Royal Institute of Technology

Master's Thesis

Portfolio management using structured products
The capital guarantee puzzle

Markus Hveem
hveem@kth.se

Stockholm, January 2011

Abstract

The thesis evaluates the concept of a structured products fund and investigates how a fund based on structured products should be constructed to be as competitive as possible. The focus lies on minimizing the risk of the fund and on capital guarantee. The difficulty with this type of allocation problem is that the available products mature before the investment horizon, thus the problem of how the capital should be reinvested arises. The thesis covers everything from naive fund constructions to more sophisticated portfolio optimization frameworks and results in recommendations regarding how a portfolio manager should allocate its portfolio given different settings. The study compares different fund alternatives and evaluates them against, competing, benchmark funds.

The thesis proposes a framework called the modified Korn and Zeytun framework which allocates a portfolio based on structured products, which have maturity prior to the end of the investment horizon, in optimal CVaR sense (i.e. appropriate for funds).

The study indicates that the most important concept of a structured products fund is transaction costs. A structured products fund cannot compete against e.g. mixed funds on the market if it cannot limit its transaction costs at approximately the same level as competing funds. The results indicate that it is possible to construct a fund based on structured products that is competitive and attractive given low commission and transaction costs.

Keywords: Structured products fund, structured products, portfolio theory, portfolio optimization, portfolio management, Conditional Value-at-Risk, transaction costs

Acknowledgements

I would like to thank my supervisor at KTH Mathematical Statistics, Filip Lindskog, for great feedback and guidance as well as my supervisors at Nordea Markets, Karl Lindqvist and William Sjöberg, for essential knowledge in the field of structured products, feedback and ideas.

Stockholm, January 2011

Markus Hveem

Table of Contents

CHAPTER 1

Introduction

Structured products are a structured form of investment vehicles that consists of a bond and a market exposed financial instrument. The popularity of investing in Equity-Linked Notes, and other structured products, has been significant exceeding 55.1 billion SEK in Sweden alone during 2009 (My News Desk, 2010, [32]). One reason for their popularity is that they possess the property of capital guarantee. Thus many investors have a belief that these products are safe and do not realize the impact of credit risk. An investor can by investing in structured products participate in the market besides the capital guaranteed part (the bond). Thus investors are protected in bear markets and participate in bull markets.

The other major risk for an investor is market risk; the key to avoid market risk is by diversifying. Market risk imposes a demand for applying a diversification framework, similar to the one of Markowitz's (1952, [30]) seminal work on basic portfolio management theory, applicable to structured products. Most studies in the area have been conducted by using a structured product, a bond, the underlying and an option on the underlying as the investment universe in a one period model (as in Martellini et al. (2005, [31])), which is inadequate to describe the investment universe for e.g. a hedge fund. The first study conducted on the subject, using a longer investment horizon than the time to maturity for the structured product, was performed by Korn and Zeytun (2009, [27]). This setup is more relevant for an investor that must rebalance its portfolio at different intermediate time points, and especially when the investor is allowed to invest in more than just one structured product.

1.1 Market risk

It has been shown in studies of the Swedish market for structured products that an investor using structured products in a diversified portfolio can achieve a fair return during bull markets and take advantage of the capital protection property (to some extent) of the structured products in the bear markets (Hansen and Lärfars, 2010, [19]; Johansson and Lingnardz, 2010, [25]).

These studies indicate that there are advantages of diversification when investing in structured products against market risk. Many investors on the Swedish market of structured products are small private investors and thus not willing to commit most of their capital in one investment category (Shefrin, 2002, [35]). Hence to offer the advantages of diversification in structured products to small investors the concept of a fund based on structured products emerges. By creating a fund based on structured products it is possible to decrease the required amount invested for each individual investor and practically gain the same advantages as if the investor had the possession itself.

1.2 Capital guarantee

The concept of market risk and diversifying using a set of numerous assets is very appealing to an investor, especially if the investment is capital guaranteed. The main issue is that the portfolio based on several structured products with different maturities will not be capital guaranteed.

When, in the setting of structured products, a product is capital guaranteed it means that the investor will at least receive the invested amount, at maturity (when disregarding the impact of defaults). In the setting of a fund the investment horizons will instead be overlapping and of approximately one to five years, thus the capital guarantee should be measured during each of these investment periods. Problem arises when the fund holds positions in products with maturity beyond the investment horizon, since all structured products can have a value of zero before maturity due to changes in interest, movement of the underlying etc. Also if products have maturity prior to the investment horizon's end the issue of how the payoff should be reinvested arises. Consider that the fund should not only be guaranteed during one investment horizon, but several, each new investor wants to have the property of capital guarantee over its own investment horizon. Thus if the fund should be capital guaranteed for each investor, with an investment horizon of τ_i years, the return of the portfolio of every single τ_i year long time period should be positive. It is obvious that it is difficult to gain the property of capital guarantee in combination with the possibility of a high return.

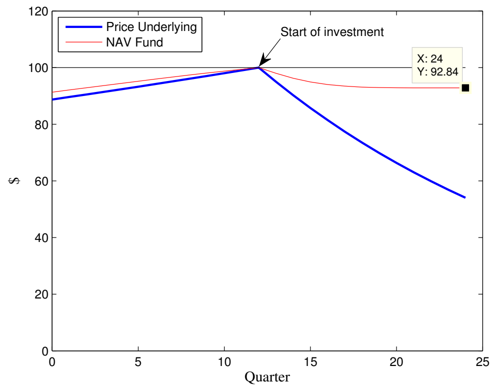


Figure 1.1: The underlying has 1% in return each quarter until quarter 12, when the investor buys a share of the fund, after quarter 12 the underlying has -5% in return quarterly, $\sigma = 0.15, r = 0.015$ p.q., hence the fund is not capital guaranteed.

The example in Figure 1.1 illustrates the value of a fund that consists of rolling capital guaranteed Equity-Linked Notes (notional amount of the bond equals to the price of the product) each quarter with a maturity of three years. Thus each product is capital guaranteed but the fund is not capital guaranteed over the three-year time horizon. The investor in this example is an investor with an investment horizon of three years and holds only capital guaranteed products (a total of 23 products, 12 at a time) with a maximum time to maturity of three years. During this time period the fund value decreases with 7.16%, hence the simple naive fund construction is not capital protected.

As the price of the underlying goes up the value of the fund increases since the options gain more in value. This implies that after twelve quarters, when the underlying has gone up a total of 12.68%, the fund's NAV consists of options in a high extent, which makes the investment more risky and erasing the capital guarantee.

1.3 Purpose

The purpose of this thesis is to investigate if it is possible to construct a portfolio based on structured products that possesses the property of capital protection. The focus lies on developing an alternative definition of capital protection, since there obviously does not exist any structured products fund (SPF) fulfilling the requirements of absolute capital guarantee. The objective is also to investigate how a fund with as low downside risk as possible, with a decent expected return (should be competitive in relation to competing funds), should be constructed. The investigation is performed numerically based on simulating trajectories of future price patterns for available structured products and by solving an optimization problem allocating the portfolio based on CVaR constraints as in Uryasev (2000, [38]). Since the thesis will only consider investing in products with the same underlying the importance of using a scheme based on Black and Litterman's (1992) studies in [8] is reduced, thus exploring the impact of active subjective views on the investment choice is left for future research, as well as the possibility of multiple underlying.

The thesis focuses on market associated risks and will not evaluate the impact of credit risk on the portfolio choice. Thus when investigating if the products are capital protected the impact of defaults is neglected, this is usually how the terminology capital protection is used in the concept of structured products.

1.4 Outline

Chapter 2 covers the most fundamental basics which the thesis has its foundation in, such as the theory of risk measures, scenario based optimization and principal component analysis. It is not necessary for the reader to go through this chapter, since it is not imperative to understand these theories to understand the result. In Chapter 3 the definition of a structured product is covered and how the individual components are priced. Chapter 4 covers the issue of capital guarantee, the definition of capital guarantee, how the portfolio should be allocated initially to attain as high degree of capital protection as possible. The chapter covers everything from naive fund constructions to benchmarking with potential competition. A scheme to minimize the downside risk is also described in the chapter.

Chapter 5 covers how the financial assets are modeled and discloses the details regarding the PCA. Chapter 6 covers three proposed allocation schemes, where a modified Korn and Zeytun framework is recommended. In the last chapter conclusions are drawn regarding the results and recommendations for future studies are given.

CHAPTER 2

Theoretical Background

This chapter covers the theoretical background that the reader should be familiar with to understand how the study is conducted. By understanding the concepts in this chapter it is also easier to understand how to replicate the results and also the assumptions affecting the results. The chapter will cover everything from risk measures and principal component analysis (which is an integral part of the modeling) to scenario based optimization and transaction costs. Notable, it is not necessary to understand the concepts discussed in this chapter to understand the results of the study.

To ensure full clarity for the reader matrices are written as bold upper case characters, vectors as bold lower case characters and transposed with superscript T, not only in this chapter but also throughout the whole paper.

2.1 Risk measures

The concept of risk has been around finance for more than fifty years. Markowitz introduced his seminal work within portfolio theory during 1952 [30] where he used standard deviation (volatility) as risk measure to find the optimal tradeoff between risk and return. Since then it has been shown that assets' log returns are not multivariate normal distributed and that the distribution of stock returns often exhibit negative skewed kurtosis (Fisher, 1999, [14]), especially around extreme events such as the 1987 stock market crash, the Black Monday 19th October when the Dow Jones Industrial Average dropped by 508 points to 1738.74, a 22.61% decrease (Browning, 2007, [11]). Such extreme events prove that measuring risk with the variance is not adequate, not even for stock returns.

A portfolio containing derivatives is not, in general, symmetric and variance (even if stock returns are relative symmetric) is thus not an adequate risk measure. Some risk measures have been introduced to capture these heavy tailed, skewed distributions such as Value-at-Risk (VaR), Conditional-Value-at-Risk (CVaR, closely related to Expected Shortfall) (Acerbi, 2002, [3]). Another risk measure, which is not as commonly used as VaR and CVaR is the absolute lower bound, which measures the worst-case outcome.

Further the net worth of the fund portfolio at time T is denoted as $\mathcal{X} = X_T - X_t$, where X_t is the value of the portfolio at time t.

2.1.1 Absolute lower bound

An acceptable portfolio is a portfolio which final net worths are those that are guaranteed to exceed a certain fixed number (e.g. a percentage of the initial investment) c, i.e. the set of acceptable portfolios must fulfill these requirements,

$$\mathcal{A} = \{\mathcal{X} \in \mathfrak{X} : \mathcal{X} \geq c\},$$

which gives the risk measure,

$$\rho(\mathcal{X}) = \min\{m : m(1+r_f) + \mathcal{X} \in \mathcal{A}\} = \min\{m : m(1+r_f) + \mathcal{X} \geq c\}.$$

$x^0 = x^0(\mathcal{X})$ is denoted as the smallest value that $\mathcal{X}$ can take, notice that,

$$\rho(\mathcal{X}) = \min\{m : m(1 + r_f) + \mathcal{X} \geq c\} = \frac{c - x^0}{1 + r_f}.$$

Thus the definition is regarding the worst possible outcome of the position at time T. When regarding absolute capital guarantee the portfolio satisfies $\rho(\mathcal{X}) \leq 0$ (Hult and Lindskog, 2009, [23]).

2.1.2 Value-at-Risk

Value-at-Risk (VaR) is one of the most important concepts within risk management and is also regulated by the FSA in many countries and by the guidelines from the second Basel Accord regarding minimum capital requirements (BIS, 2006, [5]). The idea of VaR is that $\mathrm{VaR}_\alpha(L^w)$ is the value that the portfolio's loss will be less than or equal to with a probability of α. Thus VaR is given as (a similar notation is given in [27]),

- Let L^w denote the loss of the portfolio with the portfolio weights w, and the probability of L^w not exceeding a threshold m as,

$$\psi(w, m) = P(L^w \leq m).$$

- Then Value-at-Risk $\mathrm{VaR}_\alpha(L^w)$ is defined as the loss with a confidence level of $\alpha \in [0, 1]$ by,

$$\mathrm{VaR}_\alpha(L^w) = \min\{m \in \mathbb{R} : \psi(w, m) \geq \alpha\},$$

or,

$$\mathrm{VaR}_\alpha(L^w) = \min\{m \in \mathbb{R} : P(L^w \leq m) \geq \alpha\}.$$

- Where $L^w = -R^w$, where R^w is the return associated with the portfolio vector w, and the return is defined as,

$$R^w = \frac{\text{final wealth}}{\text{initial wealth}} - 1.$$

There exists several areas of critique against VaR; one of the most common critiques is that it is not a coherent risk measure (Hult and Lindskog, 2009, [23]). Also it is not very useful for distributions that have heavy tails, since it only measures the outcomes at the quantile α. Many blame Value-at-Risk of being an integral part why a financial system may fail, since the model underestimates the risk/implication of market crashes (Brooks and Persand, 2000, [10]) and does not take into account the size of the losses exceeding VaR, thus traders have the possibility to hide risk in the tail. Also, VaR is a non-convex and non-smooth function which in many cases has multiple local maximums and minimums, thus it is very hard to construct a portfolio optimization scheme which is effective enough to still be robust and give valid results (Uryasev, 2000, [38]).

Instead of VaR many authors propose Conditional Value-at-Risk (CVaR), which is much easier to implement in portfolio optimization as proposed by Uryasev et al. in [28, 33, 38]. Also Alexander et al. (2006, [4]) have recently developed an optimization scheme which is very efficient for minimizing CVaR and VaR for portfolios of derivatives, but it is much more complex than the one proposed by Uryasev et al., and Alexander et al.'s theory will not be covered in this paper.

2.1.3 Conditional Value-at-Risk

CVaR is based on the definition of VaR since it is the expected loss under the condition that the loss exceeds or equals the VaR, i.e. it is defined as,

$$\mathrm{CVaR}_\alpha(L^w) = \mathbb{E}\left[L^w \mid L^w \geq \mathrm{VaR}_\alpha(L^w)\right].$$

CVaR is a coherent risk measure and more adequate than VaR since it discloses the expectation of the loss if the loss exceeds the VaR, which is an important concept in risk management. The goal of portfolio optimization is often to maximize the return subject to a demand on the maximum risk acceptable. The optimization problem reduces to a linear optimization problem with linear constraints, in the CVaR case. The advantage with a linear optimization problem is that it can be solved using the simplex method.

Krokhmal et al. (2002, [28]) have shown that the solutions to the optimization problems,

$$\min_{x \in X} -R(x), \quad \text{s.t. } \mathrm{CVaR}_\alpha(L^w) \leq C, \tag{2.1}$$

and,

$$\min_{x \in X} -R(x), \quad \text{s.t. } F_\alpha(w, \beta) \leq C, \tag{2.2}$$

give the same minimum value where,

$$F_\alpha(w,\beta) = \beta + \frac{1}{1-\alpha}\int_{y \in \mathbb{R}^m} \left[L(w,y) - \beta\right]^+ p(y)\, dy,$$

$$\mathrm{CVaR}_\alpha(L^w) = \min_{\beta \in \mathbb{R}} F_\alpha(w,\beta),$$

and $L(w,y)$ is the loss function associated with the portfolio vector w, $y \in \mathbb{R}^m$ is the set of uncertainties which determine the loss function. If the CVaR constraint in 2.1 is active, then (w^*, β^*) minimizes 2.2 if and only if w^* minimizes 2.1 and $\beta^* \in \arg\min_{\beta \in \mathbb{R}} F_\alpha(w,\beta)$. When $\beta^* \in \arg\min_{\beta \in \mathbb{R}} F_\alpha(w,\beta)$ reduces to a single point, then β^* gives the corresponding VaR with confidence level of α (Korn and Zeytun, 2009, [27]). Thus it is possible to solve the optimization problem 2.2 instead, which is easily transformed to a linear optimization problem (disclosed in Section 2.3), since it generates the same solution as 2.1, for the proof and more detailed information please consult [28, 33, 38].

