described, this financial contract will be summarized.

2.4.2 Credit Default Swap

The basic idea behind a CDS is to transfer the risk of a reference entity defaulting from one party (the protection buyer) to another party (the protection seller). The contract is specified on a notional principle, which is assumed to be 1 currency unit to simplify notation. Figure 2.4.1 gives an overview of all participants and payment streams of this financial contract. A CDS gives the holder the right, in the case of default of the reference entity, to be compensated for a loss of the notional principle. To obtain this right, the protection buyer pays the protection seller a fixed cash flow at a specified frequency up to the maturity of the contract or until default occurs. The sum of these payments is known as the premium leg, which has the following present value.

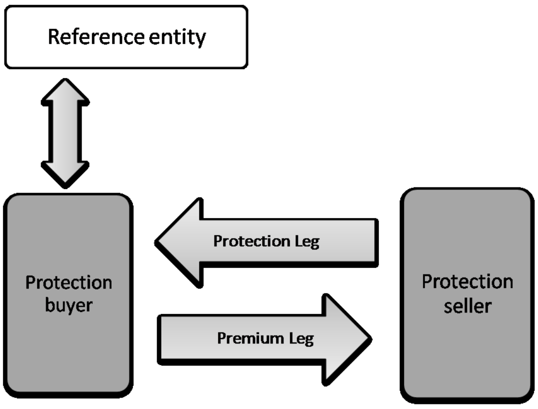


Figure 2.4.1: *Relationship between the participants of a credit default swap and payment streams*

$$\text{Premium Leg PV}(t,T) = s(t_0,T)\sum_{n=1}^{N}\Delta(t_{n-1},t_n)P(t,t_n)Q(t,t_n)$$

where:

t	the current time
t_0	the issue time of the CDS
T	the maturity of the CDS
$\Delta(t_{n-1},t_n)$	the time fraction between the two time points t_{n-1} and t_n
$s(t_0,T)$	the CDS rate contracted at time t_0 with maturity T
$P(t,T)$	the price of a zero coupon bond at time t with maturity T

The present value of the protection leg is the discounted expected value of the compensation payed to the protection buyer.

$$\text{Protection Leg PV}(t,T) = (1-R)\int_t^T P(0,s)Q(t,s)\zeta(s)ds$$

In this formulation R is the recovery rate, which is assumed to be deterministic and constant. If one assumes that the event of default can only occur on a finite number M of fixed time points during the contract time, then the integral can be replaced by a sum describing the present value of the protection leg

$$\text{Discrete Protection Leg PV}(t,T) = (1-R)\sum_{m=1}^{M}P(t,t_m)\left(Q(t,t_{m-1})-Q(t,t_m)\right)$$

At the issue time the CDS rate, also known as the credit spread, is chosen so that the value of the contract is zero, i.e. the value of the premium leg and the value of the protection leg are equal.

2.4.3 Calibration of the hazard rate term structure

This part illustrates how a piecewise constant hazard rate is calibrated to a given set of CDS rates with different maturities $T_1, T_2, \ldots, T_K$. The algorithm is based on the fact that the premium and the protection leg are equal at issue time. The hazard rate is assumed to be constant on the time interval $[T_{i-1}, T_i)$ for $i = 1, \ldots, K-1$. For time horizons beyond T_K the hazard rate matches the value between T_{K-1} and T_K. A monthly discretization is used and the premium is payed $\frac{12}{k}$ times per year. The calibrating procedure starts with the shortest maturity T_1 and determines the hazard rate ζ_{T_1} in such a way that the contract value is zero which means solving

$$s(t_0, T_1) \sum_{n=1}^{T_1/k} \Delta(t_{(k-1)n}, t_{kn}) P(t, t_{kn}) e^{-\zeta_{T_1} t_{kn}} = (1-R) \sum_{m=1}^{T_1/12} P(t, t_m) \left(e^{-\zeta_{T_1} t_{m-1}} - e^{-\zeta_{T_1} t_m} \right)$$

After the value of the hazard rate for the first interval is determined, the procedure is repeated for the next one in the same manner. The survival probability is then given as follows

$$Q(t_0, s) = \exp\left\{ -\left(\sum_{i=1}^{K-1} \zeta_{T_i} \max\left(\min\left(s, T_i\right) - T_{i-1}, 0\right) \right) - \zeta_{T_K} \max\left(s - T_{K-1}, 0\right) \right\}$$

It should be noticed that, situations can arise where the model cannot be fitted to market data. If the CDS rate is high for a given maturity and then drops dramatically for the next maturity, the fitted hazard rate tends to be negative, which is incorrect from a probability point of view. Another critical situation arises if the CDS rate has a big jump from one to the next maturity, since there might be no large enough hazard rate to make both legs equal.

3

Analysis of the rebalancing decision

3.1 Assets

This section formally introduces the assets that will be used in the portfolio construction. The terminal payoff structure of the assets and their theoretical value in the Black & Scholes setting are examined.

The market contains three different assets, which are a cash account, a PPN and a NPPC. The cash account is a risk-free asset, that pays no interest. This asset is a safe haven for the investor because it bears no risk. The investor can only have a long position in the cash account, since this would otherwise imply that he could borrow money without paying interest, which would be an unrealistic assumption. The second asset present in the market is a PPN. This structured product is a contingent claim, that entitles the holder to receive the following payoff at maturity

$$\mathrm{PPN}(T) = S_0 + \beta \max(S_T - S_0, 0)$$

where:

T	maturity of the asset
S_0	value of the underlying on the issue date
S_T	value of the underlying at maturity
β	participation rate

The PPN guarantees a predetermined minimum payoff S_0 plus an unknown payoff dependent on the value of the underlying at maturity. This structured product can be viewed as a combination of a T-year zero coupon bond with face value S_0 and β European call options with the same time to maturity as the bond and a strike price S_0. The bond is assumed to be free of default risk until otherwise specified. The price of this structured investment at time t in the Black & Scholes model can be described by the following equation

$$\begin{aligned} \mathrm{PPN}(t) &= S_0 e^{-r(T-t)} + \beta c(S_t, S_0, T-t, r, q, \sigma) \\ \text{where:} & \\ S_t & \quad \text{value of the underlying at time } t \\ r & \quad \text{risk-free interest rate} \\ q & \quad \text{dividend yield of the underlying asset} \\ \sigma & \quad \text{volatility of the underlying} \\ c(S_t, K, T, r, q, \sigma) & \quad \text{Black \& Scholes price of a European call option} \\ & \quad \text{with strike } K \text{ and time to maturity } T \end{aligned}$$

In this report the issue price of the PPN, which is the value of the PPN at time zero, is set to S_0, which implies that the participation rate can be determined by

$$\beta = \frac{S_0\left(1 - e^{-rT}\right)}{c(S_0, S_0, T, r, q, \sigma)} \tag{3.1.1}$$

The exposure of the PPN to the underlying market is controlled by β, which is a function of the underlying's volatility, the risk-free interest rate, the time to maturity and the underlying's dividend yield. The participation rate is independent of the current price of the underlying. This follows directly when inserting the Black & Scholes formula for an ATM European call option in equation 3.1.1. Figure 3.1.1 illustrates the effects of the dependent factors on the participation rate.

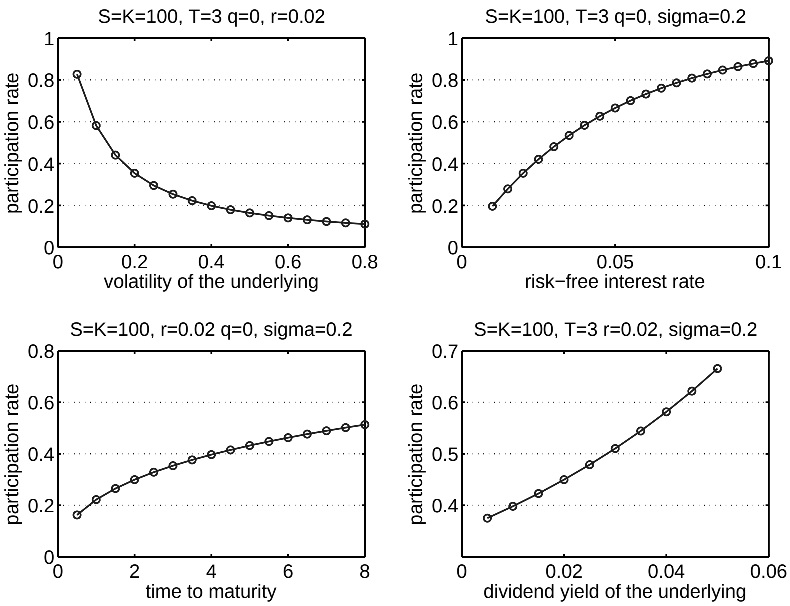


Figure 3.1.1: *The participation rate of a PPN as a function of the underlying's volatility (upper left graph), risk-free interest rate (upper right graph), time to maturity (lower left graph) and the underlying's dividend yield (lower right graph)*

If the volatility of the underlying increases, the price of the call option increases too and therefore the participation rate declines. An increase in each of the other mentioned factors results in an increase of the participation rate.

The third asset available on the market is a structured product, which consists only of the option part of the PPN and is referred to as NPPC. The payoff at maturity is equal to one European call option with strike price S_0

$$\mathrm{NPPC}(T) = \max\left(S_T - S_0, 0\right)$$

The price of the NPPC at time t is given by the Black & Scholes formula for an European call option

$$\mathrm{NPPC}(t) = c(S_t, S_0, T-t, r, q, \sigma)$$

This asset gives the investor the opportunity to increase the upside potential of his portfolio. Some market conditions such as low interest rate or high volatility reduce the participation rate of the PPN. This limits how much the investor can profit from positive developments in the underlying market. By investing in NPPC, he can increase the dependence of the portfolio value on the underlying.

3.2 Data Set

The data set, extracted from Bloomberg, contains observations of the OMX Stockholm 30 Index (OMXS30) level, interest rates implied by Swedish Treasury Bills for maturities 3,6 and 12 months and Swedish Government Bonds for maturities 2,5 and 10 years, and credit default swap (CDS) rates with the Royal Bank of Scotland Plc (RBS) as reference entity.

The OMXS30, which has Bloomberg ticker OMX Index, is a capitalization-weighted index of the 30 most actively traded stocks on the Stockholm Stock Exchange. The index is not adjusted for ordinary dividends, but the index value is corrected for all other corporate actions such as bonus issues, splits and mergers (NASDAQ OMX AB). Bloomberg also provides a total return version of the OMXS30, which means that the ordinary dividends are reinvested. This data is used to estimate the dividend yield of the index.

The Swedish National Debt Office issues Swedish Treasury Bills (in Swedish: statsskuldväxel) with maturities of 1 to 12 months. These bills have no coupon payments. Another instrument issued by this state organization is the Swedish Government Bond (in Swedish: statsobligation). The bond pays a yearly fixed coupon and is available with maturities of 2 up to 30 years (Riksgälden - Swedish National Debt Office, 2007). The government interest rates with maturities 3 months,6 months ,12 months, 2 years, 5 years and 10 years are calculated from the fixed-income instruments issued by the Swedish National Debt Office. The rates are composed by a Bloomberg service called fair market yield curve. The Bloomberg ticker for the 3 month rate is C2593M Index. For the other maturities, the fifth and the sixth character, which indicate the maturity are replaced by the respective ones.

The CDS rates are given for contracts with time to maturity of 3 years and 5 years. These maturities have the longest history of the available contracts with RBS as reference entity. The Bloomberg ticker for such an instrument is RBOS CDS EUR SR 3Y, where the last two characters denote the time to maturity of the CDS. The CDS data is only used in the part of the investigation concerning credit risk.

The data set contains weekly observations. The Swedish government interest rates and the OMXS30 level records range from the 1st of January 1999 to the 23th of April 2010. This amounts to 591 observation dates of each time series. The CDS rates are recorded from the 18th of April 2004 until the 23th of April 2010. The historical government interest rates and the OMXS30 levels are displayed in figure 3.2.1.

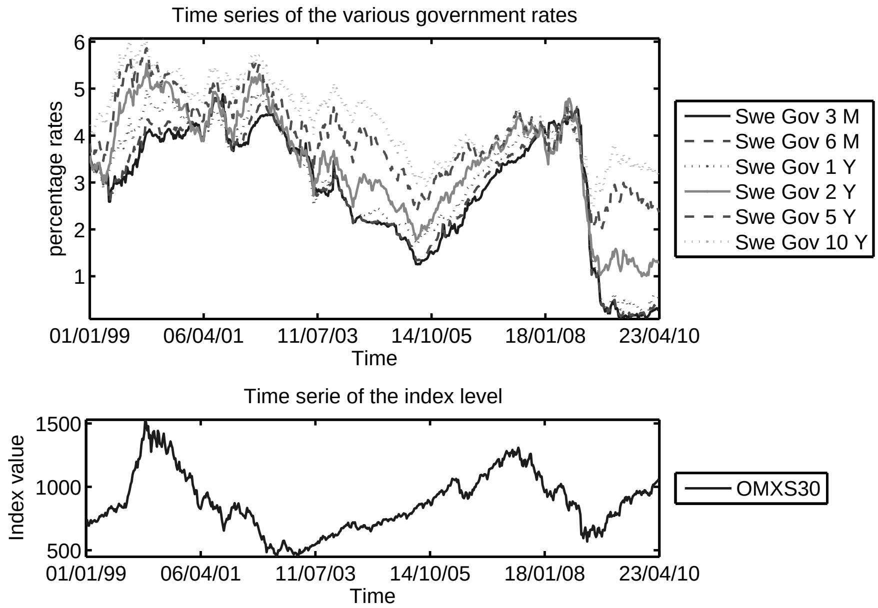


Figure 3.2.1: *Time series of the Swedish government interest rates and the OMXS30*

3.3 Overview of the model components

An overview of the components, which are needed to analyze the rebalancing situation for a portfolio of structured products, is presented in this section. First of all, the measures of risk and reward are motivated. Thereafter a description of the investment situation is presented. It follows a parameter specification of the Black and Scholes model, that is used to price the structured products. Then the construction of the return scenarios is introduced. The section closes with an outline of the procedure, that is used to compute the optimal portfolio weights.

3.3.1 Quantification of reward and risk

A portfolio of assets is characterized in terms of risk and reward, which are conflicting objectives. An investor is assumed to view reward as a positive characteristic and risk as a negative characteristic. In traditional portfolio theory (Markowitz, 1952) reward is quantified in terms of expected return and risk is measured in terms of variance. This study also uses expected return as the measure of reward an investor receives from holding a certain position. It is a generally excepted quantity, which is easy to understand and to formulate for a general investor.

Variance is a measure of risk, which relies on the assumption that the return distribution of the portfolio has a finite second moment and is symmetric around its mean, as for example the normal distribution (McNeil et al., 2002). When derivatives are included in a portfolio, the portfolio's return distribution becomes asymmetric and heavily tailed. A different measure of risk must be used to account for these features of the return distribution. VaR and CVaR are risk measures which are commonly used in risk management, when distributions show asymmetry around the mean and heavy tails.

CVaR is the measure of risk used in the report due to two reasons. First of all, CVaR can be formulated as a linear program in the setting of scenario optimization. This makes it possible to solve the optimization problem in reasonable time and ensures that the global optimum will be found. The second advantage is that it takes into account losses beyond the VaR threshold. This means that no risks can be hidden in the tail of the return distribution unobservable for the chosen risk measure. The definitions of VaR and CVaR, and how to formulate CVaR as convex function is described in section 2.1.

3.3.2 Description of the investment situation

The considered investment situations are very similar to the ones in the motivating example presented in the introduction. At the initial stage, the investor wealth can be allocated into three different assets available on the market, which are a non-interest paying cash account, a currently issued PPN and a currently issued NPPC. The portfolio is assumed to be long-only and purchasing or selling the structured products involves transaction costs, that are modeled as costs proportional to the size of the investment. Changing the position in the cash account is not penalized with any costs. After a specified period of time has passed, which is referred to as rebalancing period, the investor has the possibility to rebalance his portfolio to match his reward and risk conception. This situation is called the rebalancing stage. He is offered to reallocate his portfolio weights to either the cash account, a newly issued PPN or a newly issued NPPC. This means that the portfolio owner can only sell the previously issued structured investments, and only purchase the currently issued structured products. This restriction matches the actual market situation. Structured investments usually have a limited offer time. The rebalancing stage is repeated regularly until the time horizon of the investor is reached. At each stage the expected return and the CVaR of the portfolio is measured over the next rebalancing period. The initial stage can be viewed in the same way as the rebalancing stage, assuming that the investor previously holds a cash-only portfolio.

The rebalancing situation has to be modeled as a stochastic optimization program, since the future prices of the structured products are unknown as seen from today. The concept of scenario optimization is used to transform the stochastic program into the deterministic equivalent. Scenarios have to be generated to describe the possible outcomes of the random parameters over the next rebalancing period. Before selecting a methodology to generate scenarios, a pricing method for the structured products needs to be found and the random parameters entering this method must be identified.

3.3.3 Pricing model

The Black & Scholes formula is used to price the option part of the structured products, and the value of the bond part is determined by discounting the bond's face value using the risk-free rate. This approach is used due to its computational efficiency. The option value needs to be computed for each scenario. In order to generate a sufficient amount of scenarios and still keep a reasonable computational complexity, the method used to determine the value of the option needs to be very efficient. The Black &

Scholes option pricing formula is a closed-form solution and therefore produces option price at very small computational cost. Another advantage is that this setting only requires a few inputs, which limits the amount of risk factors, that need to be modeled. Furthermore,the model's advantages and disadvantages are well known.

As a consequence of this modeling approach, the price of the structured investments depends on the time to maturity, the future interest rate, the future price of the underlying, the future volatility of the underlying and the future dividend yield of the underlying. The following assumptions are made on these variables. The interest rate and the price of the underlying are modeled as random variables. Since the portfolio can contain structured products, that have different times to maturity, not only a single interest rate needs to be modeled, but the whole yield curve. The annual dividend yield of the underlying is set to be 2.5%,which is the average dividend yield when comparing the total return version of the OMXS30 with the standard version.

The volatility of the underlying can either be specified using implied volatility extracted from option market prices or using realized volatility. Wasserfallen and Schenk (1996) compared market prices of structured products to theoretical prices, using both volatility specifications. They did not observe any systematic differences in the theoretical prices when utilizing either implied or realized volatility. Furthermore, one can notice when examining historical option prices, that the option market of the OMXS30 is not very liquid. This causes difficulties to find a consistent methodology of extracting the volatility surface over long time spans. Moreover, the amount of data and risk factors, that need to be handled, increases rapidly when following this approach. This report uses therefore realized volatility as defined in Hull (2005). Let ς be the standard deviation of the log returns of the underlying within a given time period of past observation and let υ be the time length in years between two observations, then the realized volatility σ reads

$$\sigma = \frac{\varsigma}{\sqrt{\upsilon}}$$

The most recent 104 observations are used to compute the realized volatility of the underlying.

3.3.4 Scenario construction

Using this framework, scenarios are generated in the following way. Based on historical data a statistical factor model is built to be used for simulation of future developments of the yield curve and the underlying asset over the time span until the next rebalancing stage. The factor model and the simulation method, that is used to generate time series of future changes in the yield curve and the underlying asset, are presented in section 3.4. Each simulated time series will account for one scenario. Next, the price of the structured products at the final observation of the simulated time series is determined. The risk-free rate for pricing the option and/or the bond is chosen to match the maturity of the structured investment. If the interest rate of this particular maturity is not available, it is determined by linear interpolation of the two rates closest in terms of maturity. The return of each asset can then be computed for the scenario. Each scenario gives a realization of the unknown return of each asset over the next period. Since all scenarios are equally likely, averaging over the different outcomes gives the expected return of the asset over the next period.

3.3.5 Determining optimal portfolio weights

The scenario returns of the assets as well as the assets' expected returns and the initial portfolio weights are then used as the input for the linear program, that yields the optimal portfolio weights as a solution. This optimization problem is formulated in section 3.5. It is presented in two different variations. The investor can either choose to restrict the CVaR of the portfolio by an upper bound and maximize the portfolio's expected return, or formulate a minimum requirement on the expected return of the portfolio and minimize the portfolio's CVaR. Both the expected return of the portfolio and the portfolio's CVaR are measured over the next rebalancing period, that is set to be one year if not specified otherwise.

3.4 A factor model of the yield curve and the underlying index

Scenario optimization requires a finite set of realizations of the random parameters to formulate the deterministic equivalent of the stochastic problem. The random parameters for the rebalancing problem are the asset returns over the next rebalancing period. The asset returns depend on some random risk factors. In this section a statistical factor model is set up that can be used to simulate future yield curves and index levels. A historical and a Monte Carlo algorithm for the simulation under the physical measure is presented.

During the time span of the data set, the Swedish government interest rates and the OMXS30 have experienced up- and downturns. Since neither the index level nor the interest rates are approximately stationary, the investigation is based on relative changes for the interest rates and logarithmic returns for the index, which seem to be weakly stationary as Figure 3.4.1 indicates.

A PCA (as presented in section 2.3.1) is performed on the whole available time span of the data set, which concludes that 95.7% of the variation can be explained by the first four PCs. This deduction is in line with the findings of Litterman and Scheinkman (1991) ,who confirmed that around 96% of the variation in the yield curve can be explained by three factors. Since the data set is composed of the yield curve and the index, an additional source of randomness is added to the data set which increases the amount of necessary PCs to describe it by one. This reasoning is also sustained by the fact that the correlation between the OMXS30 and the different interest rates varies between 0.11 and 0.26. The outcome of the PCA is summarized in table 3.1 and the factor loadings of the main PCs are presented in figure 3.4.2.

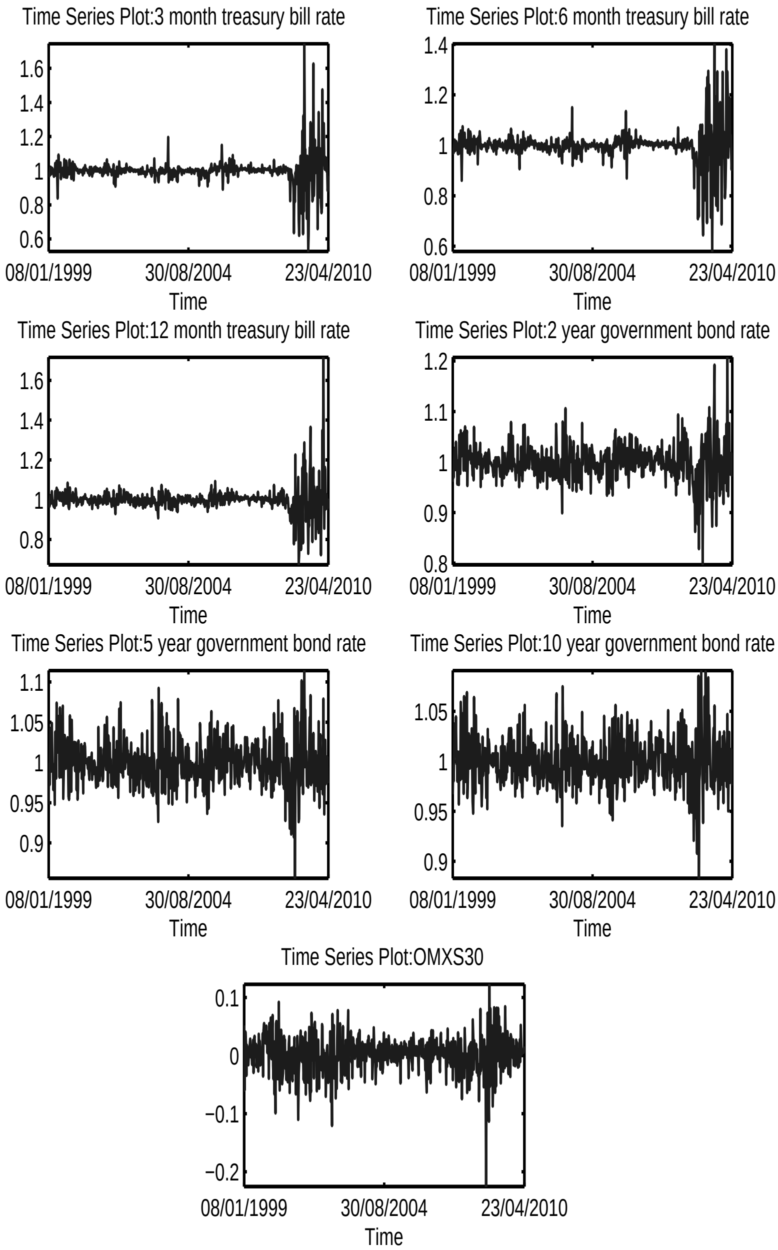


Figure 3.4.1: *Time series of weekly relative changes of Swedish government interest rates with six different maturities and weekly log returns of OMXS30*

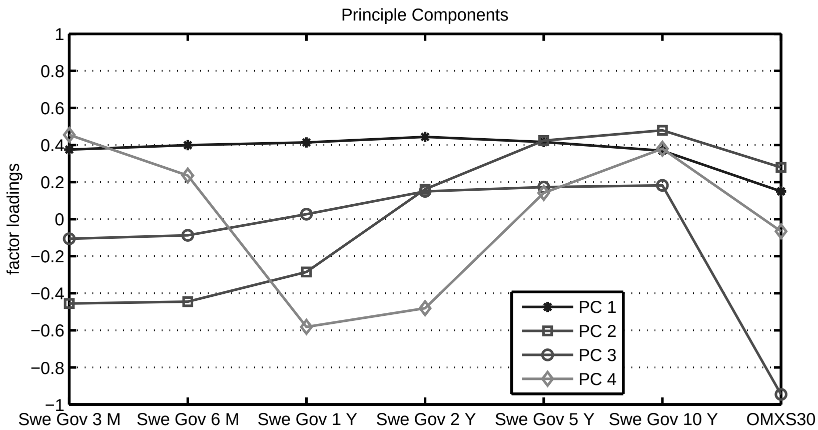


Figure 3.4.2: *The factor loadings of the first four principle components for a data set of relative changes of Swedish government interest rates with six different maturities and log returns of the OMXS30. The analysis is performed on 590 weekly observations ranging from January 1999 until April 2010.*

PC	Explained variation [%]	Cumulative explained variation [%]
1	56.4	56.4
2	20.2	76.7
3	12.8	89.4
4	6.3	95.7
5	2.3	98.0
6	1.4	99.5
7	0.5	100

Table 3.1: *Variance explained by the principle components for a data set of relative changes of Swedish government interest rates with six different maturities and log returns of the OMXS30. The analysis is performed on 590 weekly observations ranging from January 1999 until April 2010.*

In Litterman and Scheinkman (1991), the three dominating PCs of the yield curve are described as a level, steepness and curvature. Similar portrayal holds true for the first PCs of the data set investigated here. The first PC is referred to as the level, because it has no sign changes and therefore can be interpreted as a parallel shift. The second PC accounts for a change in the steepness of the yield curve. A steeping of the yield curve has a positive effect on the index level. The third PC represents a shock to the index development. This PC has a small effect on the interest rates but a large effect on the index. The fourth PC corresponds to a change in curvature of the yield curve. It has only a small effect on the index. When the PCA is performed on just the Swedish government interest rates, very similar results to those of Litterman and Scheinkman are obtained. Further illustration and discussion on this issue are omitted.

