Master's Thesis

The Analysis of Different Financial Risk Measures in Hydro-Electric Portfolio Optimization

Somar Koria

Abstracts

During the last two decades the electricity market has gone through overwhelming changes. From monopoly to market orientation and exchange markets, where contracts are traded on the spot market not only daily contracts but also forward and futures contracts. This development has made risk management an important part of the utility companies with hydroelectric assets. It is in this area that this thesis affects and it gives the development of risk management a push, which leads to higher net revenues for utility companies. The contribution of this thesis is to prove that the new formulation of CVaR implemented in Stochastic Dynamic Programming (SDP)/Stochastic Dual Dynamic Programming (SDDP), which are state-of-the-art in algorithms for mid-term hydroelectric assets operation optimization, is the best risk measure for minimizing the risk of a real electric system. The new CVaR formulation is proven to be more efficient in the sense that it gives higher value net revenues.

Acknowledgements

I would like to thank my family for their never ending support and help in everything during my years of study.
I also would like to thank my tutors PhD student Steffen Rebenneck, Distinguished Professor Panos Pardalos at University of Florida and professor Boualem Djehiche at the Royal Institute of Technology, for the interesting discussions, the many advices and the valuable comments. And at last but not least I thank my friends for making my years at college unforgettable.

Somar Koria
Stockholm, October 18, 2010

Contents

1. Introduction

During the non-competitive market period in the industry, the electricity sector was considered a natural monopoly, which was not subjected to the rules of competition in the EU treaties. The development of the electricity market in most developed countries have gone through a reform evolving from public-monopoly to a competitive de-regulated environment. Among all the EU states,the UK, the Nordic countries and Germany were able to develop functioning competitive markets. These market changes have lead to the development of the exchange markets where in the beginning it was just daily contract but later on forward and futures contracts were introduced. Which gives the means to further understand the reason of existence of the problem that is analyzed in this thesis and identification of the emergence of electricity financial markets in the various countries. The roll of the exchange market is of great importance because of the possibility of hedging against loss or low revenues. This exchange market has become centralized in regions and has help to develop a healthy competition between market players. One example of such market is the nordic NordPool. Hence, on the free competitive market the electricity industry had to adapt to the same rules that other industries had, but there are one difference and that is the property of electricity which aren't the same as other products.

With the liberalization, every energy utility company are seeking to maximize their net revenues by operate their hydroelectric assets' portfolio in a time dynamic way that is by deciding their reservoir operation and trading volume in the exchange market. The whole optimization are subject to their defined financial risk profile. One of the latest contributions to area is the state-of-the-art algorithm in mid-term hydroelectric optimization, the SDP/SDDP, with the implemented risk control process of CVaR (see [Iliadis et al.(2008)]).

It should also be mentioned that the majority of risk management models used in the industry today, measure today's risk for the entire time horizon a sum of the net revenues during the horizon. This approach leaves the company exposed to attain prohibitive levels of low net revenues during this period. A process to cope with the non-static nature of risk and avoid missing any information about the risk of a portfolio is by measuring it in various periods.

In [Iliadis et al.(2008)] it was shown that the theory and the new definition of CVaR worked for a single hydroelectric power plant. The question is, will it work in a more realistic system of hydroelectric and thermal power plants like it is in a real utility companies. We set up a more realistic system but a bit smaller than it is in reality (see figure 1) and for this system we will try to prove that the new formulation of CVaR implemented in SDP/SDDP gives us the best revenues compare to other risk measures. In this thesis, we apply the financial risk control-processes in the optimization algorithm for a single period constraining the sum of net revenues scenario at the end of the time horizon, this is done without any loss of generality or precision. We will also focus on hydroelectric assets, having access to spot and forward physical markets. The company is a price taker in our model.

The hydroelectric asset operation optimization subjected to financial risk, is characterized as a large-scale and stochastic problem, with two stochastic variables; water inflows and electricity spot prices. The later problem can be solved by using the hybrid Stochastic Dynamic Programming (SDP) / Stochastic Dual Dynamic Programming (SDDP) algorithm (see [Pereira et al. (1991)] and [Gjelsvik et al. (1996)]). An important property of the electricity is that the produced electricity has to be consumed simultaneously. This raises an important management task, how to maintain balance in production/consumption at all times. This property leads to the electricity markets' net revenue distribution have the characteristics of skewness (larger number of upsides than downsides represented by lognormal distributions) and kurtosis ("heavy tails" resulting from "price spikes"), (see [Clewlow et al. (2000)]). The latter characteristics have to be considered when applying a financial risk control process within this context. Through the developments made, we make use of risk control processes such as Rmin (see [Fleten (2000)], [Mo et al. (2001)b] and]Kristiansen (2004)]), VaR (see [Roy (1952)], [Markowitz (1959)] and [Bava (1978)]) and CVaR (see [Rockafellar et al. (2000)]). The widely used risk indicators in academia and industry are VaR and, its enhanced extension, CVaR. These two indicators had not been directly implemented as risk control processes in SDP/SDDP, until [Iliadis et al. (2008)] implemented CVaR in SDP/SDDP. Instead, companies use the already implemented non-probabilistic[1] risk control process, Rmin, as a proxy to simulate the risk control on the net revenue distribution of VaR and CVaR. Nevertheless, there are portfolio optimality issues that arise when Rmin is used as a proxy. The nature of the SDP/SDDP algorithm does not allow the direct implementation of all types of risk control-processes because of its complex mathematical formulation. Therefore, it is necessary to select one of risk control-processes that is appropriate for the electricity industry, one that can be implemented efficiently in the optimization algorithm and yield coherent results. CVaR, although is appropriate for the electricity industry, it has a multistate[2] and multistage[3] nature, thus making its implementation directly into SDP/SDDP algorithm not possible. So, in order to overcome this problem [Iliadis et al (2008)] proposed a mathematical formulation that makes the implementation of CVaR in SDP/SDDP possible. To develop the theory further it is important to prove it, for a more realistic system with the different parts of electricity production in a utility company. In [Iliadis et al. (2008)] it was shown for a single hydro electric power plant. Hence, this model will be expanded to prove that the theory still works.

In this thesis we are going to explain how the SDP, SDDP and SDDP/SDP works, but the prove we are searching for are derived from optimization in a Mixed Integer Linear Programming (MILP).

[1]The non-probabilistic nature of Rmin, renders it a non-applicable risk control process when used directly on its own

[2]We define the constraints that involve state variables across the stage as multistate

[3]We define the constraints that involve state variables across the entire horizon and in a non sequential order as multistage

2. Background and Theory

We will try to explain the general property of the operational research techniques such as SDP, SDDP and SDDP/SDP and the different risk measure. To see the formulation of CVaR implemented in the SDDP/SDP we refer to [Iliadis et al. (2008)] were the problem are presented and implemented. The main problem in this thesis is to prove that the new CVaR formulation works for bigger system (that is system bigger than only one hydroelectric plant) compared to VaR and Rmin, therefore we only will explain the theory of the operational research techniques and the risk measures.

2.1 Operational Research techniques

Because of the complexity of optimizing the operation of hydroelectric assets and the direct relation between their operation and their hedging strategy, also as a result of the application and implementation of risk control process, the computation of a hedging strategy should be made jointly with the operation. Therefore, to get a greater understanding of the optimization, we present the theory of SDP (see [Bellman R. (1957)]) and discuss it's usability for solving the mid-term hydroelectric operation optimization problem. Due to the large size of the problem examined in this thesis, we refer to the limitations of SDP and hence present and discuss SDDP and then SDP/SDDP, which is suitable for this problem class. But to begin with some background information will be presented.

The problem (60)-(76) could be solved as a large LP calculating all the decision variables at one stage. However, the actual scheduling problem can involve a planning horizon of several years, several hydroelectric plants in different cascades and a large number of joint water inflow and spot market price scenarios. Due to the exponential increase of inflow branches with time, the resulting stochastic optimization problem quickly becomes computationally infeasible. This computational infeasibility has motivated the development of solution processes based on a state-space formulation described next.
To reduce the dimension, we solve this large optimization problem by decomposing it in time stages. More specifically, we solve smaller sub-problems using the Bellman principle of optimality (see [Bellman (1957)]), which states that:

"No matter in what state of what stage one may be, in order for a policy to be optimal, one must proceed from that state and stage in an optimal manner."

The elusive principle can be reformulated as maximizing the sum of immediate benefits (IB) plus expected future benefits (FB) at each stage t. Immediate benefits (IB) take into account sales on the spot market and costs in buying from the spot market. Future benefits (FB) are the cumulated immediate respective benefits from the stage $t+1$ until the end of the planning horizon T. Since these future benefits are stochastic due to the price and hydrologic uncertainties, we take their expected value. For every stage, the FB and IB are represented by their respective functions (see Figure 1).
The derivative of the FB function (FBF), for the dimension of water volume reservoir and FB, represents the future water value and the derivative of the IB function (IBF) the immediate water value. Hence, according to the decision variable that we are interested in calculating

its sensitivity with respect to the objective function, we calculate its derivative. Therefore, for the water reservoir volume decision variable, the water values, inform on the change in the total net revenues that would result from a small change in the availability of water in the reservoir. At the optimal solution of each sub-problem, the immediate and expected future water values are equal (see Figure 1). This is the optimal trade-off between the immediate and future use of water in an uncertain environment. Based on the problem

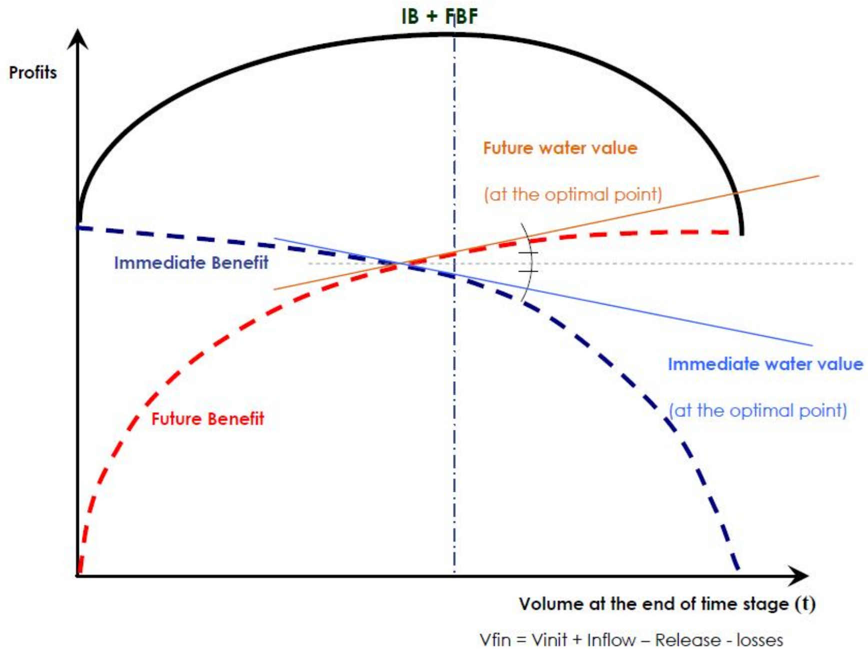


Figure 1: Water values at the optimal solution - example with the release decision for a single reservoir.

formulated in (3)-(19), given the initial storage $v_t(i)$, the inflow at stage t, $a_t(i)$, where through its stochastic inflow model is conditioned from the value at stage $t-1$, $a_{t-1}(i)$, the spot sell price at stage t, Π_t^s, the spot buy price at stage t, Π_t^b, where through its stochastic price model is conditioned from the value at stage $t-1$, Π_{t-1}^s and Π_{t-1}^b, and the (FBF) $\beta_{t+1}(v_{t+1}(i), a_t(i), \Pi_t^s, \Pi_t^b)$, the one stage hydroelectric dispatch problem is formulated as:

$$\beta_t(v_t(i), a_{t-1}(i), \Pi^s_{t-1}, \Pi^b_{t-1}) = max[RS^s_t - CS^b_t - \sum_{k=1}^{K} g_t(k)c(k) + \beta_{t+1}(v_{t+1}(i), a_t(i), \Pi^s_t, \Pi^b_t)] \quad (1)$$

subject to:

$$v_{t+1}(i) = v_t(i) - u_t(i) - w_t(i) + a_t(i)t_{sds} + \sum_{j \subset U(i)} (u_t(j) + w_t(j))\} \; for\ i = 1, ..., I \quad (2)$$

$$v_{t+1}(i) \geqslant 0 \qquad for\ i = 1, ..., I \quad (3)$$

$$a_t(i) \geqslant 0 \qquad for\ i = 1, ..., I \quad (4)$$

$$u_t(i) \geqslant 0 \qquad for\ i = 1, ..., I \quad (5)$$

$$w_t(i) \geqslant 0 \qquad for\ i = 1, ..., I \quad (6)$$

$$v_{t+1}(i) \leqslant v_{max}(i) \qquad for\ i = 1, ..., I \quad (7)$$

$$u_t(i) \leqslant u_{max}(i) \qquad for\ i = 1, ..., I \quad (8)$$

$$g_t(k) \leqslant g_{max}(k) \qquad for\ k = 1, ..., K \quad (9)$$

$$e_t(i) = \rho(i)u_t(i)\frac{t_{sdh}}{t_{sds}} \qquad for\ i = 1, ..., I \quad (10)$$

$$e_t(i) \geqslant 0 \qquad for\ i = 1, ..., I \quad (11)$$

$$RS^s_t = \Pi^s_t e^s_t \quad (12)$$

$$CS^b_t = \Pi^b_t e^b_t \quad (13)$$

$$\sum_{i=1}^{I} e_t(i) + e^b_t + \sum_{k=1}^{K} g_t(k) = e^s_t + d_t \qquad for\ i = 1, ..., I, k = 1, ..., K \quad (14)$$

The variables $v_t(i)$, $a_{t-1}(i)$ and Π_{t-1} in the equations above are the state variables of the SDP recursion. The function $\beta_t(v_t(i), a_{t-1}(i), \Pi_{t-1})$ represents the expected operational cost from stage t to the final stage T, assuming that the initial storage vector in stage t is $v_t(i)$, the observed inflow vector in the previous stage is $a_{t-1}(i)$ and the observed price in the previous stage is Π_{t-1}.
Thus, the major problem becomes the calculation of the FBF for every stage as accurately as possible. For that reason, we have recourse to the discreet state and space representation of the problem using Stochastic Dynamic Programming.