2.2 Principal component analysis

Principal component analysis (PCA) is a common tool to generate scenarios for changes in the yield curve and returns for other types of assets. The idea of PCA is that data sets of intercorrelated quantities can be separated into orthogonal variables, which explains the variance and dependence in the data in a simpler way. Thus the number of factors to simulate can be reduced drastically by reducing the number of risk factors to model. A whole yield curve can often be reduced into only three factors, or so-called principal components, thus PCA is a useful tool to generate scenarios that are parsimonious. Studies have shown that by using principal component analysis only two or three principal components are often enough to describe more than 95% of the variation in a yield curve (Barber and Copper, 2010, [6]; Litterman and Scheinkman, 1991, [29]). The main idea is to transform the data to a new orthogonal coordinate system such that the greatest variance by any projection of data comes to lie on the first coordinate (i.e. first principal component), the second greatest variance by any projection on the second coordinate and so on.

Consider a setting with m assets, n number of observations and that the asset returns are given as a $n \times m$ matrix denoted $\tilde{\mathbf{R}}$. It is important to center the returns/data by subtracting its mean to perform the PCA and also normalize the data by dividing with $\sqrt{(n-1)}$ (or by $\sqrt{n}$). Let denote $\mathbf{R} = \frac{\tilde{\mathbf{R}}-\mu}{\sqrt{n-1}}$, this type of PCA is referred to as covariance PCA since the matrix $\mathbf{R}^{\mathrm{T}}\mathbf{R}$ is a covariance matrix (the covariance matrix of the returns). It is also common with correlation PCA, in which each variable is divided by its norm, making $\mathbf{R}^{\mathrm{T}}\mathbf{R}$ a correlation matrix (most common in statistical packages such as MATLAB and R), correlation PCA will be used in this thesis (Abdi, 2010, [2]).

By using singular value decomposition $\mathbf{R}$ can be written on the following form,

$$\mathbf{R} = \mathbf{P}\boldsymbol{\Delta}\mathbf{Q}^{\mathrm{T}},$$

where $\mathbf{P}$ is a $n \times n$ orthonormal matrix, $\mathbf{Q}$ is a $m \times m$ orthonormal matrix (called loading matrix and each column corresponds to one PC) and $\boldsymbol{\Delta}$ is a $n \times m$ matrix with non-negative values on the diagonal. The PCA creates new variables called principal components, which are linear combinations of the original variables and are defined in a way such that the amount of variation associated with them are in decreasing order and orthogonal to each other. Let denote the factor scores $\mathbf{F}$ (observations of the principal components) as,

$$\mathbf{F} = \mathbf{P}\boldsymbol{\Delta},$$

thus,

$$\mathbf{F} = \mathbf{P}\boldsymbol{\Delta} = \mathbf{P}\boldsymbol{\Delta}\mathbf{Q}^{\mathrm{T}}\mathbf{Q} = \mathbf{R}\mathbf{Q},$$

which implies that the ith observation of the jth original variable is expressed as follows,

$$r_{i,j} = \mathbf{Q}^{\mathrm{T}}_{1,j}\mathbf{F}_{i,1} + ... + \mathbf{Q}^{\mathrm{T}}_{m,j}\mathbf{F}_{i,m}.$$

Next $\mathbf{R}^{\mathrm{T}}\mathbf{R}$, is investigated,

$$\mathbf{R}^{\mathrm{T}}\mathbf{R} = \left(\mathbf{P}\boldsymbol{\Delta}\mathbf{Q}^{\mathrm{T}}\right)^{\mathrm{T}}\mathbf{P}\boldsymbol{\Delta}\mathbf{Q}^{\mathrm{T}} = \mathbf{Q}\boldsymbol{\Delta}^{\mathrm{T}}\mathbf{P}^{\mathrm{T}}\mathbf{P}\boldsymbol{\Delta}\mathbf{Q}^{\mathrm{T}} = \mathbf{Q}\boldsymbol{\Delta}^{\mathrm{T}}\boldsymbol{\Delta}\mathbf{Q}^{\mathrm{T}} = \mathbf{Q}\boldsymbol{\Delta}^2\mathbf{Q}^{\mathrm{T}}.$$

$\boldsymbol{\Delta}$ is a diagonal matrix and $\boldsymbol{\Delta}^2$ equals the diagonal matrix $\boldsymbol{\Lambda}$ containing the (positive) eigenvalues $\lambda_1, ..., \lambda_n$ to $\mathbf{R}^{\mathrm{T}}\mathbf{R}$ (since $\mathbf{R}^{\mathrm{T}}\mathbf{R}$ is a positive semi-definite matrix), $\mathbf{Q}$ is an orthogonal matrix. An orthogonal matrix has the property $\mathbf{Q}^{\mathrm{T}} = \mathbf{Q}^{-1}$, thus $\mathbf{Q}^{\mathrm{T}}\mathbf{Q} = \mathbf{Q}\mathbf{Q}^{\mathrm{T}} = \mathbf{I}$. The columns in $\mathbf{Q}$, $\mathbf{q}_1, ...\mathbf{q}_m$ are the corresponding eigenvectors of $\mathbf{R}^{\mathrm{T}}\mathbf{R}$, which are orthonormal. It is possible to assume, without loss of generality, that the columns of $\boldsymbol{\Lambda}$ and $\mathbf{Q}$ are ordered such that the diagonal elements in $\boldsymbol{\Lambda}$ appear in descending order. Note that,

$$\mathrm{Cov}\left(\mathbf{F}^{\mathrm{T}}\right) = \mathbb{E}\left[\mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{R}\mathbf{Q}\right] = \mathbf{Q}^{\mathrm{T}}\mathrm{Cov}\left(\mathbf{R}\right)\mathbf{Q} = \mathbf{Q}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{R}\mathbf{Q} = \boldsymbol{\Lambda},$$

thus the components of $\mathbf{F}$ are uncorrelated and have variances $\lambda_1 \geq ... \geq \lambda_m$, in that order. It can be shown that the returns $\mathbf{R}$ are uncorrelated expressed on the orthogonal basis, which is shown by Hult et al. (2010) in [24].

The idea of PCA is as mentioned to reduce the number of variables needed to describe the data. Only the variables that add important information to the sample are interesting and withdrawn, thus the PCs that add the most variability. To investigate the contribution of variability of each PC the ratio $\frac{\lambda_i}{\sum_{j=1}^{m}\lambda_j}$ is studied. It is most often possible to describe the whole dependence structure for a data set with just the first K PCs, as when modeling the yield curve. Thus the returns are given by,

$$r_{i,j} = \sum_{k=1}^{K}\mathbf{Q}^{\mathrm{T}}_{k,j}\mathbf{F}_{i,k} + \varepsilon_{i,j}.$$

Hence to simulate new returns; simulate factor scores (PCs) and use the factor loadings to calculate the return. The sample must also be rescaled with its standard deviation and mean. For more information regarding PCA and yield curve modeling please advise [2, 29, 36].

2.3 Scenario based optimization

There are in general two approaches to portfolio optimization problems: mean-variance and scenario optimization. One of the frontrunners within mean-variance optimization was Markowitz (1952, [30]), also many others have been well awarded for their contributions within this field. As the concept of skewness and kurtosis has become more prevalent the importance of scenario based optimization has increased. Thus one of the most important uses of scenario based optimization is that it actually allows derivatives/options to be part of the product mix, which is not in general the case with a mean-variance optimization (Grinold, 1999, [17]).

This thesis considers portfolios of structured products, thus it is imperative to use scenario optimization when allocating amongst the assets. Important to mention is that the optimization is totally dependent on the scenarios, thus with the wrong assumptions or scenarios the result will most likely be sub-optimal.

The idea of scenario based optimization is to turn a stochastic problem into a deterministic problem by simulating future scenarios for all available assets. The problem stops being stochastic when the scenarios are generated and the problem is (most often) transformed to linear form, which can be solved by mathematical linear techniques.

To gain a qualitative solution adequate scenarios must be generated, in particular Scherer (2004, [34]) states that the scenarios must be:

- Parsimonious - as few scenarios as possible to save computation power, or time.
- Representative - the scenarios must be representative and give a realistic representation of the relevant problems and not induce estimation error.
- Arbitrage-free - scenarios should not allow arbitrage to exist.

There exist several different methods of simulating data, two of these are bootstrapping historical empirical data and Monte Carlo simulation, i.e. drawing samples from a parametric distribution.

When bootstrapping historical empirical data, also known as historical simulation, the user draws random samples from the empirical distribution, e.g. if the user is trying to simulate annual returns the user may draw 12 monthly returns from the empirical distribution, thus generating an annual return. The draws are performed with replacement, thus if the user is simulating 1,000 yearly returns the user draws for example 12,000 samples of monthly return from the empirical distribution. Notable is that bootstrapping leaves correlation amongst the samples unchanged, but destroys autocorrelation. Since bootstrapping is done by repeated independent draws, with replacement, the data will look increasingly normal as the number of samples increases.

Monte Carlo simulation is similar to historical simulation where the sample is, instead of drawn from the empirical distribution, drawn from a parametric distribution. The parametric distribution may be attained by fitting a statistical distribution to a historical sample of data. If the assets are independent of each other the user can fit an individual distribution to each asset. A popular way of simulating from a parametric distribution is by using e.g. copulas or autoregressive (AR) models.

In a scenario based optimization N available assets to allocate in are considered with the added feature of S possible return scenarios. To solve a scenario based optimization problem there are usually two steps to consider:

Step 1. Simulate S paths of returns for the assets $i = 1, ..., N$.

Step 2. Define a linear optimization problem on those simulated paths, which can be solved using the simplex method.

2.3.1 Conditional Value-at-Risk

The CVaR problem can be (as mentioned earlier) converted to a linear optimization problem, for more details regarding how the problem is transformed please advise [28, 33, 38]. The linear CVaR problem based on scenario simulation is defined as follows,

$$\max_{\mathbf{w},\mathbf{z},\beta} \frac{1}{S}\sum_{k=1}^{S} R^{w}_{T,k},$$

such that:

$$\begin{aligned}
&R^{w}_{T,k} = w_1 R^{1}_{T,k} + ... + w_N R^{N}_{T,k}, && k = 1, ..., S\\
&R^{w}_{T,k} + \beta + z_k \geq 0, && k = 1, ..., S\\
&\beta + \frac{1}{S(1-\alpha)}\sum_{k=1}^{S} z_k \leq C, &&\\
&z_k \geq 0, && k = 1, ..., S\\
&w_1 + ...w_N = 1, &&\\
&w_i \geq 0, && i = 1, ..., N
\end{aligned}$$

where, α is the confidence level of CVaR, β is a free parameter which gives VaR in the optimum solution of the CVaR problem, $R^{i}_{T,k}$ is the return for asset i for scenario k until time T and w_i the weight held in asset i (Krokhmal et al., 2002, [28]). The index k corresponds to which scenario, the index i corresponds to which asset, S is the number of simulated paths and N the number of assets. The problem can be solved using the simplex method, which is to prefer due to its efficiency. Thus the simulation based CVaR problem is a problem that is relative well defined since the increasing power of today's personal computers enables the possibility to solve these problems to a reasonable cost.

It is also possible to write the portfolio choice problem with the CVaR as minimization objective, resulting into the following linear approximation based on scenarios,

$$\min_{\mathbf{w},\mathbf{z},\beta} \beta + \frac{1}{S(1-\alpha)}\sum_{k=1}^{S} z_k,$$

such that:

$$\begin{aligned}
&R^{w}_{T,k} = w_1 R^{1}_{T,k} + ... + w_N R^{N}_{T,k}, && k = 1, ..., S\\
&R^{w}_{T,k} + \beta + z_k \geq 0, && k = 1, ..., S\\
&\frac{1}{S}\sum_{k=1}^{S} R^{w}_{T,k} \geq R_{\text{target}}, &&\\
&z_k \geq 0, && k = 1, ..., S\\
&w_1 + ...w_N = 1, &&\\
&w_i \geq 0, && i = 1, ..., N
\end{aligned}$$

Notable is that the two different problems generate the same efficient frontier.

2.3.2 Minimum regret

Minimum regret is an optimization scheme based on scenario optimization, which maximizes the least favorable outcome of S scenarios given a certain demand on return. It is possible to formulate the optimization problem as a linear problem based on scenarios and it is defined as follows (Scherer, 2004, [34]).

Let $R_{\min}$, be the worst possible outcome and $\overline{R}_{T,k}$ be the expected return vector over all scenarios thus,

$$\max_{\mathbf{w}\in R^N} R_{\min}$$

such that:

$$\begin{aligned}
&w_1 R^1_{T,k} + ... + w_N R^N_{T,k} \geq R_{\min}, && k = 1, ..., S\\
&\mathbf{w}^{\mathrm{T}}\overline{R}_{T,k} \geq R_{\mathrm{target}}, &&\\
&w_1 + ... w_N = 1, &&\\
&w_i \geq 0, && i = 1, ..., N.
\end{aligned}$$

By using this scheme the downside is restricted, this type of allocation is preferable for really risk-averse investors since they know the extent of their worst outcome. It is possible to modify the setup to an equivalent optimization scheme that is defined as follows,

$$\max_{\mathbf{w}\in R^N} \mathbf{w}^{\mathrm{T}}\overline{R}_{T,k}$$

such that:

$$\begin{aligned}
&w_1 R^1_{T,k} + ... + w_N R^N_{T,k} \geq R_{\min}, && k = 1, ..., S\\
&w_1 + ... w_N = 1 &&\\
&w_i \geq 0, && i = 1, ..., N
\end{aligned}$$

2.4 Transaction costs

Transaction costs are important to take into consideration since they can change the profitability of an investment. An investor will have to pay transaction costs every time it rebalances its portfolio due to several factors. The most common transaction costs are such as brokerage commission, bid-ask spread, market impact (volume etc.) Scherer (2004, [34]) suggests that transaction costs, tc, are of the following functional form,

$$tc = \text{Commission} + \frac{\text{Bid}}{\text{Ask}}\text{-Spread} + \theta\sqrt{\frac{\text{Trade volume}}{\text{Daily volume}}}.$$

The bid-ask spread is expressed as a percentage and θ is a constant that needs to be estimated from the market. The problem with this model is that data is needed on daily trading volumes, which is not always available. Instead an appropriate way to estimate transaction costs is by separating the costs into one fixed and one linear part. There are many types of optimization problems incorporating transaction costs and only those relevant to the study will be discussed. There are in general two different approaches to transactions costs; the direct approach restricts the actual cost from happening by introducing actual transaction costs, which are deducted from the return, thus having an impact on the result. The second is to put up restrictions upon actions that have transaction costs linked to them, thus preventing the reactive transaction costs, e.g. restricting turnover and/or trading constraints.