Another important aspect in order to determine a statistical factor model that can be used for simulation purposes is the stability of the model through time. This is done by investigating the stability of the factor loadings and the explanatory power of the PCs.

The estimation of the PCs is based on a sample from a larger population. Choosing a specific sample might have a strong effect on the factor loadings and the explanatory power of the PCs. For the investigation of sample dependence, weekly observations of the time period ranging from January 1999 to April 2010 are divided into quarter-yearly overlapping periods with a length of four years: This results in 30 different samples. Each of these samples contains 209 data points, which are enough observations to get stable estimates of the PCs.

The explanatory power of each of the first four PCs and their cumulative sum is displayed in Table 3.2. There are effects of sample dependence present in the explanatory power of the PCs. The first PC explains in some periods more than 68% of the variation (Jun-00 to Jun-04), while in other periods it only stands for below 55% of the variation (May-04 to May-08). There are also similar strong variations present in the other three PCs. Even though there is variation in the explanatory power of the individual factors, the sum of explanatory power of the first four factors is always above 93%. This leads to the conclusion that the first four factors account for the major part of the variation in the data set, independent of the selected sample.

The estimated factor loadings for each of the first four PCs are shown in figure 3.4.3 where each line represents a different sample. Since all plots are presented on the same scale, it is possible to observe that the sample dependence in the factor loadings for the first PC is much smaller than for the other three PCs. The most variable coefficient is the one affecting the index in the second PC. Even though the factor loadings for the PCs show sample dependence, it seems that there is a stable structure in the shape of the principle components.

Period		Explanatory power [%]				
Start	End	PC 1	PC 2	PC 3	PC 4	Cumulative Sum
Jan-99	Dec-02	60.92	16.18	13.51	5.44	96.05
Mar-99	Mar-03	60.84	16.40	13.16	5.67	96.07
Jun-99	Jun-03	62.20	16.16	12.98	4.45	95.80
Sep-99	Sep-03	63.57	14.92	12.68	4.69	95.86
Dec-99	Dec-03	65.71	15.25	11.83	4.58	97.37
Mar-00	Mar-04	67.43	14.50	11.51	4.27	97.71
Jun-00	Jun-04	68.24	14.93	10.76	3.89	97.83
Sep-00	Sep-04	68.29	14.78	10.80	3.92	97.79
Dec-00	Dec-04	68.56	14.44	10.96	3.88	97.85
Mar-01	Mar-05	67.82	15.23	10.61	4.14	97.80
Jun-01	Jun-05	66.04	16.31	10.91	4.23	97.49
Sep-01	Sep-05	65.29	16.61	11.08	4.43	97.40
Dec-01	Dec-05	65.15	16.73	10.36	4.66	96.90
Mar-02	Mar-06	61.12	18.29	11.62	4.90	95.93
Jun-02	Jun-06	60.56	18.08	12.09	4.85	95.58
Sep-02	Sep-06	59.96	18.18	12.29	4.95	95.38
Dec-02	Dec-06	59.90	18.21	12.25	4.96	95.30
Mar-03	Mar-07	59.32	18.44	12.55	4.90	95.21
Jun-03	Jun-07	58.81	18.29	12.98	5.01	95.10
Sep-03	Aug-07	57.72	18.09	13.35	5.60	94.76
Dec-03	Nov-07	55.20	19.38	13.38	6.02	93.97
Mar-04	Feb-08	55.80	19.41	12.93	5.97	94.11
May-04	May-08	54.72	20.20	12.99	6.00	93.92
Aug-04	Aug-08	54.50	20.04	13.48	5.92	93.95
Nov-04	Nov-08	58.69	19.64	11.65	5.33	95.31
Feb-05	Feb-09	60.19	18.60	12.54	5.01	96.34
May-05	May-09	58.35	20.78	12.47	4.85	96.45
Aug-05	Aug-09	58.67	20.25	12.20	5.49	96.61
Nov-05	Nov-09	58.22	20.10	12.11	5.78	96.21
Feb-06	Feb-10	57.72	19.69	11.86	6.73	96.00
Jan-99	Apr-10	56.45	20.20	12.75	6.34	95.74

Table 3.2: *Sample dependence of the explanatory power for 30 quarter-yearly overlapping time intervals consisting of 4 years of weekly observations. The last row indicates the explanatory power for the whole data set ranging from January 1999 to April 2010.*

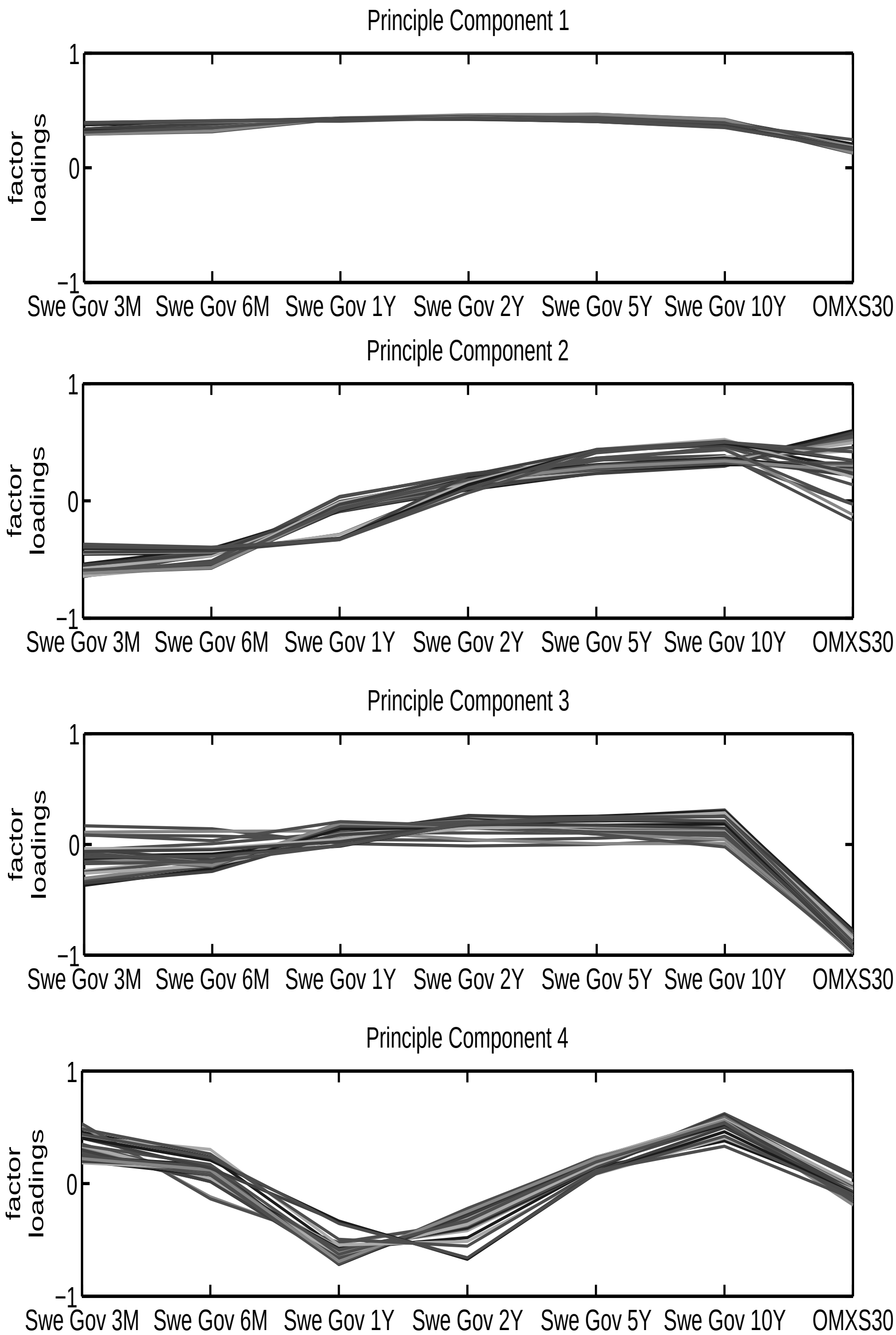


Figure 3.4.3: *Sample dependence of the factor loadings of the first four PCs. One line in a subplot represents the factor loadings of the indicated PC for one sample. The investigation is performed on 30 quarter-yearly overlapping samples consisting of 4 years of weekly observations. All samples are taken from a population of 590 weekly observations ranging from January 1999 to April 2010*

3.4.1 PCA-based simulation of future yield curves and index levels

This part focuses on the simulation of changes in the yield curve and index level. One simulation path consists of 52 weekly changes, which represent the time span of one year. The simulation is based on a four factor model where the factor loadings and the factor scores are estimated by PCA. Since the PCA method is applied on the centered observations, the simulated observations need to be rescaled with the historical mean and standard deviation of the original variables. The simulation data is conducted in two different ways. The first approach is historical simulation and the second is Monte Carlo simulation. Before looking at these sampling methods, the autocorrelation of the PCs is studied, since both approaches rely on the assumption that the observations are independently identically distributed.

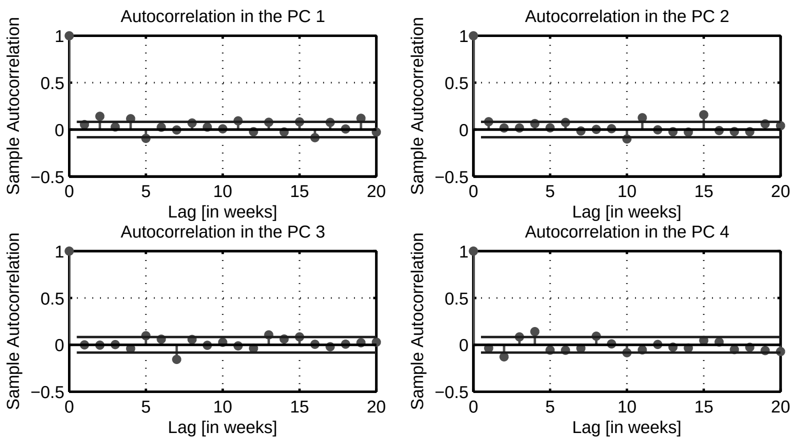


Figure 3.4.4: *Autocorrelation plot of each of the first four PCs. The sample is composed of 590 weekly observations ranging from January 1999 to April 2010*

Figure 3.4.4 shows the autocorrelation graph for the factor scores of the first four PCs. Significant autocorrelation is present when lag i correlation does not fall within the interval indicated by the two horizontal lines around the x-axis. There is hardly any significant autocorrelation present in the PCs. Therefore implying that the observations are independent is a reasonable assumption.

3.4.2 Historical simulation

One non-parametric method for simulation is bootstrapping, also known as historical simulation. To simulate new observations of the PCs, random sampling with replacement is applied to the historical observations of the PCs. The PCs are uncorrelated but this does not imply that they are independent. The sampling procedure is done in the following manner. One historical observation date is randomly picked from the sample with replacement, where each observation date has equal probability of being chosen. The following equation is used to compute the centered original variables

$$r_{i\bullet} = \sum_{k=1}^{4} q_{\bullet k}^{T} F_{ik} + \varepsilon$$

The noise term is simulated from a normal distribution with $E[\varepsilon] = 0$ and $Var[\varepsilon] = Var\left[\sum_{j=5}^{7} q_{\bullet j}^{T} F_{ij}\right]$. Finally the original variables will be rescaled with their historical mean and standard deviation.

3.4.3 Monte Carlo simulation

Monte Carlo simulation is a parametric simulation method, therefore a parametric multivariate distribution function has to be selected for the PCs. The study is preformed on the whole available time span of the data set and is carried out in two steps. First the distribution of the dominating PCs is investigated, and then the residual term is examined. To do this, the marginal distribution functions of the PCs are analyzed by quantile-quantile plots (qq plots), and the dependence structure of the marginals are investigated with the help of scatter plots.

Figure 3.4.5 shows the qq plots where the empirical distribution is plotted against a standard normal distribution for the left column of the diagrams, and against a t-distribution for the right column. The qq plots indicate that the empirical marginals are heavier tailed that a normal distribution, since the line is upward sloping for the right tail and downward sloping for the left tail. A distribution with heavier tails is believed to give a better fit to the observed data, as confirmed by the qq plots against t-location scale family. The line is fairly straight for all PCs, which indicates a good fit. The maximum likelihood estimated (MLE) parameters of the marginal distributions are given in table 3.3. The table indicates that the marginals have different degrees of freedom. However,the qq plots show that the fit is even acceptable when restricting the degrees of freedom to 3.85.

Figure 3.4.6 illustrates the pairwise scatter plots of the centered factor scores. Almost all scatter plots display a strong concentration in a circle that is centered at the origin. The scatter plot that deviates the most from this observation is the upper left one in the figure. Here the shape is rather elliptic. Since a symmetry with respect to the origin could be observed for most scatter plots, a spherical distribution would be a good fit for the centered factor scores.

	normal distribution		t-location scale		
PC	μ	σ	μ	σ	ν
1	0.0000	1.9861	-0.0360	1.1529	2.77
2	0.0000	1.1882	0.0548	0.7604	3.03
3	0.0000	0.9440	-0.0164	0.7783	6.51
4	0.0000	0.6656	-0.0041	0.2653	1.93
5	0.0000	0.3986	-0.0013	0.2235	2.58
6	0.0000	0.3173	-0.0041	0.0554	1.07
7	0.0000	0.1947	0.0035	0.1412	3.91

Table 3.3: *Maximum likelihood estimates for normal and t-location scale distributions fitted to the historical observations of the PCs*

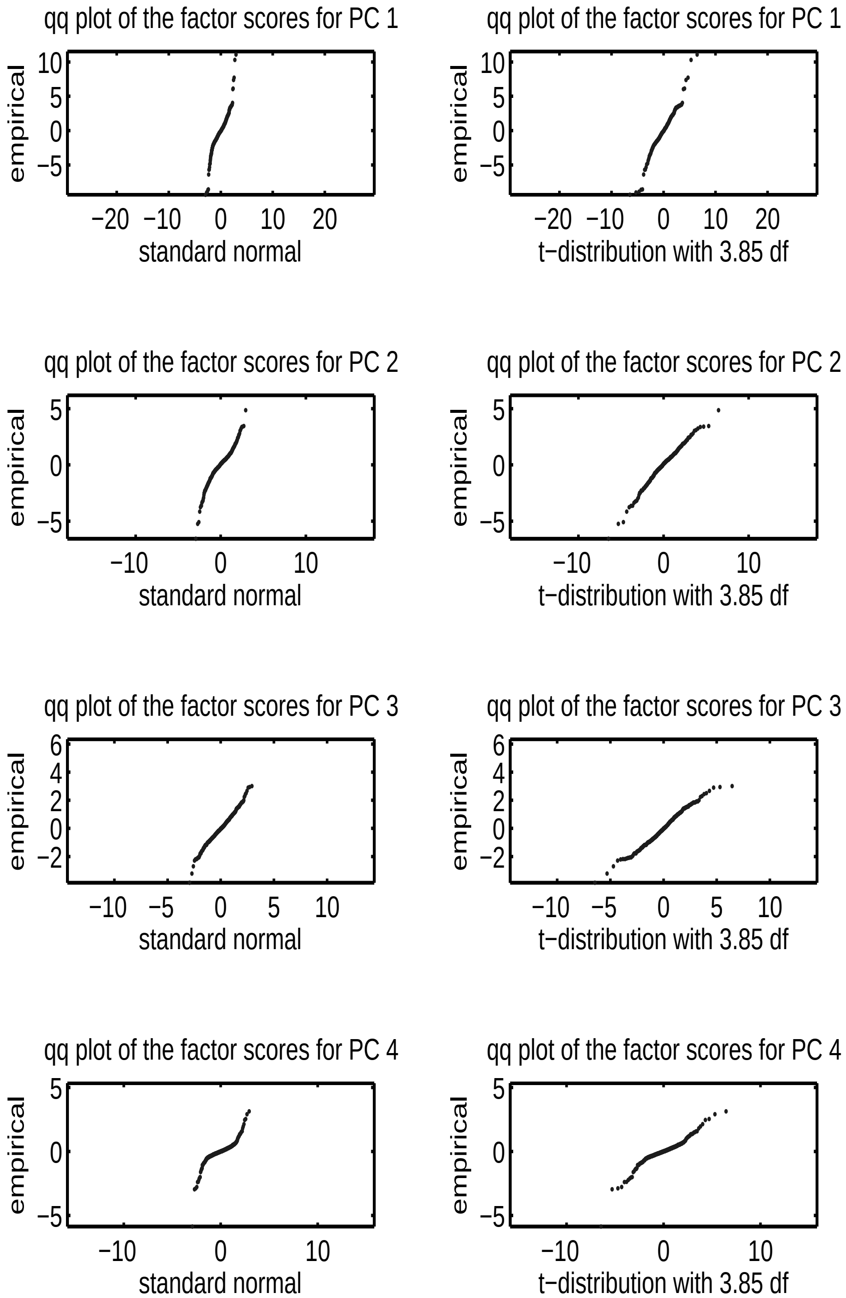


Figure 3.4.5: *Quantile-quantile plots of the marginal distribution functions of the first four PCs. The figure displays two plots for each marginal. The left one is the empirical distribution against a standard normal distribution and the right one is the empirical distribution against a t-distribution with* 3.85 *degrees of freedom.*

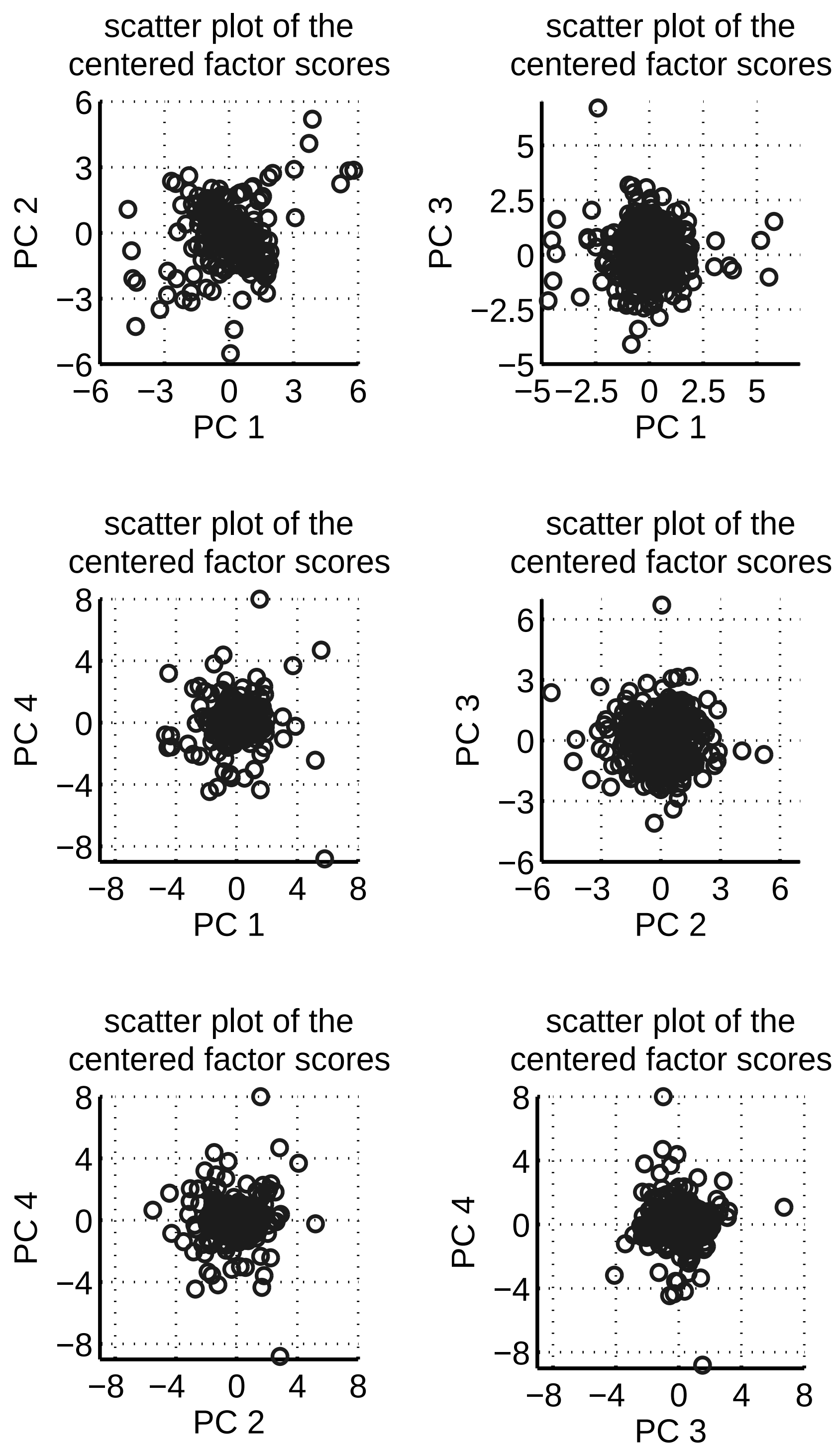


Figure 3.4.6: *Pairwise scatter plots of the centered factor scores for the first four PCs*

Taking into account both the observations made from the qq plots and the scatter plots, a multivariate t-distribution gives a good parametric model of the multivariate distribution of the first four PCs. It is not necessary to use a copula model since the degrees of freedom for the marginals are in close range. A MLE of the centered first four PCs gives a multivariate t-distribution with $\nu = 3.85$. This parameter is equal to the degrees of freedom used for the t-distribution in the displayed qq plots of the first four PCs. Instead of using the MLE estimates for the marginal distribution, the mean and standard deviation are chosen to match the empirical mean and standard deviation when each marginal has the same degrees of freedom.

The next object under investigation is the residual term. In the previous section it was assumed, that ε follows a multivariate normal distribution. Instead of making such an assumption the residual term is investigated in the same manner as the first four PCs. The only assumption made is that the residual term is independent of the first four PCs. Figure 3.4.7 shows the qq plots of the last three PCs. The t-distribution with 3.76 degrees of freedom gives a good fit for the fifth and seventh PC. The sixth PC shows slightly heavier tails. The scatter plots of the last three PCs are displayed in figure 3.4.8. In all three plots the shape looks approximately centered.

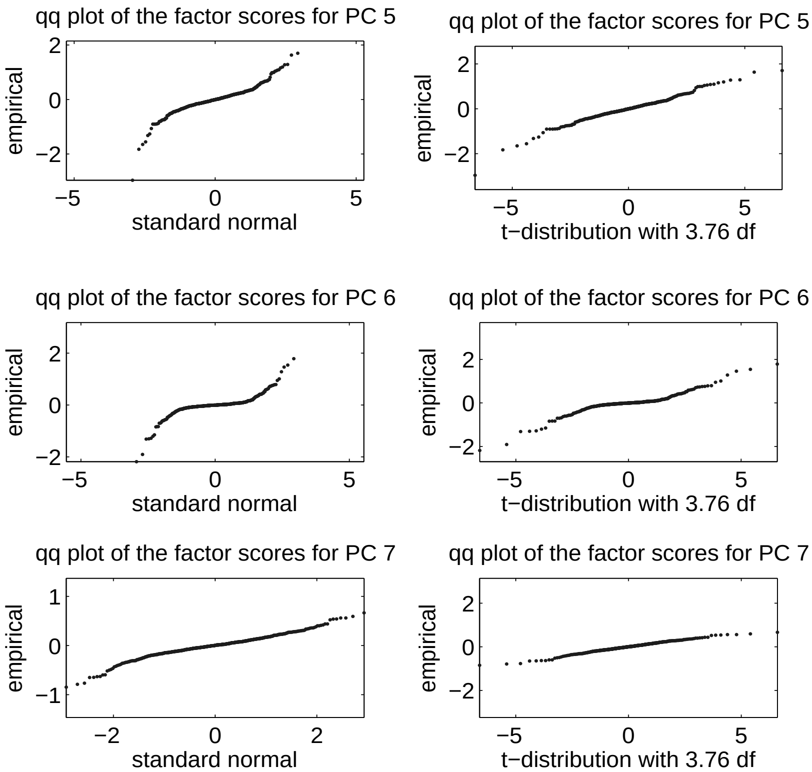


Figure 3.4.7: *Quantile-quantile plots of the marginal distribution functions of the last three PCs used as residual term. The figure displays two plots for each marginal. The left one is the empirical distribution against a standard normal distribution and the right one is the empirical distribution against a t-distribution with* 3.76 *degrees of freedom.*

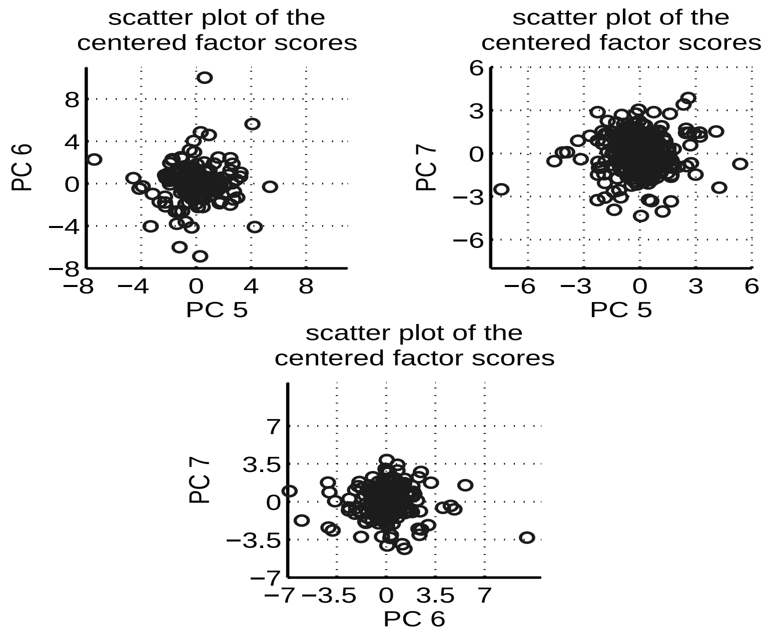


Figure 3.4.8: *Pairwise scatter plots of the centered factor scores for the last three PCs*

Since the t-distribution with 3.76 degrees of freedom gives a good fit for the marginals and the scatter plots do not reject a spherical distribution, the last three PCs are modeled by a multivariate t-distribution with 3.76 degrees of freedom. The complete parametric model reads

$$
\begin{aligned}
r_{i\bullet}^{T} &= \sum_{k=1}^{4} q_{\bullet k} F_k + \sum_{k=1}^{3} q_{\bullet k+4} \varepsilon_k \\
\text{where:} & \\
F &\sim t_{3.85}(\mathbf{0}, \Sigma_F) \\
\Sigma_F &= \begin{pmatrix} 1.8935 & 0 & 0 & 0 \\ 0 & 0.6777 & 0 & 0 \\ 0 & 0 & 0.4277 & 0 \\ 0 & 0 & 0 & 0.2127 \end{pmatrix} \\
\varepsilon &\sim t_{3.76}(\mathbf{0}, \Sigma_\varepsilon) \\
\Sigma_\varepsilon &= \begin{pmatrix} 0.0742 & 0 & 0 \\ 0 & 0.0470 & 0 \\ 0 & 0 & 0.0177 \end{pmatrix}
\end{aligned}
$$

The centered simulated risk factor changes still need to be rescaled with their historical mean and standard deviation.