2.1.1 SDP algorithm

In SDP, we use discrete reservoir volume values and solve the problem for each stage and state. Each solution represents a point in the FBF. Using interpolation techniques such as SP-lines we can define the function passing through these points (see [Tejada-Guilbert Johnson et al. (1993)] and [Johnson et al. (1993)]).
We describe the SDP recursion steps for the case of net revenue maximization. We consider for each stage and state as state variable the initial reservoir volume at this stage (equal to the ending volume of the previous stage), the water inflow and spot market price of the previous stage. For dimensionality reasons and illustration purposes, the steps presented refer to a single reservoir. That means that the figures used for the description of the SDP Recursion steps are for a single reservoirs. These steps are:

1. for each stage (t) (typically a week or month) define a set of system states indexed by $m = 1, \ldots, M$, for example, reservoir levels at 100%, 90%, etc. until 0%. The following figure (see Figure 2) illustrates the system state (nodes) definition for a single reservoir. Note that the initial state (i.e. storage levels at the beginning of the first stage) is assumed to be known.

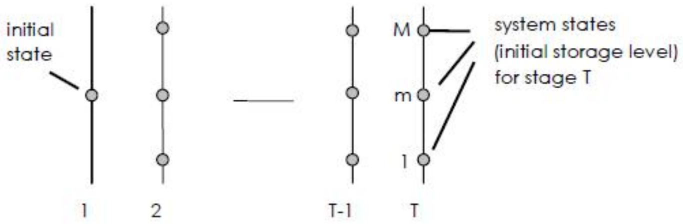


Figure 2: Definition of System States in SDP recursion.

2. start with the last stage, T, and solve the one-stage hydroelectric dispatch problem using the previous stage's inflows and prices to calculate the actual ones assuming that the initial reservoir storage corresponds to the first storage level selected in step (1) - for example, 100%. Because we are at the last stage, assume that the future benefit function is zero[4]. The procedure is illustrated in Figure 3.

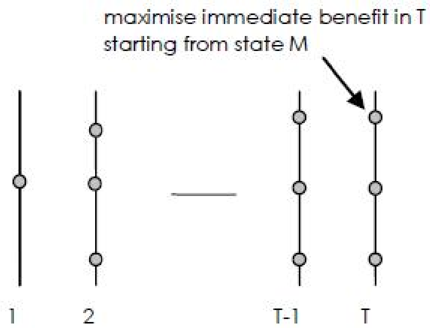


Figure 3: Optimal Strategy Calculation - Last Stage.

3. Calculate the expected operational benefit associated to storage level 100%. This is the first point of the expected future benefit function for stage $T-1$ i.e. $\beta_T(v_T(i), a_{T-1}(i), \Pi_{T-1})$. Repeat the calculation of expected operation benefits for the remaining states in stage T. Interpolate the future benefits between calculated stages, and produce the FBF $\beta_T(v_T(i), a_{T-1}(i), \Pi_{T-1})$ for stage $T-1$, as illustrated in Figure 4.

4. The process is then repeated for all selected states in stage $T-1$, as illustrated in Figure 5. Note that the objective is now to maximize the immediate operation benefit in stage $T-1$ plus the expected future benefit, given by the previously calculated FBF.

[4] This consideration is only occurring in the beginning. This however does not mean that the dam is empty, but the horizon ends there. Using as initial conditions the reservoir level of the beginning and the end of the horizon, we proceed to the system optimization. In addition, in order not to influence the optimal reservoir level of the initial and final stage we use buffer years before and after the year that we will analyze. This way the effects of the initial conditions are smoothed out.

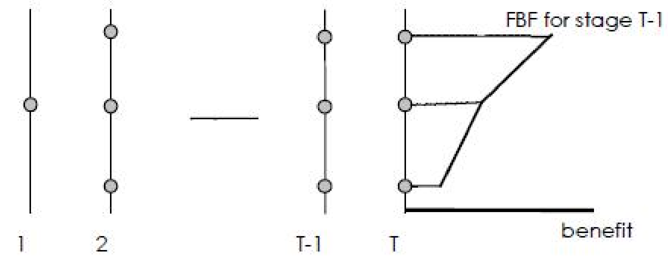


Figure 4: Calculation of the FBF for Stage T-1.

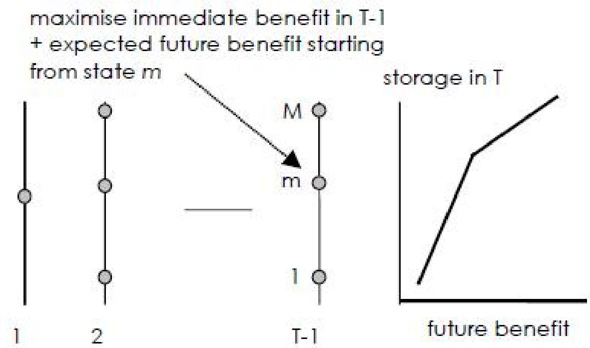


Figure 5: Calculation of Operation Benefits for Stage T-1 and FBF for stage T-2.

5. Repeat the procedure of step (3) and step (4) for the remaining stages $T-2$, $T-3$ etc.

The SDP scheme is straightforward to implement and has been used widely in most hydro-dominated countries. However, as seen above, the SDP recursion requires the enumeration of all combinations of initial storage values with previous water inflows and spot market prices. Consequently, computational effort increases exponentially with the number of reservoirs, known as "curse of dimensionality" of DP (see [Pereira et al. (1985)] and [Pereira et al. (1991)]). For this reason, it has became necessary to develop computationally feasible state-space schemes. The traditional process has been to reduce system dimensionality by the aggregating system reservoirs into one reservoir that represents the energy production capability of the cascade.

2.1.2 SDDP algorithm

In order to solve the problem of dimensionality, an extension of the SDP algorithm called Stochastic Dual Dynamic Programming (SDDP) was developed by [Pereira et al. (1991)]. The basic idea of SDDP is to construct an approximation of the FBF that does not rely on interpolation techniques to determine the value of the functions between the grid points, as in SDP, but on extrapolation based on a point and its slope (see Figure 6). Hence, as the FBF is described by a piecewise linear function (see Figure 7), the concavity of the described FBF has to be guaranteed[5]. The slope coefficient calculation consists on the partial derivative of the objective function at optimal solution, with respect to the state variable we wish to calculate the slope coefficient. Through the calculation of the abovementioned derivative and the application of chain rule, we can identify the value of the Lagrange multipliers[6].Our proposed process is to discretize the reservoir volume values state variable, as in the traditional SDP scheme, and apply the SDDP scheme while using the other state variables.

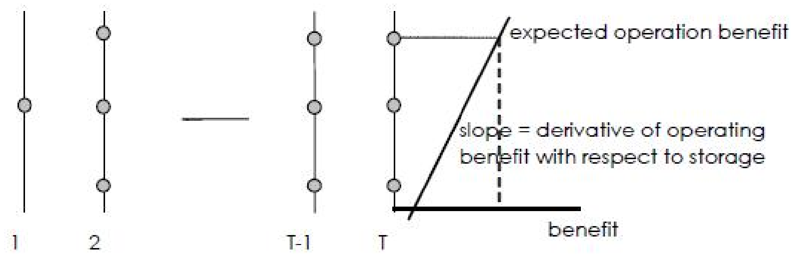


Figure 6: It illustrates, for a single reservoir (for reasons of dimensionality and illustration), the Dual DP calculation of expected operation cost and FBF slope for the last stage, initial state = 100% of reservoir level (step (3) of the traditional DP procedure).

[5]The process of a non-concave (or non-convex) function is not possible since the monotonicity is not of the same sign for the whole function. Hence, try to process a function for which the monotonicity sign changes then we cannot guarantee an envelope of piecewise linear approximation for this function.

[6]It equal the partial derivative of the objective function in the optimal solution with respect to the right-hand-side (RHS) of the constraints associated with the state variable for which we wish to calculate the slope for. This is the explanation for the use of the word "Dual" in SDDP, where the solutions comes from.

The last-stage dispatch problem is shown below (note that the future benefit function in this stage, $\beta_{T+1}(v_{T+1}(i), a_T(i), \Pi_T^s, \Pi_T^b)$, is set to zero):

$$W_T = max[RS_T^s - CS_T^b - \sum_{k=1}^{K} g_T(k)c(k)] \tag{15}$$

subject to:

$$v_{T+1}(i) = v_T(i) - u_T(i) - w_t(i) + a_T(i)t_{sds} + \sum_{j \subset U(i)} (u_t(j) + w_t(j))\} \quad for\ i = 1, ..., I \tag{16}$$

$$v_{T+1}(i) \geqslant 0 \qquad for\ i = 1, ..., I \tag{17}$$

$$a_T(i) \geqslant 0 \qquad for\ i = 1, ..., I \tag{18}$$

$$u_T(i) \geqslant 0 \qquad for\ i = 1, ..., I \tag{19}$$

$$w_T(i) \geqslant 0 \qquad for\ i = 1, ..., I \tag{20}$$

$$v_{T+1}(i) \leqslant v_{max}(i) \qquad for\ i = 1, ..., I \tag{21}$$

$$u_T(i) \leqslant u_{max}(i) \qquad for\ i = 1, ..., I \tag{22}$$

$$g_T(k) \leqslant g_{max}(k) \qquad for\ k = 1, ..., K \tag{23}$$

$$e_T(i) = \rho(i)u_T(i)\frac{t_{sdh}}{t_{sds}} \qquad for\ i = 1, ..., I \tag{24}$$

$$e_T(i) \geqslant 0 \qquad for\ i = 1, ..., I \tag{25}$$

$$RS_T^s = \Pi_T^s e_T^s \tag{26}$$

$$CS_T^b = \Pi_T^b e_T^b \tag{27}$$

$$\sum_{i=1}^{I} e_T(i) + e_T^b + \sum_{k=1}^{K} g_T(k) = e_T^s + d_T \qquad for\ i = 1, ..., I, k = 1, ..., K \tag{28}$$

It is well known from Linear Programming (LP) theory that there is a set of Lagrange multipliers associated to the set of constraints, and so is the case with the abovementioned problem. The multipliers associated to the water balance equation, $\lambda_T^L(i)$, represent the derivative of W_T with respect to a variation in initial storages $v_T(i)$:

$$\lambda_T^L(i) = \frac{\partial W_T}{\partial v_T(i)} \qquad for\ i = 1, ..., I \tag{29}$$

We see in Figure 7 that the expression above corresponds to the slope of the FBF for stage $T-1$. Figure 8 shows the calculation of operation net revenue and FBF slopes for each state in stage T for a single reservoir (for reasons of dimensionality and illustration). We see that the FBF $\beta_T(v_T(i), a_{T-1}(i), \Pi_{T-1})$ for stage (T) corresponds to the piecewise net revenue surface produced by taking the linear segment with the highest revenue value in each state (concave hull).