Let w_i be the weight invested in asset i, w_i^{initial} the weight invested in asset i prior to the reallocation, w_i^+ as a positive weight change and w_i^- as a negative weight change (asset sold). Thus the weight invested in asset i after the reallocation is given as: $w_i = w_i^{\mathrm{initial}} + w_i^+ - w_i^-$.

Proportional Transaction Costs

One of the most common types of transaction costs is proportional transaction costs, which means that the transaction costs are proportional to the amount bought or sold of the asset. Let TC_i^+ be the proportional transaction cost associated with buying asset i and TC_i^- be the cost associated with selling asset i. The budget constraint in the traditional portfolio optimization problem is $\sum_i w_i = 1$ which now must be modified since the transactions have to be paid out of the existing budget, thus instead the budget constraint is as follows,

$$\sum_{i=1}^{n} w_i + \sum_{i=1}^{n} \left(TC_i^+ w_i^+ + TC_i^- w_i^-\right) = 1.$$

By introducing transaction costs the reward function is changed from $\sum_{i=1}^{n} w_i \mu_i$ to $\sum_{i=1}^{n} w_i \left(1 + \mu_i\right)$ since $\sum_{i=1}^{n} w_i \leq 1$.

The linear CVaR problem based on scenario based optimization and proportional transaction costs can be formulated as follows,

$$\max_{\mathbf{w},\mathbf{w}^+,\mathbf{w}^-,\mathbf{z},\beta} \frac{1}{S} \sum_{k=1}^{S} \left(1 + R_{T,k}^w\right),$$

such that:

$$\begin{aligned}
&\beta + \frac{1}{S\left(1-\alpha\right)} \sum_{k=1}^{S} z_k \leq C, && \\
&1 + R_{T,k}^w = \left(1 + R_{T,k}^1\right) w_1 + ... + \left(1 + R_{T,k}^N\right) w_N, && k = 1, ..., S \\
&R_{T,k}^w + \beta + z_k \geq 0, && k = 1, ..., S \\
&z_k \geq 0, && k = 1, ..., S \\
&\sum_{i=1}^{N} w_i + \sum_{i=1}^{N} \left(TC_i^+ w_i^+ + TC_i^- w_i^-\right) = 1, && \\
&w_i = w_i^{\text{initial}} + w_i^+ - w_i^-, && i = 1, ..., N \\
&w_i \geq 0, && i = 1, ..., N \\
&w_i^+ \geq 0, && i = 1, ..., N \\
&w_i^- \geq 0, && i = 1, ..., N
\end{aligned}$$

Proportional transaction costs are popular to model with since they do not add a lot of complexity to the optimization program. Also most of the transaction costs in the market are proportional, thus making it a good and adequate choice (Krokhmal et al., 2002, [28]).

Fixed Transaction Costs

Fixed transaction costs arise as soon as a particular asset is traded. The fixed transaction costs are not dependent on the trade size, thus a trade of $1 and $1,000,000 will generate the same cost. Besides proportional transaction costs, fixed transaction costs is one of the most common type of transaction costs in the market. To be able to use fixed transaction costs integer variables $\delta_i^{\pm}$ must be introduced, which takes the value one if trading takes place in asset i ($w_i^{\pm}$ is positive) and zero otherwise. Including proportional transaction costs, the new budget constraint is given as,

$$\underbrace{\sum_{i=1}^{n} w_i}_{\text{Holdings}} + \underbrace{\sum_{i=1}^{n} \left(FC_i^+ \delta_i^+ + FC_i^- \delta_i^-\right)}_{\text{Fixed TC}} + \underbrace{\sum_{i=1}^{n} \left(TC_i^+ w_i^+ + TC_i^- w_i^-\right)}_{\text{Proportional TC}} = 1,$$

where,

$$
\begin{aligned}
&w_i^+ \leq \delta_i^+ w^{\max}, \\
&w_i^- \leq \delta_i^- w^{\max}, \\
&\delta_i^\pm \in \{0,1\},
\end{aligned}
$$

and $w^{\max}$ is a large number.

The problem of using fixed transaction costs is associated with the integer variables $\delta_i^\pm$, which transforms the linear optimization problem, which can be solved using the simplex method, to a mixed integer linear program. A mixed integer linear program has a higher complexity and takes more computation power, or time, to solve thus making it inadequate for many scenarios. Hence it is preferable to consider an optimization program that does not contain binary or integer variables. The mixed integer linear program for CVaR as constraint and the return as target function is defined as follows,

$$
\max_{\mathbf{w},\mathbf{w}^+,\mathbf{w}^-,\delta^+,\delta^-,\mathbf{z},\beta} \frac{1}{S}\sum_{k=1}^{S}\left(1+R_{T,k}^w\right),
$$

such that:

$$
\begin{aligned}
&\beta + \frac{1}{S\left(1-\alpha\right)}\sum_{k=1}^{S} z_k \leq C, \\
&1+R_{T,k}^w = \left(1+R_{T,k}^1\right)w_1 + \ldots + \left(1+R_{T,k}^N\right)w_N, && k=1,\ldots,S \\
&R_{T,k}^w + \beta + z_k \geq 0, && k=1,\ldots,S \\
&z_k \geq 0, && k=1,\ldots,S \\
&\sum_{i=1}^{N} w_i + \sum_{i=1}^{n}\left(FC_i^+\delta_i^+ + FC_i^-\delta_i^-\right) + \\
&+\sum_{i=1}^{N}\left(TC_i^+ w_i^+ + TC_i^- w_i^-\right) = 1, \\
&w_i = w_i^{\text{initial}} + w_i^+ - w_i^-, && i=1,\ldots,N \\
&w_i \geq 0, && i=1,\ldots,N \\
&0 \leq w_i^+ \leq \delta_i^+ w^{\max}, && i=1,\ldots,N \\
&0 \leq w_i^- \leq \delta_i^- w^{\max}, && i=1,\ldots,N \\
&\delta_i^\pm \in \{0,1\}, && i=1,\ldots,N
\end{aligned}
$$

CHAPTER 3

Financial Assets

This chapter covers the basics behind the financial assets, e.g. what types of instruments that are available and how they are priced. It is not necessary to read the chapter for someone who is well familiar with structured products. In this thesis three types of assets are available for the investor, these are: a risk-less asset (bond), a risky index (an Exchange Traded Fund (ETF)) and structured products (Equity-Linked Notes) with the risky index as underlying. The bond is a theoretical asset where the default probability of the issuer is zero. The ETF is assumed to follow the index perfectly, thus this market is relative theoretical. The structured products are combinations of bonds and derivatives with the index as underlying.

3.1 Definition of a structured product

Structured products are in general synthetic investment instruments that are created to meet special needs for customers that cannot be met by the current market. These needs are often focused on low downside risk on one hand and the possibility of growth on the other. This is usually achieved by constructing a portfolio consisting of securities and derivatives. Hence there are endless combinations of possible structured products, thus there exists no standard product. According to Martellini et al. (2005, [31]) was the first structured form of asset management the introduction of portfolio insurance such as the constant proportion or option based portfolio insurance strategy. Later on this has spurred the development of more exotic structures and creative constructions. The main reason for such constructions is to fit the investors' risk and aspiration preferences.

Today the most common setup for a structured product is the combination of a zero-coupon bond and an option. The type of option varies widely and most often a plain vanilla option with a certain equity index, equity, currency, etc., as underlying is used; these products are usually called Equity-Linked Notes (ELN). Since only the option part is exposed to the market, the products are often also called capital guaranteed investment vehicles. Investment structures that possess the property of capital guarantee are usually called notes, thus structured products that posses a minimum of 100% capital guarantee are often called Principal protected notes (PPN).

Examples of other types of options used in structured products are such as basket options, rainbows, look-back options, path dependent options and barrier options (HSBC, 2010, [21]). These products will not be used in this thesis due to their computational heavy pricing models involving, in many cases, Monte Carlo based option pricing methods. In the future when referring to a structure product this thesis will refer to an Equity-Linked Note.

The example in Figure 3.1 is of a structured product that consists of a zero-coupon bond and an option. The notional amount (amount paid at maturity $T = 1$) of the zero-coupon bond equals the price of the structured product at time $t = 0$. Thus the investment is capital guaranteed since the minimum amount that the investor (conditional on that the issuer of the zero-coupon bond has not defaulted) receives is the price at time $t = 0$. Note that the bond's face value is deterministic and the payoff of the option is stochastic.

The idea of the structured product is usually that most of its value is contributed by the

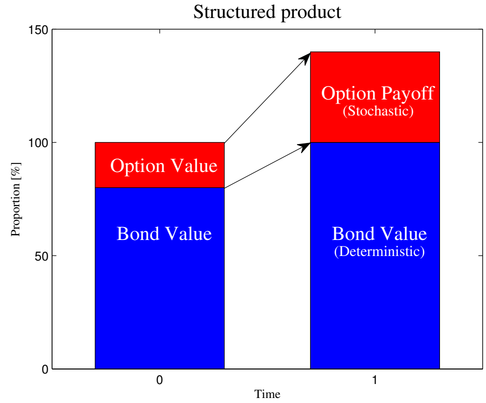


Figure 3.1: Structure of a structured product, time $t = 0$ is time of issuance and $T = 1$ is the time of maturity, the bond payoff is deterministic and the option payoff is stochastic.

zero-coupon bond. The relation between how much is invested in the zero-coupon bond and the option depends on the price of the zero-coupon bond (in theory, in practice it also depends on the fees taken by the issuer). Thus different constructions are available depending on the current market climate, e.g. in a regime with low interest rates the zero-coupon bonds are expensive and thus the amount left for buying options is relative small. The leveraged exposure to the underlying is called participation rate (HSBC, 2010, [21]). The participation rate is measured in percentage (or you might call it number of contracts).

3.1.1 Dynamics

Equity-Linked Notes in this thesis have notional amount of the bond and strike price equal to the price of the underlying when issued, i.e. the options are plain vanilla at-the-money call options. Let denote,

T	as the time of maturity
S_0	as the value of the underlying at the time of issuance
S_T	as the value of the underlying at maturity
k	as the participation rate
C	as the price of the option
B	as the price of the zero-coupon bond with notional amount S_0
r	as the risk-free interest rate

Thus the payoff at maturity of the structured product can be written as,

$$\underbrace{S_0}_{\text{Bond part}} + \underbrace{k \max\left(S_T - S_0, 0\right)}_{\text{Option part}},$$

where the participation rate, k, is given by,

$$k = \frac{S_0 - B_0}{C}.$$

Thus the amount paid out to the investor at maturity, T, in dollars per dollar invested is given by,

$$1 + k \max\left(\frac{S_T - S_0}{S_0}, 0\right),$$

i.e. k controls how much the investor will participate in the market. Notable is that k is increasing in r and decreasing in σ (volatility) as seen in Figure 3.2. The option prices are calculated with Black-Scholes formula as disclosed in Section 3.3. As the volatility increases the price of the option increases, thus decreasing the participation rate. When the interest rate increases the price of the zero-coupon bond decreases, thus increasing the participation rate.

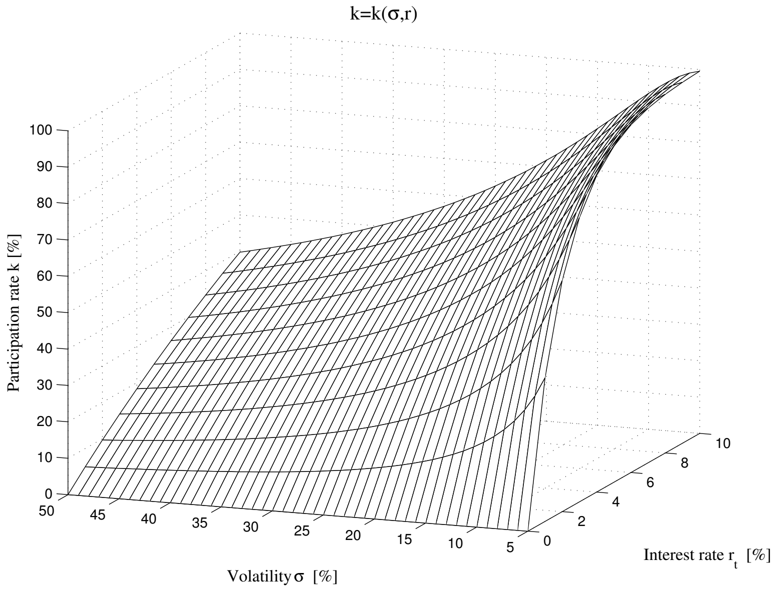


Figure 3.2: The surface plot shows the relation between the interest rate r_t, the volatility σ and the participation rate $k\left(r_t, \sigma\right)$ for $T = 1$.

3.2 Zero-coupon bond

A zero-coupon bond is a bond that pays no coupons, the only cash flow generated from a zero-coupon bond is the notional amount (face value) paid at maturity. Thus using a continuous compounding for the interest rate the price of a risk-less zero-coupon bond at time t, with maturity T and notional amount \$1 is given by,

$$B_t = e^{-r_t(T-t)},$$

where r_t is the risk-free rate at time t.

3.3 Plain vanilla call option

A plain vanilla call option, also called European call option is a product that gives the holder the right, not the obligation, at maturity, time T, to buy the underlying asset S for the predefined strike price K. The price of the asset S at time T is denoted as S_T. Since the holder does not have the obligation to exercise the option, the option will have the following payoff X at time T,

$$X = \max\left(S_T - K, 0\right).$$

The price at time t of the European call option is denoted $C\left(S_t, K, t, T, r, \sigma, \delta\right)$ where S_t is the price of the underlying at time t, K the strike price, T the time of maturity, r the risk-free rate,

σ the volatility and δ the continuous dividend yield. It is common to price these options with Black-Scholes formula, which is given below (Black and Scholes, 1973, [9]; Hull, 2002, [22]),

$$C(S_t, K, t, T, r, \sigma, \delta) = \Pi(t; X),$$

where,

$$\Pi(t; X) = S_t e^{-\delta(T-t)} N[d_1(t, S_t)] - e^{-r(T-t)} K N[d_2(t, S_t)], \tag{3.1}$$

and,

$$\begin{aligned} d_1(t, s) &= \frac{1}{\sigma\sqrt{T-t}} \left\{ ln\left(\frac{s}{K}\right) + \left(r - \delta + \frac{1}{2}\sigma^2\right)(T-t) \right\}, \\ d_2(t, s) &= d_1(t, s) - \sigma\sqrt{T-t}. \end{aligned}$$

N is the cumulative distribution function for the standard normal distribution. It is important to understand the underlying assumptions of Black-Scholes formula. These are such as that the stock returns are lognormal distributed, constant volatility, constant risk-free rate, etc. The Black-Scholes market model under the probability measure $\mathbb{P}$ is given as (Björk, 2009, [7]),

$$\begin{aligned} dS(t) &= \mu S(t)\,dt + \sigma S(t)\,dW(t), & S(0) &= s \\ dB(t) &= rB(t)\,dt, & B(0) &= 1 \end{aligned}$$

where W is a Brownian Motion (i.e. Wiener process), μ the drift and σ the volatility. Notable is that the transform under this market model to the risk-neutral probability measure Q only transforms the drift μ to r by using Girsanov's theorem, which implies that the volatility σ is the realized volatility observed at the market (Björk, 2009, [7]).

Notable is that the assumption of a continuous dividend yield is adequate in the setting of modeling an index, since it is reasonable to assume that the dividends are spread all over the year for different components of the index.