Since the multivariate t-distribution belongs to the family of normal variance mixture distributions, the simulation algorithm for this family as outlined in McNeil et al. (2002) can be utilized. Let S be a d-dimensional random variable that has a multivariate t-distribution, then S has the following representation

$$\begin{aligned} S &\stackrel{d}{=} \mu + \sqrt{W} A Z \\ \text{where:} \quad & Z \sim N_d(0, I_k) \\ & \nu / W \sim \chi^2_\nu \\ & AA^T = \frac{\nu - 2}{\nu} \Sigma \end{aligned}$$

Here μ and Σ correspond to the mean vector and the covariance matrix of the factor scores and ν is the degrees of freedom parameter of the t-distribution. It should be noticed that the covariance matrix of this distribution is only defined if $\nu > 2$. The step by step simulation algorithm reads as follows.

Simulation of normal variance mixture

1. Compute Cholesky decomposition of $\frac{\nu-2}{\nu}\Sigma$ to obtain A
2. Sample independent standard normal variables $Z = (Z_1, Z_2, \ldots, Z_d)^T$
3. Sample independently $\hat{W}$ from χ^2-distribution with ν degrees of freedom and compute $W = \nu / \hat{W}$
4. Compute $S = \mu + \sqrt{W} A Z$

The parametric approach has the advantage that the simulated observation can experience changes that have not occurred in the past. This is achieved on cost of a modeling error which is due to a misfit between the parametric distribution and the unknown distribution of the PCs. Both the non-parametric and the parametric simulation approaches can be used to generate scenarios under the physical measure. If a different sample is used, the presented analysis for choosing a parametric distribution has to be repeated to be able to insure a good fit. Since this is a time-intensive procedure, the historical simulation approach is used for all investigation in this report.

3.5 Portfolio rebalancing decision

The optimization problem for the rebalancing situation is formulated in this section. Both the maximizing expected return of the portfolio subject to an upper bound on the portfolio's CVaR case and the minimizing the portfolio's CVaR subject to a lower bound on the portfolio's expected return case are presented. The scenario optimization problem is based on the general case, which is introduced in section 2.2.5. Trading constraints are incorporated in the problem formulation so that structured products with limited offer time can be handled.

It is assumed that the investor holds a portfolio of assets at time t_i specified by the initial weights x_i^{initial}. The portfolio choice problem becomes a rebalancing problem where the positions are adjusted in order to satisfy the investor's objective and constraints on his portfolio. Furthermore, it has to be taken into account that structured products can only be purchased at the issuing date, and mature after a fixed amount of time has passed. Let asset $i = 0$ represent the non-interest paying cash account. To simplify the rebalancing situation, it is assumed that the initial portfolio consists only of structured investments that have not reached their maturity. This is not a limitation, since structured products that reach their maturity at time t_i can be handled in the following way. The value of such an asset is given by the payoff function. The investor can no longer hold this asset, so it is removed from the portfolio and its value is added to the cash account. The value of the portfolio does not change but the weight vector x^{initial} is adjusted using the described procedure. Furthermore, all structured investments that have neither a positive weight in the portfolio, nor are issued at the current time can be disregarded, since structured products can only be sold after their issuing date. The assets are enumerated as follows

$i = 0$	cash account
$i = 1, 2, \dots, m$	assets with positive weights issued prior to t_i
$i = m+1, m+2, \dots, n$	assets issued at time t_i

So using the notation introduced in section 2.2.3 the trading constraints on the weight adjustments x_i^+ and x_i^- are given by

$$\begin{array}{ll} 0 \leq x_i^- \leq x_i^{\text{initial}} & i = 0, 1, 2, \dots, m \\ x_i^+ = 0 & i = 1, 2, \dots, m \\ x_i^- = 0 & i = m+1, m+2, \dots, n \\ 0 \leq x_i^+ & i = 0, m+1, m+2, \dots, n \end{array}$$

The variables forced to be zero can be eliminated from the optimization procedure. Since the scenario optimization introduces a lot of constraints and decision variables to the portfolio choice problem, only proportional transaction costs are taken into account since they do not rely on binary variables, which would tremendously increase the complexity. The new budget and weight constraints are

$$\underbrace{\sum_{i=0}^{n} x_i}_{\text{invested wealth}} + \overbrace{\underbrace{\sum_{i=1}^{m} TC_i^- x_i^-}_{\text{cost of sells}} + \underbrace{\sum_{i=m+1}^{n} TC_i^+ x_i^+}_{\text{cost of buys}}}^{\text{transaction costs}} = 1$$

$$\begin{array}{rcll} x_0 & = & x_0^{\text{initial}} + x_0^+ - x_0^- & \\ x_i & = & x_i^{\text{initial}} - x_0^- & i = 1, \dots, m \\ x_i & = & x_i^{\text{initial}} + x_0^+ & i = m+1, \dots, n \end{array}$$

The linear program maximizing the expected portfolio return, subject to an upper bound ξ on the CVaR of the portfolio can be formulated as

$$
\begin{array}{lll}
\min\limits_{x,x^-,x^+,z,\gamma} & -\sum\limits_{i=0}^{n}(1+\mu_i)\,x_i & \\
\text{subject to:} & \gamma+\dfrac{1}{\alpha S}\sum\limits_{k=1}^{S} z_k \leq \xi & \\
& z_k \geq 0 & k=1,\ldots,S \\
& z_k \geq 1-\sum\limits_{i=1}^{n}\left(1+r_i^k\right)x_i-\gamma & k=1,\ldots,S \\
& \sum\limits_{i=0}^{n} x_i+\sum\limits_{i=1}^{m} TC_i^- x_i^- +\sum\limits_{i=m+1}^{n} TC_i^+ x_i^+ = 1 & \\
& x_0 = x_0^{\text{initial}}+x_0^+ - x_0^- & \\
& x_i = x_i^{\text{initial}}-x_i^- & i=1,\ldots,m \\
& x_i = x_i^{\text{initial}}+x_i^+ & i=m+1,\ldots,n \\
& x_i \geq 0 & i=0,\ldots,n \\
& x_i^- \geq 0 & i=0,1,\ldots,m \\
& x_i^+ \geq 0 & i=0,m+1,\ldots,n
\end{array}
$$

In the same manner the next linear program minimizes the CVaR of the portfolio, while attaining a minimum target rate of expected return r_{target}.

$$
\begin{array}{lll}
\min\limits_{x,x^-,x^+,z,\gamma} & \gamma+\dfrac{1}{\alpha S}\sum\limits_{k=1}^{S} z_k & \\
\text{subject to:} & z_k \geq 0 & k=1,\ldots,S \\
& z_k \geq 1-\sum\limits_{i=1}^{n}\left(1+r_i^k\right)x_i-\gamma & k=1,\ldots,S \\
& \sum\limits_{i=0}^{n}(1+\mu_i)\,x_i \geq 1+r_{\text{target}} & \\
& \sum\limits_{i=0}^{n} x_i+\sum\limits_{i=1}^{m} TC_i^- x_i^- +\sum\limits_{i=m+1}^{n} TC_i^+ x_i^+ = 1 & \\
& x_0 = x_0^{\text{initial}}+x_0^+ - x_0^- & \\
& x_i = x_i^{\text{initial}}-x_i^- & i=1,\ldots,m \\
& x_i = x_i^{\text{initial}}+x_i^+ & i=m+1,\ldots,n \\
& x_i \geq 0 & i=0,\ldots,n \\
& x_i^- \geq 0 & i=0,1,\ldots,m \\
& x_i^+ \geq 0 & i=0,m+1,\ldots,n
\end{array}
$$

3.6 Example of a rebalancing situation

This section investigates the computational complexity of the portfolio choice problem and the robustness of its solution. To show the consistency of the two different portfolio optimization problems, which are presented in section 3.5, an example of a rebalancing situation is investigated.

The setup is as follows. The investor holds a cash-only portfolio and can invest in either a new issued PPN with proportional transaction cost 2% or an NPPC with proportional transaction cost 4%. Both assets have 3 years time to maturity and the next rebalancing time will be in one year. 1000 scenarios are generated using the statistical factor model with the historical simulation algorithm based on 4 years of weekly observations of interest rates and the OMXS30 level. The assets' scenario returns as well as the assets' expected return are determined and given as an input to the scenario optimization problem. CVaR is measured with a confidence level of $\alpha = 0.05$. The current time point (19th of May 2006) is chosen, since it is one of the time points where the simulated risk factors give a non-negative expected asset return. The OMXS30 has a closing level of 951.59 on that day and the three year government interest rate is 3.17%. The volatility of the index is estimated to be 12.95% and the Black & Scholes call option price is 87.09 SEK. In this economic setting the participation rate of the PPN is 98.99%. The expected return computed as the average outcome for the simulated scenarios is 12.83% for the PPN and 98.03% for the NPPC. Histograms illustrating the distribution of the asset returns are presented in figure 3.6.1

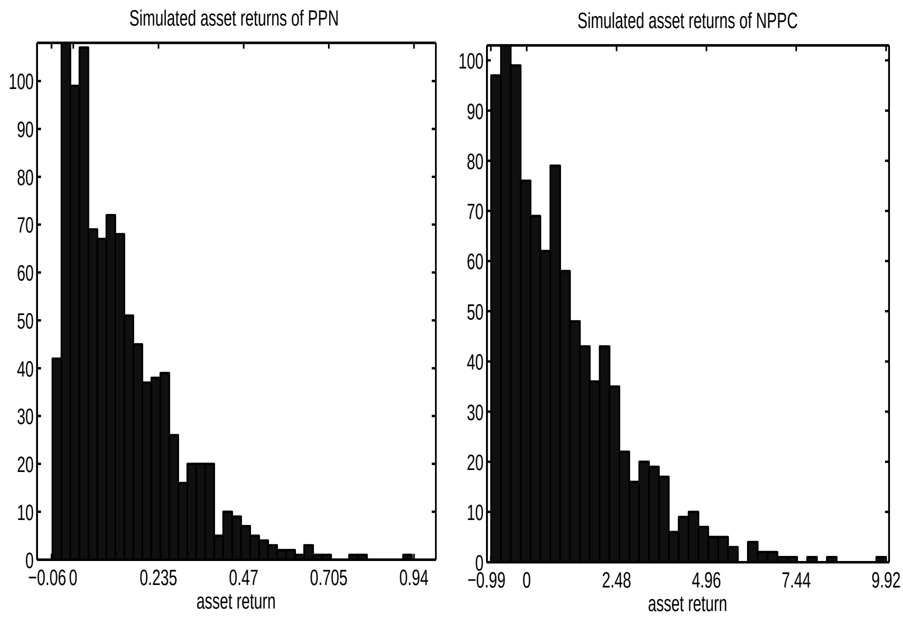
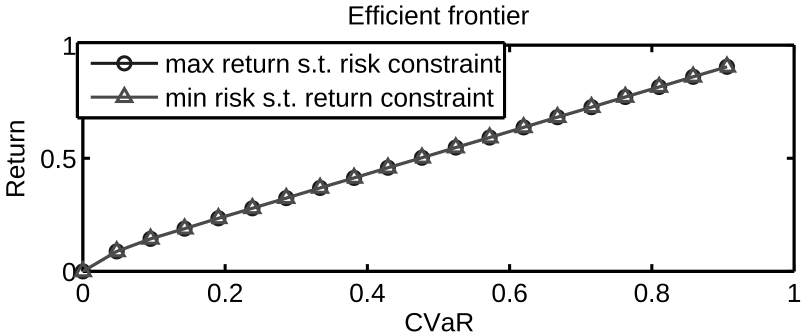
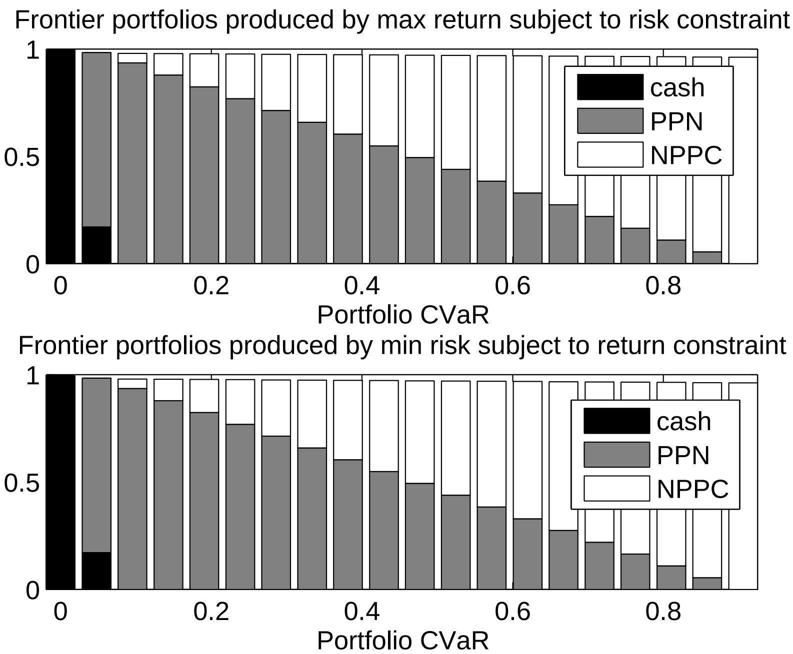


Figure 3.6.1: *The two histograms show the annual return distribution of a PPN (left diagram) and a NPPC (right diagram) as a result of 1000 simulated index and interest rate scenarios using the statistical factor model formulated in section 3.4 with the historical simulation algorithm. The PPN with* 98.99% *participation rate and the NPPC have 3 years time to maturity*



(a) Efficient frontier of the example rebalancing situation



(b) Weight allocations of the frontier portfolios

Figure 3.6.2: *Efficient frontier of the example rebalancing situation and the weight allocations of the frontier portfolios*

Figure 3.6.2 shows the efficient frontiers produced by the different optimization programs and the weight allocation of the frontier portfolios. The frontier portfolios are produced with the following approach. First, the maximum return portfolio with no CVaR restriction is determined. Next, a grid of 20 equally spaced points of the interval between zero and the CVaR of the max return portfolio is established. Each of these values is used as an upper bound on the portfolio's CVaR for which the maximum expected return portfolio is determined. The frontier computed in this manner is referred to as maximum return subject to risk constraint frontier in figure 3.6.2. To work out the second frontier called minimum risk subject to return constraint, the expected returns of the first frontier portfolios are used as lower bounds on the portfolio's expected return and then the portfolio's risk measured in CVaR is minimized.

As one should suspect must the so computed frontier portfolios have the same expected return and CVaR levels, which is confirmed by the figure. When investigating either of the weight allocation plots, one can see how the weights change when the risk or return requirements are changed. An investor who is highly risk averse holds cash-only since this is the only risk-free asset, which though has zero return. When the investor is prepared to hold a risky portfolio he first allocates his weight into the PPN. The more return the investor requires, or the more risk he is willing to take, the larger is the weight invested in the NPPC. This simple situation makes it possible to compare the optimization results with commonsense solutions to the investment problem. In the weight allocation figures the weights do not always add up to one, due to the transaction costs. Apart from confirming the correctness of the algorithms, this situation can be used to investigate differences in computational complexity of the two portfolio choice problem settings. Since computation time is a very unstable measure, the computational work is recorded as the number of iterations. To solve the linear program, the simplex algorithm is used. The iterations needed to solve the problems are displayed in table 3.4.

Expected return of the frontier portfolio	Max Return Iterations	Min CVaR Iterations
0 %	52	53
8.8 %	125	326
14 %	81	322
19 %	65	320
23 %	55	319
28 %	54	319
32 %	54	319
37 %	54	319
41 %	54	316
46 %	54	316
50 %	54	316
55 %	54	316
59 %	54	316
64 %	54	315
68 %	52	314
73 %	52	313
77 %	52	313
81 %	52	313
86 %	52	313
90 %	52	314

Table 3.4: *Iterations of the simplex method needed to find the weights of the efficient frontier portfolios in a 3 asset setting with 1000 scenarios*

In this situation, the optimization problem maximizing expected return subject to a CVaR constraint converges much quicker than the minimizing CVaR subject to an expected return constraint. The minimizing CVaR setting needs often more than 6 times the amount of the iterations used for the maximizing return problem to converge. Only for the portfolio with either zero return constraint, or zero as an upper bound of the

CVaR of the portfolio, the number of iterations are approximately equal. Also, the number of iterations seems to be very stable for various points on the frontier. When varying the number of scenarios, it could be observed that the amount of iterations changes proportionally to the number of scenarios, and that the advantage of the maximizing return setting is retained.

		Min CVaR st. lower expected return bound			Max expected return st. upper CVaR bound		
	Bound value	14.32	41.30	72.56	9.53	38.14	71.51
# Scenarios	Asset weight						
250	PPN	1.446	4.750	8.580	0.553	1.585	2.871
	NPPC	1.418	4.658	8.415	0.543	1.555	2.816
500	PPN	0.960	3.149	5.687	0.352	1.084	1.989
	NPPC	0.942	3.089	5.578	0.346	1.063	1.951
1000	PPN	0.734	2.432	4.400	0.231	0.744	1.385
	NPPC	0.720	2.385	4.315	0.227	0.730	1.358

Table 3.5: *The table presents the standard deviation in percent of the portfolio weights for 100 repeatedly solved portfolio choice problems using a new set of generated scenarios for each repetition. The investigation is done for a low, middle and high risk/return portfolio using different number of scenarios as input to the optimization program. The variation of the weight allocated to the cash account is not displayed since it turned out to be zero for all investigated cases.*

The weight allocation, that is the solution to the portfolio choice problem, depends on the generated scenarios, which are an input of the optimization program. The following study examines how robust the weights are when the optimization procedure is repeated with a new set of generated scenarios for the same factor model using the historical simulation technique. The robustness of the portfolio weights is investigated for both the minimizing the portfolio's CVaR subject to a lower bound on portfolio's expected return setting, and the maximizing the portfolio's expected return subject to an upper bound on the portfolio's CVaR setting. The necessary upper and lower bounds are determined as follows. Three portfolios are selected from the efficient frontier displayed in figure 3.6.2. Counting from left to right the third, the ninth and the sixteenths portfolio were chosen which correspond to a low, medium and high risk/return portfolio. The risk/return value of the portfolio is used as a bound in the respective optimization setting. These portfolio choice problems are solved a 100 times with a new set of scenarios for each repetition. The standard deviation in the weights is then investigated for sets consisting of 250, 500 and 1000 scenarios. Table 3.5 displays the results of the investigation, which lead to three conclusions. First of all, larger numbers of scenarios yield more robust weights. Secondly, the maximizing expected return setting produces more stable weights than the minimizing CVaR setting. The third deduction is that lower risk/return portfolios have more robust weights than high risk/return portfolios.

As a consequence of the advantages of the maximizing expected return setting over the minimizing CVaR setting in terms of weight robustness and required number of iterations to determine a solution, the maximizing expected return setting is used for all further investigation. Furthermore, the number of generated scenarios given as an input to the optimization program is set to 1000 for most of the investigations to be conducted, since it produced stable weights, while requiring reasonable computation time.

4 Investigations

4.1 Investigation of the rebalancing strategy on historical data

This section investigates how well the rebalancing strategy performs on a set of historical data. The main focus is on examining the effects of the confidence limit and the CVaR upper bound on the capital allocation at the rebalancing times and the historical performance of the strategy.

For the historical tests, a data set of 10 years of weekly observations of the Swedish government interest rates and the OMXS30 ranging from the 14th of April 2000 to the 2nd of April 2010 is chosen. The selected time span includes both up- and downturns of the underlying of the structured products and the Swedish government interest rates. The allocation strategy starts with the first investment on the 9th of April 2004 and is rebalanced every 52 weeks, which is approximately one year. The final value of the strategy is measured on the 2nd of April 2010. At each investment time point, maximizing the portfolio's expected return subject to an upper bound on the portfolio's CVaR measured with respect to a given confidence level is used to determine the portfolio weights. The optimization problem works with 1000 scenarios, which are generated by the historical simulation algorithm based on a statistical factor model for the yield curve and the index. The factor model takes into account the most recent four years of weekly observations. The market is assumed to be frictionless in these investigations , i.e. the proportional transaction costs of all assets are equal to zero. The investment situation at the different rebalancing points is as follows. At the first rebalancing point, which is the initial investment situation, the investor's wealth can be allocated to a cash account, a currently issued PPN and a currently issued NPPC. The structured products have 3 years time to maturity at their date of issue. At any later rebalancing point, a new issued PPN and NPPC are available to the investor. An overview of the market conditions and the selected rebalancing points are given in figure 4.1.1.

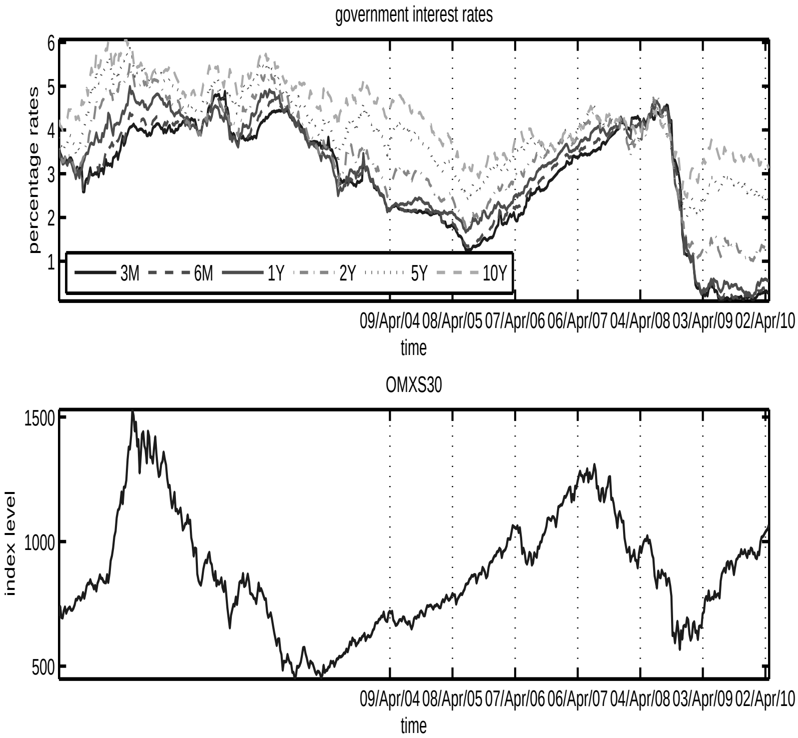


Figure 4.1.1: *Historical observations of Swedish government interest rates and OMXS30. The rebalancing times and the final evaluation time are indicated by the horizontal dotted lines and the dates on the x-axis.*

4.1.1 Historical strategy performance and weight allocation for various upper bounds on the portfolio's CVaR

A portfolio with a high upper bound on CVaR is a more risky investment compared to a portfolio with a low CVaR constraint. In a well functioning market, an investor, who takes higher risk, is compensated by greater return potential. This means that in an economic setting favorable to the risky investment, the profit accomplished should be higher than the one of a low risk strategy. In an unfavorable economic setting, the loss of a risky investment will though be much higher compared to a strategy with lower risk.

In this investigation an increase in the index level can be identified as a favorable economic situation, while a decrease in the index level represents an unfavorable economic situation for the investment strategy. Based on the reasoning presented earlier, a strategy with a higher CVaR bound should outperform a strategy with a lower CVaR bound when the index experiences an upturn. In the case of a downturn, should the performance of a strategy with a lower CVaR bound be superior to a strategy with a higher CVaR bound.