The hydroelectric dispatch for the stage $T-1$ is represented as a LP problem:

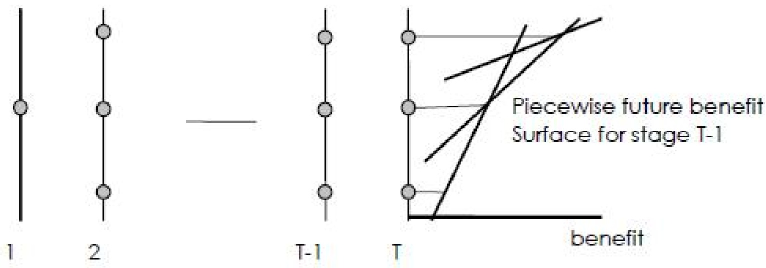


Figure 7: Calculation of a Piecewise FBF for Stage T-1.

$$\beta_{T-1}(v_{T-1}(i), a_{T-2}(i), \Pi^s_{T-2}, \Pi^b_{T-2}) = max[RS^s_{T-1} - CS^b_{T-1} - \sum_{k=1}^{K} g_{T-1}(k)c(k) + \beta_T(v_T(i), a_{T-1}(i), \Pi^s_{T-1}, \Pi^b_{T-1})] \quad (30)$$

subject to:

$$v_T(i) = v_{T-1}(i) - u_{T-1}(i) - w_{T-1}(i) + a_{T-1}(i)t_{sds} + \sum_{j \subset U(i)} (u_t(j) + w_t(j))\} \quad for\ i = 1, ..., I \quad (31)$$

$$v_T(i) \geqslant 0 \quad for\ i = 1, ..., I \quad (32)$$

$$a_{T-1}(i) \geqslant 0 \quad for\ i = 1, ..., I \quad (33)$$

$$u_{T-1}(i) \geqslant 0 \quad for\ i = 1, ..., I \quad (34)$$

$$w_{T-1}(i) \geqslant 0 \quad for\ i = 1, ..., I \quad (35)$$

$$v_T(i) \leqslant v_{max}(i) \quad for\ i = 1, ..., I \quad (36)$$

$$u_{T-1}(i) \leqslant u_{max}(i) \quad for\ i = 1, ..., I \quad (37)$$

$$g_{T-1}(k) \leqslant g_{max}(k) \quad for\ k = 1, ..., K \quad (38)$$

$$e_{T-1}(i) = \rho(i)u_T(i)\frac{t_{sdh}}{t_{sds}} \quad for\ i = 1, ..., I \quad (39)$$

$$e_{T-1}(i) \geqslant 0 \quad for\ i = 1, ..., I \quad (40)$$

$$RS^s_{T-1} = \Pi^s_{T-1} e^s_{T-1} \quad (41)$$

$$CS^b_{T-1} = \Pi^b_{T-1} e^b_{T-1} \quad (42)$$

$$\sum_{i=1}^{I} e_{T-1}(i) + e^b_{T-1} + \sum_{k=1}^{K} g_{T-1}(k) = e^s_{T-1} + d_{T-1} \quad for\ i = 1, ..., I, k = 1, ..., K \quad (43)$$

$$\beta_T \leq \tau^m_T + \kappa^m_T(i)v_T(i) + \varepsilon^m_T(i)a_T(i) \quad for\ m = 1, ..., M \quad (44)$$

where:
$\tau^m_t(i)$: intersection of segment m of the hyper-plane with the y-axis at stage t - [kEUR]
$\kappa^m_t(i)$: Slope coefficient of the segment m for the $v_t(i)$ state variable at stage t for reservoir i - [-]
$\varepsilon^m_t(i)$: Slope coefficient of the segment m for the $a_t(i)$ state variable at stage t for reservoir i - [-]
m: number of linear segments of the FBF - [-]

In the FBF described above, we should have also included a term for the price state variable. Nevertheless, as it will be shown in the following paragraph, the price cannot be accounted for a state variable but it will be transformed to a parameter. The FBF is represented by the scalar variable β_T and M linear constraints $\beta_T \leq \tau_T^m + \kappa_T^m(i)v_T(i) + \varepsilon_T^m(i)a_{T-1}(i)$, where m is the number of linear segments. As shown in Figure 13, for the dimension of v_T (single reservoir) the inequalities $\beta_T \leq \tau_T^m + \kappa_T^m(i)v_T(i) + \varepsilon_T^m(i)a_{T-1}(i)$ represent the piecewise characteristic of this function (for any v_T, the segment with the lowest value $\kappa_t^m v_t + \tau_t^m$ will always be binding).

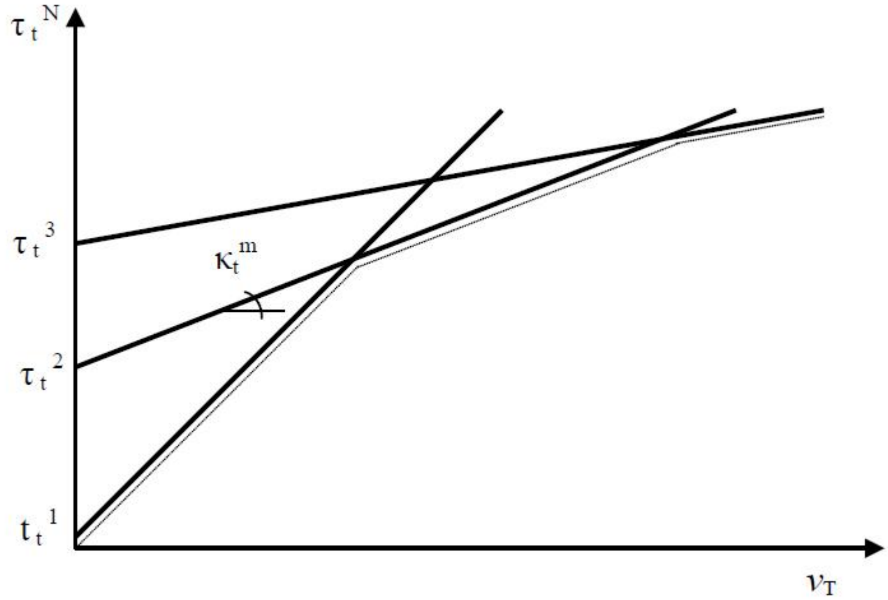


Figure 8: Piecewise linear (envelop) of the future benefit function for a single reservoir.

Hence, summarizing the algorithm, the main steps would be:

1. Start with some lower bound of the FBFs.
2. Backward optimize[7] (improve FBF and find upper bound of the problem).
3. Make forward simulation[8] (realistic state space, find lower bound of the problem).
4. Iterate (2) and (3) until stochastic convergence of the upper with the lower bound.

The results are the FBFs (operating policies) and the operating decisions per stage.

[7] The word backward optimization is used for the process of the policy calculation. Since in dynamic programming the policy is calculated through the backward recursion, for that reason the word backward is used.

[8] The forward simulation is the process where, the inflow and price scenarios are used in the calculated policy of the backward optimization, and calculate the optimal solution for each scenario.

2.1.3 Hybrid SDP/SDDP algorithm

The development of SDP/SDDP began with experimentation with nested Benders' decomposition (see [Benders (1962)]) in electricity models this was used and developed further by ([Pereira et al. (1985)], [Jacobs et al. (1995)] and [Morton (1996)]). The problem is that the SDDP scheme cannot be applied directly to the net revenue maximization recursion because the FBF $\beta_{t+1}(v_{t+1}(i), a_t(i), \Pi_t)$ is saddle-shaped (see [Cvatal (1983)]), i.e. concave on the dimensions for storage and past inflows, and convex on the dimension of spot price[9]. Hence, the approximation of this multidimensional surface using Bender's cuts (hyper-planes) is not possible anymore. The problem being considered as such, we have to calculate the Lagrangian multipliers for a function that is convex for one variable (prices) and concave for the other (volume of the reservoir) forming a saddle shaped point. Therefore, we would have to process simultaneously a surface from its upper and lower part. In order to overcome the latter problem the price were transformed from a state variable to a parameter, discretizing thus the problem on the price component. The discretization process is in practice the application of the SDP algorithm for the dimension of price while still maintaining the application of the SDDP algorithm in the dimension of reservoir volume and the other state variables. The latter method applied is called hybrid SDP/SDDP, where we transform the electricity spot market prices to a discrete state variable equivalent to a parameter to the problem. In the new problem, price is the discrete state variable of an SDP algorithm in which each sub-problem for the volume state variable of the latter algorithm is solved by SDDP (see [Gjelsvik et al. (1996)], [Fosso et al. (1999)] and [Pereira et al. (2000)]).
Hydroelectric operation optimization models are very sensitive to the way electricity price forecasting and water inflow forecasting are calculated. More specifically, the inflow process is multidimensional and has strong seasonal components. Forecasting the inflows and capturing the structure of the process and their degree of predictability is of vital importance to hydroelectric scheduling models (see [Tejada-Guilbert Johnson et al. (1995)]). Inside the SDP/SDDP algorithm, the water inflow uncertainty is captured using an autoregressive (AR) model of variable lag for each stage and incremental inflow point, taking into consideration spatial cross correlation. The AR(P) model is calibrated using the historical inflow series that we enter as input in order to generate the synthetic forecasted scenarios. In the formulation below the inflows will be considered through an autoregressive model of order 1, that is AR(1) model:

$$a_t(i) = \omega_{1t}(i)a_{t-1}(i) + \omega_{2t}(i)\xi + \omega_{3t}(i) \tag{45}$$

where, $\omega_{1t}(i)$, $\omega_{2t}(i)$ and $\omega_{3t}(i)$ are model parameters, and ξ is a random variable. As there is one inflow model for each stage of the year (month/week), the index t of the parameter ω is related to the model at stage t.

The electricity prices inside SDP/SDDP model is input in scenarios, which are calculated externally using sophisticated price-forecasting models (see [Mo et al. (2001)] and [Haugstad

[9]The fact that the FBF in the dimension of prices is convex and in the dimension of volumes is concave comes directly form the theory of LP (see [Cvatal (1983)]). Since the prices are a part of the objective function, when they vary, the resulting function is convex. On the contrary, the initial reservoir volumes are on the RHS (right-hand-side) of the constraints and when they vary, the resulting function is concave.

et al. (1998)]). Since the price process is represented using an SDP formulation and discreet prices, we have to guarantee that the prices follow the stochastic process between two stages. Inside the model, the price uncertainty is perceived via a Markov chain process (autoregressive of order 1). In order to calculate the transition probabilities of two consecutive stages we use a clustering method and calculate price clusters for every stage (t), for the electricity prices scenarios. Out of the joint scenarios, we detach the price scenarios and we cluster them. The objective functions described by β, FBF, at each stage, are constructed during the SDDP algorithm iterations for each price cluster (see [Iliadis et al. (2008)]). At the same time all features of the SDDP algorithm such as piecewise functions are still applicable and the overall computational effort is not affected by the representation of the spot price state variable. It should also be mentioned that the spot price does not increase the number of combinations as it is considered as a parameter and not as a state variable (the reason for that is the assumption of the number of price clusters are smaller than the number of hydrological scenarios).

As described, the SDP/SDDP algorithm offers the possibility of taking into account the stochastic water inflows and electricity prices, and considering all technical details of the system while solving the problem within acceptable computational times. The importance of considering a stochastic model can be seen directly in the operation results. Although schedulers might feel more comfortable with deterministic models, the solutions reached underestimate the true operating profits and the risk of spilling water. Deterministic models do not see any value difference between waiting and releasing water in order to learn more about future inflows and prices (see [Araripe Neto et al. (1985)] and [Philbrick et al. (1999)]). Other reason why hydroelectric optimization is computed through the complex algorithm of SDP/SDDP is to tackle problems with large number of hydroelectric assets. In lately, the algorithm based on SDP/SDDP has been applied in several countries[10], and more and more are using it.

Once more we repeat the main steps and calculate the Upper and Lower bound (see [Pereira et al.(1985)] and [Pereira et al.(1991)]):

1. Start with some lower bound of the FBFs.
2. Backward optimize (improve FBF and find upper bound of the problem). In the Dual DP scheme, the piecewise linear segments can be used to extrapolate the FBF values, i.e. it is not necessary to use all combinations of points to obtain a complete (although approximate) FBF. Moreover, if a smaller number of initial storage values are used, a smaller number of linear segments will be generated. As seen in Figure 8, the resulting FBF, which is based on the maximum value over all segments, will then be an upper bound to the "true" function.
3. Make forward simulation (realistic state space, find lower bound of the problem). A lower bound can be obtained by the forward simulation of the system operation, using the set of FBF produced by the recursion scheme. This is because the only FBF that can result in the optimal expected operation net revenue is the optimal function itself; all others, by definition, have to result in lower operation revenues.

[10] in South and Central America, USA, New Zealand, Spain and Norway

4. Iterate (2) and (3) until stochastic convergence of the upper with the lower bound. If the lower bound and the upper bound do not satisfy the convergence criteria defined by the user for the approximation of the FBF, then the backward recursion is executed again with an additional set of state variable values. The natural candidates for the new values are these produced by the forward simulation step. We should note that the linear segments calculated in the previous iteration are retained, because the piecewise FBF is given by the maximum over all segments. In other words, it is possible to improve gradually the FBFs representation.

2.2 Risk Measures

For further understanding, of the different risk control processes, we present the properties of the different risk measures (3.1.1. Rmin, 3.1.2. VaR and 3.1.3. CVaR).