3.3.1 A note regarding the critique against B & S

It is commonly known that the market model above has sustained a lot of critique since Black and Scholes proposed it in 1973 [9]. The main area of criticism is the assumption of lognormal returns and thus the dependence of the normal distribution (Björk, 2009, [7]). Instead many authors such as Heston (1993, [20]) and Hagan et al. (2007, [18]) propose models that incorporate stochastic volatility and the volatility-smile. Empirical evidence has shown that asset returns are skewed and exhibit kurtosis. Since the Black-Scholes formula seems inadequate to price plain vanilla options using the original definition with realized (historical) volatility, practitioners usually use the so-called implied volatility instead. Implied volatility is the volatility so that Black-Scholes formula gives the correct market price.

Since the thesis is regarding structured products the pricing of these options is only relevant in the context of structured products. Wasserfallen and Schenk (1996, [39]) found that the prices of structured products are not affected systematically by using either realized volatility or implied volatility. The study compared the theoretical value of the structured products (using both realized and implied volatility) with the observed price at the primary and secondary market.

Thus the effect of using realized volatility in comparison to implied volatility in the pricing of the structured products can be disregarded in this thesis since the error in comparison with the primary and secondary market is negligible (Wasserfallen and Schenk, 1996, [39]). Hence Black-Scholes formula is an adequate choice for pricing the ELNs.

CHAPTER 4

The Capital Protection Property

One of the most appealing properties of Equity-Linked Notes is the capital guarantee. Most Equity-Linked Notes are constructed such that the notional amount of the bond equals the issue price. Thus the investor is guaranteed to not loose any capital. Investors suffer from a syndrome called loss-aversion, which means that they often make irrational decisions just to avoid loosing any capital (Shefrin, 2002, [35]). This is one main factor why people invest in structured products, the neat construction of market participation and capital guarantee.

The concept of a structured products fund is very appealing since the investor gains diversification amongst the assets, with different strikes and maturities (and by several underlying, which is excluded in this paper). As mentioned in the introduction, several studies recent years have shown that structured products actually are a good investment choice for rational investors, as long as the investor uses a diversification framework. The problem with structured products is that they usually require that the investor invests a minimum amount in each structured product. The market for structured products is quite large and during 2009 the total market for ELNs was 55.1 billion SEK in Sweden alone, a large proportion of these investors are small private investors. If an investor should diversify its portfolio amongst structured products the invested amount increases to relative high levels, since each product has a required minimum invested amount.

Investors do not, according to Shefrin (2002, [35]), prefer to invest a huge percentage of their total capital in one single type of investment. Thus the only reasonable way for small investors to diversify in structured products, without requiring a huge amount of capital, would be to invest in a fund that is based on structured products. Problem arises when trying to construct a fund in a way such that the property of capital guarantee is retained.

This chapter covers a relative broad spectrum of topics. First of all the definition of capital guarantee is discussed and how it relates to a fund. A large part of the chapter is used to discuss so called naive fund constructions. These constructions are simple forms of funds based on structured products that are used to evaluate how the fund should be constructed so that they carry as little downside risk as possible. The chapter also covers a similar study conducted for option portfolios, where it is investigated how an investor should allocate amongst ATM call options, at issuance, to limit the downside risk as much as possible. The results from these two sections are then combined to investigate certain limits for the possession allowed in structured products, to limit the downside risk. In the end alternative definitions of capital protection are discussed to decide under which risk measure the structured products fund should be allocated.

4.1 Capital guarantee

When describing a fund based on capital guaranteed products it is quite easy to believe that the fund itself also would be capital guaranteed, it is not as easy as that. A capital guaranteed product is considered to be capital guaranteed over a certain investment period, e.g. three years. Thus the investment horizon is finite and fixed for every investor. When considering a fund, the investor base is widely varying and most investors have different investment horizons and investment times. This means that for a fund to be capital guaranteed it needs to be capital guaranteed for all maturities and all overlapping time periods. Most financial products can have a value of zero prior to their maturity based on different market risk factors, thus the value of the fund at intermediate time points may converge to zero with a positive probability. It is now quite obvious that a fund that holds these properties is a fund practically only investing in the risk-free rate, which is not a desirable result.

Thus it is now quite clear that a fund based on structured products will not hold the property of capital guarantee for all investors, maybe not for even anyone. This implies that the terminology must be changed from capital guarantee to capital protection. Since capital protection is a more diffuse definition that only states that the capital is protected, not guaranteed. Thus a fund that uses the terminology capital protection in its marketing campaign only needs to show that some of the capital base is protected. Thus it is important to notice the difference between capital guarantee and capital protection, while smaller private investors may not notice the difference, the difference is significant.

In the following sections it will be covered how a fund should be constructed to attain as much capital protection as possible. This includes naive fund constructions and investigations of option portfolios etc.

4.2 Naive fund constructions

This section covers naive fund constructions; these constructions are created to understand more of the dynamics of funds constructed of only structured products. The naive fund constructions are probably the simplest possible funds based on structured products, thus called naive fund constructions. By understanding more of the factors affecting the return of the naive fund, it is possible to gain knowledge of how a portfolio should be allocated amongst different structured products, to have as much capital protection as possible.

This section covers two different naive fund constructions and how changes in the different risk factors affect the return of the fund over an investment period. These different changes are stressed through using predefined scenarios, not simulation. The reason why simulation is not used is that the outcomes should be independent on the market model, as far as possible. Many more scenarios, than the ones disclosed in this thesis, have been tested but only some the most unfavorable scenarios are disclosed, since these scenarios are the only interesting scenarios when regarding capital guarantee.

All structured products, in this chapter, have a time to maturity of three years at issuance; a new structured product is issued each quarter. Thus it takes twelve quarters until the first product has matured, thus it is for simplicity assumed that the fund is launched to the public during quarter twelve. It is assumed that investors have an investment horizon of three years, thus the scenarios will be for six years, three prior to the investment (since the price of structured products are path dependent) and three years after the investment.

The first fund construction is a fund that is rolling capital guaranteed structured products. Hence the notional amount of the bond equals the price of the structured product at issuance. Thus the fund is just rolling capital guaranteed products with twelve different maturities. A new product is bought every quarter with the payoff of the product that matures the same quarter.

The second fund construction is a fund that buys a standardized ELN at the start of the fund and buys a new customized structured product each following quarter. The new customized structured product has the same ratio of option:bond value as the fund prior to the rebalancing, thus maintaining the fund's ratio options:bonds relatively intact.

4.2.1 Naive fund construction number 1

A new structured product, that is 100% capital guaranteed, is issued every quarter where its price equals the notional amount of the bond as well as the price of the underlying. Thus the ratio options:bonds value is determined by the interest rate and the volatility, each product has maturity three years after they are issued. The structured products follow the dynamics given in Section 3.1.1.

Naive fund construction number one is a fund that is rolling the available structured products. Every structured product matures after three years, which means that the fund buys the newly issued structured product every quarter with the payoff of the matured product (it is assumed that the fund can hold infinitesimal fractions of the structured products). Hence the fund will hold a maximum of twelve products each quarter.

The fund has to start somewhere and since the product prices are path dependent it is necessary to start the fund three years prior that the investor invests in it (in this setting). Below follows a more detailed description of the fund. Denote,

i	as the quarter the product was issued
β_t^i	as the value of the bond issued at quarter i at time t
O_t^i	as the value of the ATM call option issued at quarter i at time t
S_t	as the price of the underlying at time t
V_t	as the value of the fund at time t
$c(S_t, K, t, T, r, \sigma)$	as the value of a call option at time t with strike K, maturity time T
k^i	as the participation rate in the option issued at quarter i
r_t^f	as the risk-free interest rate at time t
r_t^i	as the return of product i between $t-1$ and t
$\mathbf{r}_t$	as the return vector between $t-1$ and t
σ_t	as the volatility at time t
w_t^i	as the weight allocated in the product issued at quarter i at time t
$\mathbf{w}_t$	as the weight vector at time t

As mentioned above, the structured products follow the dynamics given in Section 3.1.1 thus the following formulas describe the prices of the structured products, the bonds are priced as,

$$\beta_t^i = \begin{cases} S_i e^{-r_t(i+12-t)}, & \text{if } i \le t \le i+12 \\ 0, & \text{otherwise} \end{cases}$$

the participation rate of the option issued at quarter i is given as,

$$k^i = \frac{\left(S_i - \beta_i^i\right)}{c\left(S_i, S_i, i, i+12, r_i, \sigma_i\right)} = \frac{S_i\left(1 - e^{-12r_i}\right)}{c\left(S_i, S_i, i, i+12, r_i, \sigma_i\right)},$$

the value of the call option issued at quarter i at time t is given as,

$$O_t^i = \begin{cases} k^i c\left(S_t, S_i, t, i+12, r_t, \sigma_t\right), & \text{if } i \le t \le i+11 \\ k^i \max\left(S_t - S_i, 0\right), & \text{if } t = i+12 \\ 0, & \text{otherwise} \end{cases}$$

Hence the return between $t-1$ and t for the structured product issued at quarter i is given as,

$$r_t^i = \frac{\beta_t^i + O_t^i}{\beta_{t-1}^i + O_{t-1}^i} - 1.$$

Now that the return for each structured product between each time period is known the attention can again be turned towards the fund (the return of each product is the only necessary information to calculate the return of the fund). The portfolio weights will be given differently between quarters 0-11 and 12-24, since there is only one product available at quarter 0, only two products available at quarter 1 and so on. Thus the portfolio sells of some of its capital each quarter to allocate this in the newly issued product, until there are twelve products. At quarter 0 the fund buys the newly issued product, at quarter 1 the fund sells of 50% of its possession to allocate in the newly issued product, at quarter 2 the fund sells of 33.33% of its possession to allocate in the newly issued product, at quarter 3 the fund sells of 25% of its possession to allocate in the newly issued product and so on until the first product matures, thus the portfolio weight for asset i at time t between quarters 0-11 is given as,

$$w_t^i \mid 0 \leq t \leq 11 = \begin{cases} \dfrac{\left(1+r_t^i\right) w_{t-1}^i}{\left(\mathbf{1}+\mathbf{r_t}\right)^{\mathrm{T}} \mathbf{w}_{t-1}} \dfrac{t}{t+1}, & \text{otherwise} \\ 1/(i+1), & \text{if } i = t \\ 0, & \text{if } i > t \end{cases}$$

The first product matures during quarter twelve, thus there is a different allocation scheme to consider from quarter twelve and onwards. The payoff of the matured product is as from quarter twelve reinvested in the newly issued product (it is only rolling over the product). Thus the portfolio weight for asset i at time t from quarter twelve and onwards is given as,

$$w_t^i \mid t \geq 12 = \begin{cases} \dfrac{\left(1+r_t^i\right) w_{t-1}^i}{\left(\mathbf{1}+\mathbf{r_t}\right)^{\mathrm{T}} \mathbf{w}_{t-1}}, & \text{otherwise} \\ \dfrac{\left(1+r_t^{i-12}\right) w_{t-1}^{i-12}}{\left(\mathbf{1}+\mathbf{r_t}\right)^{\mathrm{T}} \mathbf{w}_{t-1}}, & \text{if } i = t \\ 0, & \text{if } i \leq t-12 \text{ or } i > t \end{cases}$$

The fund's return is the weighted return of all the assets' returns, thus the value of the fund is given as,

$$V_t = \begin{cases} S_t, & \text{if } t = 0 \\ V_{t-1}\left(\mathbf{1}+\mathbf{r_t}\right)^{\mathrm{T}} \mathbf{w}_{t-1}, & \text{otherwise} \end{cases}$$

Scenarios, underlying

The next step is to investigate some of the most important scenarios for the fund construction. These scenarios should stress the fund construction such that weaknesses are disclosed. By disclosing the worst-case scenarios it is possible to counter these characteristics by changing the fund construction.

Note that only some of the most unfavorable scenarios are disclosed in this thesis, less unfavorable scenarios are not disclosed, since they are not of importance (see Appendix A for more scenario examples).

As mentioned earlier, it is assumed that the investors have an investment horizon of three years and invest after three years (when the fund is announced on the market). It is important that the investment horizon coincides with the time to maturity of the structured products, since the individual structured product issued at the start of the investment provides absolute capital guarantee for the investors and is their alternative investment. The scenarios are constructed to stress the negative outcomes and as shown in Section 4.2.4 the underlying is the most prevalent risk, thus the scenarios are mainly conducted for changes in the underlying. Therefor the other parameters are set to be constant, $\sigma = 0.10$ and $r^f = 0.015$ per quarter, for all quarters with a flat yield curve.

Scenario 1

The first scenario is a scenario where the underlying has a continuous return of 10% during the first twelve quarters, then a continuous return of -15% the last twelve quarters; the result is shown in Figure 4.1.

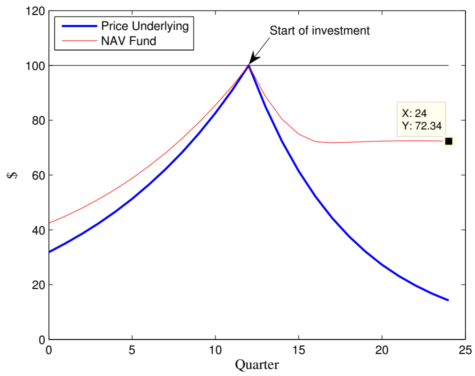


Figure 4.1: Scenario 1, fund construction number 1: The value of the fund deteriorates as the price of the underlying decreases, as the options are decreasing in value. The fund reallocates to more options and cannot utilize the effect of the growing value of the bonds in the same extent as a regular structured product, since the fund buys new structured products each quarter. Fund return quarters 12-24: -27.66%.

Figure 4.1 indicates that the price of the fund increases steadily during the quarters 1-12, since all of the options end up in the money and increase in value. This implies that the ratio between the value contained in options and bonds increases. Thus an investor who starts its investment at quarter 12 will buy a high proportion of options, much higher than an investor who bought the fund at quarter 0, hence the investor has no capital guarantee. On the other hand an investor who invested at quarter 7 will actually receive a slight capital protection on its previous positive return. The fund invests each quarter in a new structured product, thus decreasing the ratio of options:bonds prior quarter 12, thus increasing the capital protection for existing investors.

Figure 4.1 discloses that the value of the options decreases, towards zero, as the price of the underlying continues to decrease. The problem with this allocation scheme is that the ratio options:bonds increases as new products are introduced to the market, thus allocated in the fund. The result indicates that the fund always strives to attain the original ratio by rebalancing from bonds to options (or from options to bonds in the increase case), resulting in a further loss on the option part. Thus there exists no capital guarantee since the fund is always buying new options.

The portfolio has lost most of its value in options, so how does the portfolio respond to an increase in the price of the underlying after quarter 24? Figure 4.2 shows that there are still some options left, but their value was really small prior to the increase, the portfolio still has the same level of participation rate for the structured products.