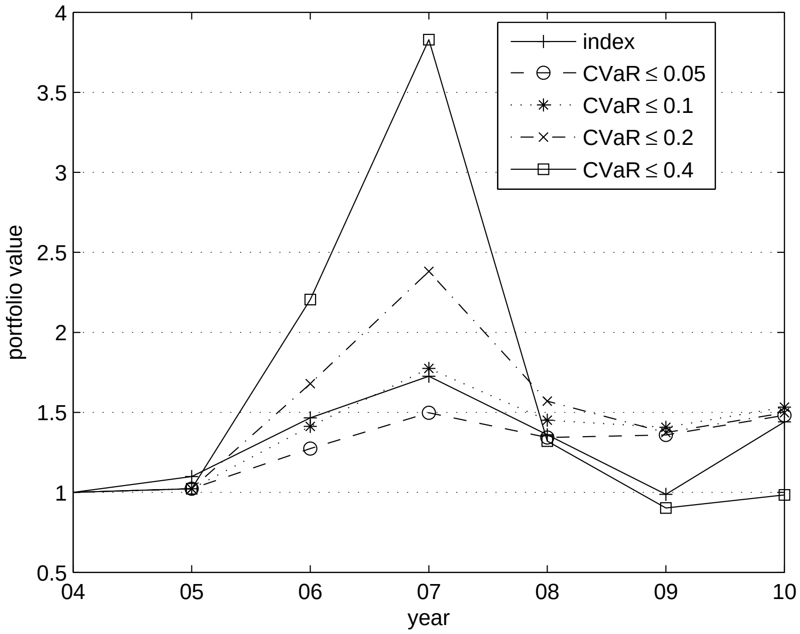
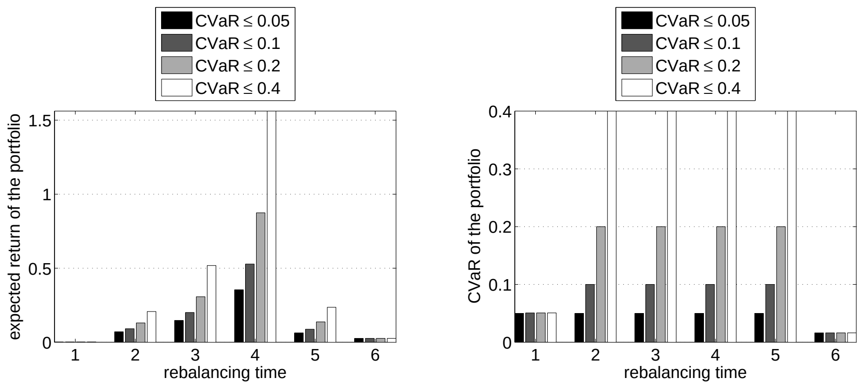


Figure 4.1.2: *Historical portfolio value development for maximizing the expected portfolio return over the next rebalancing period with subject to an upper bound on portfolio's CVaR measure at 0.05 confidence level. The figure displays the four strategies' portfolio value at the rebalancing times and at the final evaluation time with different upper CVaR bounds compared to a direct investment in the underlying index.*

Figure 4.1.2 displays the realized developments of the portfolio values for four investment strategies using different CVaR bounds. CVaR is measured over the next rebalancing period at the 0.05 confidence level. On the one side, the portfolios with a higher CVaR bound experience a much larger increase in value compared to the portfolios with a lower CVaR bound during the upturn of the index from 2005 to 2007. On the other side, the high CVaR bound portfolios decrease far more in value compared to the low CVaR bound portfolios during the downturn between rebalancing points of 2007 and 2009. The result is in line with how portfolios with different CVaR bounds should behave in various economic situations.

To analyze the performance in more detail, one needs to examine the expected returns and the CVaR of the composed portfolios, which are shown in figure 4.1.3, and their weight allocations, which are displayed in figures A.1.1 to A.1.4 in the appendix.



(a) Expected return of the composed portfolios (b) CVaR of the composed portfolios

Figure 4.1.3: *The expected return and CVaR of the composed portfolios using different upper bounds on the portfolio's CVaR. The confidence level for all CVaR measurements is set to be 0.05.*

At the first rebalancing point all portfolios have almost equivalent expected return and CVaR. This is due to the fact, that the PPN issued at the first rebalancing time is the only asset available at that time point with a positive expected return over the next rebalancing period. Any portfolio will have a CVaR less than or equal to the CVaR of the PPN. The resulting portfolio values at the next time point are very close, because the capital allocations are nearly equal. A similar situation appears at rebalancing time 6, where again the newly issued NPPC has a negative expected return over the coming rebalancing period. At the other rebalancing points do both the newly issued PPN and NPPC have positive expected returns. All portfolios do reach their respective upper CVaR bound at these time points. This means that one should only compare the portfolio value developments for the years 2006 to 2009 in figure 4.1.2 to investigate the impact of the upper CVaR bound. Examining the capital allocation diagrams, one can recognize that for portfolios with higher CVaR bounds, the proportion of wealth invested in NPPCs is greater than for portfolios with lower CVaR bounds. The rebalancing behavior seems to be unaffected of the CVaR bound. During the upturn of the index (rebalancing times 2, 3 and 4), all existing positions are sold at each rebalancing point and the complete wealth is reinvested in newly issued structured products. After the first downturn (rebalancing time 5), the structured investments issued at rebalancing time 4 partially remain in the portfolio and the value received for selling fractions of formerly purchased structured products is invested in PPN 5. An important observation that this historical test points out is, that capital allocated to the NPPCs increases more and more, as the index experiences an upturn, and decreases again when the index takes a downturn. Since the investor's risk conception is kept fixed for each rebalancing strategy, one can conclude that the risk characteristics of the assets are influenced by trends in the time series of the risk factors.

4.1.2 Historical strategy performance and weight allocation for various confidence levels at which the portfolio's CVaR is measured

The previous investigation is repeated, but instead of the upper bound of the portfolio's CVaR is the confidence level for the CVaR measurements alternated. The upper bound of the portfolio's CVaR is held constant at 0.10. If two portfolios have the same upper bound on CVaR, then the portfolio for which CVaR is measured at a higher confidence level can be identified as the risky one. Figure 4.1.4 shows that the portfolios with a higher confidence level outperform the ones with a lower confidence level during the upturn of the index, and that the portfolios with a lower confidence level give better results than the portfolios with a higher confidence level during the downturn of the index.

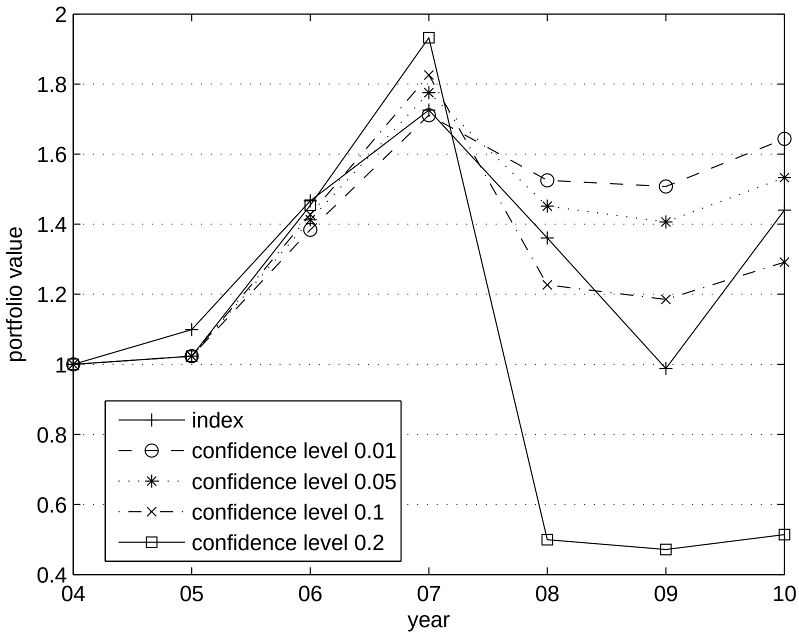
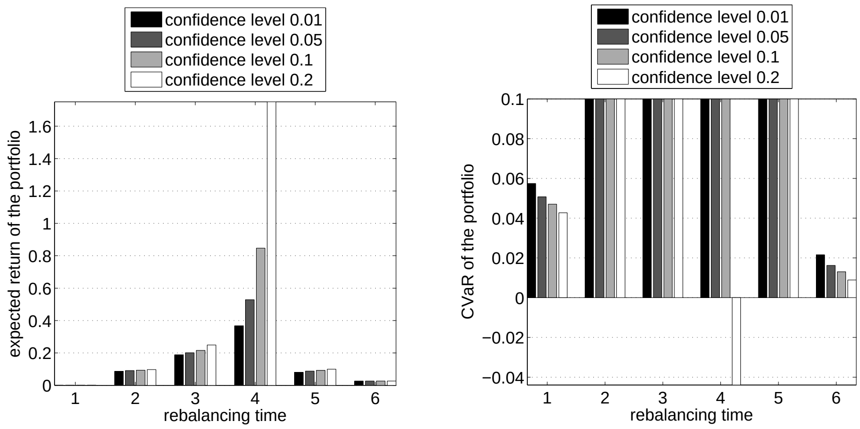


Figure 4.1.4: *Historical portfolio value development for maximizing expected portfolio return of the next rebalancing period with subject to an upper bound on the portfolio's CVaR equal 0.10. The figure displays the portfolio value at the rebalancing times and at the final evaluation time of four strategies with different confidence levels for measuring CVaR compared to a direct investment in the underlying index.*



(a) Expected return of the composed portfolios (b) CVaR of the composed portfolios, measured at the respective confidence level

Figure 4.1.5: *The expected return and CVaR of the composed portfolios using different confidence levels for the CVaR measurements. The upper bound on portfolio's CVaR is equal to 0.10 for all portfolios.*

Figure 4.1.5 shows the expected return and CVaR of the portfolios. Due to the limited availability of investment alternatives with a positive expected return at rebalancing times 1 and 6, are the expected returns of the portfolio very similar for those periods. For the other rebalancing times does a portfolio with a CVaR measured on a higher confidence level reach a higher expected return compared to one for which CVaR is measured on a lower confidence level. One can notice that the differences in expected return of the portfolios significantly increase from rebalancing time 2 to 4. At rebalancing time 4 the CVaR for the portfolio with confidence level 0.2 is even negative. The weight allocation diagrams are not displayed in the report, because they do not yield any new information. As mentioned earlier, do the more risky portfolios, which are the ones with higher confidence level, have a larger part of the wealth allocated to NPPCs.

4.1.3 Conclusions from the historical tests

Both the confidence level for CVaR measurements and the upper bound on the portfolio's CVaR successfully control the riskiness of the investigated portfolios, as the tests using historical data indicates. A low risk portfolio does not profit from favorable economic developments at the same degree as a high risk portfolio. However, the portfolio value of a low risk portfolio does not decrease as rapidly as the one of a high risk portfolio in unfavorable economic situations.

The investigation revealed that historical trends have a large effect on evaluating the riskiness of an asset using the formulated scenario generation approach. When the index has a large positive trend, the riskiness of both the NPPC and the option part of the PPN reduces significantly, because for most scenarios the call option will be in-the-money at the next rebalancing time. This results in a higher expected return of these assets. Historical trends in the interest rates have also an effect on the riskiness and the expected return of the structured products.

It is important to be aware of the historical trends' influence on the generation of future return scenarios, because one can not just expect history to repeat itself. A methodology of controlling trends in the risk factor changes has to be established. The next section presents an approach to this problem.

One should notice that the realized portfolio values and the weight allocations of the rebalancing strategy also depend on the chosen set of rebalancing dates. However, the observations and conclusions made from the investigated set of rebalancing dates were also established for other sets of rebalancing dates.

4.2 The investor's view as a second source of information

The section adds a new source of information to the portfolio selection process. The previous section concluded that the historical trends of the risk factor changes create a strong bias in the scenario generation. To control this effect, the investor can specify his own view on the expected future development of the risk factors. The present approach combines both the information from the historical data, and the investor's opinion to generate scenarios that are less influenced by historical trends and more representative with respect to the investor's view on the expected future developments of the risk factors

4.2.1 Formulation of a subjective view

Up to this point, all information entering the model for simulating future developments of the yield curve and the index is solely based on historical observations. One consequence of this modeling approach is that the generated scenarios match the historical data input closely in terms of the mean and the standard deviation of each return series. In this context a return series refers to a series of risk factor changes. The centered simulated returns are realizations of a random vector, that has zero mean and a covariance matrix with ones on the main diagonal. These centered returns are then rescaled by the historical mean and standard deviation. As a result of the law of large numbers does the sample mean and sample standard deviation of the simulated return series converge to the sample mean and sample standard deviation of the historical return series.

Such a behavior is undesirable since it implies that if a return series had a positive mean in the past, it will have a positive mean in the future if the number of scenarios is sufficiently large. Since the weights of the portfolio optimization should be robust, the number of generated scenarios is large so that there is hardly any deviation between the mean and the standard deviation of the simulated return series and the historical return series. The scenarios are only representative if the market is expected to have a similar behavior as in the past.

Market climates change though and there is no reason to believe that a return series will experience a positive trend in the future because it had a positive trend in the past. Since historical data gives no reliable information about the direction of the market developments, other sources of information have to be used. One way to do this, is to simply ask the investor about his subjective belief on the expected future developments of the return series. If an investor expects the index to grow at a high rate, he should probably allocate his capital in assets, that profit from this scenario. It is only rational to make investment decisions based on ones own subjective believes. A price of an asset reflects the consensus forecast of its future performance but this forecast can differ from ones own opinion. This difference makes an asset either attractive or unattractive for an investor.

The investor's view as a stochastic trend

The investor's view on the return series is expressed by the random variable $\mu_{\text{view}} \sim N(\hat{\mu}_{\text{view}}, \Omega)$. The expected returns the investor presumes are specified in the vector $\hat{\mu}_{\text{view}}$ and the uncertainty in the view is formulated in $\Omega = \text{diag}\{s_{11}, s_{22}, \ldots, s_{nn}\}$ where s_{ii} is the variance in the ith view. The views are assumed to be independent of each other. The new rescaling equation reads

$$\begin{aligned} \tilde{r}^T &= \Lambda(\hat{\mu}_{\text{view}} + \varepsilon_{\text{view}}) + (I - \Lambda)\mu_{\text{data}} + S_{\text{data}}\, r^T \\ \text{where:} \quad & \varepsilon_{\text{view}} \sim N(0, \Omega) \\ & \Lambda = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\} \text{ with } \lambda_i \in [0,1] \end{aligned}$$

μ_{data} is the sample mean vector of the historical return series and S_{data} is a diagonal matrix with the sample standard deviation of the historical return series on the main diagonal. λ_i is the weight the investor puts on the ith view. If λ_i is 0, the investor's opinion is disregarded and the incorporated information comes only from historical data. The case when Λ is the zero matrix is referred to as an uninformed view. If λ_i is 1 the historical mean is not taken into account and the trend is solely estimated on the investor's view. It might be a difficult task for an investor to express the uncertainty of his view in terms of variance. But if he can specify an uncertainty interval for each of his views, then the variance can be determined from it. The following example clarifies the idea.

The investor's ith view indicates that the index will have a mean weekly return of $\bar{r} = 0.20\%$ which is around 11% annual return. He is uncertain about his forecast but he thinks that with 80% chance the mean weekly return will lie between $r_L = 0.18\%$ and $r_U = 0.22\%$. For this two-sided confidence interval the variance of the view on the index is

$$s_{ii} = \left(\frac{r_L - \bar{r}}{\Phi^{-1}\left(\frac{1 - \text{confidence level}}{2}\right)} \right)^2 \tag{4.2.1}$$

where $\Phi^{-1}(x)$ is the inverse of a standard normal distribution. For this example the variance would be 0.000244.

To see how the choice of $\Lambda, \hat{\mu}_{\text{view}}$ and Ω influences the simulation of the return series, the mean and covariance of $\tilde{r}$ are determined.

$$\begin{aligned} E\left[\tilde{r}^T\right] &= \Lambda\hat{\mu}_{\text{view}} + (\mathbf{1} - \Lambda_i)\,\mu_{\text{data}} \\ \text{Cov}\left[\tilde{r}^T\right] &= \Lambda^2 \text{Cov}\left[\varepsilon_{\text{view}}\right] + S_{\text{data}} \text{Cov}\left[r^T\right] S_{\text{data}} \end{aligned}$$

The new mean of the return series is a convex combination of the $\hat{\mu}_{\text{view}}$ and μ_{data}. Only the elements on the main diagonal of the historical covariance matrix $S_{\text{data}}\text{Cov}\left[r^T\right] S_{\text{data}}$ are going to be changed. The term $\lambda_i^2 s_{ii}$ is added to the ith diagonal element. This increases the variance of each return series and decreases the correlation between the different return series. An investor, who puts a lot of weight on his uncertain subjective view, actually implies that the future market movements will be more rapid than in the past and that the return series will have a lower dependence on each other. The investor's subjective view and its effect are very different, since the way in which the two sources of information are combined is not appropriate.

Historical data and the investor's view combined by using statistical distance

A different approach has to be formulated. The information from the investor's view and the historical observation should be combined in such a way, that the new expected return vector differs as little as possible from the one suggested by either source. The statistical distance is used as a distance measure to accomplish this objective. This approach is also used with respect to asset returns in the Black-Litterman model. A detailed description of this model can be found in Mankert (2006).

μ_{data} is the estimate of the expected future developments suggested by the historical data, which has the covariance matrix $\frac{1}{m}\Sigma$ where m is the number of observations and Σ is the covariance matrix of the observed returns. The investor can formulate k either absolute or relative views on the expected developments of the return series. An absolute view for example is, that the expected weekly log return of the index is 0.20%, and a relative view is that the weekly relative change in the 2-year Swedish government interest rate is 0.002 higher than the weekly relative change in the 10-year Swedish government interest rate. To express these k views a $k \times n$ matrix P is introduced with only three different elements, that are $-1, 0$ and 1. An absolute view has only one positive element in the row, which is situated at the respective column entry. A relative view has a zero row sum. The risk factors that are compared have a 1 and -1 entry at their representative entries. The formulation of all k views reads

$$P\mu = q + \varepsilon_{\text{view}}$$

q is a k dimensional vector that holds the investor's views and $\varepsilon_{\text{view}}$ is a random k dimensional vector expressing the uncertainty of the views. The views are assumed to be independent and therefore the covariance matrix of $\varepsilon_{\text{view}}$ is the diagonal $k \times k$ matrix Ω. Let μ_{comb} denote the estimate of the future expected returns, then the statistical distances are defined as

$$\begin{array}{ccc} (P\mu_{\text{comb}} - q)^T & \Omega^{-1} & (P\mu_{\text{comb}} - q) \\ (\mu_{\text{comb}} - \mu_{\text{data}})^T & m\Sigma^{-1} & (\mu_{\text{comb}} - \mu_{\text{data}}) \end{array}$$

The weight put on the subjective view is $\lambda \in (0,1)$. Notice that λ is the weight for all views. μ_{comb} is then the solution to the following optimization problem

$$\begin{aligned} \mu_{\text{comb}} &= \arg\min_{\mu} \{\lambda\,(P\mu_{\text{comb}} - q)^T\,\Omega^{-1}\,(P\mu_{\text{comb}} - q) + \\ &\quad + (1-\lambda)\,(\mu_{\text{comb}} - \mu_{\text{data}})^T\,m\Sigma^{-1}\,(\mu_{\text{comb}} - \mu_{\text{data}})\} \\ &= \arg\min_{\mu} \{(P\mu_{\text{comb}} - q)^T\,\Omega^{-1}\,(P\mu_{\text{comb}} - q) + \\ &\quad + (\mu_{\text{comb}} - \mu_{\text{data}})^T\,\frac{1-\lambda}{\lambda} m\Sigma^{-1}\,(\mu_{\text{comb}} - \mu_{\text{data}})\} \end{aligned}$$

To simplify notation set $\tau^{-1} = \frac{1-\lambda}{\lambda} m$. This optimization problem can be classified as an unconstrained quadratic optimization program. The matrices Ω^{-1} and $m\Sigma^{-1}$ are positive semidefinite as a result of Ω being a diagonal matrix and Σ being a covariance matrix, which both are positive semidefinite matrices. The inverse of a positive semidefinite matrix is also a semidefinite matrix. So both statistical distances are convex functions and a non-negative weighted sum of convex function is also a convex function. Since the function to be minimized is convex, any local optimum is also a global optimum. Setting the first derivative with respect to μ equal to zero yields the solution to the problem.

$$\mu_{\text{comb}} = \left(P^T \Omega^{-1} P + (\tau\Sigma)^{-1}\right)^{-1} \left(P^T \Omega^{-1} q + (\tau\Sigma)^{-1} \mu_{\text{data}}\right)$$

The centered simulated returns are then rescaled using μ_{comb} and S_{data}.

$$\tilde{r}^T = \mu_{\text{comb}} + S_{\text{data}}\, r^T$$

This formulation has the advantage that the covariance matrix is preserved and only the estimation of the future trends uses both sources of information.

4.2.2 Rebalancing problem with subjective view

While in the previous section the only source of information is the historical data, in this section the investor's view enters as a second source of information. Both information sources are combined in the way described in section 4.2.1 to get a new estimate of the expected development in the underlying return series. The goal is to investigate how the allocation among the assets changes depending on q, λ and Ω.

The analysis is done for a single absolute view on the expected log return of the OMXS30 in two steps. First λ and Ω are kept fixed and q is modified. In the second part q is constant and λ and Ω are varied. The investment situation investigated and all parameter settings are equivalent to section 3.6. The investor is maximizing the expected return of the portfolio over the next year keeping the one year-CVaR of the portfolio below 20% of his initial wealth. All computations are performed with the same random stream to eliminate any effects due to random variations of the simulation process.

Effects of varying q

To analyze the impact of q on the estimation of the expected changes, λ is set to be 0.5 and the uncertainty in the view is chosen so that it is equal to the historical variance of the log returns of the OMXS30. The historical mean weekly return of the index is 0.0018. The parameter q takes on values between -3 and $+3$ times the historical mean.

The effect of a single view on the expected log return of the index is shown in figure 4.2.1. The expected relative changes of the interest rates and the expected log return of the index experience a linear dependence on the parameter q, in the case of a single absolute view on the expected log return of the index. The effect on the expected interest rate changes is an interesting observation. The impact is dependent on the covariance between the respective interest rate and the index. The 3 month interest rate is the rate with the smallest positive covariance. The slope of the graph is rather flat. The 5 year rate has the largest covariance and therefore the slope is rather steep.

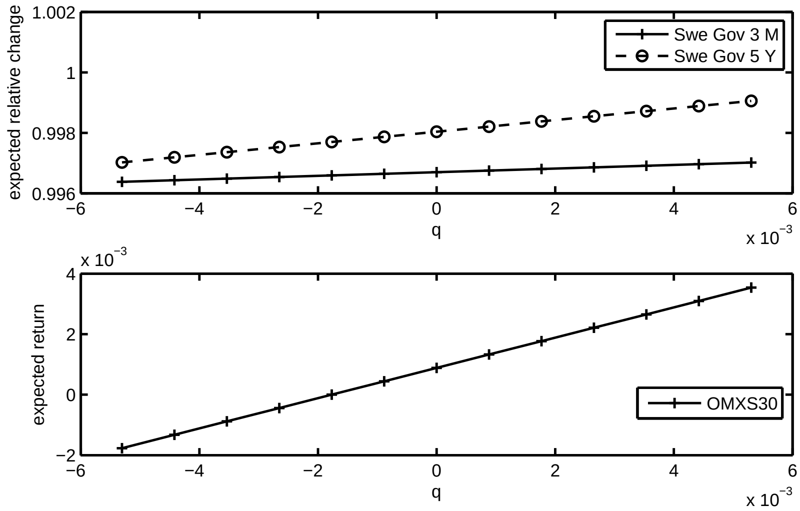


Figure 4.2.1: *The effect of a single absolute view on the expected log return of the OMXS30. The upper graph shows the dependence of the expected relative changes in the interest rates on q. The 3M rate is the interest rate with the smallest covariance with the index and the 5Y rate is the one with the largest covariance with the index. The lower graph shows how the expected return of the index changes depending on q*

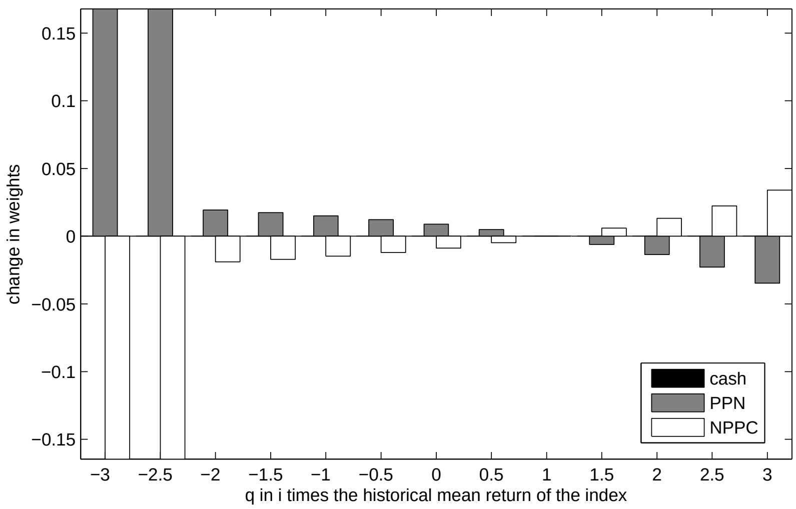


Figure 4.2.2: *The changes in portfolio weights depending on a single subjective absolute view q of the investor on the expected log return of the OMXS30, compared to the no-view situation of section 3.6*

The resulting impact on the weight allocation is displayed in figure 4.2.2.The investor holds no cash and the weight put into the PPN and the NPPC is dependent on the subjective view. In the original no-view case, 81.73% of the initial wealth is invested in the PPN and 15.99% are allocated to the NPPC. The remaining 2.28% of the funds are used to pay for the transaction costs. When the investor has the lowest presumption on the expected future performance of the index, the wealth invested in the certificate is reduced to zero and instead all funds are allocated to the PPN. When the investor has a strong positive view on the market's expected future development, the NPPC's weight in the portfolio is increased by more than 3%. This parameter study indicates a strong dependence of the weight allocation on the investor's subjective view.

Effects of varying λ and Ω

The investor's view on the expected return of the index is fixed to investigate the effects of varying λ and Ω. The OMXS30 has a historical mean annual log return of 9.20%, estimated on four years of weekly observations prior to the 19th of May 2006. The investor believes the index will have an expected annual log return of 4% above the historical mean annual log return. This corresponds to $q = 0.0025$ and $\mu_{\text{data}} = 0.0018$. In the following analysis λ varies in the interval between 0 and 1, where 0 corresponds to the no-view situation. Equation (4.2.1) is used to specify different levels of uncertainty. The confidence level for all views is set to be 0.9 and $\overline{r} = q$. To create views with different uncertainty, r_L is varied. The case, where r_L is close to $\overline{r}$, corresponds to a certain view, while the case, where r_L is much smaller than r, corresponds to an investor, who is uncertain in his view. Figure 4.2.3 shows how the expected values for three selected return series change depending on λ and Ω. First of all, one can notice that a formulation of a view on the expected index log return also has an impact on the expectation of relative changes for the interest rates, as already observed when varying λ. This is a consequence of the statistical distance approach, which takes into account the covariance between the random variables. The shape of all surfaces is very similar, but the slopes differ a lot. If Ω is small, the impact of the subjective view on μ_{combo} is large. When Ω is large, the effect of the investor's view on the expectations of the changes in the risk factors is rather small, compared to the no-view situation. A λ close to one puts more weight on the subjective view, while a λ close to zero, puts more weight on the historical mean. The slope is largest in the graph concerning the OMXS30. Comparing the graphs of the selected interest rates with one another leads to the conclusion, that the slope of the 3 month rate surface is rather flat compared to the 5 year surface. This is due to the greater covariance between the relative changes in the 5 year government interest rate and the log returns of the index, causing a bigger impact of the subjective view formulated on the index on this rate.