2.2.1 Rmin

The risk control process Rmin (see [Fleten (2000)], [Mo et al. (2001)b] and [Kristiansen (2004)]) is characterized by the minimum acceptable net revenue imposed on a probabilistic distribution of scenarios. Rmin implies that all the scenario revenues that are under the desired level will have to be improved in order to get as much as the imposed level of Rmin or higher. This can be achieved through the simultaneous optimization of the reservoir operation and contracting decisions. If it is implemented as a hard constraint[11], it might lead to an unfeasible solution of the problem due to the impossibility of guaranteeing the desired minimum revenues for a set of scenarios. Moreover in the industry, the minimum net revenue with probability P=1 is neither interesting nor realistic to be achieved[12]. In any case where a minimum net revenue level has to be guaranteed for all scenarios, the designed hedging policy will be costly due to the high-risk aversion of the portfolio and therefore excessive hedging in the market. Thus, in order to allow the Rmin constraint to be violated in certain occasions and, 1) to avoid the above-mentioned feasibility problems, and 2) to simulate a probabilistic risk control process, such that $P \neq 1$, a relaxation technique is applied. In this case, instead of using the hard constraint we use a violation decision variable [13], which takes into account the difference between the imposed level and the solution level (if not achieved). The new constraint including the violation decision variable is relaxed using a Lagrangian Relaxation where the constraint is inserted in the objective function and multiplied by a penalty coefficient. This can be seen as a risk control process using a penalization technique (see [Haneveld Klein et al. (2003)]).
Until now Rmin was the only risk control process that was implemented in SDP/SDDP, and used by the utility companies. Through the description of Rmin we have observed that Rmin is neither a true risk control process nor a probabilistic one. Thus through the Rmin level and the penalization coefficient adjustments, probabilistic constraints such as VaR and its enhanced extension, CVaR, can then be simulated, in the case were SDP/SDDP optimization

[11] Hard constraints represent absolute limitations imposed on a system

[12] In this case, the level of the risk aversion is equivalent to imposing a VaR_{100} equal to a specific level

[13] A violation decision variable is used to represent the difference between the RHS (Right Hand Side) and the LHS (Left Hand Side). Relaxing a decision variable, we allow to the constraint to be violated. According to the severity of the penalty coefficient of the Lagrangian Relaxation, we can increase or reduce the level of the constraint violation

algorithm is used. Hence, the transformation of the hard constraint into a relaxed one inside the objective function using a penalty coefficient aims at making it the simulation of probabilistic constraints. This process is fastidious and computationally consuming process to yield results when used as a proxy. It is not appropriate to be used as a proxy to simulate the VaR risk control process for the net revenues distribution. In the same context it does not guarantee an optimal portfolio, when used as a proxy to simulate the CVaR risk control process on the revenues distribution with respect to the desired CVaR level. This occurs mainly because the two-abovementioned risk control-processes have not been implemented yet in the SDP/SDDP algorithm. The use of Rmin as a risk control process might lead to exceedingly risk adverse portfolios involving costly hedging programs. In [Iliadis et al. (2006)], a series of results, using the tool mentioned in the paragraph above, placing in evidence the weaknesses of the Rmin [14] risk control process and its optimality issues when applied in mid-term hydroelectric assets portfolio operation optimization.

2.2.2 Value-at-Risk

We now introduce the widely used risk measure known as Value-at-Risk (VaR), which is recommended by (see [Basel Committee on Banking Supervision (2004)]). In [Hult and Lindskog (2007)], the Value-at-Risk is defined as such:
Definition: *Given a loss L and a confidence level $\alpha \in (0,1)$, $VaR_\alpha(L)$ is given by the smallest number l such that the probability that the loss L exceeds l is no larger than $1-\alpha$, i.e.*

$$\begin{aligned} VaR_\alpha(l) &= inf\{l \in \mathbb{R} : P(L > l) \leqslant 1-\alpha\} \\ &= inf\{l \in \mathbb{R} : 1 - F_L(l) \leqslant 1-\alpha\} \\ &= inf\{l \in \mathbb{R} : F_L(l) \geqslant \alpha\} \end{aligned} \tag{46}$$

The VaR is defined as the α:th percentile of the portfolio return. In the mid-term hydroelectric portfolio operation optimization imposing a level of VaR for an α probability, leads to guarantying that the net revenues scenarios situated in the upper part of the distribution (belonging to the α:th percentile interval) are higher than the desired level defined. That is, VaR measures the worst expected loss of a portfolio to a specific confidence level during a given period of time under normal market conditions. VaR is widely used as risk management indicator, it has become a standard (see [Risk Metrics (1996)] and [Jorion (2000)]). The reason for this is its simple interpretation and allows the user to focus its analytic resources on "normal market conditions" (see [Basak et al. (2001)]). In addition, [Wang (1999)] shows that VaR can be applied in multi-period optimization of financial portfolios. Hence, the latter is very important since hydroelectric portfolio optimization has to be considered in a multi-period way.

[14] Rmin stands for minimum revenues and is a risk control process that guarantees a desired level of minimum revenues in the portfolio's net revenues distribution. Rmin was the first approach of implementing a risk control process within the SDP/SDDP algorithm. Rmin is implementing using a Lagrangian Relaxation through a penalty coefficient in the objective function of the model. The nature of its implementation allows the use of Rmin as a proxy for other similar risk control process, which are not at all or not directly implementable in the algorithm of SDP/SDDP

For a maximization problem where we seek to maximize $p_{sc}Cx_{sc}$ subject to a VaR level the formulation is:

$$Max \sum_{sc=1}^{SC} p_{sc}Cx_{sc} \tag{47}$$

$$s.t. a_{sc}x_{sc} \leqslant b_{sc} \tag{48}$$

$$a_{sc}x_{sc} - b_{sc} + M\chi_{sc} \geqslant \eta_{VaR} \tag{49}$$

$$\sum_{sc=1}^{SC} p_{sc}\chi_{sc} \leqslant (1-\alpha) \tag{50}$$

$$\chi_{sc} \in \{0,1\} \tag{51}$$

Where p_{sc} is the probability for each scenario, x_{sc} is a decision variable for the weight of each scenario, C, is a coefficient matrix of the different scenarios, in (kEUR), χ_{sc} is a binary decision variable for the selection of each scenario, a_{sc} and b_{sc} are coefficients for each scenario, M is a very large number, η_{VaR} is a number representing the imposed level of VaR, and α is a probability. As observe, the implementation of VaR in a problem formulation requires MILP (Mixed Integer Linear Programming).
The information of how much the loss will be if we find ourselves outside the specified probability is not given. In this case, these events can lead to an undesirable stretch of the lower part of the distribution exceeding VaR. This lack of information is crucial because the characteristics of the electricity markets are skewness and heavy-tailed distribution. In addition, when VaR is used as a risk control process in electricity markets, the resulting portfolio remains exposed to risk as this risk indicator does not control the lower part of the revenues distribution (see [Larsen et al. (2001)]). VaR has also been criticized for its lack of sub-additivity and lack of convexity (see [Artzner et al. (1999)]). The lack of sub-additivity can be problematic in the consideration of a set of portfolios inside a company. A very wide example is in the consideration of a VaR over a department and the individual portfolios of every asset manager. The lack of convexity makes the implementation of VaR as a risk control process in various algorithms using LP, such as SDP/SDDP, not possible. In addition, VaR requires MILP in all the algorithms where it is implemented, increasing thus the overall computational time due to the nature of the solving process while constraining our modeling capabilities. The implementation of the VaR for non-normal distributions, is showing that the resulting scenarios under VaR are riskier (see [Rockafellar et al. (2002)] and [Duffie et al. (1997)]). , hence resulting in lower scenarios than in the initial case, where expected net revenues are not constraint by a risk control process [15]. The latter characteristic is inherent to VaR risk control process.

2.2.3 Conditional Value-at-Risk

Conditional VaR (CVaR) is a more recent extension of VaR that gains steadily appreciation in the industry. The enhanced extension of VaR is defined as following, [Hult and Lindskog (2007)]:
Definition: *For a loss L with continuous loss distribution function F_L the expected CVaR at*

[15] In the case of VaR and for a non-normal distribution there is no control for the lower part of the distribution because of lack of symmetry. This characterizes VaR as a risk control process where the losses are not already incorporated in it

confidence level $\alpha \in (0,1)$ *is given by*

$$CVaR_\alpha(L) = E(L|VaR_\alpha(L)) \tag{52}$$

Hence, CVaR measure the conditional expected value of the distribution lower the VaR level. More specifically, CVaR is a non-symmetrical risk control process and it controls the lower part of the VaR of the revenues distribution (accounting for the skewness and kurtosis). Unlike VaR, CVaR has the desired property of focusing on the size of the loss and not just the probability that the loss will not exceed a certain level. Therefore it is possible to penalizes large and less probable losses, which is a desired characteristic for a financial risk control process applied in the electricity market. Also, for its capacity to account for distribution skewness and asset optionality. The latter is a result of the nature of hydroelectric assets, which is directly related to the existence of a reservoir and their capacity to store water and use it according to the system operation optimization. The skewness in the distribution of profit and loss caused by the price spikes and the options built into any electricity contract needs an asymmetric risk control process that can really penalize extreme events, that is why CVaR is interesting. In addition, the CVaR is characterized as a multi-period coherent risk indicator (see [Artzner et al. (2004)]) which is important in the electricity asset portfolio operation optimization for the cases of multi-period portfolio optimization, coherence includes qualities such as convexity and sub-additivity. Another property of CVaR is that, it has a multistate and multistage nature, thus making it's implementation in SDP/SDDP algorithm impossible. Hence, in order to overcome the latter problem [Iliadis (2008)] proposed a mathematical formulation that makes the implementation of CVaR in SDP/SDDP possible.

In comparison of VaR and CVaR constraints on portfolio selection, for a given confidence level, a CVaR constraint is tighter than a VaR constraint, if the CVaR and VaR bounds coincide (see Alexander et al. (2004)). Furthermore, the formulation of CVaR when solved (optimal solution) provides us with the information about the level of VaR (η_{CVaR} decision variable) for the probability that was defined. Hence bounding CVaR with the objective to control VaR is possible but not optimal.

Until [Rockafellar et al. (2000)] and [Rockafellar et al. (2002)], CVaR had to be considered through the calculation of VaR and hence involving MILP. The latter involvement made it laborious and often impossible to utilize CVaR in several optimization algorithms. Through the development of [Rockafellar et al. (2000)], a significant contribution was made by proposing a practical technique of optimizing CVaR applied as a constraint and the corresponding VaR is calculated, at the same time. [Uryasev (2000)] proposed that a CVaR minimization process can be easily handled using LP optimization techniques . This advantage comes from the fact that CVaR is convex thus making the calculations through LP feasible. Based on [Rockafellar et al. (2000)], [Krokhmal et al. (2001)] showed that this process can also be used for maximizing expected returns under CVaR constraints, as opposed to minimizing CVaR for a specific level of returns. Moreover, it is possible to impose many CVaR constraints with different confidence levels and shape the net revenue (or loss) distribution according to the preferences of the decision-maker. For a maximization problem where we seek to maximize the expected value of $p_{sc}Cx_{sc}$ subject to a CVaR level the formulation is:

$$Max \sum_{sc=1}^{SC} p_{sc}Cx_{sc} \tag{53}$$

$$s.t. Cx_{sc} \leqslant b_{sc} \tag{54}$$

$$Cx_{sc} - b_{sc} - \eta_{CVaR} \geqslant y_{sc} \tag{55}$$

$$\eta_{CVaR} + \frac{\sum_{sc=1}^{SC} p_{sc}y_{sc}}{1-\alpha} \geqslant \Psi_{CVaR} \tag{56}$$

$$y_{sc} \leqslant 0 \tag{57}$$

Where p_{sc} is the probability for each scenario, x_{sc} is a decision variable for the weight of each scenario, C, is a coefficient matrix of the different scenarios, in [kEUR], y_{sc} are auxiliary decision variables to CVaR, for each scenario, η_{CVaR} is an auxiliary decision variable associated to the CVaR constraint, b_{sc} is a coefficient for each scenario related to constraints, Ψ_{CVaR} is a number representing the imposed level of CVaR, and α is a probability. As we can observe from the formulation above, the way CVaR is formulated makes it possible to implement it in an optimization algorithm that uses LP.
Hence, hydroelectric assets have to be modeled within an algorithm, so that the implementation of a risk control process is possible, and which is stochastic, time-dependent and sequential, in order to account for the specificities of hydroelectric assets. At the same time the features of CVaR through LP, the convexity and tractable computational, do indeed strengthen its position in terms of implementation to an optimization algorithm. One such optimization algorithm is SDP/SDDP, which can accommodate risk measures that are based in LP and that are convex.

The nature of the SDP/SDDP algorithm does not allow the direct implementation of all types of risk control processes because of its mathematical formulation complexity. Therefore, it is necessary to select the appropriate risk control process for the electricity industry, one that can be implemented efficiently in the optimization algorithm and yield coherent results. The property of CVaR with its multistate and multistage nature, thus making it's implementation in SDP/SDDP algorithm impossible. To overcome the problem with multistate and multistage nature of CVaR, [Iliadis (2008)] proposed a mathematical formulation that makes the implementation of CVaR in SDP/SDDP possible. As a solution to the multistate problem, a Lagrangian Relaxation (see [Held et al. (1970)] and [Held et al. (1971)]) were used for the specific CVaR constraint components that could not be formulated when using SDP/SDDP. These components will be entered into the objective function with a penalty coefficient. We observe that the calibration of the latter coefficient is a simple process and possible to automate due to its scalar nature. As a solution to the multistage problem, we will insert a series of additional state variables in the FBF of the SDP/SDDP algorithm that will account for their information throughout the whole horizon. Hence, in every stage the FBF will include all the information needed in order for the algorithm to take into account the impact of the state variable for the current stage to the future state variables.