An interesting feature with this portfolio construction is that; after an extreme decline a new investor actually gains a degree of capital protection, as disclosed in Figure 4.3. In this setting each product that matures is invested in the newly issued structured products, which means that the amount invested in the new structured product is the notional amount of the previous product, which implies that both products have the same notional amount. Thus during a large

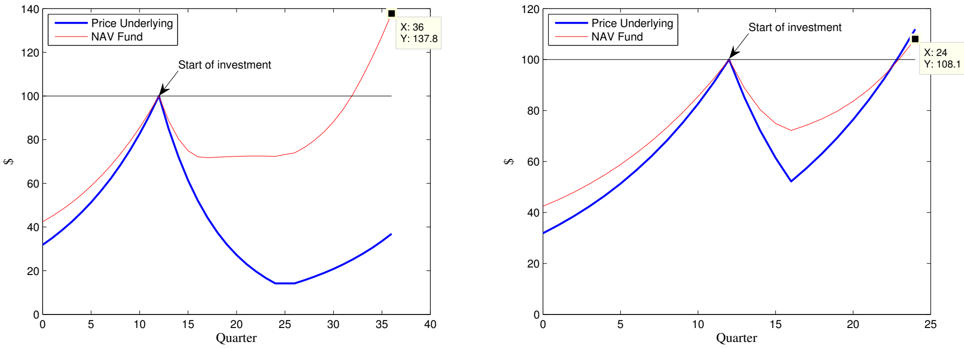


Figure 4.2: Extensions of scenario 1: The fund still has a lot of options held in the portfolio at quarter 24, thus an appreciation in the underlying generates a high return even though the previous decrease in the price.

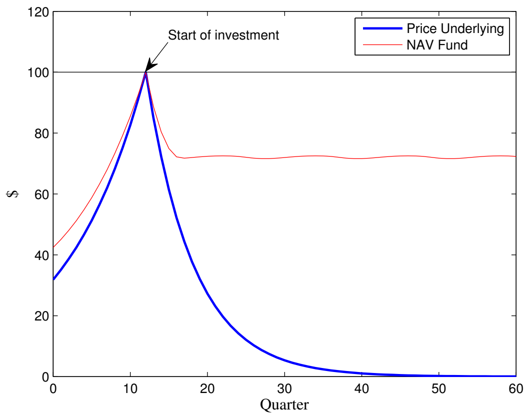


Figure 4.3: The fund has a floor which it does not cross (given a flat and constant yield curve) since the portfolio is in practice just rolling bonds.

decline the fund's value has a period of 12 quarters, where the value repeats itself. This means that an investor at quarter 25 or 37 will in practice hold an identical position in bonds and options as seen in Figure 4.3 (note that this is a special case with the flat and constant yield curve), thus the portfolio is in some sense capital guaranteed over the twelve month period in this case assuming a flat and constant yield curve, given the previous decline in the underlying. On the other hand this happens since the investor buys almost only bonds and a small fraction of options, thus the upwards potential is limited during the first quarters of a bull market.

Scenario 2

The second scenario is a scenario where the underlying has zero in return during the first twelve quarters, then a continuous return of -15% the last twelve quarters; the result is shown in Figure 4.4. It can be expected that the outcome of this scenario should be very similar as the previous one, since the scenarios do not differ a lot from each other.

The fund value increases slightly during quarters 0-12 due to the interest of the bonds, while the option value declines due to the theta value of the options. As the underlying crashes during quarters 12-24 so does the option value, thus most of the value is contributed by the bonds at

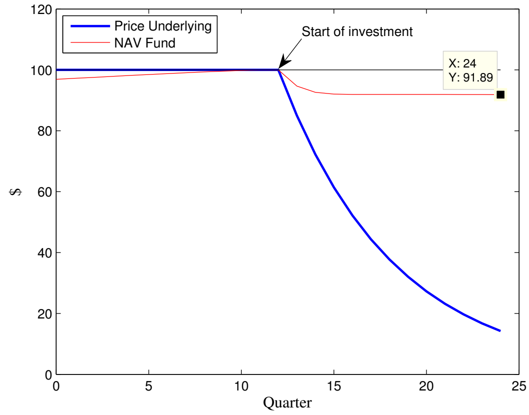


Figure 4.4: Scenario 2, fund construction number 1: 0% in return of the underlying for twelve quarters, then continuous return of -15% each quarter. Fund return quarters 12-24: -8.11%.

quarter 15. This implies that, as the underlying continues to drop in price, there is not much value left in options to affect, thus after a few quarters the fund is almost rolling bonds (since the decrease in option value each quarter reflects the increase of the bond value proportion of the total value).

Thus the portfolio actually has, in this scenario, a restricted downside over the time horizon. Hence the key to capital protection is to avoid holding products that are far in the money, since they have a large percentage of value contained in the options. Therefor the portfolio should be rebalanced such that these assets are underweighted.

4.2.2 Naive fund construction number 2

The naive fund construction number 2 differs slightly from the previous one. The first product follows the same dynamics as in the previous subsection (100% capital guaranteed) but all the other products are not standardized. Instead the structured products issued after quarter 0 are customized in a way such that the ratio between the value contained in options and in bonds is maintained relative stable for the fund. Thus every new structured product that is issued has a participation rate such that the ratio between its option value and bond value, at issuance, equals the fund's ratio between option value and bond value at this quarter. The price of the structured product equals the price of the underlying at issuance, which is also the option's strike price.

Every structured product matures three years after issuance, which means that the fund buys the newly issued structured product every quarter with the payoff of the matured product (it is assumed that the fund can hold infinitesimal fractions of the structured products). Hence the fund will hold a maximum of twelve products each quarter.

The fund has to start somewhere and since the product prices are path dependent it is necessary to start the fund three years prior that the investor invests in it (in this setting). Below follows a more detailed description of the fund (the same notations as in the previous subsection will be used).

The bonds' notional amounts depend on both the price of the underlying and the ratio options:bonds in the fund. The fund's percentage of capital held in bonds at quarter i is given as,

$$\sum_{j=0}^{i-1} \frac{\beta_i^j}{\left(\beta_i^j + O_i^j\right)} w_i^j, \text{ and the percentage held in options as, } \sum_{j=0}^{i-1} \frac{O_i^j}{\left(\beta_i^j + O_i^j\right)} w_i^j$$

thus the bond prices are given as,

$$\beta_t^i = \begin{cases} S_i e^{-r_t(i+12-t)}, & \text{if } i = 0 \\ \displaystyle\sum_{j=0}^{i-1} \frac{\beta_i^j}{\left(\beta_i^j + O_i^j\right)} w_i^j S_i e^{12r_i - r_t(i+12-t)}, & \text{if } i \leq t \leq i + 12 \\ 0, & \text{otherwise} \end{cases}$$

the participation rate of the option issued at quarter i is given as,

$$k^i = \begin{cases} \dfrac{\left(S_i - \beta_i^i\right)}{c\left(S_i, S_i, i, i+12, r_i, \sigma_i\right)} = \dfrac{S_i\left(1 - e^{-12r_i}\right)}{c\left(S_i, S_i, i, i+12, r_i, \sigma_i\right)}, & \text{if } i = 0 \\ \displaystyle\sum_{j=0}^{i-1} \frac{O_i^j}{\left(\beta_i^j + O_i^j\right)} w_i^j \frac{S_i}{c\left(S_i, S_i, i, i+12, r_i, \sigma_i\right)}, & \text{otherwise} \end{cases}$$

the value of the call option issued at quarter i at time t is given as,

$$O_t^i = \begin{cases} k^i C\left(S_t, S_i, t, i+12, r_t, \sigma_t\right), & \text{if } i \leq t \leq i + 11 \\ k^i \max\left(S_t - S_i, 0\right), & \text{if } t = i + 12 \\ 0, & \text{otherwise} \end{cases}$$

Hence the return between $t-1$ and t for the structured product issued at quarter i is given as,

$$r_t^i = \frac{\beta_t^i + O_t^i}{\beta_{t-1}^i + O_{t-1}^i} - 1.$$

Now that it is known how the return for each structured product between each time period is calculated the attention can again be turned towards the fund and how the portfolio weights are calculated (since the return of the products each quarter depends on the weights the previous quarter). The portfolio weights will be given differently between quarters 0-11 and 12-24, since there is only one product available at quarter 0, only two products available at quarter 1 and so on. Thus the portfolio sells of some of its capital each quarter to allocate this in the newly issued product, until there are twelve products. At quarter 0 the fund buys the newly issued product, at quarter 1 the fund sells of 50% of its possession to allocate in the newly issued product, at quarter 2 the fund sells of 33.33% of its possession to allocate in the newly issued product, at quarter 3 the fund sells of 25% of its possession to allocate in the newly issued product and so on until the first product matures, thus the portfolio weight for asset i at time t between quarters 0-11 is given as,

$$w_t^i \mid 0 \leq t \leq 11 = \begin{cases} \dfrac{\left(1 + r_t^i\right) w_{t-1}^i}{\left(\mathbf{1} + \mathbf{r_t}\right)^{\mathrm{T}} \mathbf{w}_{t-1}} \dfrac{t}{t+1}, & \text{otherwise} \\ w_t^i = 1/(i+1), & \text{if } i = t \\ w_t^i = 0, & \text{if } i > t \end{cases}$$

The first product matures during quarter twelve, thus there is a different allocation scheme to consider from quarter twelve and onwards. The payoff of the matured product is as from quarter twelve reinvested in the newly issued product. Thus the portfolio weight for asset i at time t from quarter twelve and onwards is given as,

$$w_t^i \mid t \geq 12 = \begin{cases} \dfrac{\left(1+r_t^i\right) w_{t-1}^i}{\left(\mathbf{1}+\mathbf{r_t}\right)^{\mathrm{T}} \mathbf{w}_{t-1}}, & \text{otherwise} \\ \dfrac{\left(1+r_t^{i-12}\right) w_{t-1}^{i-12}}{\left(\mathbf{1}+\mathbf{r_t}\right)^{\mathrm{T}} \mathbf{w}_{t-1}}, & \text{if } i=t \\ 0, & \text{if } i \leq t-12 \text{ or } i>t \end{cases}$$

The fund's return is the weighted return of all the assets' returns, thus the value of the fund is given as,

$$V_t = \begin{cases} S_t, & \text{if } t=0 \\ V_{t-1}\left(\mathbf{1}+\mathbf{r_t}\right)^{\mathrm{T}} \mathbf{w}_{t-1}, & \text{otherwise} \end{cases}$$

Scenarios, underlying

The next step is to investigate some of the most important scenarios for the fund construction. These scenarios should stress the fund construction such that weaknesses are disclosed. By disclosing the worst-case scenarios it is possible to counter these characteristics by changing the fund construction.

Note that only some of the most unfavorable scenarios are disclosed in this thesis, less unfavorable scenarios are not disclosed, since they are not of importance.

As mentioned earlier, it is assumed that the investors have an investment horizon of three years and invest after three years (when the fund is announced on the market). It is important that the investment horizon coincides with the time to maturity of the structured products, since the individual structured product issued at the start of the investment provides absolute capital guarantee for the investors and is their alternative investment. The scenarios are constructed to stress the negative outcomes and as shown in Section 4.2.4 the underlying is the most prevalent risk, thus the scenarios are mainly conducted for changes in the underlying. Therefor the other parameters are set to be constant, $\sigma = 0.10$ and $r^f = 0.015$ per quarter, for all quarters with a flat yield curve.

Scenario 1

The first scenario is a scenario where the underlying has a continuous return of 10% during the first twelve quarters, then a continuous return of -15% the last twelve quarters; the result is shown in Figure 4.5.

Figure 4.5 shows that the value of the fund increases a lot as the underlying increases, due to the increase of the options and their convexity. Thus an investor that buys the fund at quarter twelve buys a high proportion of options instead of bonds, since the fund maintains its ratio of options:bonds relative stable when rebalancing. The fund's proportion held in options converges towards zero after seventeen quarters, thus bonds constitute almost all of the value. Figure 4.5 discloses that the fund actually has a positive return after quarter 17, due to the positive return of the bonds, even though the market continues to decrease, as the value of the option part is already zero.

If the market goes down during twelve consecutive months an increase in the market would not have any affect on the value of the fund, since it would not contain any options anymore. This fund construction creates a certain level of capital protection, but not in the desired way

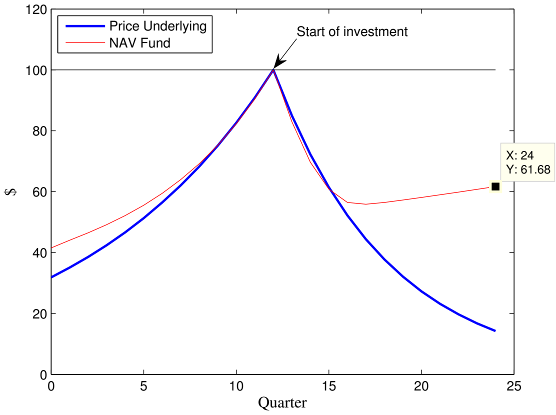


Figure 4.5: Scenario 1, fund construction number 2: Similar result as in the previous fund construction. As the price of the underlying increases the ratio options:bonds increases heavily to approximately 1:1 from 16.47:83.53, thus a new investor at quarter twelve can loose approximately half of its investment. Fund return quarters 12-24: -38.32%.

since it is useless after a huge decline. It is also too risky since gains from the options are not reallocated to bonds, creating a huge downside after a bull market. Also, when the market turns bust during consecutive quarters the portfolio reduces its participation heavily with the market, which can imply that the fund is only allocating in bonds and that it is no longer a structured products fund.

Scenario 2

The second scenario is a scenario where the underlying has zero in return during the first twelve quarters, then a continuous return of -15% the last twelve quarters; the result is shown in Figure 4.6.

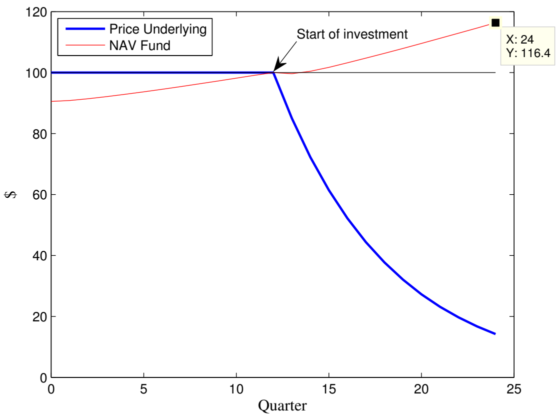


Figure 4.6: Scenario 2, fund construction number 2: The ratio options:bonds is very low since the price of the underlying does not move until quarter 12. Thus the fall of the price of the underlying does not really affect the value of the fund a lot. Fund return quarters 12-24: 16.36%.

All options end out of the money, thus the option part's value quickly converges towards zero and the portfolio gains a positive return from the remaining bonds.

4.2.3 Analysis and summary

The most preferable fund construction is construction number one since it has less extreme outcomes. Also fund construction number one will continue to participate in the market even though the market has decreased a lot. The fund also provides capital protection, when the market has gone up, for investors who experienced the bull-market.

Based on these results it is easy to conclude that there is a big advantage to rebalance the portfolio at intermediate time points so that the ratio options:bonds does not exceed a certain limit. This implies that it is possible to protect new investors from for example scenario 1. It is also desirable to increase the ratio options:bonds in the same way as it is done in fund construction number 1 after a market crash, if the fund wants to increase the possibility of appreciation, but this should not be conducted when trying to minimize the risk. Using the allocation scheme in fund construction number 1 implies that the fund slowly rebalances towards its original ratio options:bonds. On the other hand rebalancing up the ratio options:bonds each quarter when it has decreased under a certain limit would imply that the portfolio's risk increases i.e. there are pros and cons with every decision.