Figure 4.2.4 shows the changes in weight allocation. The surface for cash account is not presented since the investor does not allocate any funds into this asset in any situation. Since the q is greater than μ_{data}, the more certain or the more weight is put on the the investor's view, the more weight is allocated from the PPN towards the NPPC. If Ω is small, the effect of varying λ is almost non-existing. The same observation is made for varying Ω when λ is chosen close to 1. Therefore one cannot separate these two parameters when specifying a single subjective view. However, when defining several views, Ω can assist in adjusting the uncertainty in the different views. The more uncertain or the less weight is put on the investor's view, the smaller the changes are in the portfolio weights compared to the no-view situation.

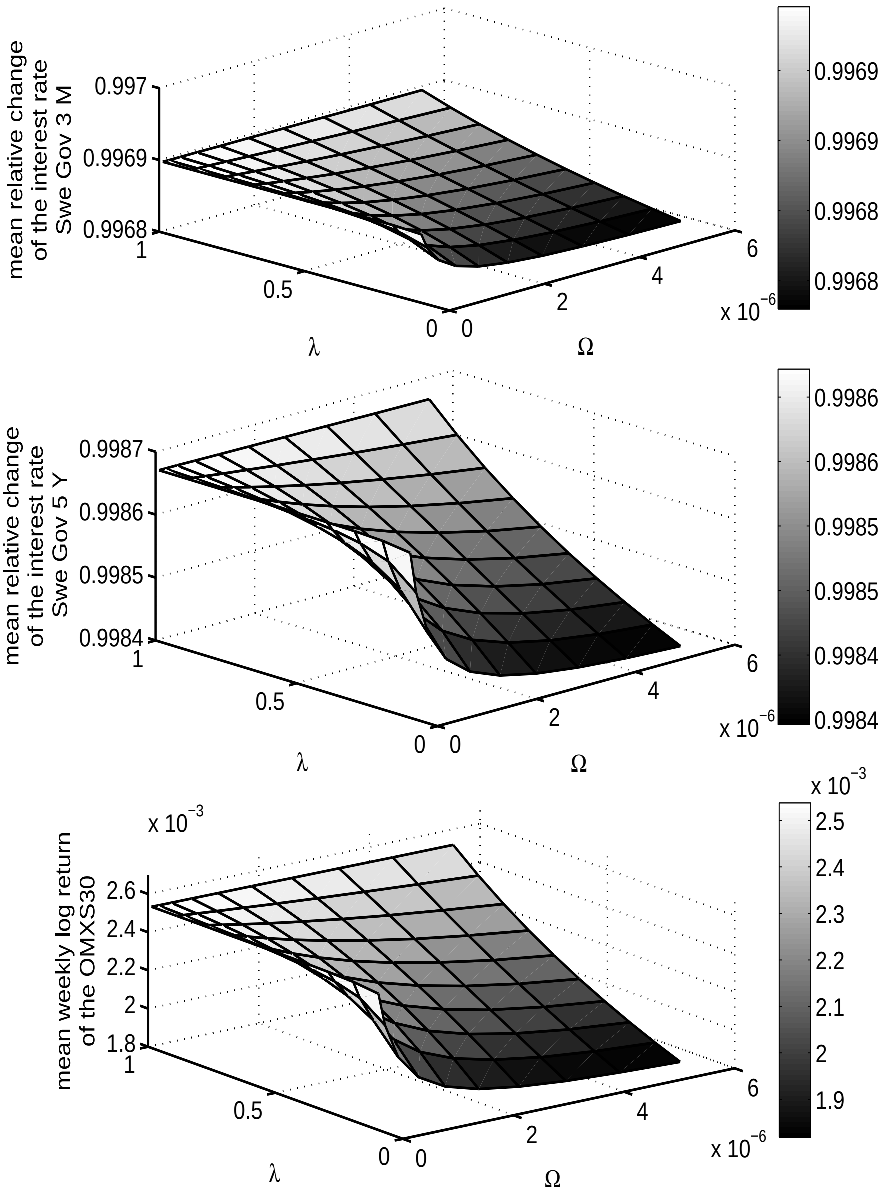
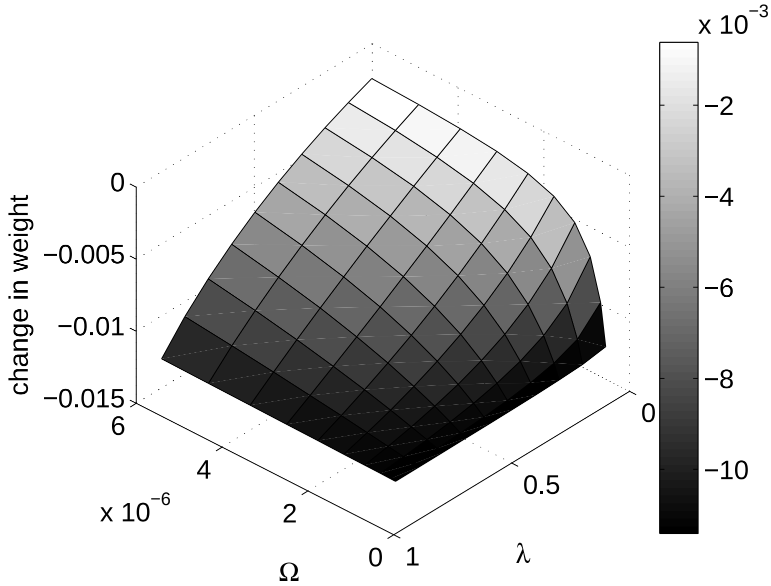
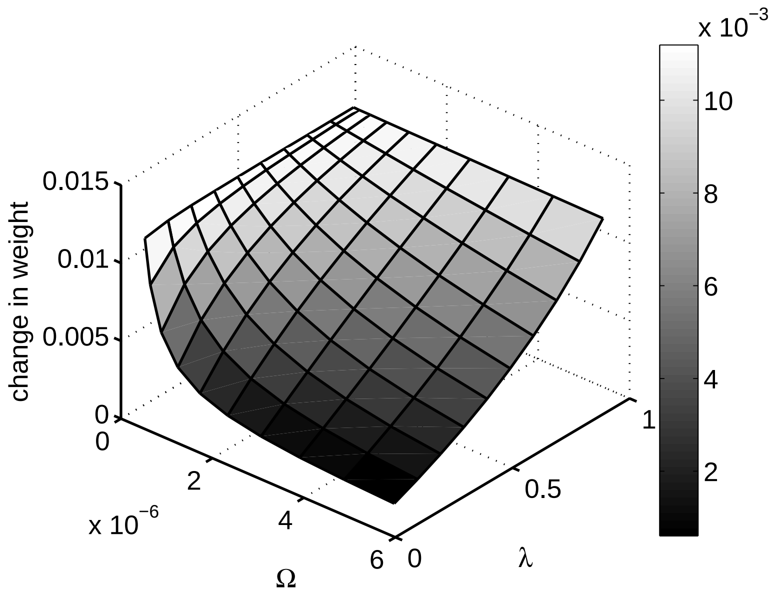


Figure 4.2.3: *The figure displays the effects of a single absolute view formulated on the expected log return of the OMXS30 with* $q = 0.0025$ *and varying* λ *and* Ω *on the combined view of the expected weekly relative change in the 3 month government interest rate (upper graph), the expected weekly relative change in the 5 year government interest rate (middle graph) and the expected weekly log returns of the OMXS30 (lower graph). The historical mean relative change of the 3 month government interest rate is 0.9968 and of the 5 year government interest rate is 0.9984. The historical mean log return of the OMXS30 is 0.0018*



(a) weight changes in the PPN; In the no-view case, the investor allocates 81.26% of his wealth into the PPN



(b) weight changes in the NPPC; In the no-view case, the investor allocates 16.46% of his wealth into the NPPC

Figure 4.2.4: *The optimal portfolio weights are compared to the no-view situation from section 3.6. The investor has an absolute view on the expected log return of the index with* $q = 0.0025$. λ *and* Ω *take various values. For all situations, the resulting position in the cash account is equal to zero and therefore the cash account surface is not displayed.*

4.2.3 Effect of the subjective view on the portfolio performance

The goal of this part is to illustrate how taking into account a subjective view can enhance the performance of the rebalancing strategy. If an investor has a good guess about what is going to happen in the future, he should hold a portfolio, that takes advantage of this superior information. Since a rational investor would only allocate his wealth in assets, that according to his option should give positive return, the performance of the portfolio should increase when the subjective view becomes more accurate.

To test if this common sense hypothesis is observable, the same experiment as in section 4.1 is repeated with a subjective view and a fixed specification of the CVaR upper bound equal to 0.20 and a 0.05 confidence level for the CVaR measurements. To get a good view, a synthetic absolute view on the expected relative changes of each interest rate and the expected log return of the index is introduced. The view is simply the actual relative change for the interest rates and respectively the actual log return for the OMXS30. The quality of the views is controlled through the covariance matrix of the views Ω. The variance of each view is calculated, as previously described, by using confidence intervals. The confidence level for all confidence intervals used in this section is 0.9. Since the confidence level is fixed, the lower confidence limit is used to alter the variance of each view. To refer to the different qualities of views the term uncertainty level is used. A view with uncertainty level v has the following specifications. The difference $r_L - \overline{r}$ is for a view on the expected relative change of an interest rate equal to $-v\,|q_i - 1|$ where q_i is the expected weekly relative change in the interest rate according to the subjective view. For a view on the expected log return of the index the difference $r_L - \overline{r}$ is equal to $-v\,|q_j|$ where q_j is the expected weekly log return of the index according to the subjective view. The reason for the different settings for the interest rates and the index is, that the uncertainty in each view should be of approximately same order of magnitude. This corresponds to saying, that the investor has equally good information on interest rates and the OMXS30. Since the uncertainty of the view is a multiplicative of q_i, the q_i have to be scaled. The q_i for interest rates lies close to one while the q_i for the index is close to zero. Subtracting one from the view on the expected relative change of the interest rate brings all uncertainties close to each other. Notice that defining the view in this way implies, that the subjective view is more uncertain on random variables that have a large change, and less uncertain on random variables that only experience a small change. Large and small refers here to the absolute change. To scale both covariance matrices to approximately equal order of magnitude λ is chosen to be 0.99.

Figure 4.2.5 illustrates how μ_{combo} is influenced by the uncertainty in the subjective view. To keep the figure clear, only two elements of μ_{combo} for two different uncertainty levels: 0.1 and 1 are displayed. When the uncertainty level is low μ_{combo} is close to q. As the uncertainty level for the subjective view increases μ_{combo} is adjusted towards μ_{data}. It is also observable that a large $|q|$ introduces more uncertainty. If $|q|$ is small μ_{combo} for uncertainty level 0.1 is very close to q, but when $|q|$ is large μ_{combo} is not as much adjusted towards q given the same uncertainty level.

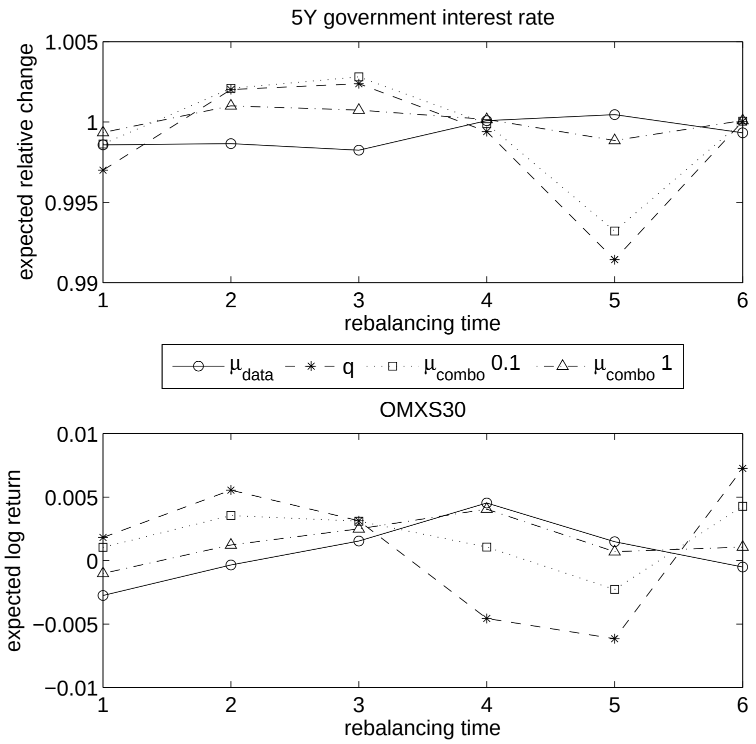


Figure 4.2.5: *The upper plot illustrates how the expected relative change of the 5 year government rate for different rebalancing times is situated for uncertainty level 0.1 and uncertainty level 1 compared to q and μ_{data}. The lower plot shows the same graph for the expected log return of the OMXS30*

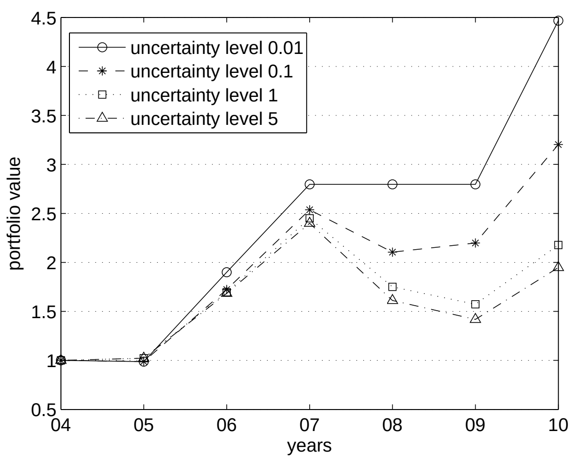


Figure 4.2.6: *Development of portfolio value for the different strategies depending on the uncertainty level in the synthetic subjective view. The chosen confidence level for the CVaR measurements is 0.05 and the upper bound on the portfolios CVaR is 0.20*

The weight allocation diagrams for the different uncertainty levels are displayed on pages 82 to 83 and the resulting portfolio values are illustrated in figure 4.2.6. First of all, one can notice that a good view enhances the performance of the strategy. In the no-view situation, the portfolio value at the final evaluation time for a strategy with the same confidence level and upper CVaR bound is around 1.5 times the initial value. When using the synthetic view with the lowest uncertainty level, the portfolio value after an investment period of six years reaches almost 4.5 times the initial value. The strategy with the uncertainty level 0.01 has only one period with a decrease in portfolio value. This is the period from 2004 to 2005. The decrease is due to a sharp decline in volatility of the index, which lowers the value of the option so dramatically that even the event of the option going into the money cannot recover the loss in the option value. One can notice that the performance relates inverse proportional to the uncertainty level of the view. Since the view is the actual future outcome this result is in line with common sense reasoning. When examining figures A.2.1 to A.2.4, it is observable that the weight allocation is highly dependent on the subjective view. Since the elements of μ_{combo} are used as the mean of the relative changes and the log returns of the risk factors, the expected return and the riskiness of an asset over the next period change a lot, depending on this parameter. At the first three rebalancing times the weight allocations are rather similar. The low uncertainty level portfolios have a greater exposure to the index development through purchasing also NPPC 1, while the high uncertainty portfolios place the entire funds in the PPN 1. At rebalancing times 2 and 3 the chosen assets are the same for all portfolios, but again the low uncertainty portfolios have more exposure to the index. Rebalancing times 4 and 5 cause the significant differences in the performance of the portfolios. The portfolio with the most accurate subjective view sells all risky positions and therefore retains the portfolio value, while all other portfolios decrease their value during this period due to investing in a bear market. The greater the uncertainty in the synthetic view is, the greater is the exposure to the index in these two periods. This is caused by the great positive historical mean of the log returns of the index, which introduces a large positive bias in the scenario generation. At rebalancing time 6 the portfolio weights are similar again.

One should keep in mind that a PPN only has a lower bound equal to the issuing price at time of maturity. The intermediate values are dependent on both the development of the interest rates and the index.

It became clear that the better the quality of the subjective view, the greater the performance enhancement. Furthermore, it is interesting to see how dependent the performance increase is on the number of absolute views. The weight allocation and realized performance is determined of a selection of different numbers of views where all views have the uncertainty level 0.01. The first setting has views on all interest rates and the index. In the second setting the number of views is reduced to two, one on the 12 month interest rate and one on the index. The third setting has only a view on the index and the fourth setting has only a view on the 12 month interest rate. Figure 4.2.7 shows the portfolio values for the different view settings. The weight allocation diagrams can be found on pages 84 to 85 in the appendix.

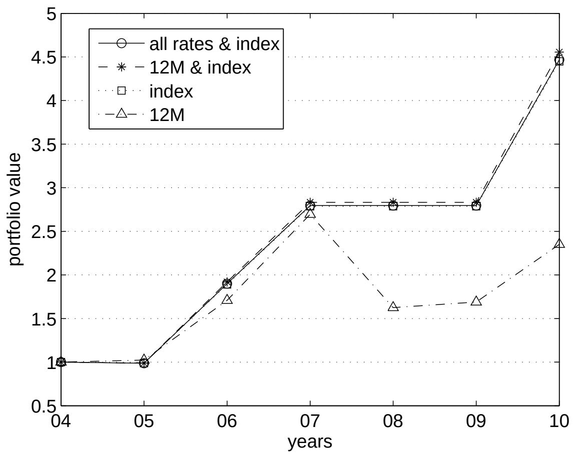


Figure 4.2.7: *Development of portfolio value for the different selection of synthetic subjective view settings with uncertainty level 0.01. The chosen confidence level for the CVaR measurements is 0.05 and the upper bound on the portfolios' CVaR is 0.20*

The weight distribution among the different assets and the performance of the strategy hardly changes for the first three settings. A big difference occurs when the view on the index is removed. The strategy with only a view on the 12 month interest rate has a much weaker performance than the other view settings, but still outperforms the no-view setting. This experiment indicates that the view on the expected log return of the index seems to be most important for the performance, though a good view on the expected relative changes of the interest rates can add some extra enhancement on the performance of the strategy.

4.2.4 Conclusions on the effects of the investor's subjective view

The investor's subjective view reduces the effect of historical trends on the scenario generation and increases the representativeness of the scenarios according to the investor's opinion on the future behavior of the risk factors. The modified approach of generating scenarios incorporates information from both historical data and the investor's view on the expected changes in the risk factors over the next rebalancing period.

The historical tests indicated, that an accurate subjective view can enhance the performance of the rebalancing strategy. The view on the index turned out to add most enhancement to the portfolio in the investigated situation.

It has to be mentioned that embedding a subjective view not automatically leads to better investment decisions. A view that badly describes the expected future behavior of the risk factors, leads to non-representative scenarios with respect to the unknown future market developments. This causes a wrong evaluation of the assets' riskiness and reward potential, which will then result into unprofitable investment decisions.

The conducted investigation only indicates that the investment decisions made using the presented approach can take advantage of information about the expected changes of the risk factors available to the investor.

Moreover, one can notice that all investigated rebalancing strategies have large turnover. In the actual market situation, this would cause high transaction costs and reduce the portfolio performance. The next section takes into account transaction costs in the investment decision.

4.3 Impact of transaction costs on the rebalancing strategy

Transaction costs are an important factor in portfolio rebalancing. This section investigates the impact of the transaction costs on the rebalancing behavior. The historical test with a synthetic view on the expected risk factor changes is used for this investigation. Both the effects of transaction costs for buying and for selling structured products are examined. The historical test has the same setup as in section 4.2.3. CVaR is measured at the 0.05 confidence level and the upper bound of the portfolio's CVaR is 0.20. The synthetic view used in this part has an uncertainty level of 0.1 and is composed of absolute views on all risk factors.

4.3.1 Transaction costs in the primary market

When an investor purchases a structured product, he usually has to pay some brokerage commission. Regarding structured products on the Swedish market, the commission is a proportion of the invested capital, which varies between 1% to 3% depending on the product type and the issuer. Table A.1 in the appendix A.3 presents the brokerage commissions for a selection of structured products from different issuers. In contrast to the investigation from section 4.2.3, the assumption of a frictionless market is abandoned, and the effects of transaction costs for buying structured investments are examined on the performance of the strategy, the expected return and the CVaR of the optimal portfolios and the weight allocation behavior. The choice of proportional buying cost levels are 0%, 1%, 2% and 3%. The actual portfolio value development under the different conditions is shown in figure 4.3.1.

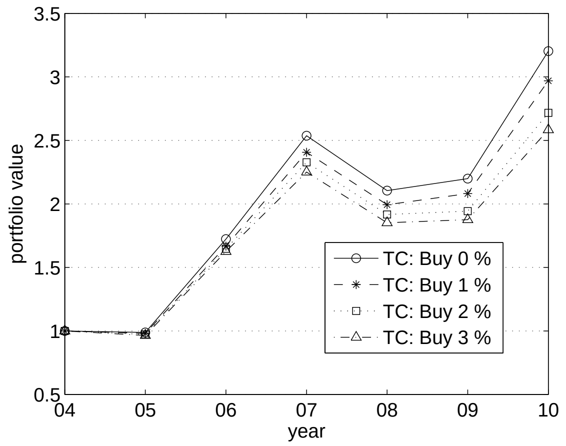


Figure 4.3.1: *Historical portfolio value development in the presence of cost for buying structured products. The Portfolio CVaR is measured at the 0.05 confidence level and the upper CVaR bound for the portfolios is 0.20. All portfolios use the synthetic view on all risk factors with an uncertainty level of 0.1*

The transaction costs imposed for purchasing structured products have a significant impact on the achieved portfolio values during the investment period from 2004 to 2010. The portfolio value in 2010 in the case of 1% buying cost is 7.3% less than in the frictionless case. For a buying cost of 2% and 3%, the final portfolio values are 15.2% respectively 19.3% less compared to the no transaction costs setting. Table A.2 in the appendix A.3 reveals the expected return and the CVaR of the optimal portfolios. At all rebalancing points a higher buying cost results in a lower expected return portfolio, while the portfolios' CVaR are equal. The fifth rebalancing time is a little bit exceptional, since the CVaR constraint is not active. This is due to the negative view on the index, which causes the expected returns of the newly issued products to be either very small or negative. Examining the weight allocation diagrams for the different transaction costs settings, that are displayed in figure A.3.1 to A.3.4 in appendix A.3 concludes two main effects. First of all, the cost of buying structured products decreases the attractiveness of the assets, that are not included in the current portfolio. The transaction costs shift the return distribution of the individual asset to the left. This decreases the expected return of the asset and increases the CVaR of the asset. Therefore, some structured products do not enter the portfolio any longer than compare to the frictionless case. An example of such an asset is PPN 5 at the beginning of the fifth rebalancing period (S5), which is in the presence of transaction cost substituted for cash. Secondly, transaction costs reduce the turnover in the portfolio. Assets, that are held for only one year in the frictionless market, are held for two years, such as PPN 1 and PPN 3 in the 2% buying cost case, or even until their maturity, as for example PPN 4 in the case of 3% buying cost, when a cost of buying structured products is imposed. In other words, transaction costs create some kind of barrier, that a newly issued structured investment has to overcome to be included in the portfolio. The higher the cost of buying structured products is the higher the barrier becomes.

4.3.2 Transaction costs in the secondary market

Structured investments can be sold prior to their maturity. This can either be conducted on an exchange for listed structured product or in form of an OTC trade, where often the issuer acts as the market maker. Since the secondary market is not very liquid compared to trading large cap stocks on an exchange, ignoring bid-ask spread would be unrealistic. The previous study is repeated with a fixed proportional cost of 2% for buying structured products and in addition a cost for selling structured investments prior to its maturity is introduced. Figure 4.3.2 displays the historical portfolio value development for different levels of transaction costs for selling structured products prior to their maturity.

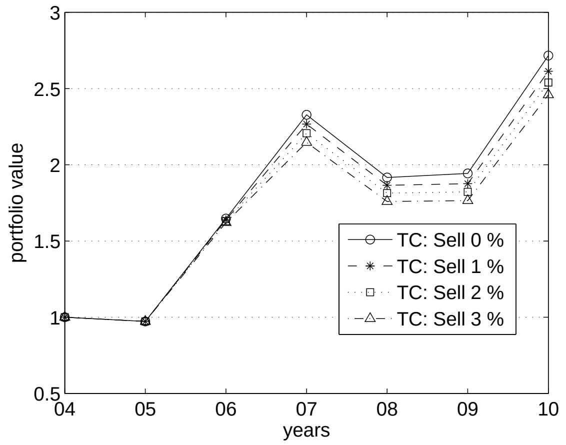


Figure 4.3.2: *Historical portfolio value development in the presence of transaction costs. The proportional cost of buying structured investments is fixed to be* 2% *for all cases. The proportional cost for selling structured products is varied between* 0% *and* 3%. *The portfolio CVaR is measured at the 0.05 confidence level and the upper CVaR bound for the portfolios is 0.20. All portfolios use the synthetic view on all risk factors with an uncertainty level of 0.1*

In the case of 1% selling cost, the portfolio value in 2010 is 3.8% less compared to the situation with no selling cost. Selling costs of 2% and 3% give portfolio values in 2010, that are 6.5 % repectively 9.5 % below the zero selling cost setting. The effect of the selling cost on the portfolio value development is not as dramatic as for the buying cost. This might be due to the fact, that one can escape this cost by holding the structured investment until its maturity. However, the investigation leads to the conclusion, that the portfolio performance decreases when higher selling cost is imposed. Table A.3 in the appendix A.3 shows that also the expected return of the portfolios is decreased, starting from the second rebalancing time. When portfolio weights are adjusted, the working capital is reduced and therefore part of the expected return, that is computed on the basis of portfolio value before adjustments have taken place, is used to recover the transaction costs. When the transaction costs are increased, a larger part of the expected return is needed to replace the loss in capital due to market friction.