We can observe in figure (Figure 9) the positioning of the three risk control-processes in a distribution. Rmin is represented here as a soft constraint that can take any position (dashed orange line) on the distribution curve, and according to the level of the penalty coefficient, controls the lower-than-the-imposed Rmin level of the distribution. VaR controls the upper part of the distribution according to the probability and the level that is imposed. CVaR corresponds to the expected value of the part under the probability and level of VaR. As a risk control process, it controls the expected value of the distribution's lower part according to the probability and the level imposed and by constraining CVaR we immediately create a lower bound for VaR. The CVaR risk control process is very similar in it's risk approach to the Rmin from a distribution control perspective, that is both the CVaR and the Rmin control the lower part of this percentile. Compared to VaR, CVaR is in a maximization problem always smaller than VaR and by

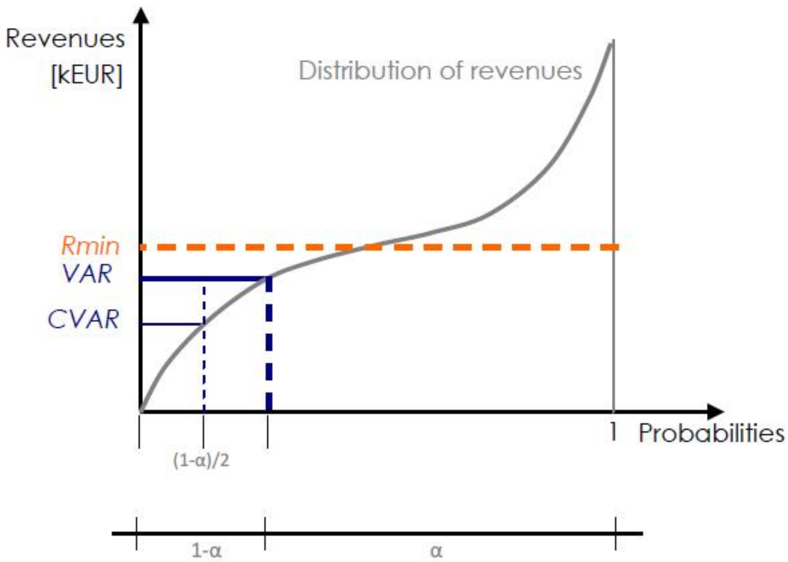


Figure 9: Representation of the Rmin, VaR and CVaR for a given cumulative distribution.

3. The Model

3.1 Geographic Topology

In [Iliadis et al. (2008)] it was proven that the risk measure CVaR implemented in the state-of-the-art algorithm in mid-term hydroelectric optimization, SDP/SDDP, gave the highest revenues. In this case, it was only proven for a single hydroelectric plant and for three stages. In this thesis we are going to prove that the theory also works for a more extended model. Figure 10 shows the schematic diagram of a hydroelectric plant $(\mathbf{i})$ where $\mathbf{v(i)}$, in $[\mathbf{m}^3]$, is the total volume of the reservoir and $\mathbf{h(v(i))}$, in $[\mathbf{m}]$, is the height of the reservoir as a function of the volume. In the Figure 11, the illustrations of the Hydroelectric model are presented numerically and with its geographic topology. We will use a system with two independent cascades and two independent inflow points and two thermal units, one nuclear power plant as a baseload unit and one Open Cycle Gas Turbine (OCGT) as a peak load unit. The two inflow points present a correlation among them due to their proximity and similarity of soil. In the first cascade there are two reservoirs and in the second cascade one reservoir. Each cascade has one hydroelectric unit. In the table 1, the characteristics of the energy system model are presented. The total time horizon considered is 8 months divided into one-month time stages. A time stage at the end of the horizon is used as a buffer where the final reservoir level conditions will be imposed. Each hydroelectric system is characterized by the installed capacity of its generator, its production coefficient, the capacity of its reservoir and its maximum water release (see Figure 11 and table 1). Each thermoelectric unit is characterized by its installed capacity and its marginal cost of operation.

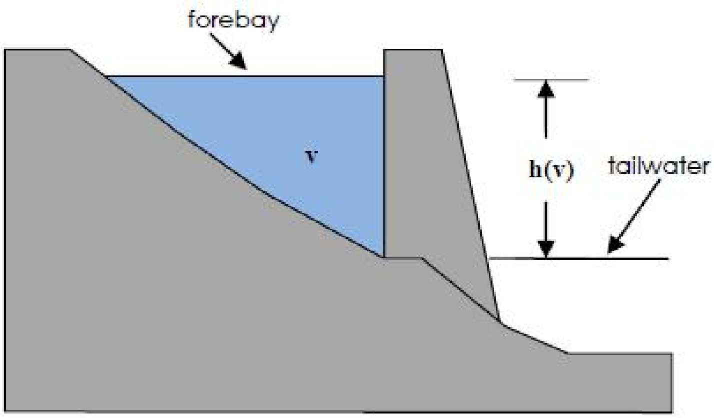


Figure 10: Schematic diagram of a hydroelectric plant.

3.2 The optimization model

The stochastic element is considered through joint scenarios of electricity prices, water inflows and load contract demand with a random correlation between them. If the market is hydro-dominated and there is correlation between prices and inflows, then the price and inflow scenarios can be combined accordingly. The electricity price scenarios considered, represent the spot market. From these scenarios and the transition probabilities between the stages, the forward price for each node can be calculated. The load contract demand represents a load obligation that has to be supplied under any conditions. We assume that

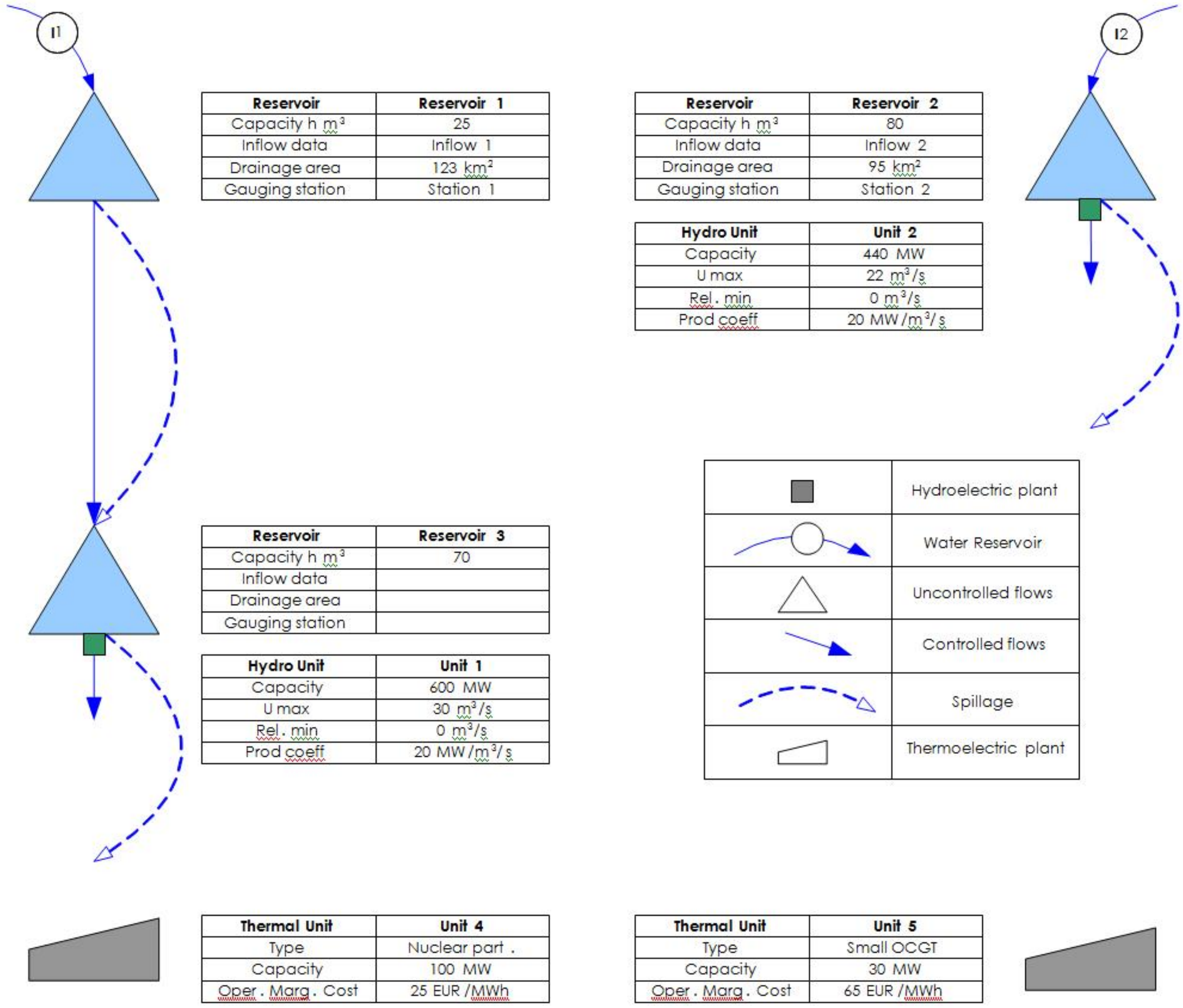
<table>
<tr><th colspan="4">System Model Characteristics</th></tr>
<tr><td>Number of Water Cascades: [-]</td><td colspan="3">2</td></tr>
<tr><td>Number of Water Inflow Points: [-]</td><td colspan="3">2</td></tr>
<tr><td>Number of Water Reservoirs: [-]</td><td colspan="3">3</td></tr>
<tr><td>Hydroelectric Units: [-]</td><td colspan="3">2</td></tr>
<tr><td>Thermoelectric Units: [-]</td><td colspan="3">2</td></tr>
<tr><td>Time Horizon: [months]</td><td colspan="3">8</td></tr>
<tr><td>Time stage length: [months]</td><td colspan="3">1</td></tr>
<tr><td>Total Time stages: [-]</td><td colspan="3">7+1</td></tr>
<tr><td>Total Scenarios: [-]</td><td colspan="3">128</td></tr>
<tr><td rowspan="2"></td><td colspan="3">Hydroelectric System</td></tr>
<tr><td colspan="2">System 1</td><td>System 2</td></tr>
<tr><td>Generator installed capacity: [MW]</td><td colspan="2">600</td><td>440</td></tr>
<tr><td>Production coefficient: [MW/m^3/s]</td><td colspan="2">20</td><td>20</td></tr>
<tr><td>Number of Reservoirs: [-]</td><td colspan="2">2</td><td>1</td></tr>
<tr><td>Water Reservoirs capacity: [hm^3]</td><td>R1: 25</td><td>R3: 70</td><td>R2: 80</td></tr>
<tr><td>Unit maximum release: [m^3/s]</td><td colspan="2">30</td><td>22</td></tr>
<tr><td>Initial Reservoirs Volume:</td><td colspan="2">50%</td><td>50%</td></tr>
<tr><td>Final Reservoirs Volume:</td><td colspan="2">20%</td><td>20%</td></tr>
<tr><td rowspan="2"></td><td colspan="3">Thermoelectric System</td></tr>
<tr><td colspan="2">Unit 1</td><td>Unit 2</td></tr>
<tr><td>Unit Type:</td><td colspan="2">Nuclear participation</td><td>Small Open Cycle Gas Turbine</td></tr>
<tr><td>Generation intalled capacity: [MW]</td><td colspan="2">100</td><td>30</td></tr>
<tr><td>Operation Marginal Cost: [EUR/MWh]</td><td colspan="2">25</td><td>65</td></tr>
</table>

Table 1: Hydroelectric model for numerical illustrations.

the power generation company is a price-taker agent in a liberalized competitive electricity environment. Hence, prices are considered as an exogenous input in the scenarios. This market has a functioning and liquid electricity spot and forward market prices. However, in order to correctly represent the reality and consider the market financial risk premium, a spread of 3 [EUR/MWh] is assumed between the buy and the sell spot market prices. For the forward market, an additional transaction fee of 1 [EUR/MWh] is considered. This transaction fee reflects the cost that incurs a company that builds-up and maintains a trading floor. The outputs of the model are the operating decisions of the hydroelectric plant, and the contracting decisions to the spot and forwards market.
The model is formulated as a multistage Stochastic Mixed Integer Linear Program that uses joint scenarios of water inflows, electricity spot market prices and load contract demand.The stochastic demand in the load contract will be implemented only in the part were we want to test the risk measure in the tree formulation, and not in the decomposed version and SDP/SDDP. The latter occurs because of the specific development required which is beyond the scope of this thesis. The stochastic nature of the model is structured by using a tree representation, as shown in Figure 11[16].