Fund construction number 2 is not desirable at all since it is a very risky investment strategy and does not hold the desired characteristics.

Concluded, it is important that the allocation scheme has a focus on rebalancing the rate options:bonds, where there exists a maximum acceptable limit. This can be done by for example solving a linear optimization problem.

The question now arises regarding how the option portfolio should be constructed, i.e. how the weights in the different options (and thus also the structured products) should be constructed to retain as much capital protection as possible. The next section investigates how an investor should allocate the option portfolio to have as little downside risk as possible. The reason why it is possible to generalize the result of options to the structured products is that a change in the underlying is the most prevalent risk of the structured products, which will be discussed in the next subsection.

4.2.4 Prevalent risk factors

This short subsection discusses the most prevalent risk factors for an ELN. It is important to determine the most prevalent risk factors to be able to construct a portfolio of structured products such that the risk is minimized.

The most volatile part of a structured product is the option, the bond is less sensitive to movements affecting its price, i.e. the yield curve. Consider an Equity-Linked-Note with a price of \$100, price of the underlying of \$100, $y_{tm} = 0.06$, $\sigma = 0.2$ p.a. and a time to maturity of three years. The ratio options:bonds would be 16.47:83.53, a shift of the underlying downwards of 50% would induce a decrease of the product's value of 15.9743. The yield to maturity would have to increase to 13.08% to cause a price drop that big. There is not a huge risk with changes in the yield curve that are not extreme since if the yield curve goes up bonds will be less expensive and the bonds bought will generate a higher return, as seen in Figure 4.7 which depicts a huge change in the interest rate. Also the interest rate risk is very small for buy and hold strategies, since the investor is guaranteed a fixed amount in the future, thus a fixed yield.

Figure 4.8 depicts the sensitivity of an ELN w.r.t. the underlying and the interest rate, the ELN is much more sensitive to changes in the underlying than in the yield curve. Thus it is more important to focus on changes in the underlying than the interest rate. When considering a buy and hold portfolio (which most funds practice, it is reasonable to assume that the whole portfolio will not rebalanced each quarter due to transaction costs) it is a much larger risk that the option is out of the money close to maturity than that the bond has a low price close to

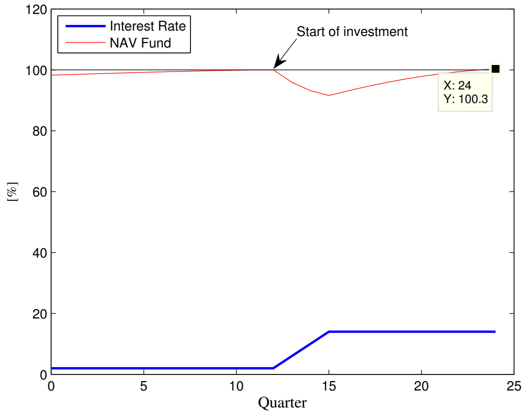


Figure 4.7: The interest rate goes from 2% p.a. to 14% p.a. during just a few quarters. The investor does not have a negative return over the whole period. The big risk is if a change occurs just prior to the end of the investment horizon, but as it can be seen, the impact is still limited.

maturity. Thus the interest rate risk of holding an existing bond is much smaller than holding the option.

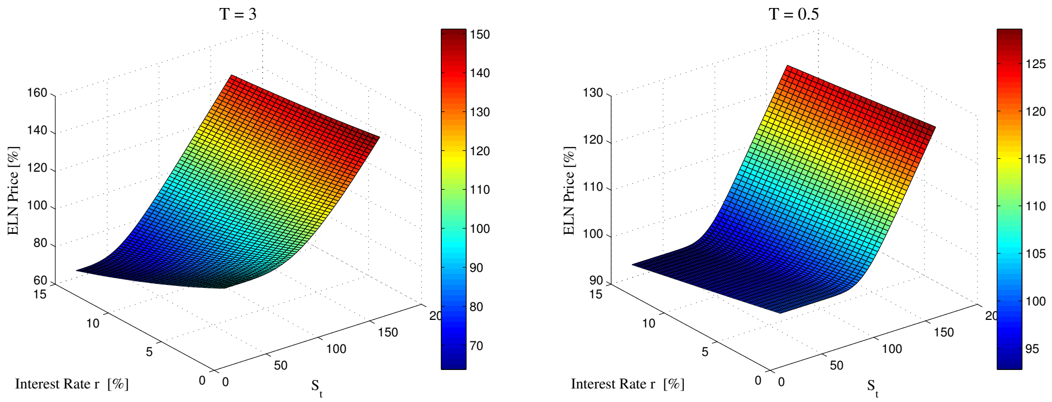


Figure 4.8: The surface plots describe how changes in the yield and price of the underlying affect the price of an ELN. $\sigma = 0.2$, $K = 100$, as time decreases so does the sensitivity to changes in the yield curve.

There may also be a risk with a decrease in volatility. But, as a consequence of the definition of volatility, the volatility increases as the price goes down (since the drift is in general positive), thus the two risk factors counter each other's impact on the price. Thus a decrease in volatility is not a big concern since it happens when the price of the underlying goes up, which affects the option price more than the volatility as seen in Figure 4.9.

Hence it is possible to generalize the optimal allocation of options, in min risk sense, to a portfolio of structured products since the options in the ELNs are significantly more risky than the bonds. The next section will therefor investigate how the option portfolio should be allocated to minimize the negative outcomes.

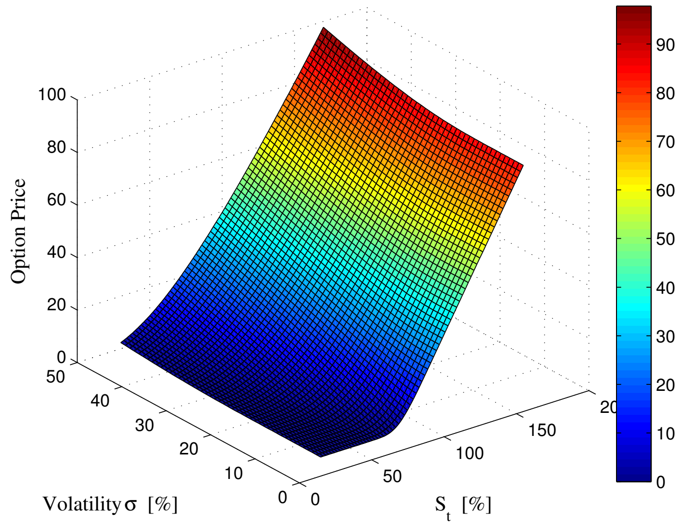


Figure 4.9: The surface plot describes the relationship between changes in volatility and the price, price is the dominant factor. Strike $K = 100$, $r = 0.06$, $T = 3$.

4.3 Investigation of the option portfolio

The purpose of this section is to investigate how the option portfolio should be allocated to minimize the negative outcomes in sense of the underlying, since the minimum-risk option portfolio should in most cases yield the minimum-risk ELN portfolio.

The options are priced with Black-Scholes formula as given in Equation 3.1. The option prices are assumed to not be smaller than 0.5% of the price of the underlying, before maturity, to prevent unrealistic results. Without setting such boundaries some options may have returns in the size of 10^{30} between two quarters.

The study covers different portfolio constructions (only those relevant are shown) and how the scenarios are modeled to stress the different portfolio allocations. The section also generates a recommendation regarding how an option portfolio should be allocated such that the risk is minimized.

4.3.1 Portfolio construction and modeling

The portfolios that are investigated consist of only plain vanilla at-the-money call options at issuance on the underlying index. To test which portfolio setups that are best in min risk sense the study will conduct Monte Carlo simulation. It is not important to replicate a certain index, since the study is looking for patterns, which should be independent as much as possible of the assumptions of the underlying. The index OMXS30 at Nasdaq OMX serves as a base for the simulation, thus daily log returns are gathered between 1st of October 2009 to 30th of September 2010. The log returns are fitted to a student's t-distribution to attain slightly heavier tails than for the normal distribution, as well as a better fit. Since the investigation is investigating patterns and worst-case outcomes it does not really matter what the drift μ is set to be, thus μ is for simplicity set to zero, the other fitted parameters for the t-distribution are: $\sigma = 0.0105$, $\nu = 5.69$ (on a days basis, it is not important what the actual parameters are since the focus lies on patterns, which should not be sensitive to the distribution of the underlying).

The following parameters are also used: $r_f = 6\%$ p.a. and the volatility used when pricing the options is the actual realized volatility during the period of 20.5% p.a.

A new ATM call option is issued each quarter with maturity in three years, e.g. the option issued at quarter t has strike $K^t = S_t$, where S_t is the price of the underlying at time t. Thus there are at each intermediate time point twelve options available to invest in, with different strikes and maturity times.

Monte Carlo simulation is used to create outcomes of the underlying, a total of 60,000 three-year periods are simulated (excluding the three-year time period used to create the necessary price paths for the options).

The weight in option j held by the portfolio i is denoted as w_j^i where j is the option that has a maximum of j quarters left to maturity, $j = 1, 2, 3, ..., 12$. The portfolio weight vector of portfolio i is denoted as $\mathbf{w}^i$.

The best result is measured in the sense of worst outcome (since intrinsic capital protection is investigated in this chapter, another option would be CVaR), it is also desirable that the distribution of the outcomes is positive skewed, thus imposing less risk to an investor. The reason for measuring the risk of holding the option portfolio as the worst-case outcome and not by just using a measure such as the volatility is that the return distributions of derivatives (e.g. options) often are skewed and exhibit kurtosis.

The next subsection describes the different relevant fund constructions and discloses the result of them as well. It turns out that the products with the longest time to maturity exhibit the lowest risk.

4.3.2 Portfolios and results

Eight portfolios are investigated in this section using the simulation described above, these portfolio are some of the most relevant constructions (for more construction please see Appendix B). The option portfolios are as follows,

The first portfolio has portfolio weights that are equal to each other, thus the portfolio is totally diversified over all available options.

$$\mathbf{w}^1 = \begin{pmatrix} \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} & \frac{1}{12} \end{pmatrix}^{\mathrm{T}}$$

The second portfolio is a portfolio that has the first eleven portfolio weights equal to each other, the last one equal to zero.

$$\mathbf{w}^2 = \begin{pmatrix} \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & 0 \end{pmatrix}^{\mathrm{T}}$$

The third portfolio is a portfolio that has the last eleven portfolio weights equal to each other, the first one equal to zero.

$$\mathbf{w}^3 = \begin{pmatrix} 0 & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} \end{pmatrix}^{\mathrm{T}}$$

The fourth portfolio is a portfolio that has the first eleven portfolio weights equal to zero, and the last one equal to one.

$$\mathbf{w}^4 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}^{\mathrm{T}}$$

The fifth portfolio is a portfolio that has decreasing portfolio weights, with $w_j^5 = 13 - j$.

$$\mathbf{w}^5 = \begin{pmatrix} \frac{12}{78} & \frac{11}{78} & \frac{10}{78} & \frac{9}{78} & \frac{8}{78} & \frac{7}{78} & \frac{6}{78} & \frac{5}{78} & \frac{4}{78} & \frac{3}{78} & \frac{2}{78} & \frac{1}{78} \end{pmatrix}^{\mathrm{T}}$$

The sixth portfolio is a portfolio that has increasing portfolio weights, with $w_j^6 = j$.

$$\mathbf{w}^6 = \begin{pmatrix} \frac{1}{78} & \frac{2}{78} & \frac{3}{78} & \frac{4}{78} & \frac{5}{78} & \frac{6}{78} & \frac{7}{78} & \frac{8}{78} & \frac{9}{78} & \frac{10}{78} & \frac{11}{78} & \frac{12}{78} \end{pmatrix}^{\mathrm{T}}$$

The seventh portfolio is a portfolio that has equal portfolio weights in assets 2-10, zero otherwise.

$$\mathbf{w}^7 = \begin{pmatrix} 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 & 0 \end{pmatrix}^{\mathrm{T}}$$

The eighth portfolio is a portfolio that has equal portfolio weights in assets 2-11, zero otherwise.

$$\mathbf{w}^8 = \begin{pmatrix} 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 \end{pmatrix}^{\mathrm{T}}$$

The results for the different option portfolios are disclosed in Figure 4.10, there are also additional option portfolios disclosed in Appendix B.

Figure 4.10 and Appendix B show that the portfolios where the options closest to maturity are underweighted have “better” distributions since they exhibit less negative skewness. Also the worst-case outcome is much better for portfolios that have underweighted the options closest to maturity. Fewer number of assets in the portfolio results in, in most cases, a more risky portfolio, both in worst-case and in distribution. Notable is that the portfolio that has the best worst-case outcome is the portfolio that holds 100% of its holdings in the option with the longest time to maturity (Portfolio 4).

Figure 4.12a shows that options with a longer time to maturity are less sensitive to the changes in time ($\theta = \frac{\partial P}{\partial t}$). Thus the value reduction coming in form of the reduction in time, ceteris paribus, is less if the investor holds options with a long time to maturity. Also the delta ($\Delta = \frac{\partial P}{\partial s}$) for options that are in the money is higher for options close to maturity than options with a long time to maturity as depicted in Figure 4.12b.

As a fund manager it would be quite unreasonable to allocate all the capital in the same asset, thus it is adequate to demand that the fund diversifies itself, to not overweight. Usually a portfolio manager is not allowed to invest more than a certain percentage (say 30%) in a particular asset, it is in this case beneficial to diversify along almost all options, except the one closest to maturity (which has the highest risk, since it might be “all or nothing”). The results in Figure 4.10, B.1 and B.2 indicate that it does not really matter how the portfolio diversifies amongst the assets since the results are quite similar as long as the portfolio holds a relative large proportion of the holdings in products with a longer time to maturity. Thus it might be reasonable to diversify by holding only four options with a long time to maturity as in Portfolio 16 in Appendix B or for example diversify over the whole spectrum and attain approximately the same results.

It would be desirable if the return distribution for the portfolio of options over 36 months would look somewhat similar to the return distribution of a single option from issue to maturity as seen in Figure 4.11. The results shown in the figures indicate that it is not possible to attain the same type of distribution as for a single option when investing in several options over 36 months.

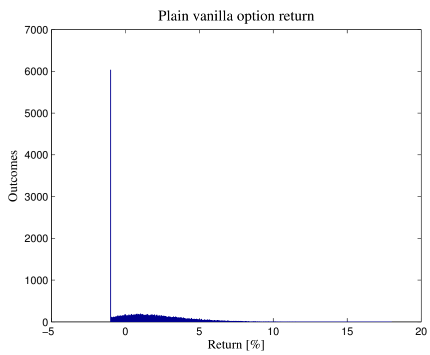
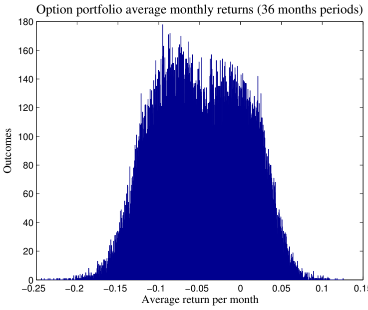
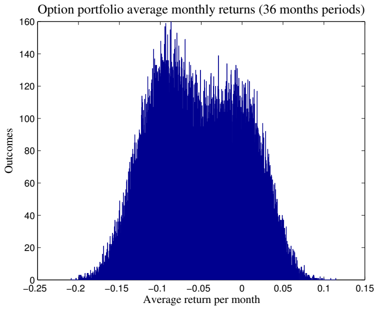


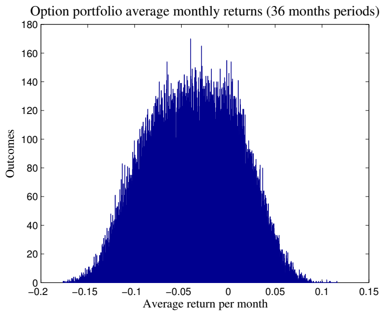
Figure 4.11: The histogram shows the distribution of a plain vanilla option's one-year return, issued ATM ($T = 1$).