The weight allocation diagrams, presented in figures A.3.5 to A.3.8 in appendix A.3 lead to two observations. First of all, the selling cost introduces some kind of barrier for an asset to leave the portfolio. This can be seen in the following case. PPN 3 is sold at the end of the fourth period and the gained capital is placed in the cash account in the case of no selling cost. This reallocation is triggered by the small negative return of the PPN 3 over the next period. In the presence of selling cost in the market, it is a better option to hold an asset with a negative return smaller than the proportional cost of selling it. This is exactly what this example shows, since in all cases with selling cost the PPN 3 remains in the portfolio after the fourth rebalancing time. Secondly, one can observe that the holding period of the structured products increases with increasing selling cost. In case of 2% buying cost and 0% selling cost, no structured investment is held until its maturity. When selling costs are increased, two structured products (PPN 3 and PPN 4) remain in the portfolio until they reach their maturity.

4.3.3 Conclusions on the impact of transaction costs

Transaction costs have a significant influence on the rebalancing behavior. The turnover in the portfolio decreases when the frictionless market assumption is abandoned. Newly offered structured products need to outperform the ones included in the portfolio notably to make up for the transaction costs, that have to be payed. In the investigated setting, transaction costs can decrease the achieved portfolio values over an investment period of six years by up to 23.2%.

The historical tests reveal that, in the presence of transaction cost, structured products are also sold before they have reached their maturity and the received cash flow is reinvested in the newly issued structured investments.

4.4 Effects of credit risk on portfolio optimization with structured products

This section introduces credit risk that is an important risk factor that needs to be considered when investing in structured products. Most structured investments are traded OTC and therefore carry credit risk. If the issuer of a structured product defaults, the complete value might not be recovered and in the worst case the contract might be worthless. In recent years credit risk has gained a lot of recognition in the financial world. Especially since the collapse of Lehman Brothers and three major Icelandic banks, it is confirmed that financial institutions can file for bankruptcy and in the course of this, default their outstanding payments. This section is outlined as follows. First credit risk is implemented in the pricing of the assets and in the scenario generation. Thereafter the effects of credit risk on the portfolio weights are investigated. The section closes with conclusions of the effect of credit risk for portfolio optimization with structured products.

4.4.1 Modeling credit risk

Credit risk is defined as the risk of the value of an asset or a portfolio changing due to unexpected changes in the credit quality of the counterpart. So not only default of the counterpart has an effect on the portfolio value, but also a change in credit rating may influence the value of an asset or of a portfolio. The first step in order to incorporate credit risk in the investment situation is to determine the fair price of a PPN and a NPPC, taking into account the possibility of the contract issuer's default.

Pricing defaultable structured products

Previously in this study, the pricing of structured products is done using the Black & Scholes model under the assumption, that the issuer of a structured investment is always able to fulfill its obligations. This assumption is now abandoned. The random time of default is denoted τ. At this time point, the issuer can no longer meet its obligations. If this situation occurs, the owner of the structured products is assumed to be compensated by a fraction of the structured product's payoff at maturity. This fraction, denoted $R \in [0,1]$, is an exogenous, fixed parameter and known as the recovery rate of the entity. The credit default event is assumed to be independent of all other economic quantities. The reduced-form approach is used, describing the event of an entity defaulting by the first jump of a Poisson process. This model is summarized in section 2.4.1. More information on credit risk models can be found in McNeil et al. (2002) or Ammann (2001).

The value of a structured product at time t, given that default of the issuer has not occurred prior to time t, is the discounted expected payoff at maturity in the case that the issuer survives up to the asset's maturity plus the discounted expected compensation the holder receives in the case of default during the lifetime of the asset. Under the risk-neutral measure this can be expressed as

$$1_{\{\tau>t\}}\pi^D(t) = 1_{\{\tau>t\}}E_t\left[1_{\{\tau>T\}}P(t,T)\pi(T) + 1_{\{\tau\leq T\}}P(t,T)R\,\pi(T)\right]$$

where:

$\pi^D(t)$	price of a defaultable structured product at time t
$\pi(t)$	price of a default-free structured product at time t
$P(t,s)$	price of a default-free zero-coupon bond with face value 1 and maturity s at time t

Using the mutually independence of the default event, the interest rate and the price of the underlying, and the fact that $\pi(t) = P(t,T)E_t[\pi(T)]$ and $Q(t,T) = E_t\left[1_{\{\tau>T\}}\right]$ gives, that the price at time t of a defaultable structured investment with maturity T under credit risk, given that default has not occurred prior to time t is

$$1_{\{\tau>t\}}\pi^D(t) = 1_{\{\tau>t\}}\left(R\,\pi(t) + Q(t,T)(1-R)\pi(t)\right)$$

The price of a structured product is the sum of the value of the claim, given that default has occurred prior to maturity, and the expected additional cash flow received in case of the issuer meeting its obligations (Ammann, 2001). To be able to price a defaultable structured investment issued by a specific entity, one needs to determine the survival probability of the issuer $Q(t,T)$ given R. The survival probability under the risk-neutral measure can be retrieved using market prices of corporate bonds or CDS rates. Section 2.4.3 illustrates the procedure using CDS rates under the assumption of a piecewise linear hazard rate. This approach is used for the investigations concerning credit risk. Since the value of the defaultable structured product is non-negative, the owner of it bears the credit risk. He should be compensated by a lower price for bearing more risk. So far the price of a defaultable structured investment is determined under the condition that default of the issuer has not yet occurred. Though if the counterparty has defaulted, the value of the claim is the recovery rate times the value of the default-free claim. Both cases can be summarized as follows

$$\pi^D(t) = \begin{cases} (R + Q(t,T)(1-R))\,\pi(t) & \text{if } \tau > t \\ R\,\pi(t) & \text{if } \tau \leq t \end{cases} \tag{4.4.1}$$

Scenario generation under credit risk

The equation (4.4.1) can be used to determine the fair price of PPN and NPPC under credit risk. The next step is to generate the return scenarios. Following the same approach as for the yield curve and the index level, the evolutions of the CDS rates are simulated under the physical measure. From the CDS rates, the survival probability under the risk-neutral measure can be determined by the calibration procedure. The hazard rate is assumed to be piecewise constant, which requires CDS rates with different contract maturities for calibration. Only two contract maturities are selected, due to the limited amount of historical observations for most contract maturities. The

CDS rates for contracts with RBS as reference entity and 3 respectively 5 years time to maturity are used. This results in a hazard rate term structure with two levels. The short term hazard rate up to 3 years and the long term hazard rate for horizons beyond 3 years. It is assumed that the CDS buyer makes quarterly premium payments. To value the protection leg a grid with four equally spaced time points per year is used. Furthermore, the 30/360 day count convention is considered when calibrating the hazard rate. As mentioned in section 2.4.3, there are some limitations on the calibration procedure. Therefore are scenarios, that result in situations where no well-defined hazard rate can be fitted, replaced by new simulated scenarios.

In order to generate the return scenarios, the statistical factor model is extended to also describe the CDS rates for two different maturities. Figure A.4.1 in the appendix shows the historical CDS rates and the weekly log returns of the CDS rates. The log returns seem to be weekly stationary. One can notice though that the magnitude of the returns has increased in the more recent observations. Until 2007, the CDS rates for both horizons fluctuated around 10 basis points (bp). Since then the CDS rates have experienced a significant increase. In 2010, the rates are at around 125 bp for the 3 year time horizon and 150 bp for the 5 year time horizon. The new factor model uses the first six PCs in order to have a similar explanatory power as the previous model. The selected PCs describe 96.92% of the variation in the data. The stability of both the explanatory power and the factor loadings is investigated in the same manner as presented in section 3.4 and the results are displayed in table A.4 and figure A.4.2. The first six PCs describe more than 95% of the variation in all selected time periods. The explanatory power of the single PCs experiences quite strong variation. The factor loadings of the first four PCs show a similar stability as in the previously used factor model. The variations in the factor loadings for the fifth and sixth PC are much greater compared to the first four PCs. Two time periods show significant differences in these two PCs. They correspond to the time spans from April 2003 to April 2007 and July 2003 to July 2007. Those time intervals only include periods where the CDS rates were very low, which could be the reason for the difference in the factor loadings. In order to use the historical simulation approach, the autocorrelation of the new PCs needs to be investigated. Figure A.4.3 shows that there is hardly any autocorrelation present in the factor scores of the PCs when using weekly observations. The procedure of finding a parametric model for the PCs is not repeated, instead the generation of risk factor changes for the scenarios relies solely on the historical simulation approach.

4.4.2 Effects of credit risk on the return distribution of structured products

The following example illustrates how credit risk changes the return distribution of structured products. The extended statistical factor model and the historical simulation approach are used to generate 2000 simulated one-year evolutions of the risk factors. For each scenario the returns of the structured products are calculated. All these return scenarios then determine the one-year return distributions of a PPN and a NPPC. Three different settings are investigated. The first one represents the returns of default-free assets. The second one corresponds to the return distribution in the case, where credit risk enters the pricing formula. The third one incorporates credit risk in the pricing method and generates default scenarios. RBS has Aa3 long term credit rating determined by Moody's [1]. For this class the probability of default occurring during the next year was

[1] http://www.investors.rbs.com/debt_securitisation/ratings.cfm; date:23th of May 2010

1.424% in 2008 (Emery and Ou, 2009). This probability is used to create the actual default scenarios in the third setting. It is assumed that the event of default under the physical measure is independent from all other random variables. Since no recovery rate estimates for the reference entity could be found, the average of the defaulted debt recovery rate estimates for European institutions from O'Kane and Turnbull (2003) is used as the estimate of the recovery rate. The average rate for different classes of seniority is 22%. The economic setting is summarized in table 4.1. The most recent 4 years of weekly observation are given as input for the factor model and the historical simulation algorithm.

current time point	15 February 2008
current OMXS30 level	938.79
current 3-year interest rate	3.75%
current 3-year CDS rate	92.87 bp
current volatility	18.83%
risk-neutral 3-year survival probability	96.50%
time to maturity of the assets	3 years

Table 4.1: *Current economic setting used for the investigation of the credit risk's effects on the return distribution of structured investments*

The scenarios are generated taking into account a subjective view, that has the following specifications: The expected relative change of all interest rates is set to one. The expected weekly log return of the CDS rates is equal to zero and the expected weekly log return of the index is 0.0018, which corresponds to a 10% annual log return. The variances of the absolute views are set to match the variances of μ_{data}. λ is chosen to be 0.99. The histograms of the return scenarios for the structured investments are shown in figure 4.4.1.

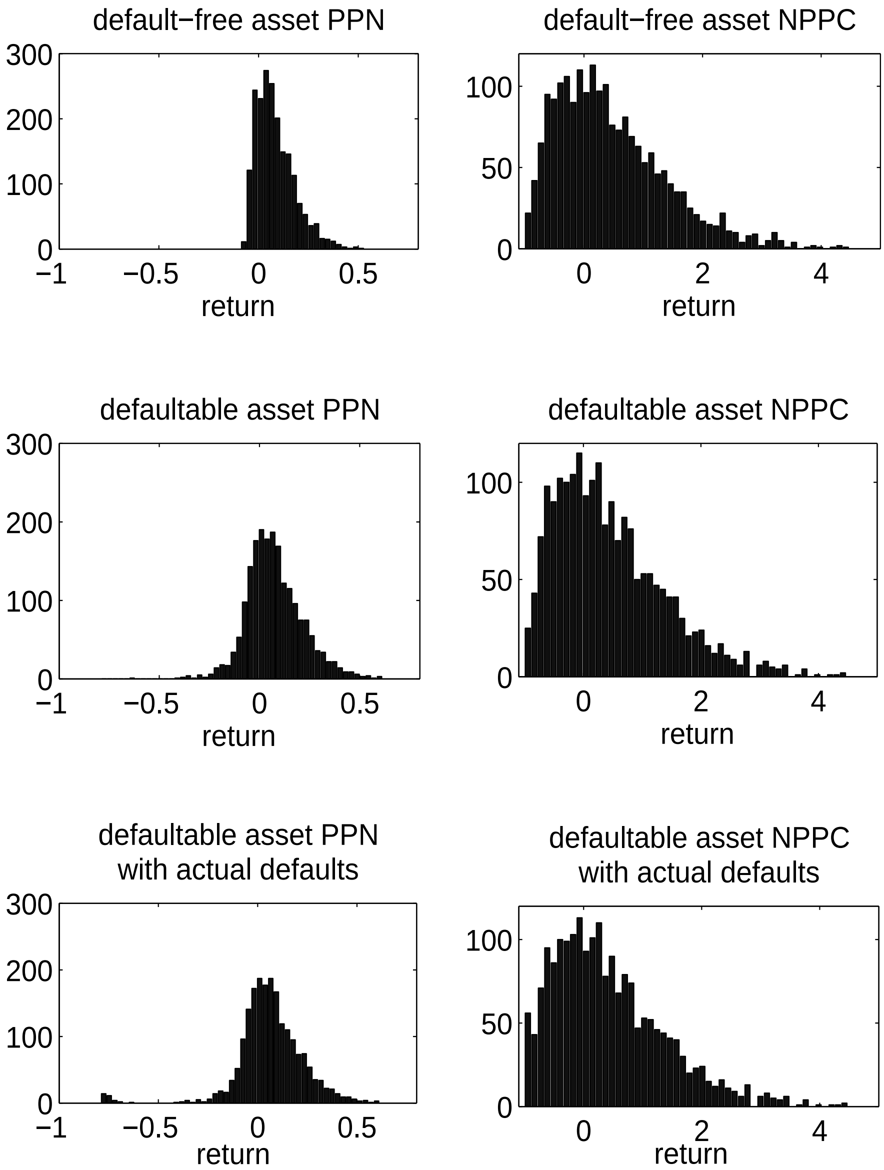


Figure 4.4.1: *Return distribution of a PPN and a NPPC in three different settings. The default-free asset case corresponds to structured products without credit risk. The defaultable asset setting represents structured products that carry credit risk. The defaultable asset case with actual default corresponds to a structured product that carries credit risk and where actual default of the issuer is simulated. Each histogram is composed of 2000 scenarios that are generated by historical simulation. The expected risk factor changes are computed using a subjective view on all risk factors. The investor believes that the expected weekly relative change of each interest rate is 1, the expected weekly log return of each CDS rate is 0 and the expected weekly log return of the index is 0.0018. The uncertainty of the views is chosen so that it matches the variances of the return series' expected returns.* λ *is 0.99.*

The histograms for the PPN in the specified settings show noticeable differences. When comparing the default-free PPN returns to the defaultable PPN returns, one can see that the scenario returns become more symmetrically distributed in the defaultable asset case. In the default-free asset setting only very few negative return scenarios occur and they have only very small absolute return values. When credit risk is considered, the scenario returns are wider spread. This is due to the additional source of randomness in form of credit risk. The right tails of the distributions are similar in shape. In the case of the defaultable PPN, the right tail has experienced a small shift to the right. This effect can be explained by the higher participation rate of the defaultable PPN compared to the default-free PPN. Credit risk reduces both the price of the bond part and the price of the option. Since the PPN has a price equal to the current index level at the issue date, the participation rate of the defaultable version must be higher. In this example the participation rates are 78.29% for the default-free PPN and 98.99 % for the defaultable PPN. In the case of a defaultable PPN with actual defaults, some scenarios with returns between -0.8 and -0.7 occur. The scenarios correspond to the 36 actual default cases. Apart from these 36 observations, this histogram is identical to the one of the defaultable PPN.

The histograms of the NPPC hardly show any differences. This is due to the fact that in a large number of scenarios big losses already occur in the default-free setting. One can though see that the left tail of the distribution becomes slightly heavier for the defaultable NPPC compared to the default-free NPPC. The same observation is noticeable when comparing the setting without actual defaults to the case with actual default scenarios.

The following conclusions can be drawn from this experiment: Credit risk changes the return distributions of structured investments. The changes are more significant for PPNs than for NPPCs. When CVaR is chosen as the measure of risk, the observed differences in the left tail of the return distribution will have an effect on the riskiness of the assets. In the case of defaultable assets the CVaR will be higher compared to the default-free asset. When also actual default scenarios are introduced the CVaR will increase even more.

4.4.3 Comparison of portfolio optimization with default-free and defaultable structured products

This section investigates the impact of credit risk on the investment decision. Due to the limited amount of historical observations of CDS rates, this study cannot be performed over the same time length as the previous historical investigations. Instead, only the last two rebalancing times can be analyzed. Figure 4.4.2 gives an overview of the two rebalancing periods considered in this study. The input for the factor model is always the most recent four years of weekly observations of the return series. For each investment situation, 2000 scenarios are generated with the historical simulation approach. The proportional cost of buying and selling structured investments is 2%. The upper bound on the portfolios' CVaR is 0.20 and CVaR is measured at the 0.05 confidence level. Instead of using a synthetic view, this study utilizes a constant subjective view in the same manner as in section 4.4.2. The composed portfolios are analyzed in the case of default-free structured products and defaultable structured investments. The case with actual default scenarios is not investigated. Table 4.2 summarizes the returns of the assets, the risk and return specifications of the optimal portfolios and the realized development in the portfolio values. Figure 4.4.3 displays the weight allocation diagrams.

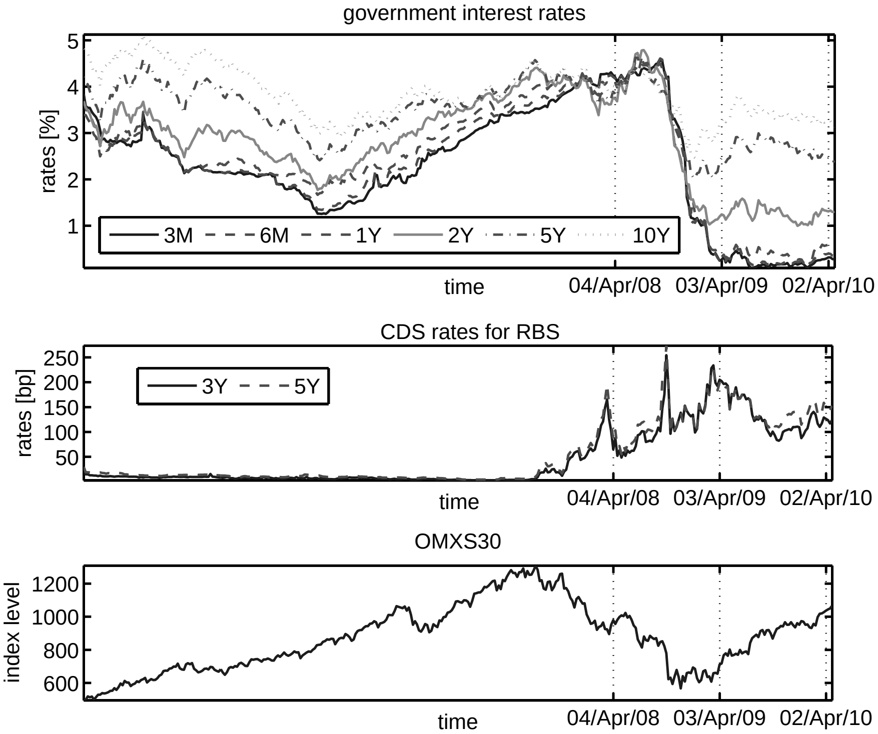
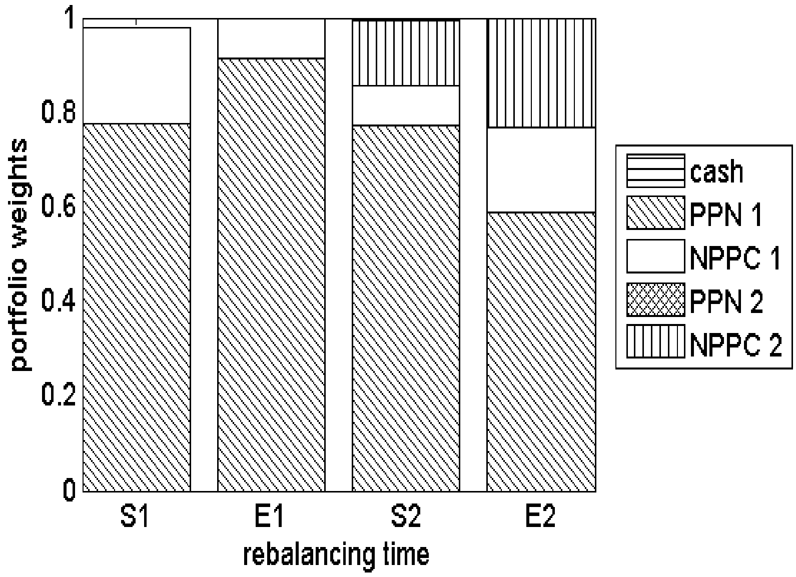


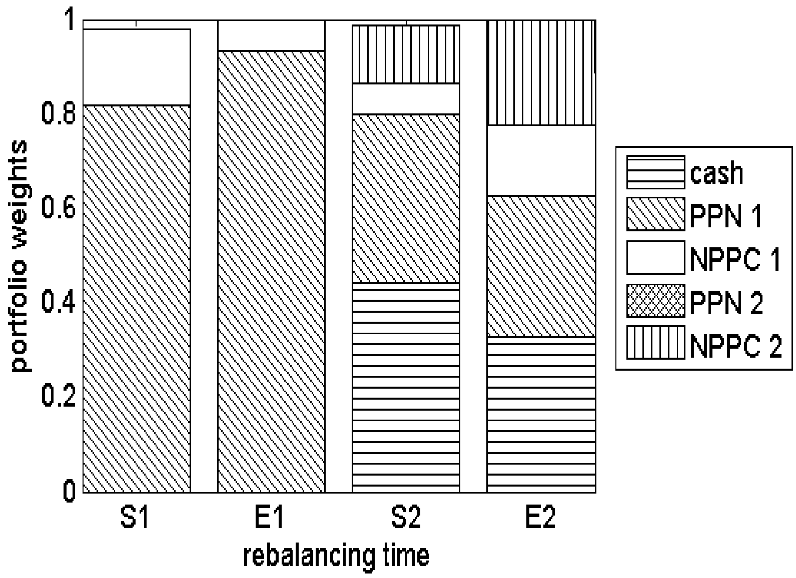
Figure 4.4.2: *Historical observations of Swedish government interest rates, CDS rates on RBS and OMXS30. The rebalancing times and the final evaluation time are indicated by the horizontal dotted lines and the dates on the x-axis.*

	default-free assets			defaultable assets		
year	2008	2009	2010	2008	2009	2010
	Expected asset returns					
PPN1	7.86%	4.11%	-	8.22%	3.31%	-
NPPC1	40.29%	52.48%	-	43.62%	52.88%	-
PPN2	-	4.42%	-	-	5.28%	-
NPPC2	-	42.46%	-	-	43.20%	-
	Optimal portfolio characteristics					
Expected return	12.43%	12.98%	-	11.78%	8.49%	-
CVaR	20.00%	20.00%	-	20.00%	20.00%	-
realized value	1	0.8594	1.2296	1	0.8604	1.1628

Table 4.2: *Overview of the structured products' expected returns at the rebalancing times in the case of default-free and defaultable assets. Furthermore, the table presents the risk and return characteristics of the optimal portfolios and the realized development in the portfolios' value for both cases.*



(a) Portfolio weight allocation in the case of default-free structured products



(b) Portfolio weight allocation in the case of defaultable structured products

Figure 4.4.3: *Portfolio weight allocation in the cases of default-free and defaultable structured products. The used subjective view has the following specifications. The expected relative change of each interest rate is 1, the expected log return of each CDS rate is 0 and the expected weekly log return of the index is 0.0018. The uncertainty of the views is chosen so that it matches the variances of the return series' expected returns.* λ *is 0.99. The chosen confidence level for the CVaR measurements is 0.05 and the upper bound on the portfolios' CVaR is 0.20*

When comparing the weight allocations in the cases of default-free and defaultable structured products, the following observations are made. Credit risk changes the return distribution of the structured investments and therefore has an impact on the weight allocation of the portfolio. At the first rebalancing time, one can notice that a smaller proportion of the funds are allocated to the NPPC 1 and a larger proportion is allocated to the PPN 1 in the defaultable asset setting, compared to the default-free asset setting. At the second rebalancing time the return potential of the PPN 1 significantly reduces, due to the low interest rates and the low index level, that pushes the option out-of-the-money. The large rise in the CDS rates from rebalancing time 1 to time 2 increases the riskiness of all defaultable structured products. The previous section showed, that the effects are much more significant on the PPNs compared to the ones on the NPPCs. In the defaultable asset case at the second rebalancing time, a large part of the weight invested in PPN 1 is sold and the received cash flow is allocated partly to the cash account and to the NPPC 2 in order to meet the investor's risk requirements and to maximize the reward potential of the portfolio. In the default-free asset setting, the riskiness of the PPN hardly changes. To increase the return potential, part of the PPN 1 is sold and the received cash flow is allocated to the NPPC 2.

When examining the expected return and the CVaR of the optimal portfolios given in table 4.2, one can notice that all portfolios reach the upper CVaR bound. At the first rebalancing time the difference in expected return is around 0.6%. In the case of defaultable structured products, the loss in the portfolio's upside potential due to a smaller weight in NPPC 1 can nearly be recovered due to the higher participation rate PPN 1, compared to the default-free setting. At the second rebalancing time the difference in the expected return of the portfolios is more than 4%. This can be explained by two reasons. A part of the portfolio has to be invested in the risk-free cash account in order to satisfy the upper CVaR bound. Since the cash account has zero return over the next time period, the expected return is reduced. Furthermore, structured products have to be sold in order to create this cash position. This action gives rise to transaction costs, which also lower the expected return of the portfolio.

The realized portfolio values are similar at the second rebalancing time. In the defaultable asset case, the loss is a little smaller compared to the default-free case. This is due to the smaller portfolio weight in the NPPC 1 in the defaultable asset setting. NPPC 1 lost more than 60% of its value in both cases. At the final evaluation time the default-free portfolio outperforms the defaultable portfolio by a 5% higher portfolio value. The default-free portfolio has more exposure to the index, due to larger weights in all structured investments, and can therefore profit much more from the positive market developments.