We present the formulation for each of the problems corresponding to the risk control-processes that we have mentioned above. We start by stating the formulation of the basic

[16] In this figure, the stochastic tree for a multistage problem is depicted



Reservoir	Reservoir 1
Capacity h m^3	25
Inflow data	Inflow 1
Drainage area	123 km^2
Gauging station	Station 1

Reservoir	Reservoir 2
Capacity h m^3	80
Inflow data	Inflow 2
Drainage area	95 km^2
Gauging station	Station 2

Hydro Unit	Unit 2
Capacity	440 MW
U max	22 m^3/s
Rel. min	0 m^3/s
Prod coeff	20 MW/m^3/s

Reservoir	Reservoir 3
Capacity h m^3	70
Inflow data	
Drainage area	
Gauging station	

Hydro Unit	Unit 1
Capacity	600 MW
U max	30 m^3/s
Rel. min	0 m^3/s
Prod coeff	20 MW/m^3/s

Thermal Unit	Unit 4
Type	Nuclear part .
Capacity	100 MW
Oper . Marg . Cost	25 EUR /MWh

Thermal Unit	Unit 5
Type	Small OCGT
Capacity	30 MW
Oper . Marg . Cost	65 EUR /MWh

Figure 11: Hydroelectric model for numerical illustrations.

problem and then we mention, for each particular risk-control process formulation, the expressions that have to be modified relatively to this reference case. We present the general formulation of the mid-term hydroelectric operation optimization problem without considering any risk control process to begin with and then present risk control process constraints.

We let:

- **tn** be the node we are considering
- **t'n'** be the parent node of tn from a previous stage
- **t"n"** be the descendent node of tn in a next stage
- **(t'n')** be the set of parent nodes of tn that belong to the same scenario
- **(t"n")** be the set of descendent node of tn in a next stage

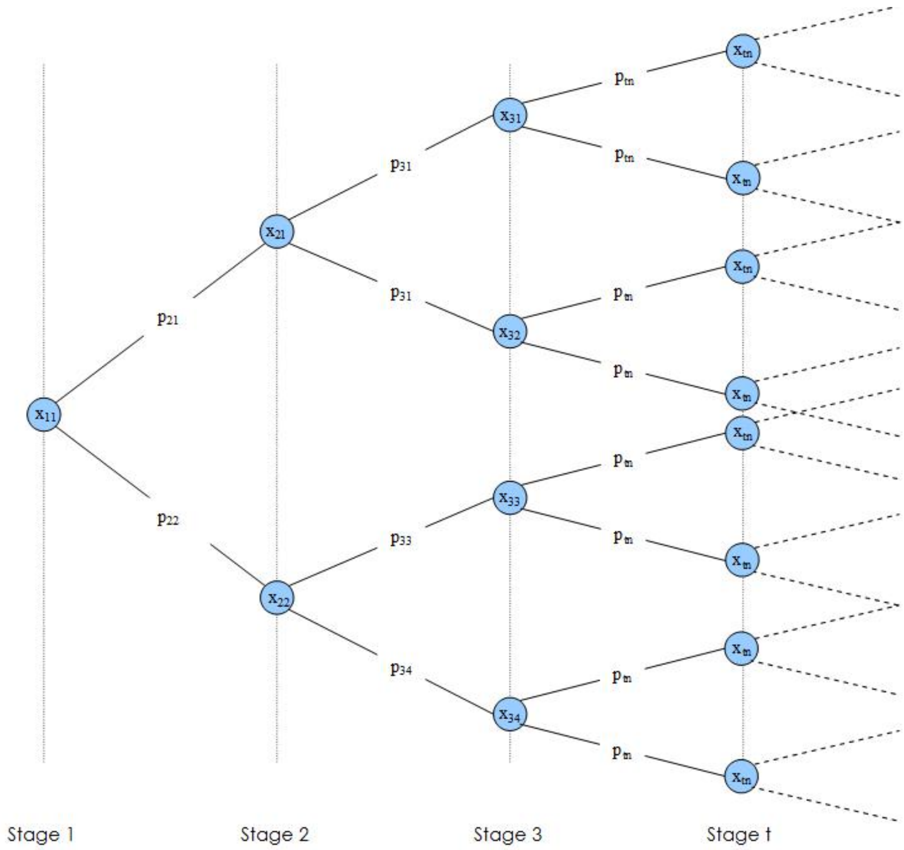


Figure 12: Representation of the tree structure of the stochastic model.

- **t** be the stage we are considering
- **t'** be the parent stage of tn from a previous stage

The first set of variable, in the forward contracting decision variable, indicates the node where the decision variable is calculated (that is, t stands for the stage and n stand for the node in stage t). The second variables, in the forward contracting decision variable, indicates the stage and the node, where the decision is accounted for. Hence, $EF(t'n';tn)$, is defined as the: forward contracted energy that is calculated in a previous stage node $t'n'$ but is accounted for in node tn of stage t.
In the forward market price state variables, the first set of variables indicates the node where the state variable is conditioned from and the second set of variables, in the forward market state variables, indicates the node, or the stage and the node, where it is defined for. Hence, $f^s(t'n';tn)$, is defined as the forward price as perceived from the node $t'n'$ of a previous stage, and will be accounted for in node tn of stage t.

Before going through the model it is of importance that some stuff are being pointed out and clearly defined. One to be defined is the occurrence probabilities which is a time-homogeneous Markov chain ([Enger and Grandell (2003)]) from the initial node to the node that we are looking for. Hence, P_{tn}, of a node tn, the product of transition probabilities,

p_{tn}, from node 11 to the node tn.

$$P_{tn} = \prod_{11}^{tn} p_{tn} \tag{58}$$

Another thing is the the forward contract. The forward market prices are calculated as the expected value of the spot market prices from each specific state that is linked through the tree branches to a future stage. In case of a foreseen distortion between the expected spot market price and the actual forward market price, the forward market prices can be input exogenously into the model. Nevertheless, in this case a pure arbitrage between the expected spot price and the forward market prices might be of an issue.

The forward price $f^s(t'n';tn)$ definition consists of the product of the interest rate times the time difference (stage difference), from the node $t'n'$ to the node tn (that is, from stage t' to stage t) and the spot price Π^s_{tn}, of the node tn ([Bjork (2004)]).
Hence, we have:

$$f^s(t'n';tn) = e^{r(t-t')}\Pi^s_{tn} \tag{59}$$

We can observe from equation (59), that the forward market prices are equal (see Figure 13).

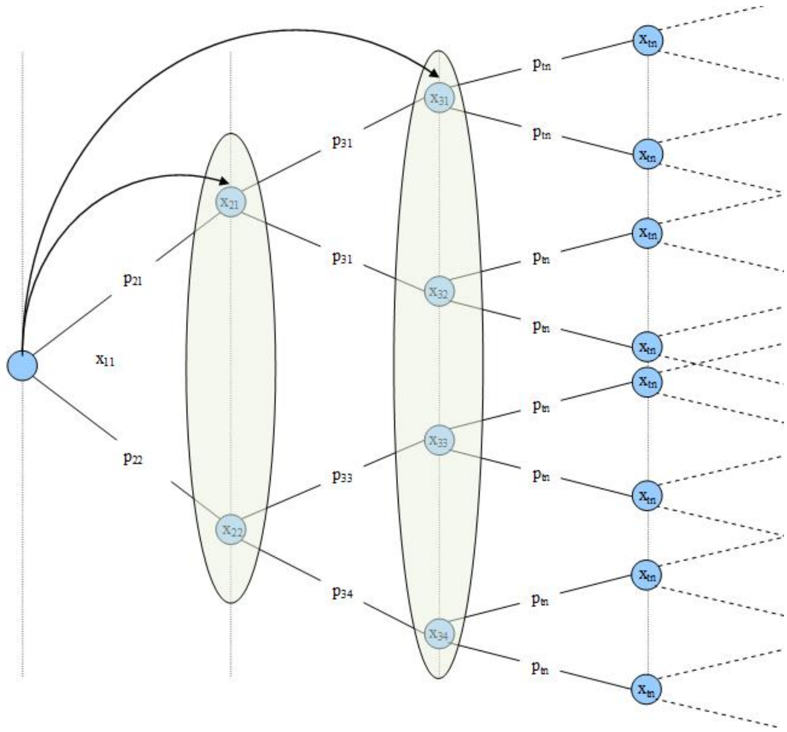


Figure 13: Example of forward price perception from node 11 for nodes 21, 22 and 31, 32, 33, 34. Circled nodes have equal forward market prices as perceived from node 11.

The model to be presented will prove that CVaR risk-control process in terms of higher revenues. The new formulation of CVaR that [Iliadis (2008)] presented will be used in this model now.

Objective function

The objective function maximizes the net revenues in [kEUR]. This consists of the summation of the net revenues of every node tn considering the occurrence probability, P_{tn}, of each node. More specifically, the latter summation consists of the revenues from selling in the spot market, RS^s_{tn}, the revenues from selling in the forward market, $f\Pi_{t'n'}(t)_{tn}EF_{t'n'}(t)_{tn}$, the cost for buying in the spot market, CS^b_{tn}, and the cost of operations of the thermal assets, $g_{tn}(k)c(k)$.

$$max\{\sum_{\forall tn} P_{tn}[(RS^s_{tn} - CS^b_{tn}) + \sum_{t'n' \in (t'n')} (f^s(t'n';tn)EF(t'n';tn)) - \sum_{k=1}^{K} g_{tn}(k)c(k)]\} \quad (60)$$

Water Balance constraints

The water balance constraints consist of the final reservoir water volume of node tn, which is defined as the initial water reservoir volumes of all nodes $t+1\,n''$ that belong to the set $(t+1\,n'')$, where all descendent nodes $t+1\,n''$ of tn belong, $v_{t+1n}(i)$, in [m^3], of the initial storage of water, $v_{tn}(i)$, in [m^3], in the reservoir, the volume of water inflows, $a_{tn}(i)$, in [m^3/s], entered the reservoir, during the time, t_{sds}, in [s], of the stage t, the volume of water, $u_{tn}(i)$, in [m^3], used to produce electricity, and the volume of water spilled, $w_{tn}(i)$, in [m^3], and the volume of water and spillage from a set $U(i)$ of upstream plants j, $u_{tn}(j)$, and $w_{tn}(j)$, in [m^3]. Each water balance constraint is applied at every node tn.

$$v_{t+1n''}(i) = v_{tn}(i) + a_{tn}(i)t_{sds} - u_{tn}(i) - w_{tn}(i) + \sum_{j \in U(i)} (u_{tn}(j) + w_{tn}(j))\}$$
$$for\ all\ nodes\ t+1\,n'' \subset (t+1\,n'')\ \&\ for\ i=1,...,I,\ for\ all\ nodes\ tn \quad (61)$$
$$v_{t+1n''}(i) \geqslant 0 \qquad for\ i=1,...,I, for\ all\ nodes\ t+1\,n \quad (62)$$
$$a_{tn}(i) \geqslant 0 \qquad for\ i=1,...,I, for\ all\ nodes\ tn \quad (63)$$
$$u_{tn}(i) \geqslant 0 \qquad for\ i=1,...,I, for\ all\ nodes\ tn \quad (64)$$
$$w_{tn}(i) \geqslant 0 \qquad for\ i=1,...,I, for\ all\ nodes\ tn \quad (65)$$

Technical characteristic constraints

The technical characteristic constraints consist of the maximum volume of the water, $v_{max}(i)$, in [m^3/s], that can be contained in the reservoir and the maximum volume of the water, $u_{max}(i)$, in [m^3/s], that can be used to produce electricity, and the maximum generation capacity, $g_{max}(k)$, in [MWh], which is generated by the thermal assets. Each technical characteristic constraint is applied at every node tn.

$$v_{tn}(i) \leqslant v_{max}(i) \qquad for\ i=1,...,I, for\ all\ nodes\ tn \quad (66)$$
$$u_{tn}(i) \leqslant u_{max}(i) \qquad for\ i=1,...,I, for\ all\ nodes\ tn \quad (67)$$
$$g_{tn}(k) \leqslant g_{max}(k) \qquad for\ i=1,...,I, for\ all\ nodes\ tn \quad (68)$$

Hydro generation constraints

The hydroelectric generation constraints consist of the energy generated, $e_{tn}(i)$, in [MWh], the coefficient of production, $\rho(i)$, in [MW/m^3/s], the volume of the water, $u_{tn}(i)$, in [m^3/s], that will be used to produce electricity, the time of each stage duration, $t_{sdh}(i)$, in [h], and the time of each stage duration, $t_{sds}(i)$, in [s]. Each hydroelectric generation constraint is applied at every node tn for every reservoir i.

$$e_{tn}(i) = \rho(i)u_{tn}(i)\frac{t_{sdh}}{t_{sds}} \qquad for\ i = 1, ..., I, for\ all\ nodes\ tn \tag{69}$$

$$e_{tn}(i) \geqslant 0 \qquad for\ i = 1, ..., I, for\ all\ nodes\ tn \tag{70}$$

Spot Market Revenue constraints
The constraints of revenue from selling to the spot market, RS^s_{tn}, in [kEUR], consist of the product of the spot market price, Π^s_{tn}, in [EUR/MWh], and the electricity that will be sold in the spot market, e^s_{tn}, in [MWh]. Each spot revenue constraints is applied at every node tn.

$$RS^s_{tn} = \Pi^s_{tn}e^s_{tn} \qquad for\ all\ nodes\ tn \tag{71}$$

$$RS^s_{tn} \geqslant 0 \qquad for\ all\ nodes\ tn \tag{72}$$

Spot Market Cost constraints
The constraints of cost from buying from the spot market, CS^b_{tn}, in [kEUR], consist of the product of the spot market price, Π^b_{tn}, in [EUR/MWh], and the energy that will be bought in the spot market, e^b_{tn}, in [MWh]. Each spot revenue constraints is applied at every node tn.