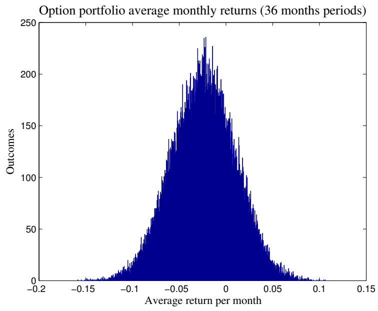
(a) Portfolio 1, −22.37%.



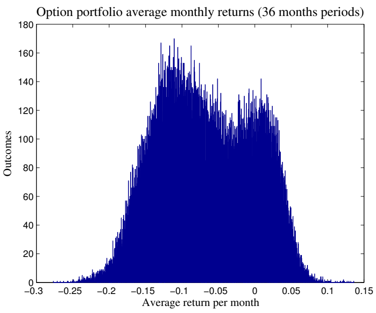
(b) Portfolio 2, −21.88%.



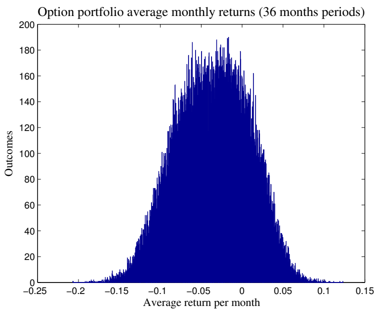
(c) Portfolio 3, −17.64%.



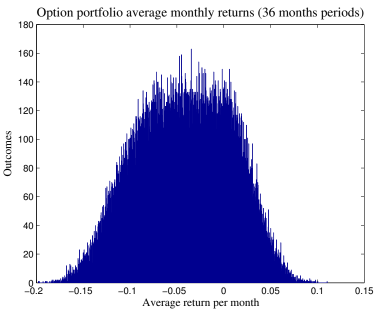
(d) Portfolio 4, −16.15%.



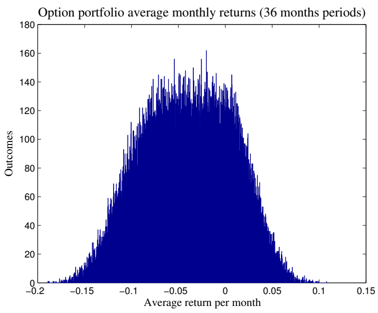
(e) Portfolio 5, −27.69%.



(f) Portfolio 6, −20.71%.



(g) Portfolio 7, −19.80%.

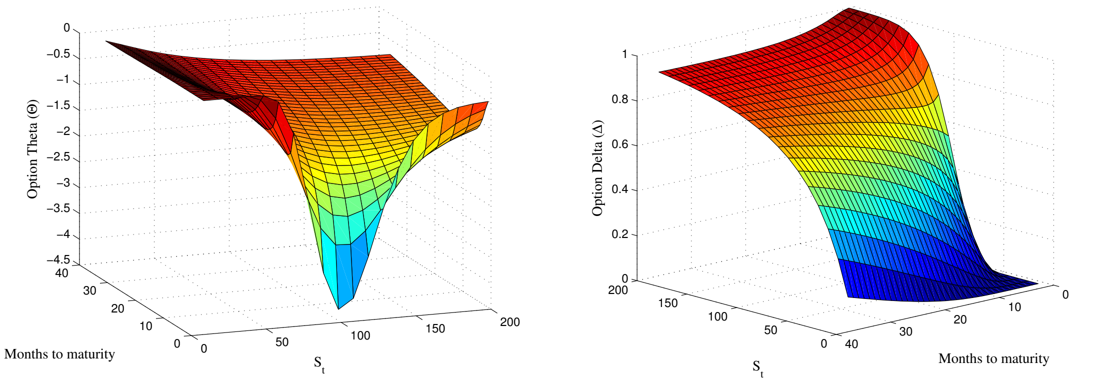


(h) Portfolio 8, −18.91%.

Figure 4.10: Histograms over the outcomes for option portfolios 1-8, the worst-case average 36 month return is given in the caption.

An optimal portfolio would be positive skewed, with a limited downside. The most beneficial construction fulfilling these criteria is Portfolio 3, as indicated in Figure 4.10c (as well as considering the worst-case outcome). Hence it is important to diversify over a large set of assets to reduce the risk, when constructing a portfolio only consisting of options. On the other hand as the time to maturity decreases, the risk increases (the theta and delta (given that the option is in the money) in particular, as disclosed in Figure 4.12). Thus when constructing a portfolio with structured products it is important to not overweight in the shorter tenors.

The main result is that the fund should diversify over a number of assets and also avoid investing in the products with a short time to maturity. Next follows an analysis of how the results found in Section 4.2 and in this section should be combined to construct a desirable structured products fund.



(a) The plot describes a plain vanilla call option's theta (time value), the theta is heavily decreasing in time, implying that the holder looses a lot of capital from a time change close to maturity, ceteris paribus.

(b) The plot describes a plain vanilla call option's delta, options closer to maturity are more sensitive when they are in the money than options with a longer time to maturity.

Figure 4.12: The Figure a) illustrates an option's theta value, the Figure b) illustrates an option's delta value for different levels of in the money and time to maturity.

4.3.3 Analysis and implications

In this subsection follows a combination of a summary, an analysis and a description of the implications from the studies in this chapter so far.

It is known from Section 4.2 that it is desirable to rebalance the portfolio when the ratio options:bonds exceeds a predetermined level (which depends on the risk profile of the fund) to limit the downside risk. The reason why the ratio options:bonds suddenly exceeds a certain level is (most often) due to either an increase in the price of the underlying (increasing the value of the options) or in the yield curve (reducing the value of the bonds). Thus by rebalancing the portfolio the downside risk is limited since the proportion of options is reduced. This is an important concept since it protects new investors, the fund must be able to guarantee new investors that they do not buy more than a certain percentage of options. Without this guarantee the purpose of the fund is lost, by appealing to investors' loss aversion (Shefrin, 2002, [35]).

By rebalancing the portfolio after a period of growth the value held in bonds increases. Thus the existing investors will receive a certain level of capital protection on their gains. The tradeoff is that the portfolio at the same time limits the upside of the investment (the tradeoff between risk and return). These characteristics appeal to psychological concepts within Behavioral Finance, where investors tend to sell assets too early during growth, since they are conservative and people do not expect the market to continue appreciating (disposition effect) (Frazzini, 2006, [15]; Kahneman and Tversky, 1979, [26]). Thus investors want to rebalance their portfolio after a period of appreciation.

During bear markets, when the portfolio decreases in value, it may be desirable to rebalance the portfolio's ratio of options:bonds towards a desired level. The rebalancing should not be conducted in the same manner as when the ratio exceeds a certain level, instead it may be desirable to slowly re-establish the portfolio's ratio options:bonds to the original level. Without this reallocation the fund will not have as much upside for new investors. Thus it is a balance act for the fund how it should change the ratio options:bonds in a way that both new investors and current investors appreciate. The next section will investigate how this ratio/level should be chosen and its implications on downside risk.

Since Equity-Linked Notes are considered in this paper the options are far more risky than the bonds, thus by allocating the portfolio according to the minimum risk choice w.r.t. the options should also yield the minimum risk choice for the structured products.

As stated previously, there is no need to investigate changes of volatility, since the volatility and the price of the underlying usually counter each other (Glot, 2005, [16]). Thus the impact of volatility can be neglected when choosing portfolio weights. The inflation goes up when the market goes up, thus also interest rates. When markets go down, investors flee to safer instruments such as bonds, thus pushing up the price and decreasing the yield. This implies that the price changes of bonds and call options are negatively correlated.

As seen in Section 4.3 the best choice to minimize the downside risk for a portfolio of options is by diversification in the longer (and middle) tenors. Thus it is not desirable to hold positions in options that have a short time to maturity.

When considering a portfolio of structured products it might be non-desirable to overweight the newly issued products during bear markets since they will have a substantial proportion of option value in them compared to the other products. Thus if the portfolio overweights/allocates to the newly issued ELNs it will always reallocate capital to newly issued options, which is not desirable in bear markets. Hence instead of allocating according to Portfolio 3, as when only considering an option portfolio, a higher degree of capital protection during bear markets will be attained by investing using either Portfolio 7 or 8. Thus it is reasonable to invest using Portfolio 3 in general and after drops in the index allocate the portfolio according to Portfolio 7.

It is desirable to rebalance the portfolio such that the ratio options:bonds are held under control, but how should this be conducted, for which levels etc? These issues will be investigated in the next section where a comparison with existing mixed funds results in optimal choices of the ratio options:bonds.

4.4 Comparison with competition

A lot of assumptions have to be made in able to construct the fund. One way to make adequate assumptions is to relate to the fund's competition. Investors will not invest in the fund if it is not capable of beating its competition, at least at some aspects. The comparison will be based on the results found in the previous sections. Thus the purpose of this section is to investigate how a potential structured products fund (SPF) performs in relation to its competition (mixed funds) during extreme scenarios. The analysis results in appropriate boundaries for the percentage of value in the SPF contributed by options given a certain competing/benchmark fund.

A competing fund is a fund that has the same client base as the SPF. Thus it is imperative to establish the SPF's client base, so that a competing benchmark fund can be identified. After that the competing fund has been identified the study investigates how different SPFs can be constructed, such that the SPF provides a higher degree of capital protection than the benchmark fund.

The different constructions are both stress tested with predefined extreme scenarios as well as simulated scenarios. The different SPF strategies are also backtested against historical data.

The section covers two types of SPFs, one where the weights are based on the possession in the structured products and one SPF where the weights in the structured products are based on the possession in the options. It turns out that the second one is more theoretical than the first one. The result in this section indicates that a structured products fund should try to have a possession of 5-25% option value attained in the portfolio, with a target level of 15% to be more attractive for investors in min risk sense than a competing mixed fund.

4.4.1 Client base

The fund based on structured products has a high focus on capital protection. Thus the investors that it is suitable for are those who are looking for investments with a limited downside, or a small risk, as well as a possibility to have a return connected to the market. These are risk-aversive investors who usually are looking for a combination of bonds and stocks. Today many of these investors invest in mixed funds, which diversify their portfolios in both debt and equity instruments to lower the risk level, hence it is this alternative which is an adequate benchmark. Thus mixed funds are identified to be the competition to the SPF, hence the structured products fund must provide a lower downside risk than competing mixed funds to be an attractive investment alternative.

Next follows a more detailed description of the benchmark fund.

4.4.2 Benchmark fund

It is assumed that a reasonable benchmark fund is a mixed fund that aims at holding 30% ($\varepsilon^{\text{bench}}$) of its capital in stocks and 70% in bonds with up to three years until maturity (to match the SPF). The benchmark fund rebalances its position every three months, if the level of stocks held is not within $30\% \pm 10\%$ (δ^{bench}). The investment within stocks is considered in this study to be conducted in the index (or an ETF), which is also the underlying for the SPF. An example of a price development of this type of benchmark fund is disclosed in Figure 4.13. The figure shows that the benchmark fund gains value as the market is appreciating and does not decrease significantly as the market takes a turn for the worse. Thus it has some of the desired properties for a SPF, imposing an adequate benchmark. The focus is now turned to the construction of the structured products fund.

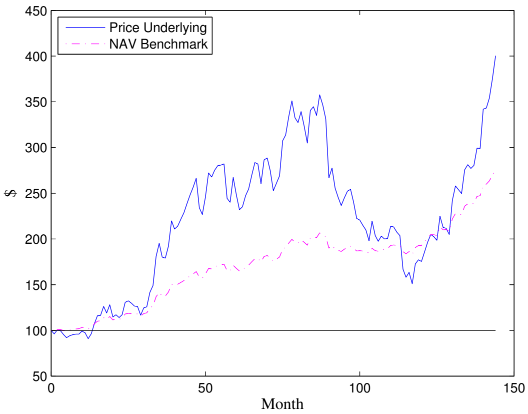


Figure 4.13: The co-movements of the underlying and the benchmark fund, for $\epsilon^{\text{bench}} = 0.3$, $\delta^{\text{bench}} = 0.1$ and $r = 0.06$ p.a, random scenario.

4.4.3 Structured products fund 1

The fund constructions in this section are based on the results gained in Section 4.2 and 4.3, thus the portfolio diversifies broadly amongst the available products. Portfolio 7 will be used, since the test is conducted for bear markets (in Appendix C results for Portfolio 3 are disclosed), thus the portfolio holds equal weights in products $i = 2 - 10$ where i denotes the maximum number of quarters until maturity for the product as previously.

A new capital guaranteed structured product is issued each quarter with maturity in three years, price equal to the price of the underlying and also the notional amount of the bond, as in Section 4.2.1. The fund also has the possibility of investing in short-term debt, in this case bonds that have maturity the next quarter. Thus the fund can by investing in the short-term debt reduce the percentage of value placed in options. A target level of options held in the fund's portfolio is also introduced, this target level is the desired percentage of the fund's value that should be constituted by options, let denote this target level as ε^{SPF}. Notable is that the fund is not allowed to short the short-term debt.

The fund rebalances its portfolio of structured products each quarter as previously, according to the weights and maturity amongst the structured products, and also the proportion placed in additional bonds.

The fund adjusts the proportion placed in the short-term debt subject to its level of option value as a percentage of the total fund. The portfolio is allowed to deviate a percentage δ^{SPF} from its target level of options. Thus the portfolio is rebalanced if the percentage of value contributed by the options exceeds $\varepsilon^{\text{SPF}} + \delta^{\text{SPF}}$ (or is less than $\varepsilon^{\text{SPF}} - \delta^{\text{SPF}}$) by allocating in short-term debt rather than the structured products, thus attaining the target level.