Table 4.2 also displays the expected returns of the default-free and defaultable structured products at the two rebalancing times. It is noticeable that the defaultable structured investments have a higher expected return over the next year at their issue date compared to the default-free ones. This is in line with the argument that the investor has to be compensated by a higher expected return for bearing credit risk. Furthermore, this example also shows the impact of a sudden increase in credit risk on the expected returns of a PPN. At the second rebalancing time the defaultable PPN 1 looses its superior expected return characteristic over the default-free one.

4.4.4 Conclusions on the impact of credit risk

Credit risk has significant effects on return and risk characteristics of structured products. The effects are more significant on PPNs than on NPPCs. The changes in the asset characteristics result in differences in the capital allocation when comparing the portfolio weights in the cases of default-free and defaultable structured investments. It seems that more rebalancing activity is necessary to satisfy the risk requirements when considering credit risk. To investigate this hypothesis in more detail, one has to look at a larger variety of rebalancing situations over the lifetime of defaultable structured investments.

Modeling credit risk based on CDS data should be investigated in more detail. First of all, a CDS is an OTC traded asset and therefore carries credit risk in itself. This issue is ignored in the presented approach. Secondly, factors such as liquidity, spread, and market supply and demand typically lead to an overestimation of the hazard rate (O'Kane and Turnbull, 2003). One possibility for evaluating the quality of the hazard rate estimates is to compare the ones extracted from CDS data to the ones determined from corporate bond prices. Moreover, the reduced-form model could be replaced by a firm-value model in order compare the model prices of OTC traded structured products.

5 Conclusions

The report implements theoretical concepts to solve investment situations concerning PPNs and NPPCs with the OMXS30 as the underlying asset. The investor's portfolio is analyzed in terms of expected return and CVaR, and the optimal weights are determined in a two-stage procedure. The first stage corresponds to generating return scenarios and the second stage determines the optimal portfolio weights. The fact that both stages are independent of each other makes this approach very flexible.

The optimization problem can handle trading constraints, such as limited offer time and short-selling restriction, and proportional transaction costs, while still retaining in the class of linear optimization programs. This has the advantage that the problem is solvable in reasonable time and that the optimal solution is going to be determined, provided that it exists. Two alternative optimization formulations are established. The maximizing the expected portfolio return subject to an upper bound on the portfolio's CVaR formulation outperforms the minimizing portfolio's CVaR subject to a lower bound on the portfolio's expected return formulation in terms of robustness of the solution and solution time. Experiments using the former approach conclude that exposure to market risk can be controlled by the upper bound on the portfolio's CVaR. Moreover, tests varying the confidence level, at which the portfolio's CVaR is measured, point out that a lower confidence level reduces the the riskiness of the portfolio when the upper CVaR bound is kept unchanged.

Investigation of investment situations on the historical data revealed, that the risk and reward characteristics of the assets determined by scenario generation under the physical measure are heavily influenced by historical trends in the risk factor changes. In order to reduce this dependence, the investor's subjective view on the expected future evolution of the risk factors is introduced. The representative quality of the scenarios depends though on how well the subjective view and the historical data describe future behavior in the development of the risk factors. Investigations on historical data using synthetic views of different quality confirmed that a more accurate view on future behavior of the risk factors leads to more profitable investment decisions.

Transaction costs are identified as an important factor for rebalancing decisions. The presented experiment points out that transaction costs significantly reduce the return performance of the used investment strategy. Higher transaction costs increase the holding period of structured products and reduce the occurrence of situations where it is profitable to sell structured products prior to their maturity.

Structured products are in general traded OTC. For the types of structured products examined in this study, the credit risk, which is present in every OTC trade, is exclusively carried by the investor. The model of the investment situation can be extended in order to take into account credit risk. Credit risk has a significant impact on the return distribution of the structured products. This impact causes differences in the capital allocation, which is shown in an experiment that compares portfolios of default-free structured products and portfolios of defaultable structured products. In order to investigate the effects of credit risk on the optimal portfolio weights, further studies are necessary.

All conducted experiments showed that rebalancing at intermediate time points is necessary in order to meet the investor's risk requirement and to maximize the reward potential of his portfolio. This indicates that even in the presence of high transaction costs, situations arise where it is profitable for an investor to rebalance his portfolio. The rebalancing decision is though very complex and influenced by various factors. The presented approach can assist in making objective rebalancing decisions for an investor, who evaluates the riskiness and reward potential of his portfolio on short term basis. Additionally, the investigated experiments give a presentation of the factors influencing the rebalancing decision when investing in structured products.

Bibliography

Abdi, Hervé and Lynne J. Williams. *Principal component analysis*, Wiley Interdisciplinary Reviews: Computational Statistics **2**, in press 2010. http://www.utdallas.edu/~herve/abdi-wireCS-PCA2010-inpress.pdf.

Ammann, Manuel. 2001. *Credit risk valuation*: *methods,models and applications*, 2nd ed., Springer, Berlin Heidelberg.

Basel Committee on Banking Supervision. 2004. *International convergence of capital measurement and capital standards*, Bank for International Settlements. http://www.bis.org/publ/bcbs128.pdf.

Birge, John R. and François Louveaux. 1997. *Introduction to Stochastic Programming*, Springer, 175 Fifth Avenue, New York.

Cornuejols, Gerard and Reha Tütüncü. 2007. *Optimization methods in finance*, Cambridge University Press, The Edinburgh Building, Cambridge, United Kingdom.

Emery, Kenneth and Sharon Ou. 2009. *Moody's global credit policy - corporate default and recovery rates, 1920-2008*, Moody's Investors Service. http://www.moodys.com/cust/content/content.ashx?source=StaticContent/Free%20Pages/Credit%20Policy%20Research/documents/current/2007400000578875.pdf.

Hull, John C. 2005. *Options, futures and other derivatives*, 6th ed., Prentice Hall, Upper Saddle River, New Jersey.

Krokhmal, Pavlo, Jonas Palmquist, and Stanislav Uryasev. 2002. *Portfolio optimization with conditional value-at-risk objective and constraints*, Journal of Risk **4**, no. 2, 11–27.

Lindskog, Filip. 2009. *Risk and portfolio analysis*: *principles and methods*, KTH The Royal Institute of Technology, Division of Mathematical Statistics.

Litterman, Robert B. and José Scheinkman. 1991. *Common factors affecting bond returns*, Journal of Fixed Income **1**, no. 1, 54–61.

Mankert, Charlotta. 2006. *The black-litterman model - mathematical and behavioral finance approaches towards its use in practice*, Ph.D. Thesis, KTH The Royal Institute of Technology,Department of Industrial Economics and Management.

Markowitz, Harry. 1952. *Portfolio selection*, The Journal of Finance **7**, no. 1, 77 –91.

McNeil, Alexander J., Rodiger Frey, and Paul Embrechts. 2002. *Quantitative risk management*: *concepts, techniques, and tools*, 2nd ed., Princeton University Press, 41William Street, Princeton, New Jersey.

NASDAQ OMX AB. *Rules for the construction and maintenance of the omx stockholm 30 index*, 1.1. https://indexes.nasdaqomx.com/docs/methodology_OMXS30.pdf.

Nyman, Mats. 2009. *Kapitalskyddade placeringar: Historisk avkastning, riskhantering och placeringsstrategier*, Handelsbanken. http://hcm.handelsbanken.se/struktureradeprodukter/Start/Kapitalskyddade-placeringar/Broschyrer/.

Jarrow, Robert A. and Stuart Turnbull. 1995. *Pricing Derivatives on Financial Securities Subject to Credit Risk*, The Journal of Finance **50**, no. 1, 53–85.

O'Kane, Dominic and Stuart Turnbull. 2003. *Valuation of credit default swaps*, Lehman Brothers - Fixed Income Quantitative Credit Research. http://www.grahamanddoddsville.net/wordpress/Files/SecurityAnalysis/Valuation/19601876-Lehman-Brothers-Valuation-of-Credit-Default-Swaps.pdf.

Riksgälden - Swedish National Debt Office. 2007. *Handbok om statspapper*. https://www.riksgalden.se/templates/Secure/Page____512.aspx.

Scherer, Bernd. 2007. *Portfolio construction and risk budgeting*, 3rd ed., Risk Books, 28 Haymarket, London.

Tsay, Ruey S. 2005. *Analysis of financial time series*, 2nd ed., John Wiley & Sons, Inc., Hoboken, New Jersey.

Uryasev, Stanislav. 2000. *Conditional value-at-risk: Optimization algorithms and applications*, Financial Engineering News **14**, 1–5.

Wasserfallen, Walter and Christoph Schenk. 1996. *Portfolio Insurance for the Small Investor in Switzerland*, The Journal of Derivatives **3**, no. 3, 37–43.

A

Appendix

A.1 Weight allocation diagrams for the rebalancing strategies using various upper bounds on the portfolio's CVaR

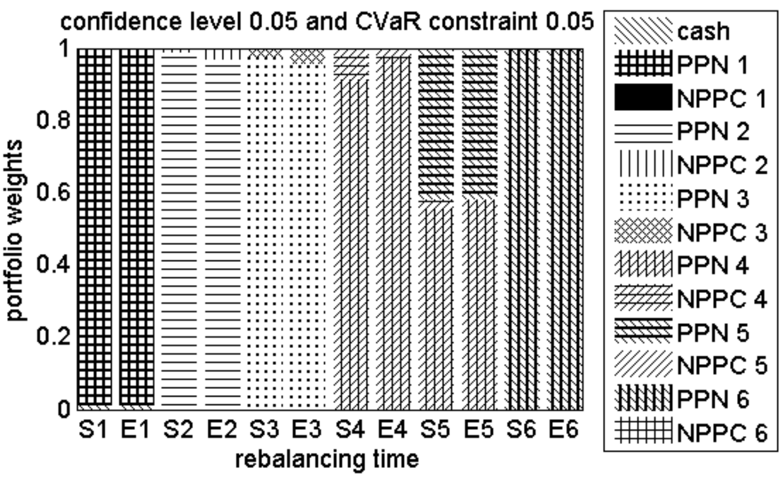


Figure A.1.1: *Weight allocation for a rebalancing strategy with a 0.05 upper bound on the portfolio's CVaR measured at the 0.05 confidence level. Si indicated the weight at the beginning of the i period, while Ei indicates the weight at the end of the period, when the assets are priced in the updated economic setting.*

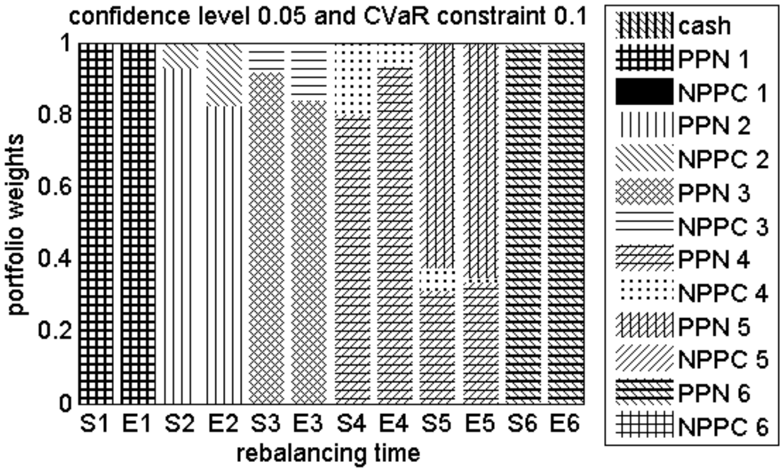


Figure A.1.2: *Weight allocation for a rebalancing strategy with a 0.1 upper bound on the portfolio's CVaR measured at the 0.05 confidence level. Si indicated the weight at the beginning of the i period, while Ei indicates the weight at the end of the period, when the assets are priced in the updated economic setting.*

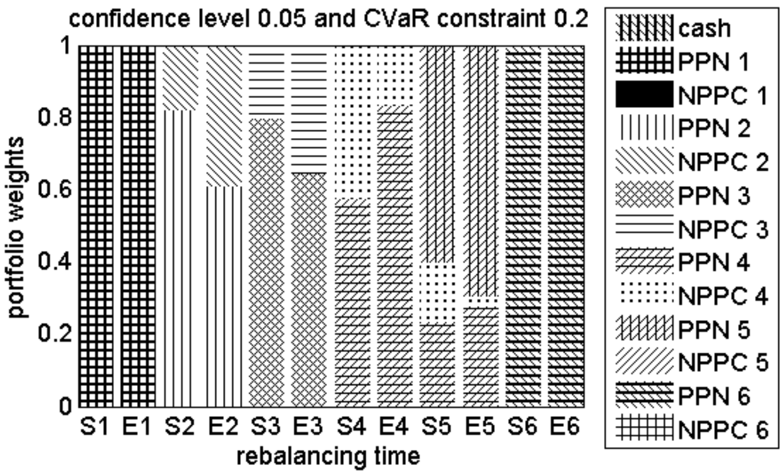


Figure A.1.3: *Weight allocation for a rebalancing strategy with a 0.2 upper bound on the portfolio's CVaR measured at the 0.05 confidence level. Si indicated the weight at the beginning of the i period, while Ei indicates the weight at the end of the period, when the assets are priced in the updated economic setting.*

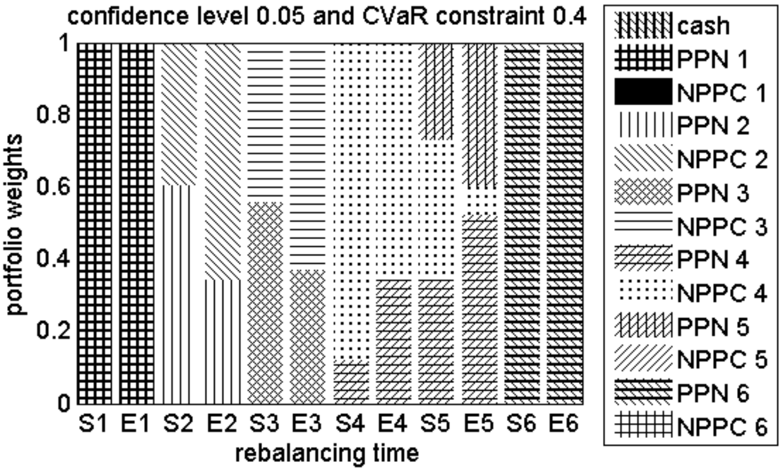


Figure A.1.4: *Weight allocation for a rebalancing strategy with a 0.4 upper bound on the portfolio's CVaR measured at the 0.05 confidence level. Si indicated the weight at the beginning of the i period, while Ei indicates the weight at the end of the period, when the assets are priced in the updated economic setting.*

A.2 Weight allocation diagrams for the rebalancing strategy on historical data using a synthetic subjective view

A.2.1 Weight allocation diagrams using a synthetic subjective view with different uncertainty levels

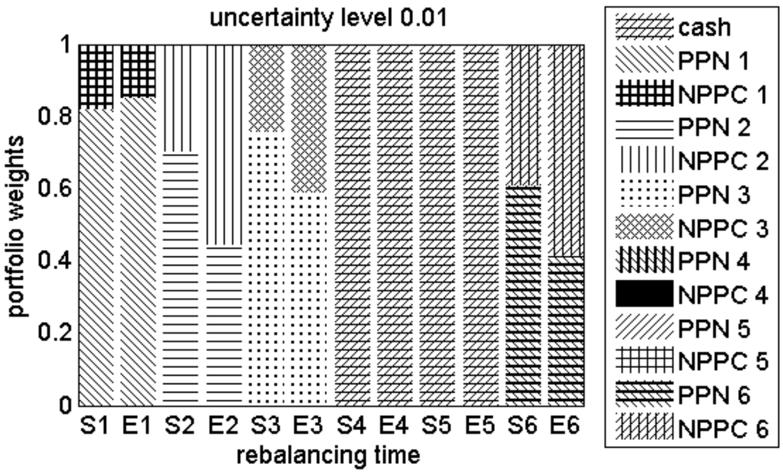


Figure A.2.1: *Weight allocation using an synthetic subjective view with uncertainty level 0.01. The chosen confidence level for the CVaR measurements is 0.05 and the upper bound on the portfolios CVaR is 0.20*

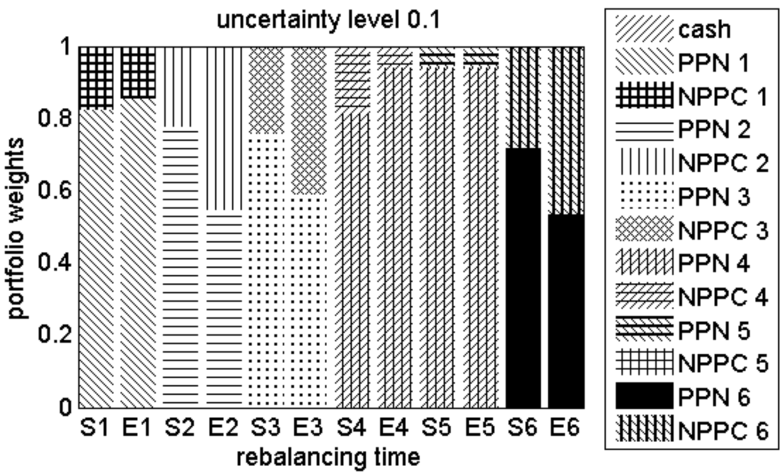


Figure A.2.2: *Weight allocation using an synthetic subjective view with uncertainty level 0.1. The chosen confidence level for the CVaR measurements is 0.05 and the upper bound on the portfolios CVaR is 0.20*

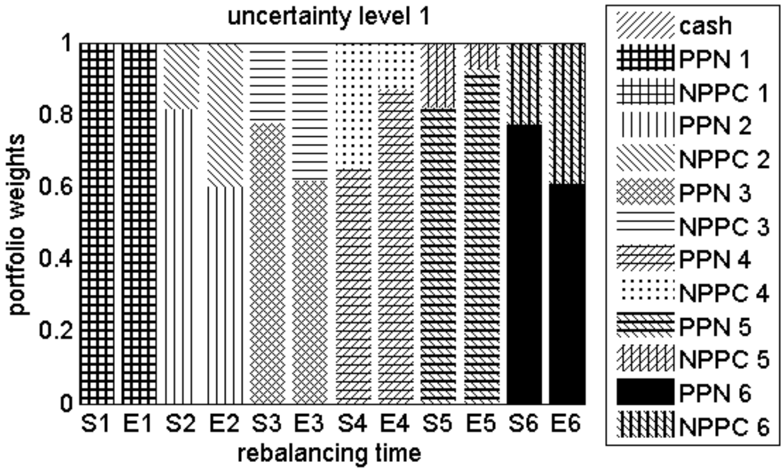


Figure A.2.3: *Weight allocation using an synthetic subjective view with uncertainty level 1. The chosen confidence level for the CVaR measurements is 0.05 and the upper bound on the portfolios CVaR is 0.20*

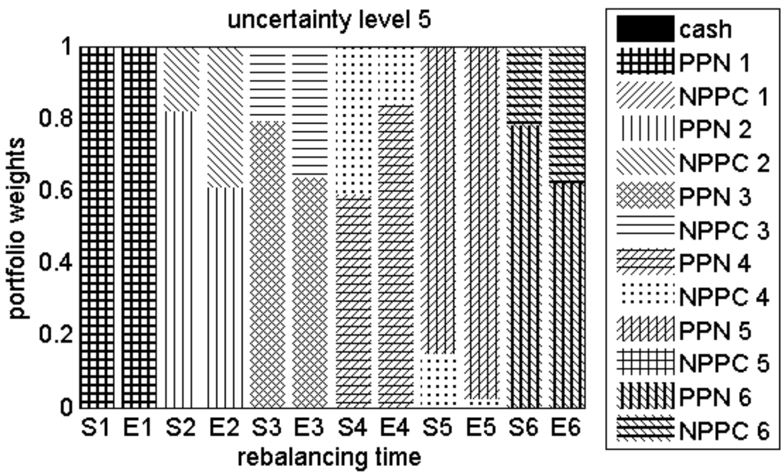


Figure A.2.4: *Weight allocation using an synthetic subjective view with uncertainty level 5. The chosen confidence level for the CVaR measurements is 0.05 and the upper bound on the portfolios CVaR is 0.20*

A.2.2 Weight allocation diagrams using different numbers of absolute views

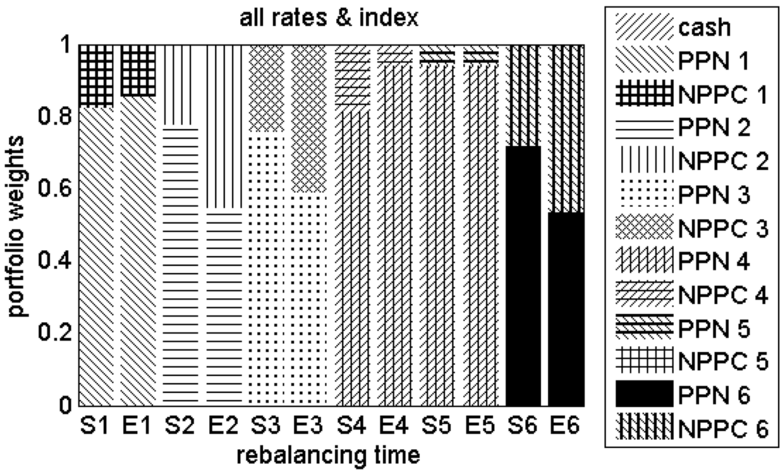


Figure A.2.5: *The weight allocation at the rebalancing points using absolute views on all government interest rates and the OMXS30. The synthetic view has an uncertainty level of 0.01. The chosen confidence level for the CVaR measurements is 0.05 and the upper bound on the portfolios CVaR is 0.20*

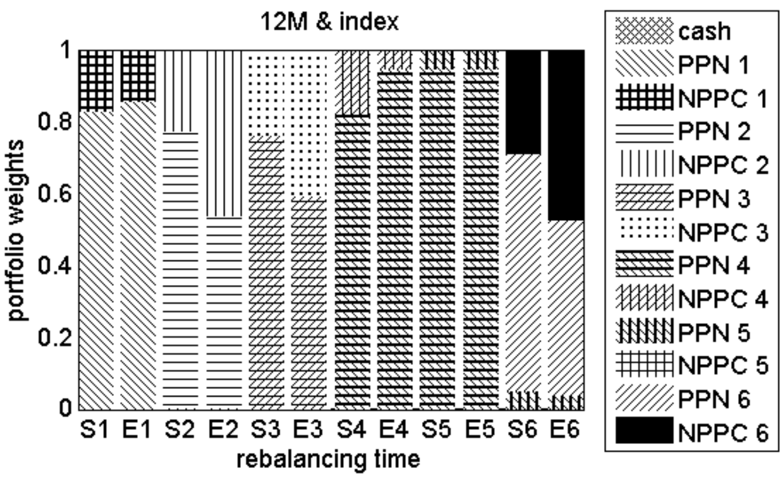


Figure A.2.6: *The weight allocation at the rebalancing points using a view on the 12 month government interest rate and the OMXS30. The synthetic view has an uncertainty level of 0.01. The chosen confidence level for the CVaR measurements is 0.05 and the upper bound on the portfolios CVaR is 0.20*

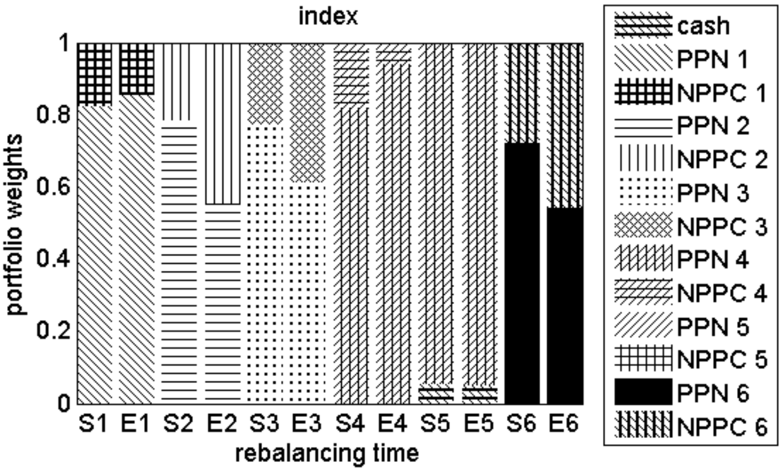


Figure A.2.7: *The weight allocation at the rebalancing points using only a view on the OMXS30. The synthetic view has an uncertainty level of 0.01. The chosen confidence level for the CVaR measurements is 0.05 and the upper bound on the portfolios CVaR is 0.20*

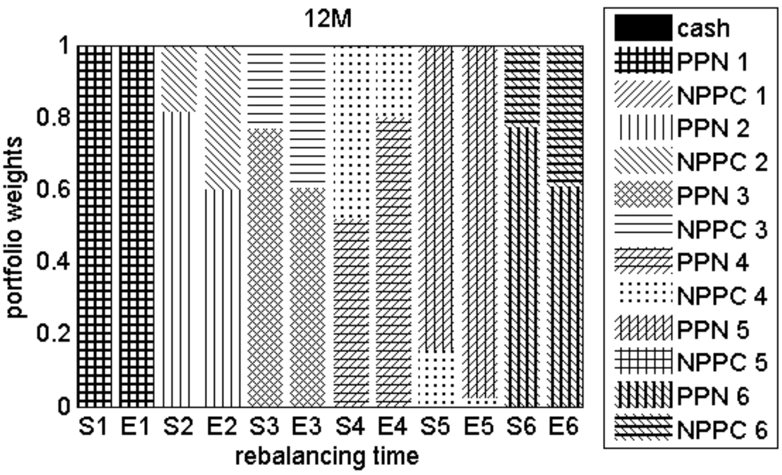


Figure A.2.8: *The weight allocation at the rebalancing points using only a view on the 12 month government interest rate. The synthetic view has an uncertainty level of 0.01. The chosen confidence level for the CVaR measurements is 0.05 and the upper bound on the portfolios CVaR is 0.20*