$$CS^b_{tn} = \Pi^b_{tn}e^b_{tn} \qquad for\ all\ nodes\ tn \tag{73}$$

$$CS^b_{tn} \geqslant 0 \qquad for\ all\ nodes\ tn \tag{74}$$

Energy Balance constraints
The energy balance constraints consist of the energy generated, $e_{tn}(i)$, in [MWh], for the sum of reservoirs i, the energy bought from the spot market, $e^b_{tn}(i)$, in [MWh], the energy sold in the spot market, $e^s_{tn}(i)$, in [MWh], the energy produced by the thermal assets $g_{tn}(k)$, in [MWh], the energy required from the load contract d_{tn}, in [MWh], and the energy contracted $EF_{t'n'}(t)_{tn}$, in [MWh] from all the nodes $t'n'$ to the node tn of stage t that belong to the set $(sc\,tn)$, where all nodes $t'n'$ of the same scenario belong with tn. Each energy balance constraint is applied at every node tn. The energy delivered in node (11) is null since there are no parent nodes to contract energy for node 11. Nevertheless, we could consider, if required, already contracted energy in the forward market.

$$\sum_{i=1}^{I} e_{tn}(i) + e^b_{tn} + \sum_{k=1}^{K} g_{tn}(k) = e^s_{tn} + \sum_{t'n' \subset (t'n')} EF(t'n'; tn) + d_{tn} \quad for\ all\ nodes\ tn \tag{75}$$

$$EF(t'n'; 11) = 0 \tag{76}$$

Risk measures - Financial Risk Control constraints
The risk measure constraints, both the non-relaxed version and the relaxed version, will here be presented. It should be mentioned now that in order for CVaR to be implemented SDP/SDDP it has to be relaxed, more about this later on (also for more information please see [Iliadis et al. (2008)]). With the relaxation, the objective function is being changed therefore we will write out the objective function for each time.

Rmin constraints
This formulation contains the Rmin risk control process as hard constraint. When considering the Rmin risk control process, the problem formulation becomes, in the non-relaxed

formulation, the objective function remains the same as in the basic problem formulation.

$$max\{\sum_{\forall tn} P_{tn}[(RS^s_{tn} - CS^b_{tn}) + \sum_{t'n' \in (t'n')} (f^s(t'n'; tn)EF(t'n'; tn)) - \sum_{k=1}^{K} g_{tn}(k)c(k)]\} \quad (77)$$

The constraints describing the core of the problem remain the same as in the basic problem formulation. Subject to:
(61) – (76)

The financial risk control process constraints consist of the sum of net revenues, the imposed level of minimum revenue, R_{min} , in [kEUR], and the decision violation variable, z_{sc}, in [kEUR], applying to each node tn that pertains in a scenario sc. We define as scenario each branch of the tree starting from the first node, (11), and ending at one of the last nodes of the tree.
In addition the decision violation variables, z_{sc}, must be greater or equal than zero for all scenarios in order to force the net revenues to be greater or equal than the imposed level of minimum net revenue R_{min}. We have formulated the financial risk control constraints using a decision violation variable in order to prepare the ground for the next formulation that will use a Lagrangian relaxation.

$$\sum_{\forall tn \in (sc)} [(RS^s_{tn} - CS^b_{tn}) + \sum_{t'n' \in (t'n')} (f^s(t'n'; tn)EF(t'n'; tn)) - \sum_{k=1}^{K} g_{tn}(k)c(k)] - R_{min} \geqslant z_{sc}$$

$$for\ all\ scenarios\ sc \quad (78)$$

$$z_{sc} \geqslant 0 \qquad for\ all\ scenarios\ sc \quad (79)$$

Relaxed Rmin constraints
This formulation contains the Rmin financial risk control process as constraints, which is achieved by penalization inside the objective function. The latter is implemented by relaxing the constraint (78) using Lagrangian Relaxation (see [Held M. et al. (1970)] and [Held M. et al. (1971)]).
In the relaxed formulation, the objective function consists of the net revenues of all nodes tn, the unconditional probability, P_{tn}, and the probability occurrence, p_{sc} of each scenario, the penalty coefficient $\alpha_{R_{min}}$, without units, and the decision violation variable, z_{sc}, in [kEUR]. The penalty coefficient defines the severity of the constraint ("hardness") where if it is set to zero, then the Rmin level is not fulfilled, hence the constraint is violated, and if it is set to infinity (large number that depends on the problem), then the Rmin level is fully fulfilled as if it was a hard constraint. All intermediate values of the penalty coefficient are used in order to achieve the desired financial risk level by simulating a probabilistic financial risk control process. Hence, as an example, in a case where one hundred scenarios of net revenues exist and we wish that ninety percent out of them shall be greater than a Rmin level, then we should experiment with various values of penalty coefficients until we succeed with the desired result.

$$max\{\sum_{\forall tn} P_{tn}[(RS^s_{tn} - CS^b_{tn}) + \sum_{t'n' \in (t'n')} (f^s(t'n'; tn)EF(t'n'; tn)) - \sum_{k=1}^{K} g_{tn}(k)c(k)] + \sum_{\forall sc} p_{sc}\alpha_{R_{min}} z_{sc}\} \quad (80)$$

The constraints describing the core of the problem remain the same as in the basic problem formulation. Subject to:
(61) – (76)

The constraint (78) is the same as in the case of non-relaxed formulation. As we can observe the constraint that was relaxed and inserted in the objective functions have changed the constraint (79). Nevertheless, as the objective is maximized, in some cases the z_{sc} variable can take positive values and that way degrading the solution by leading to non-optimal solutions (forward contracting when not needed). Hence, the variables have to be bound in it's negative part through the additional constraint (81) for all scenarios sc.

$$\sum_{\forall tn \in (sc)} [(RS^s_{tn} - CS^b_{tn}) + \sum_{t'n' \in (t'n')} (f^s(t'n';tn)EF(t'n';tn)) - \sum_{k=1}^{K} g_{tn}(k)c(k)] - R_{min} \geqslant z_{sc} \quad \textit{for all scenarios sc} \tag{81}$$

$$z_{sc} \leqslant 0 \quad \textit{for all scenarios sc} \tag{82}$$

VaR constraints

The objective function remains the same as in the basic problem formulation.

$$max\{\sum_{\forall tn} P_{tn}[(RS^s_{tn} - CS^b_{tn}) + \sum_{t'n' \in (t'n')} (f^s(t'n';tn)EF(t'n';tn)) - \sum_{k=1}^{K} g_{tn}(k)c(k)]\} \tag{83}$$

The constraints describing the core of the problem remain the same as in the basic problem formulation. Subject to:
(61) – (76)

In order to formulate the VaR risk control process in the optimization problem we use MILP[17]. The financial risk control process constraints consist of the net revenues, the binary variable, χ_{sc}, a large positive number, M, and the desired level of VaR, η_{VaR}, in [kEUR], for each node tn that pertains to a scenario sc and for all scenarios sc.
The probabilistic selection of scenarios that should satisfy the desired level of VaR is implemented through the constraint (84) that consists of each scenario's occurrence probability, p_{sc}, the binary variable of each scenario χ_{sc} and the percentile $1-\alpha^{VaR}$ of the scenarios that will have a lower net revenue value than the desired VaR. Hence through the maximization of the problem the binary variable, corresponding to the percentile of the scenarios that will not satisfy the desired VaR, will equal to one and thus these scenarios will take any value.
The constraint (84) defines the χ_{sc} as a binary variable for all scenarios sc.

$$\sum_{\forall tn \in (sc)} [(RS^s_{tn} - CS^b_{tn}) + \sum_{t'n' \in (t'n')} (f^s(t'n';tn)EF(t'n';tn)) - \sum_{k=1}^{K} g_{tn}(k)c(k)] - M\chi_{sc} \geqslant \eta_{VaR} \quad \textit{for all scenarios sc} \tag{84}$$

$$\sum_{\forall sc} p_{sc}\chi_{sc} \leqslant (1 - \alpha^{VaR}) \tag{85}$$

$$\chi_{sc} \in \{0, 1\} \quad \textit{for all scenarios sc} \tag{86}$$

[17] The large number M is part of the formulation of the "Big M" technique [Wolsey L.A. (1998)] used typically in binary MILP.

CVaR constraints
This formulation contains the CVaR risk control process as a hard constraint using its LP formulation. In the non-relaxed formulation, the objective function remains the same as in the basic problem formulation.

$$max\{\sum_{\forall tn} P_{tn}[(RS^s_{tn} - CS^b_{tn}) + \sum_{t'n' \in (t'n')} (f^s(t'n';tn)EF(t'n';tn)) - \sum_{k=1}^{K} g_{tn}(k)c(k)]\} \quad (87)$$

The constraints describing the core of the problem remain the same as in the basic problem formulation. Subject to:
(61) – (76)

The financial risk control constraints, consist of net revenues per scenario sc, the decision variable, η_{CVaR}, in [kEUR], and the scenario decision variable, y_{sc}, in [kEUR], for each scenario sc. The latter decision variable is used as an auxiliary variable for the transformation of the CVaR formulation from MILP to LP. At the end of the optimization and if an optimal solution is found, the decision variable is equal to the level of VaR for this problem. The constraint (88) consists of the decision variable, η_{CVaR}, each scenario's probability p_{sc}, the scenario auxiliary decision variable, y_{sc}, for all scenarios sc, and the inverse of the probability of the scenarios that will have a higher net revenue expected value than the desired CVaR, ψ, in [kEUR]. The constraint (89) defines the y_{sc} as a negative variable for all scenarios sc. Hence forcing the latter decision variable being negative, the constraint (88) guarantees that all scenarios that satisfy it, will belong to the percentile of the scenarios with the expected value of the net revenues that is greater or equal to the desired CVaR level.

$$\sum_{\forall tn \in (sc)} [(RS^s_{tn} - CS^b_{tn}) + \sum_{t'n' \in (t'n')} (f^s(t'n';tn)EF(t'n';tn)) - \sum_{k=1}^{K} g_{tn}(k)c(k)] - \eta_{CVaR} \geqslant y_{sc}$$

$$for\ all\ scenarios\ sc \quad (88)$$

$$\eta_{CVaR} + \frac{\sum_{\forall sc} p_{sc} y_{sc}}{(1 - \alpha^{CVaR})} \geqslant \psi \quad (89)$$

$$y_{sc} \leqslant 0 \qquad for\ all\ scenarios\ sc \quad (90)$$

Relaxed CVaR constraints
The formulation of this problem is implemented using Lagrangian Relaxation. The reason for relaxing this constraint is to expose the similarity with the Rmin formulation. Moreover, we use the formulation with relaxation of the constraint, in order to prepare the ground for the formulation of CVaR inside the SDP/SDDP algorithm. At this level, we can observe that the superiority of CVaR comes from its directly probabilistic formulation.
In the relaxed formulation, the objective function consists of the net revenues of all nodes tn, their respective occurrence probability, P_{tn}, and the penalty function. The penalty functions consists of the constraint (88) and the penalty coefficient, υ, in [kEUR/kEUR]. The penalty coefficient defines the severity of the constraint ("hardness") where if it set to zero, then the CVaR level, ψ, is not fulfilled, hence the constraint is violated, and if it is increased, then we achieve respectively higher values of CVaR level, ψ, for the defined probability level. The intermediate values until achieving the desired CVaR level, in contrast with Rmin, result

in optimal portfolios in terms of expected value and CVaR. Hence, a calibration must take place. The important difference between the calibration process of CVaR and Rmin is that for the latter we have to achieve a level of CVaR for a defined probability, using two non-probabilistic parameters, where in the CVaR formulation, the probabilistic level is explicitly set and hence we have to calibrate only one coefficient.

$$max\{\textstyle\sum_{\forall tn} P_{tn}[(RS_{tn}^s - CS_{tn}^b) + \sum_{t'n' \in (t'n')}(f^s(t'n';tn)EF(t'n';tn)) - \sum_{k=1}^{K} g_{tn}(k)c(k)] \\ +\upsilon(\eta_{CVaR} + \frac{\sum_{\forall sc} p_{sc} y_{sc}}{(1-\alpha^{CVaR})} - \psi)\} \tag{91}$$

The constraints describing the core of the problem remain the same as in the basic problem formulation. Subject to:
(61) – (76)

The constraint (91) and (92) are the same as in the case of non-relaxed formulation. As we can observe the constraint that was relaxed and inserted in the objective function is no longer present. In this case, the decision variable η_{CVaR}, representing the VaR in the optimal solution, can not take any feasible values and render the constraint negative. This can be explained by the definition of VaR and CVaR where VaR is always greater than or equal CVaR.

$$\sum_{\forall tn \in (sc)} [(RS_{tn}^s - CS_{tn}^b) + \sum_{t'n' \in (t'n')} (f^s(t'n';tn)EF(t'n';tn)) - \sum_{k=1}^{K} g_{tn}(k)c(k)] - \eta_{CVaR} \geqslant y_{sc} \quad for\ all\ scenarios\ sc \tag{92}$$

$$y_{sc} \leqslant 0 \qquad for\ all\ scenarios\ sc \tag{93}$$

4. Results

To get a better understanding of the newly formulated CVaR we have implemented it in a MILP, the reason is that the possibility of implementing VaR also (that is not possible with SDP/SDDP).

In Tablet 2 the expected net revenue are presented together with the values of $VaR_{0.99}$ and $CVaR_{0.99}$ with no risk constraint involved.

E[Net revenue]:	119340
$VaR_{0.99}$:	13173.9
$CVaR_{0.99}$:	13042.5

Table 2: The result from the model without any risk constraint, in [Euro]

The Results in Table 3 presents the system optimization using a VaR constraint and Table 4 presents the system operation optimization considering the Rmin risk control process as a proxy to achieve the desired VaR constraint. Using a series of combinations of Rmin and penalty coefficients, we have achieved the desired level of VaR that was imposed.

E[Net revenue]:	119064
$\mathbf{VaR}_{0.99}$:	13956.8

Table 3: The result from the model with $VaR_{0.99}$ risk constraint, in [Euro]

We can observe that the expected net revenues, in the case of applying directly the VaR risk control process (see Table 3), are lower than the net revenues in the case without any financial risk constraint (see Table 2). This occurs because of the approach of VaR to control risk, leaving that way the distribution's lower part uncontrolled.

E[Net revenue]:	118962
$\mathbf{VaR}_{0.99}$:	13956.8

Table 4: The result from the model with Rmin risk constraint used as a proxy instead of the $VaR_{0.99}$, in [Euro]

As we can see in Table 4, the desired $VaR_{0.99} = 13956.8$ [EUR] is achieved using Rmin as a proxy to simulate VaR through the combination of $(Rmin, \alpha_{Rmin}) = (13956.8, 1.5)$. Nevertheless, the latter example results in a lower portfolio expected net revenues value, when compared to Table 3.
In the same way we compared the values of CVaR and a Rmin proxy of CVaR. By comparing the Table 5 (direct CVaR implementation)and Table 6 (Rmin as a proxy to simulate CVaR). It was stated before that there is equivalence between the Rmin and the CVaR control-processes problems formulation in terms of control of the distribution. Nevertheless, similar results in risk levels in terms of CVaR measures, can yield different portfolio expected net revenues, when we use Rmin as a proxy to simulate CVaR.

Comparing Table 6 with Table 5, we observe in all the examples the same CVaR level is achieved. Nevertheless, in Table 6, where we use Rmin, through the parameters combi-

E[Net revenue]:	119317
$\mathbf{CVaR}_{0.99}$:	13131.4

Table 5: The result from the model with $CVaR_{0.99}$ risk constraint, in [Euro]

E[Net revenue]:	119281
$\mathbf{CVaR}_{0.99}$:	13131.4

Table 6: The result from the model with Rmin risk constraint used as a proxy instead of the $CVaR_{0.99}$, in [Euro]

nation of $(Rmin, \alpha_{Rmin}) = (13131.4, 5.5)$, as a proxy to simulate the desired CVaR level, the portfolio expected net revenues is lower than in Table 5, where the CVaR risk control process is directly applied. Hence, we deduct that the solution of the example in Table 6 is not optimal with respect to the CVaR desired level and the portfolio net revenue expected value. The optimal solution can be obtain for a portfolio subjected directly to the CVaR risk control process, using the Rmin risk control process as a proxy through an adjustment of $(Rmin, \alpha_{Rmin})$ parameters. In the best case, the adjustment of $(Rmin, \alpha_{Rmin})$ can give the same expected net revenues as the case where CVaR control process is directly applied. However, what is important, is the fact that more than one combinations of $(Rmin, \alpha_{Rmin})$ parameters, when using the Rmin risk control process as a proxy to simulate CVaR, might lead to a portfolio with the desired CVaR but with a different net revenue expected values. Hence, although the same level of CVaR and percentile are simulated, the resulting expected net revenues can be lower in one example. Furthermore, the process of obtaining the original CVaR solution through $(Rmin, \alpha_{Rmin})$ adjustments requires an uncertain number of iterations with no guarantee of success.

As seen, the results from our function aren't as big as (see [Iliadis et al. (2008)]) proposed due to my input values. By changing the input values we were able to get a lot of results in the expected net revenues. The Tables above is one of the examples of values we got.

5. Conclusion

In order to compare the three risk control-processes, [Iliadis et al. (2008)] formulated accordingly, and we implemented each one of them in the context of mid-term hydroelectric assets portfolio operation optimization problem. The hydroelectric assets have to be modeled within an algorithm that the implementation of a risk control process is possible, and which is stochastic, time-dependent and sequential, in order to account for the specificities of hydroelectric assets.
The new formulation of CVaR has been proven that it work first for one hydroelectric plant (see [Iliadis et al. (2008)]) and now by me for a small but realistic model. CVaR is an appropriate risk measure in the electricity market for its capacity to account for distribution skewness and asset optionallity. The skewness in the distribution of profit and loss caused by the price spikes and the options built into any power contract makes an asymmetric risk measure that can really penalize extreme events, such as CVaR, interesting. The characteristics of distribution skewness and asset optionality are even more pronounced in the operation optimization of hydroelectric assets. The latter is a result of the nature of hydroelectric assets, which is directly related to the existence of a reservoir and their capacity to store water and use it according to the system operation optimization. The convexity and tractable computational features of CVaR through LP do indeed strengthen its position in terms of implementation to an optimization algorithm. More specifically, as described in the paragraphs above of this chapter, hydroelectric optimization is computed through the complex algorithm of SDP/SDDP in order to tackle problems with large number of hydroelectric assets and with the stochastic variables of water inflows and prices. The latter algorithm can accommodate risk measures that are based in LP and that are convex. These two conditions are obligatory since SDP/SDDP is based on the approximation of the FBF and hence MILP and lack of convexity would lead to incoherent results. Moreover, because of the complexity of optimizing the operation of hydroelectric assets and the direct relation between their operation and their hedging strategy, as a result of the application of risk control process, the computation of a hedging strategy shall be made jointly with the operation. Hence, it is imperative the risk control process to be implemented in the operation optimization algorithm. To se the real formulation and and implementation we refer to [Iliadis et al. (2008)] where he has written it out.
we also want to point out that my only assignment in this thesis was to try to prove that the new formulated CVaR works in real life system. By the result we proved above we showed that was the case!

6. References

[Alexander et al. (2004)] Alexander G.J., Baptista A.M. A comparison of VAR and CVAR constraints on portfolio selection with the mean-variance model Management Science 2004.

[Araripe Neto et al. (1985)] Araripe Neto T de A., Cotia C.B., Pereira M.V.F., Kelman J. Comparison of stochastic and deterministic processes in hydrothermal generation scheduling In proceedings of IFAC symposium on planning and operation of electric energy system - Rio de Janeiro 1985.

[Artzner et al. (1999)] Artzner P., Delbaen F., Eber J-M., Heath D. Coherent measures of risk Mathematical Finance 1999

[Artzner et al. (2004)] Artzner P., Delbaen F., Eber J-M., Heath D., Ku H. Coherent Multiperiod Risk Adjusted Values and Bellman's Principle Annals of Operations Research 2004

[Basel Committee on Banking Supervision (2004)] Basel Committee on Banking Supervision International Convergence of Capital Measurement and Capital Standards Bank for International Settlements Press & Communications 2004

[Bava (1978)] Bava V. S. Safety first, stochastic dominance, and optimal portfolio choice Journal of Financial and Quantitative Analysis 1978

[Bellman (1957)] Bellman R. Dynamic programming Publication of Princeton university press 1957

[Benders (1962)] Benders J.F. Partitioning procedures for solving mixed-variables programming problems Numerische Mathematik 1962

[Bjork (2004)] Bjork T. Arbitrage Theory in Continuous Time (second eition)

[Cvatal (1983)] Cvatal V. Linear Programming Series of Books in the Mathematical Sciences 1983

[Clewlow et al. (2000)] Clewlow L. and Strickland C. Energy Derivatives - Pricing and Risk Management Lacima Publications 2000

[Duffie et al. (1997)] Duffie D., Pan J. An Overview of Value at Risk Journal of Derivatives 1997

[Enger and Grandell (2003)] Enger J. and Grandell J. Markov processes and queuing theory 2003

[Fleten (2000)] Fleten S-E. Portfolio management emphasising electricity market applications - A stochastic programming process Doctorate Thesis in Norwegian University of Science and Technology 2000

[Fosso et al. (1999)] Fosso O.B., Gjelsvik A., Haugstad A., Mo B., Wangensteen I. Generation scheduling in a deregulated system IEEE Transactions on power systems 1999

[Gjelsvik et al. (1996)] Gjelsvik A., Wallace S.W. Methods for stochastic medium-term scheduling of hydro-thermal power systems Norwegian electric power research institute 1996

[Haneveld Klein et al. (2003)] Haneveld Klein W.K., van der Vlerk M.H. Stochastic Programming Lecture notes of University of Groningen 2003

[Haugstad et al. (1998)] Haugstad A., Rismark O. Price forecasting in an open electricity market based on system simulation In Proceedings of EPSOM '98, Zurich Switzerland 1998

[Henrik , Lindskog] Mathematical modeling and statistical methods for risk management.

[Iliadis (2008)] Iliadis N.A. Financial risk modelling in electricity portfolio optimisation; a Stochastic Dual Dynamic Programming approach for hydroelectric assets

[Jacobs et al. (1995)] Jacobs J., Freeman G., Grygier J., Morton D.P., Schultz G., Staschus K., Stedinger J.R. SOCRATES - A system for scheduling hydroelectric generation under uncertainty Annals of Operations Research 1995

[Jorion (2000)] Jorion P. Value at Risk - 2nd edition Mc Graw-Hill 2000

[Johnson et al. (1993)] Johnson S.A., Stedinger J.R., Shoemaker C.A., Li Y., Tejada-Guilbert Johnson S.A. Numerical Solution of Continuous-State Dynamic Programs Using Linear and SP-line Interpolation Operation Research 1993

[Kristiansen (2004)] Kristiansen T. Financial risk management in the electric power industry using stochastic optimisation Advanced Modeling and Optimisation 2004

[Krokhmal et al. (2001)] Krokhmal P., Uryasev S., Palmquist J. Portfolio optimisation with conditional value-at-risk objective and constraints University of Florida - Research report 2001

[Larsen et al. (2001)] Larsen N., Mauser H., Uryasev S. Algorithms for Optimisation of Value at Risk In Book: Financial Engineering, e-commerce and supply chain 2001

[Markowitz (1959) Markowitz H. M. Portfolio Selection: Efficient Diversification of Investments Wiley, New York 1959

[Mo et al. (2001)] Mo B., Gjelsvik A., Grundt A., Karesen K. Optimisation of hydropower operation in a liberalised market with focus on price modelling 2001 Porto power tech proceedings 2001

[Morton (1996)] Morton D.P. An enhanced decomposition algorithm for multistage stochastic hydroelectric scheduling Annals of Operations Research 1996

[Olsen et al. (2007)] Olsen O.J. Johnsen T.A. Pakkanen M. Are households better off in liberalized electricity markets - the Nordic case conference of the European Group of Public Administration, Madrid 2007

[Pereira et al. (1985)] Pereira M.V.F., Pinto L.M.V.G. Stochastic optimisation of a multireservoir hydroelectric system - a decomposition process Water resources research 1985

[Pereira et al. (1991)] Pereira M.V.F., Pinto L.M.V.G. Multi-stage stochastic optimisation applied to energy planning Mathematical programming 1991

[Pereira et al. (2000)] Pereira M.V.F., Campodonico N., Kelman R. PSR Report 2000

[Philbrick et al. (1999)] Philbrick C.R. Kitanidis P.K. Limitations of deterministic optimisation applied to reservoir operations Journal of water resources planning and management - ASCE 1999

[Rockafellar et al. (2000)] Rockafellar R.T., Uryasev S. Optimisation of Conditional Value-at-Risk Journal of Risk 2000

[Rockafellar et al. (2002)] Rockafellar R.T., Uryasev S. Conditional value-at-risk for general loss distributions Journal of Banking and Finance 2002

[Roy (1952)] Roy A. D. Safety first and holding of assets Econometrica 1952

[Tejada-Guilbert Johnson et al. (1993)] Tejada-Guilbert Johnson S.A., Johnson S.A., Stedinger J.R. Comparison of Two Processes for Implementing Multireservoir Operating Policies Derived Using Stochastic Dynamic Programming Water resources research 1993

[Tejada-Guilbert Johnson et al. (1995)] Tejada-Guilbert Johnson S.A., Stedinger J.R. The value of hydrologic information in stochastic dynamic programming models of a multireservoir system Water resources research 1995

[Uryasev (2000)] Uryasev S. Conditional Value-at-Risk: Optimisation Algorithms and Applications Financial Engineering News 2000

[Wang (1999)] Wang T. A Class of Dynamic Risk Measures Working paper in UBC 1999

[Wolsey (1998)] Wolsey L.A. Integer Programming Wiley-Interscience Series in Discrete Mathematics 1998