A.3 Illustrations on the impact of transaction costs on rebalancing strategy

A.3.1 Overview of brokerage fees for structured products

Issuer	Product name	Brokerage Fees	Time to maturity
ErikPenser	Accelerator 10 Tillväxt	2%	3 years
ErikPenser	Sverige 18 Tillväxt	2%	3 years
ErikPenser	Accelerator 9 Tillväxt	2%	5 years
ErikPenser	Accelerator 9 Trygghet	2%	5 years
ErikPenser	Kina 20 Tillväxt	2%	3 years
SHB	Kanada Balans 923AK	2%	4 years
SHB	Sverige Balans 923AS	2%	5 years
SHB	Sverige Balans 923AX	2%	5 years
SHB	Svenska Aktier 923AZ	1%	2 years
SHB	Svenska Aktier 923AY	1%	2 years
SHB	Tillväxtmarknader Balans 923AT	2%	5 years
SHB	Index Balans 923RI	2%	3 years
HQ	Brasilien Riskkontroll	3%	5 years
HQ	Indien / Kina Tillväxt 2	3%	5 years
HQ	Sverige Max / Min 2	2%	3 years
HQ	Trend Emerging Markets Riskkontroll 9	3%	4 years
HQ	Trend Total Riskkontroll 4	3%	5 years
HQ	Trend Total Riskkontroll Tillväxt 4	3%	5 years
RBS	Autopilot Bull & Bear 2 Trygghet	2%	5 years
RBS	Autopilot Bull & Bear 2 Tillväxt	2%	5 years
RBS	Autopilot Vector 13 Trygghet	2%	5 years
RBS	Sharpener Accumulator 3	2%	5 years
RBS	Etiska Tillväxtmarknader 2	2%	5 years
RBS	Global Trygghet	2%	5 years
RBS	Global Tillväxt	2%	5 years
RBS	Kina Ostasien 2	2%	3 years
RBS	Brasilien Total	2%	4 years
SEB	Brasilien/Kina 005B	2%	4 years
SEB	Brasilien/Kina 005C	2%	4 years
SEB	Sverige Kupong 005K	2%	5 years
SEB	Råvaror 005P	2%	4 years
SEB	Råvaror 005R	2%	4 years
SEB	Sverige 10 Bolag 005S	2%	4 years
SEB	Sverige 10 Bolag 005T	2%	4 years

Table A.1: *The tables shows a variety of structured products from different issuers. The table is taken from Nordnet's web page on the 11 of May 2010.*

A.3.2 Expected return and CVaR of the optimal portfolios under transaction costs for buying structured products

portfolio property	buying cost	Rebalancing time 1	2	3	4	5	6
Return	0%	7.14%	60.60%	60.98%	12.17%	1.99%	30.67%
Risk		20.00%	20.00%	20.00%	20.00%	-0.21%	20.00%
Return	1%	5.99%	57.04%	57.41%	10.79%	1.97%	28.39%
Risk		20.00%	20.00%	20.00%	20.00%	-0.42%	20.00%
Return	2%	4.85%	55.45%	53.91%	10.26%	0.62%	26.15%
Risk		20.00%	20.00%	20.00%	20.00%	-0.13%	20.00%
Return	3%	3.74%	54.71%	50.48%	9.81%	0.59%	24.91%
Risk		20.00%	20.00%	20.00%	20.00%	-0.13%	20.00%

Table A.2: *Expected returns and CVaR of the optimal portfolios at the six rebalancing times under different proportional costs of buying structured products. CVaR is measured at the 0.05 confidence level and the upper CVaR bound of all portfolio is 0.20*

A.3.3 Expected return and CVaR of the optimal portfolios under transaction costs for buying and selling structured products

portfolio property	selling cost	rebalancing time 1	2	3	4	5	6
Return	0%	4.85%	55.45%	53.91%	10.26%	0.62%	26.15%
Risk		20.00%	20.00%	20.00%	20.00%	-0.13%	20.00%
Return	1%	4.85%	54.69%	50.38%	9.79%	0.54%	26.09%
Risk		20.00%	20.00%	20.00%	20.00%	0.59%	20.00%
Return	2%	4.85%	53.94%	46.84%	9.37%	0.47%	26.10%
Risk		20.00%	20.00%	20.00%	20.00%	0.65%	20.00%
Return	3%	4.85%	53.19%	43.30%	9.09%	0.41%	26.10%
Risk		20.00%	20.00%	20.00%	20.00%	0.70%	20.00%

Table A.3: *Expected returns and CVaR of the optimal portfolios at the six rebalancing times under 2% proportional costs of buying and varying proportional cost for selling structured products. CVaR is measured at the 0.05 confidence level and the upper CVaR bound of all portfolio is 0.20*

A.3.4 Weight allocation diagrams under transaction costs for buying structured products

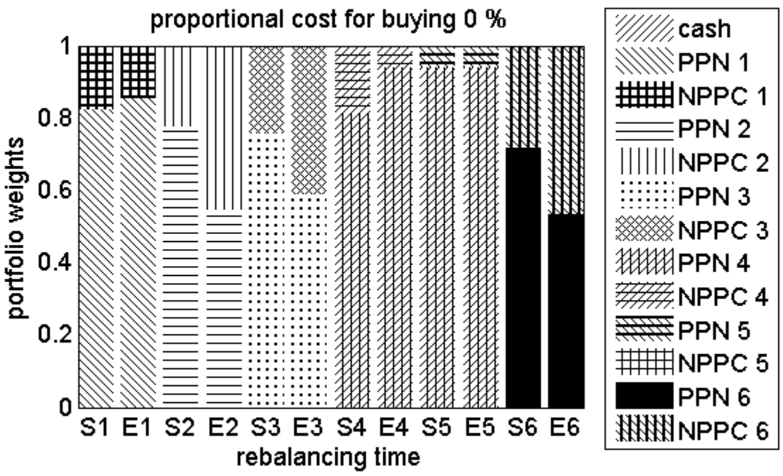


Figure A.3.1: *Weight allocation for a view on all government interest rates and the OMXS30 with uncertainty level 0.1 with no transaction costs for buying and for selling PPNs and NPPCs*

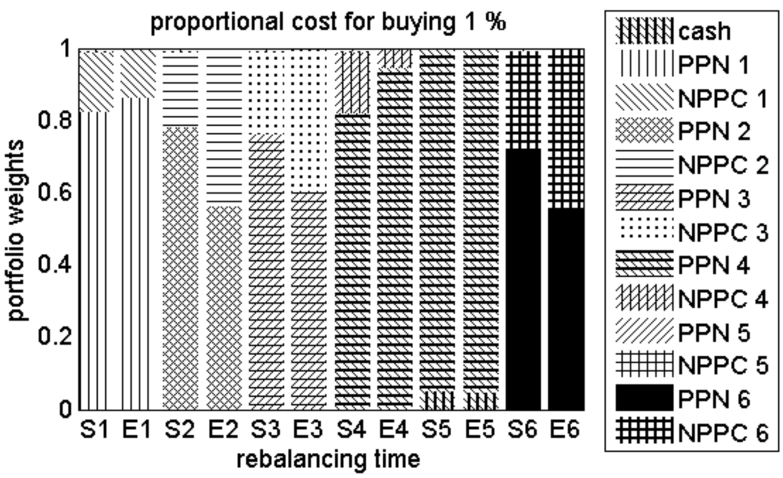


Figure A.3.2: *Weight allocation for a view on all government interest rates and the OMXS30 with uncertainty level 0.1 with* 1% *transaction costs for buying and zero transaction costs for selling PPNs and NPPCs*

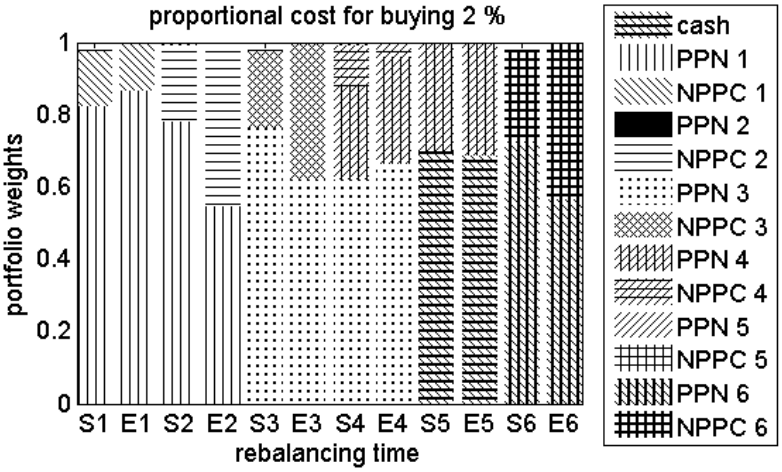


Figure A.3.3: *Weight allocation for a view on all government interest rates and the OMXS30 with uncertainty level 0.1 with* 2% *transaction costs for buying and zero transaction costs for selling PPNs and NPPCs*

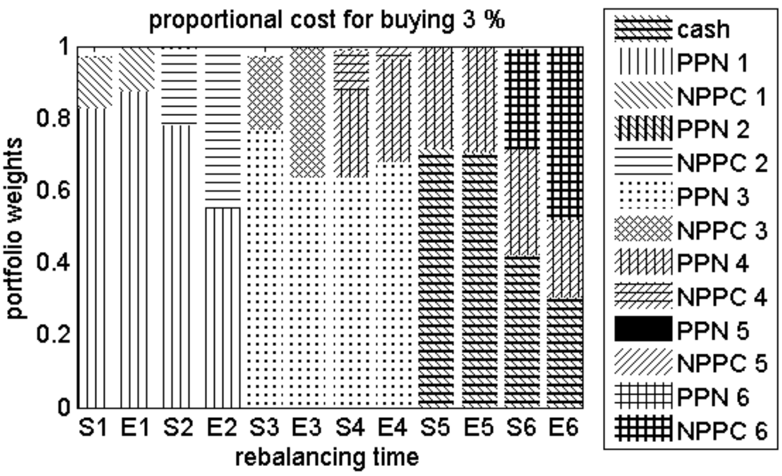


Figure A.3.4: *Weight allocation for a view on all government interest rates and the OMXS30 with uncertainty level 0.1 with* 3% *transaction costs for buying and zero transaction costs for selling PPNs and NPPCs*

A.3.5 Weight allocation diagrams under transaction costs for buying and selling structured products

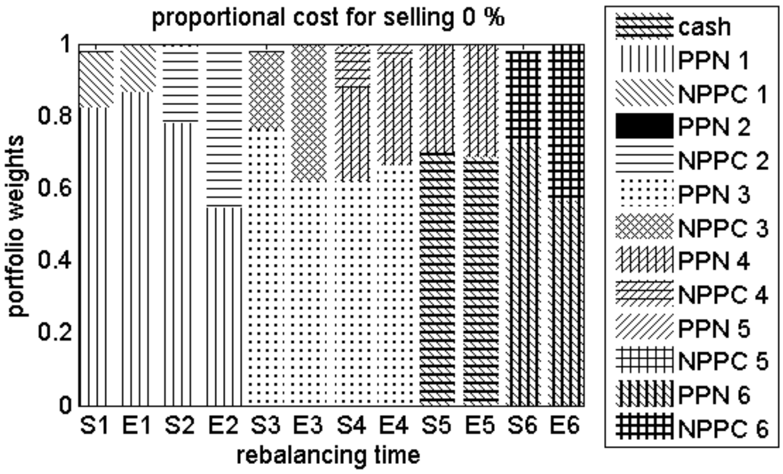


Figure A.3.5: *Weight allocation for a view on all government interest rates and the OMXS30 with uncertainty level 0.1 with* 2% *transaction costs for buying and no cost for selling PPNs and NPPCs*

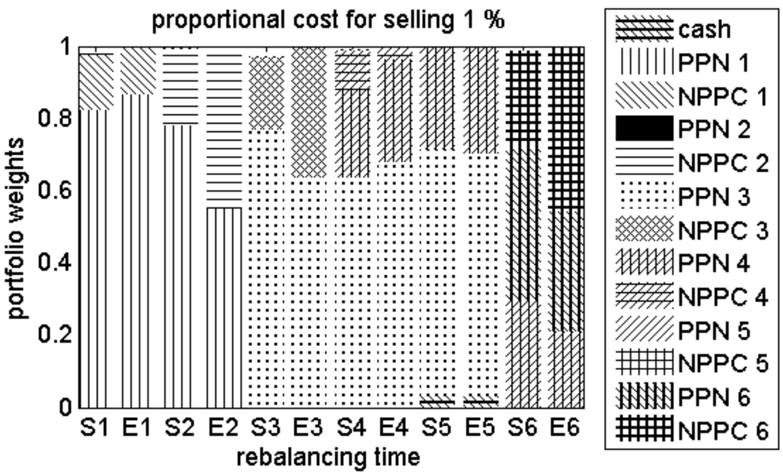


Figure A.3.6: *Weight allocation for a view on all government interest rates and the OMXS30 with uncertainty level 0.1 with* 2% *transaction costs for buying and* 1% *transaction costs for selling PPNs and NPPCs*

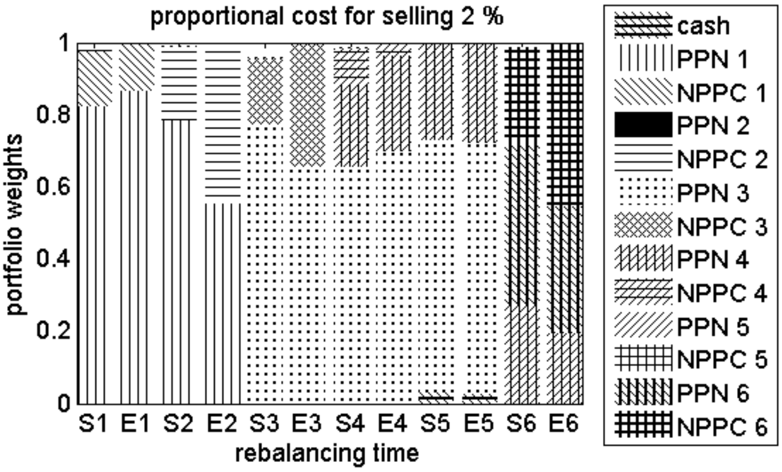


Figure A.3.7: *Weight allocation for a view on all government interest rates and the OMXS30 with uncertainty level 0.1 with* 2% *transaction costs for buying and* 2% *transaction costs for selling PPNs and NPPCs*

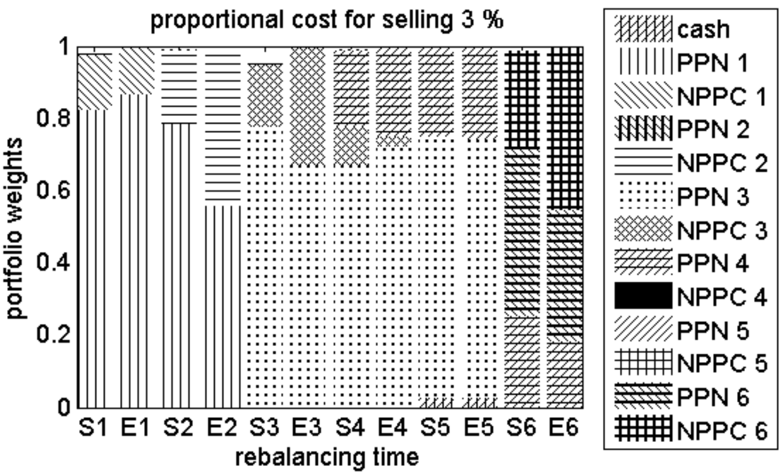


Figure A.3.8: *Weight allocation for a view on all government interest rates and the OMXS30 with uncertainty level 0.1 with* 2% *transaction costs for buying and* 3% *transaction costs for selling PPNs and NPPCs*

A.4 Factor model for changes in interest rate, CDS rates and index level

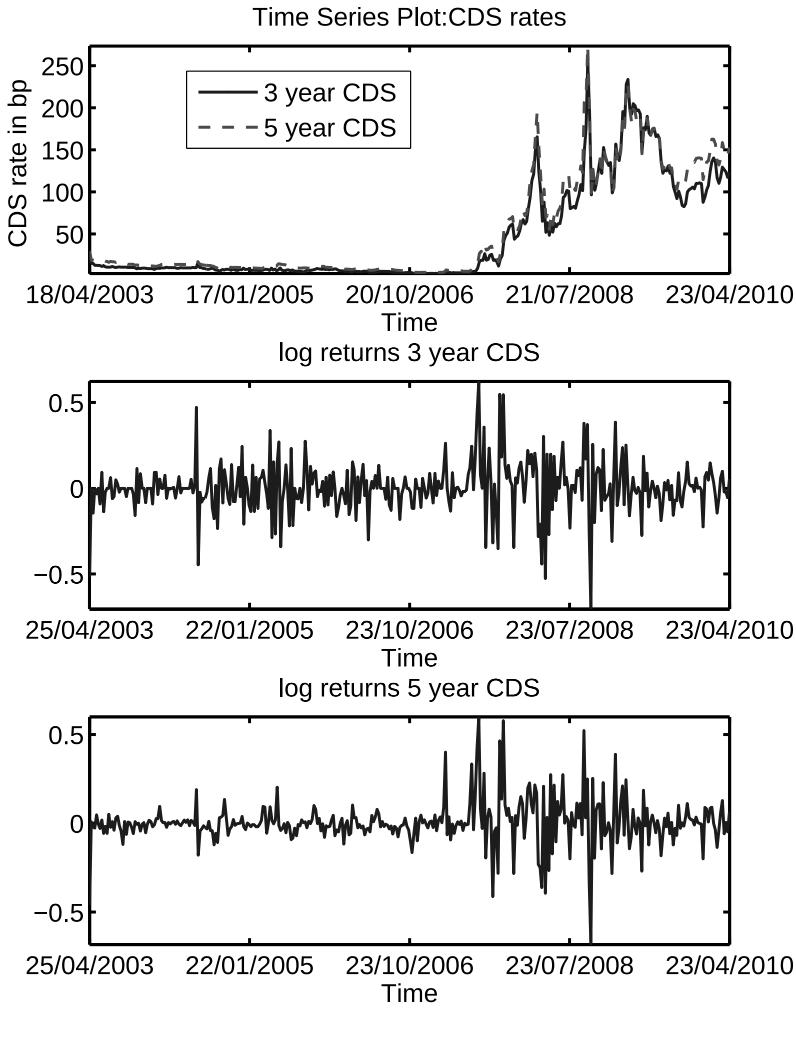


Figure A.4.1: *Time series of CDS rates and weekly log returns of CDS rates for RBS with 3 and 5 years time to maturity recorded of the time span from April 2003 to April 2010*

Period		Explanatory Power						
Start	End	PC 1	PC 2	PC 3	PC 4	PC 5	PC6	Sum
Apr-03	Apr-07	46.21	17.30	13.95	10.21	4.84	3.77	96.29
Jul-03	Jul-07	45.48	16.59	14.08	10.45	5.23	4.27	96.10
Oct-03	Oct-07	45.14	18.80	13.79	10.19	4.61	3.41	95.94
Jan-04	Jan-08	44.07	19.97	14.32	9.54	4.77	2.80	95.48
Apr-04	Apr-08	43.74	20.67	14.47	9.20	4.79	2.75	95.61
Jul-04	Jul-08	44.80	21.13	14.02	8.58	4.64	2.72	95.89
Oct-04	Oct-08	45.65	21.26	14.01	8.77	4.40	2.29	96.38
Jan-05	Jan-09	47.74	21.91	11.41	9.18	4.98	1.97	97.19
Apr-05	Apr-09	45.72	22.35	14.92	9.00	3.77	1.90	97.66
Jul-05	Jul-09	47.07	22.60	13.46	9.08	4.12	1.56	97.89
Oct-05	Oct-09	47.38	22.88	12.86	8.94	4.27	1.64	97.97
Jan-06	Jan-10	46.40	23.40	12.32	8.97	4.93	1.78	97.81
Apr-06	Apr-10	46.29	23.69	12.06	8.83	5.31	1.75	97.93
Apr-03	Apr-10	45.26	22.34	13.29	9.11	5.04	1.88	96.92

Table A.4: *Sample dependence of the explanatory power for 13 quarter-yearly overlapping time intervals consisting of 4 years of weekly observations. The last row indicates the explanatory power for the whole data set ranging from April 2003 to April 2010.*

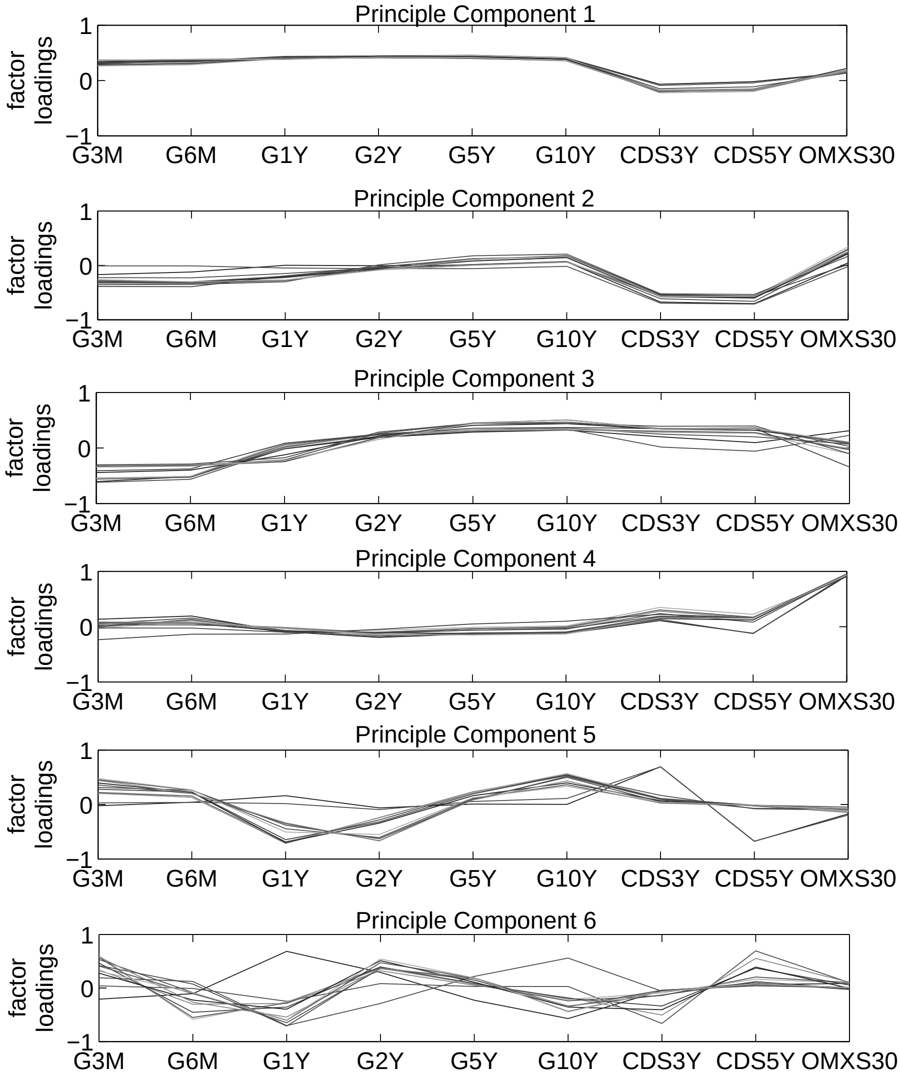


Figure A.4.2: *Sample dependence of the factor loadings of the first six PCs. One line in a subplot represents the factor loadings of the indicated PC for one sample. The investigation is performed on 13 quarter-yearly overlapping samples consisting of 4 years of weekly observations. All samples are taken from a population of 367 weekly observations ranging from April 2003 to April 2010*

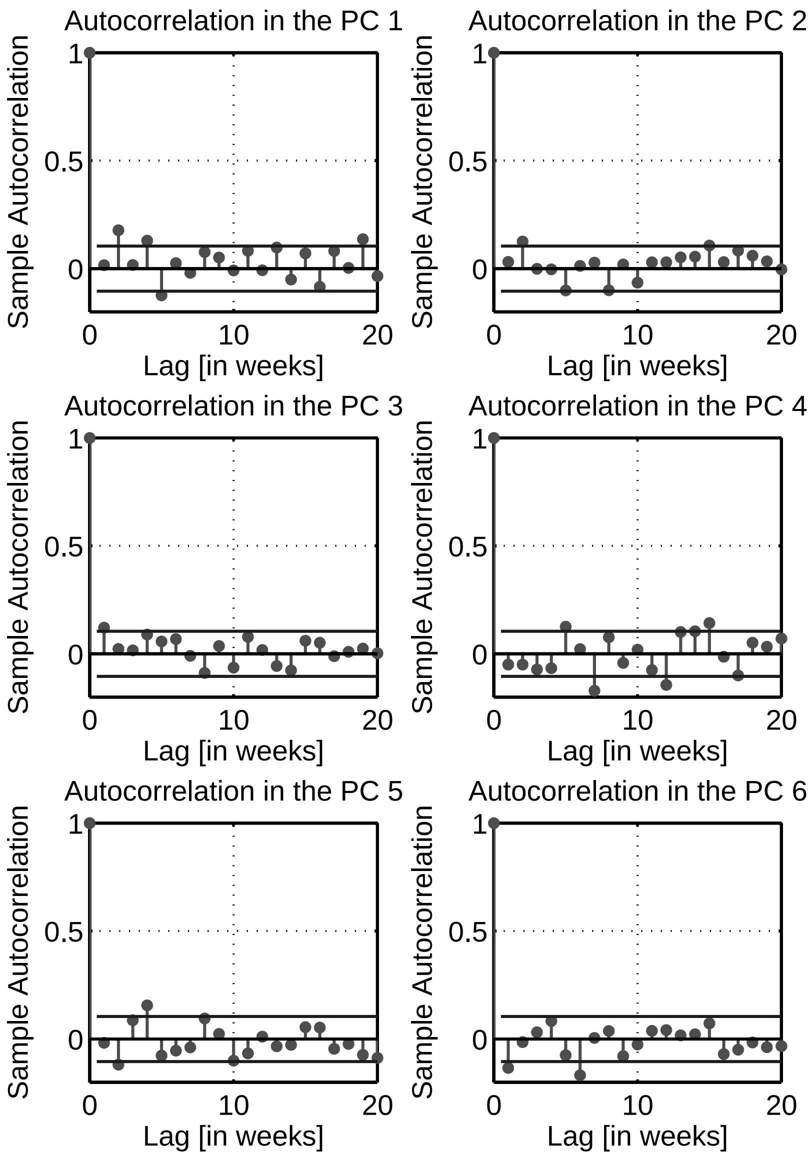


Figure A.4.3: *Autocorrelation plot of each of the first six PCs. The sample is composed of 367 weekly observations ranging from April 2003 to April 2010*