$\tilde{\mathbf{w}}$ denotes the weight vector for the structured products excluding the short-term debt (the weights amongst the structured products), the weight in the short-term debt at time t is denoted as w_t^b. Thus the actual weights (except w_t^b) for each time-period is given by:

$$\mathbf{w}_t = \left(1 - w_t^b\right) \tilde{\mathbf{w}},$$

and since the focus lies on Portfolio 7 from Section 4.3 the following weights are used,

$$\tilde{\mathbf{w}} = \begin{pmatrix} 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 & 0 \end{pmatrix}^{\text{T}}.$$

Note that $w_0^b = 0$ and $\mathbf{w}_0 = \tilde{\mathbf{w}}$, the proportion of value of the fund contributed by bonds is denoted α^b, and α^o is the proportion of value of the fund contributed by options. Which gives us the following definitions,

$$\alpha_t^b = w_t^b + \left(1 - w_t^b\right) \sum_{i=1}^{n} \tilde{w}^i \frac{\beta_t^i}{\beta_t^i + O_t^i},$$

$$\alpha_t^o = \left(1 - w_t^b\right) \sum_{i=1}^{n} \tilde{w}^i \frac{O_t^i}{\beta_t^i + O_t^i},$$

where n is the number of available products (12 in this case), thus the portfolio is rebalanced each quarter if,

$$\alpha_t^o \leq \varepsilon^{\text{SPF}} - \delta^{\text{SPF}} \quad \text{or} \quad \alpha_t^o \geq \varepsilon^{\text{SPF}} + \delta^{\text{SPF}}.$$

The portfolio is rebalanced with the aim that α^o should equal ϵ^{SPF}, this is not always possible since it is not allowed to short the short-term debt, which implies that the new portfolio weights are given as,

$$\mathbf{w}_t^{\text{new}} = \begin{cases} \mathbf{w}_t \dfrac{\varepsilon^{\text{SPF}}}{\alpha_t^o}, & \text{if } \displaystyle\sum_i w_t^i \frac{\varepsilon^{\text{SPF}}}{\alpha_t^o} \leq 1 \\ \tilde{\mathbf{w}}, & \text{if } \displaystyle\sum_i w_t^i \frac{\varepsilon^{\text{SPF}}}{\alpha_t^o} > 1 \end{cases}$$

these weights are the weights excluding the short-term debt, thus the new allocation in the short-term debt is as follows,

$$w_t^b = 1 - \mathbf{1}^{\mathrm{T}}\mathbf{w}_t^{\mathrm{new}}.$$

The fund is not allowed to short the short-term debt (to minimize the risk), thus it cannot hold negative positions in w_t^b. This implies that the short-term debt is used as a safe haven to avoid a too high exposure to the market.

The purpose with the following subsections is to determine the adequate levels for $\varepsilon^{\mathrm{SPF}}$ and δ^{SPF} such that the fund has a greater protection (towards extreme downside risk) than the benchmark portfolio(s), and still has a good upside potential. The levels will be determined such that the SPF provides a greater capital protection for market crashes of -20% p.a. and more.

4.4.4 Scenarios

This subsection describes the scenarios that the SPF and benchmark fund are stressed with. The scenarios are not simulated since it is desirable that the SPF provides capital protection for some specified market crashes.

The scenarios should stress the SPF and the benchmark fund towards negative outcomes, thus expose the funds in an extent where they disclose weakness (which is a market crash immediately after a strong increase in the market, as seen in the previous sections). The focus lies on scenarios with an up-going market followed by a market crash (as the worst-case scenarios in Section 4.2), five different scales of market crashes are investigated, $-80\%, -50\%, -30\%, -20\%$ and -10% p.a. during four years. Two different settings prior to the crash will be investigated, the first setting with a bull market with a return of 30% in return p.a. between months 0-48 (Scenario A). The second scenario is quite similar, 30% return p.a. during months 0-48 and a return of 50% each month for months 48-50 (Scenario B). The idea is that since the funds can only rebalance to short-term debt every three months that the funds can be stressed even more if an extreme increase occurs during a period where the funds cannot rebalance. Thus these two types of scenarios should cover the worst types of extreme outcomes for the funds (for more extreme scenarios please consult Appendix C).

Hence the market crash in Scenario A is between months 48-96 and in Scenario B between months 50-98.

After the crash the market experiences an increase of 30% p.a. until month 144 (12 years). An annual interest rate of 6% is assumed for the bonds as well as the short-term debt (flat yield curve is assumed for simplicity), the scenarios can be seen in Figure 4.14.

The reason why predefined stressed scenarios are studied instead of simulation in this subsection is that the study is trying to stress test the funds in particular scenarios and see if the SPF has the desired properties. Also it is imperative that the results gained from this study are independent of the models selected for the underlying and that they can be generalized to any distribution or market model, thus it is appropriate to select scenarios without simulation.

The next subsection covers the results over how the funds perform in the different scenarios and how the results should be interpreted.

4.4.5 Results, structured products fund vs benchmark

The focus lies on the worst-case outcomes of the SPF (based on Portfolio 7) and the benchmark fund given different scenarios and values for ε and δ. The objective is to construct a SPF such that it has more favorable worst-case outcomes than the benchmark portfolio.

Various results are disclosed in Figure 4.14, Table 4.1 and 4.2. Table 4.1 discloses the result regarding the SPF, its worst-case outcome given the different scenarios, $\varepsilon^{\mathrm{SPF}}$ and δ^{SPF}. Table 4.2 discloses in the same manner the result regarding the benchmark fund, its worst-case outcome given the different scenarios, $\varepsilon^{\mathrm{bench}}$ and δ^{bench}.

Return Index Months 48-96 / 50-98 p.a.	ε^{SPF}	δ^{SPF}	Scenario A Worst-Case Return	Scenario B Worst-Case Return
-80%	1	1	-18.53%	-56.07%
-80%	0.3	0.1	-18.53%	-37.62%
-80%	0.25	0.1	-11.95%	-32.18%
-80%	0.20	0.1	-11.35%	-22.62%
-80%	0.15	0.1	-2.85%	-15.20%
-80%	0.15	0.05	-2.54%	-19.22%
-80%	0.10	0.05	-1.06%	-9.59%
-50%	1	1	-17.46%	-48.50%
-50%	0.3	0.1	-17.46%	-33.04%
-50%	0.25	0.1	-11.57%	-25.42%
-50%	0.20	0.1	-11.03%	-18.39%
-50%	0.15	0.1	-3.42%	-11.48%
-50%	0.15	0.05	-6.20%	-16.71%
-50%	0.10	0.05	-1.82%	-7.59%
-30%	1	1	-16.64%	-37.40%
-30%	0.3	0.1	-16.64%	-22.49%
-30%	0.25	0.1	-13.24%	-19.26%
-30%	0.20	0.1	-11.31%	-13.63%
-30%	0.15	0.1	-5.00%	-5.41%
-30%	0.15	0.05	-7.32%	-10.23%
-30%	0.10	0.05	-3.45%	-3.54%
-20%	1	1	-14.60%	-26.84%
-20%	0.3	0.1	-14.60%	14.64%
-20%	0.25	0.1	-11.13%	-10.59%
-20%	0.20	0.1	-9.72%	-6.15%
-20%	0.15	0.1	-3.95%	0.34%
-20%	0.15	0.05	-6.09%	-3.38%
-20%	0.10	0.05	-2.55%	2.90%
-10%	1	1	-9.60%	-14.08%
-10%	0.3	0.1	-9.60%	-4.75%
-10%	0.25	0.1	-6.78%	-1.81%
-10%	0.20	0.1	-5.81%	0.43%
-10%	0.15	0.1	-1.65%	3.82%
-10%	0.15	0.05	-3.11%	1.23%
-10%	0.10	0.05	0.22%	4.79%

Table 4.1: SPF 1: Worst-case outcomes for SPF 1 based on Portfolio 7 given scenarios A and B, ε^{SPF} and δ^{SPF}, the worst-case period is always the 36 month following the initiation of the crash.

Return Index Months 48-96 / 50-98 p.a.	$\varepsilon^{\text{bench}}$	δ^{bench}	Scenario A Worst-Case Return	Scenario B Worst-Case Return
-80%	0.7	0.1	-94.49%	-94.48%
-80%	0.5	0.1	-87.35%	-87.80%
-80%	0.3	0.1	-62.94%	-63.04%
-80%	0.20	0.1	-43.27%	-44.37%
-80%	0.10	0.05	-17.36%	-18.17%
-50%	0.7	0.1	-72.62%	-72.16%
-50%	0.5	0.1	-54.86%	-55.73%
-50%	0.3	0.1	-34.13%	-33.07%
-50%	0.20	0.1	-15.61%	-15.26%
-50%	0.10	0.05	-0.37%	0%
-30%	0.7	0.1	-47.71%	-47.22%
-30%	0.5	0.1	-31.84%	-32.5%
-30%	0.3	0.1	-15.68%	-12.84%
-30%	0.20	0.1	-2.59%	-2.39%
-30%	0.10	0.05	7.45%	7.94%
-20%	0.7	0.1	-32.85%	-31.70%
-20%	0.5	0.1	-18.36%	-18.91%
-20%	0.3	0.1	-6.66%	-4.61%
-20%	0.20	0.1	2.58%	4.78%
-20%	0.10	0.05	10.24%	11.56%
-10%	0.7	0.1	-15.57%	-13.80%
-10%	0.5	0.1	-4.76%	-5.03%
-10%	0.3	0.1	1.58%	4.75%
-10%	0.20	0.1	7.87%	9.48%
-10%	0.10	0.05	13.09%	14.24%

Table 4.2: Benchmark fund: Worst-case outcomes given scenarios A and B, $\varepsilon^{\text{bench}}$ and δ^{bench}.

The structured products fund provides, as the tables indicate, (in general for the same levels of ε and δ as the benchmark fund) a much higher safety in extreme downturn markets (-80% drop), since it does not short bonds to reallocate its portfolio. It is not a huge difference between the worst-case outcomes for different extreme scenarios (-30 to -80% drops) when regarding the SPF (Table 4.1), since the value of the options does not decrease a lot more in the -80% case than in the -30% case. Table 4.2 shows that the benchmark fund on the other hand suffers to a high extent when the index has huge negative returns since it reallocates every three-month to the index, thus not providing a limit where it cannot lose more capital. It is important to notice that the SPF appreciates quite a lot when the market starts to go up again, which is very desirable as seen in Figure 4.14. The SPF has a greater upside for huge increases than the benchmark fund (Figure 4.14). Notable is that it takes longer time for the SPF than the benchmark fund to appreciate after the market turns bust, it is this effect which also creates the capital protection property, since it takes a few quarters to increase the participation rate.

Table 4.1 shows that the SPF is in general more sensitive to a large drop in the stock market after an extreme increase without having the possibility to rebalance the portfolio (Scenario B) than to Scenario A. The SPF is less sensitive to Scenario B if it is only followed by a correction in the market (return of -10% to -20% p.a.). This is due to that the SPF does not suffer a lot of the decline between months 50-51 and the portfolio is rebalanced at month 51 to lower levels, thus making it less sensitive to a market crash. On the other hand the portfolio does not benefit so much of the rebalancing possibility during month 51 in the case of a huge decline between months 50-51, such as in the -80% drop scenario (Table 4.1). Notable is that the results are quite similar if there is a drop of -80% per month between months 50-51, the results for this scenario are disclosed in Appendix C[1].

Since the fund rebalances its portfolio at month 48 a sense of upper loss limit, which the investor cannot lose more than if it starts its investment horizon at quarter 48, is created. This implies that if the fund experiences a total market crash (-100%) the investor still holds its position in bonds, while the market linked assets would by worth zero. Thus the investor loses a "maximum" (in sense of the underlying) of $\varepsilon + \delta$ which is applicable to both the SPF and benchmark fund. But the value invested in options is convex and more volatile than investing in the underlying. Thus a decline with 50% may erase almost all value attained in the options. Hence the criteria $\varepsilon^{\text{SPF}} + \delta^{\text{SPF}} < \varepsilon^{\text{bench}} + \delta^{\text{bench}}$ must hold if the SPF should have a higher capital protection for declines of the scale 30-50%.

One interesting property that is not disclosed in the tables is that the fund can limit its downside significantly using two different δ^{SPF}, one for the upside (δ_+^{SPF}) and one for the downside (δ_-^{SPF}). By increasing the downside limit δ_-^{SPF} the fund has a higher tolerance for holding short-term debt than when having a symmetrical δ^{SPF}. Important to notice is that as δ_-^{SPF} increases it will be harder for the SPF to appreciate after a bear market since it accepts higher levels of short-term debt and does not reallocate in a high extent to structured products.

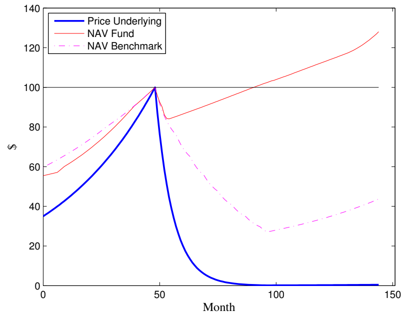
Next follows an analysis regarding the results and an interpretation regarding their implications for a portfolio manager.

4.4.6 Analysis and summary

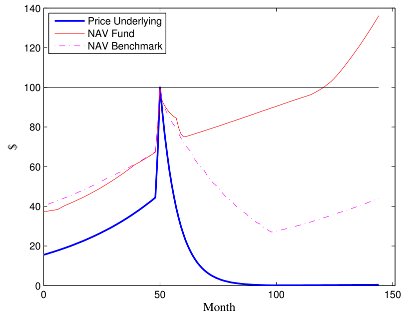
As mentioned earlier an adequate benchmark fund is a fund which has $\varepsilon^{\text{bench}} = 0.3$ and $\delta^{\text{bench}} = 0.1$. Thus the desired weights of ε^{SPF} and δ^{SPF} should provide better capital protection against downside risk than the benchmark fund. It is reasonable to demand that the SPF should be less sensitive to market drops of -20% p.a. or more (it is difficult to find portfolio weights for the fund that would be less sensitive at a -10% market drop[2], as seen in Table 4.1 and 4.2). By inspecting Table 4.1 and 4.2 it is easy to notice that the adequate levels for ε^{SPF} and δ^{SPF} are 0.15 respectively 0.1 (notable is that these boundaries also provides a better worst-case outcome

[1]The scenarios considered in the appendix are worse, but if those scenarios would occur the investor would have, most certainly, bigger problems to consider than its position in the SPF.

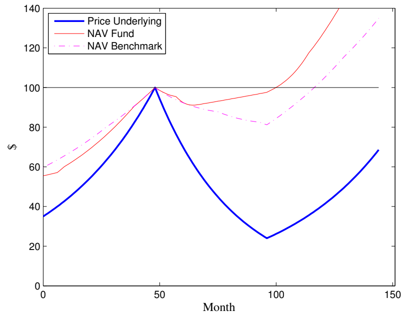
[2]Since the price of the options is more volatile than for stocks and indices.



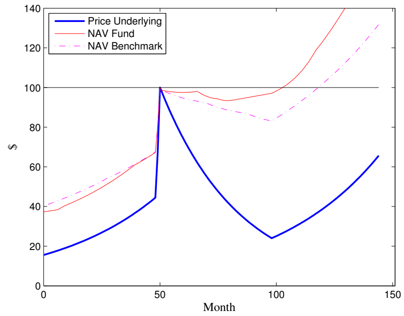
(a) Scenario A, −80% market crash.



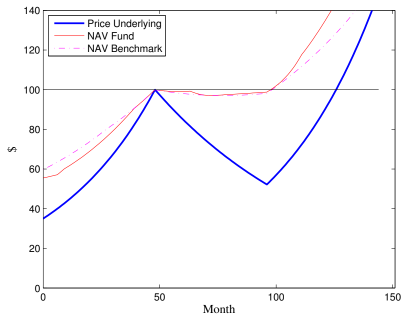
(b) Scenario B, −80% market crash.



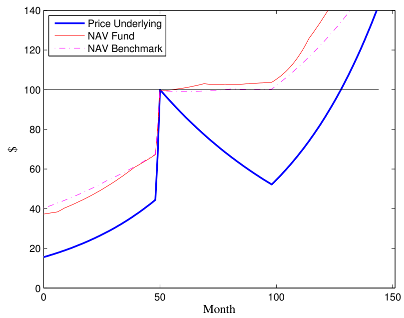
(c) Scenario A, −30% market crash.



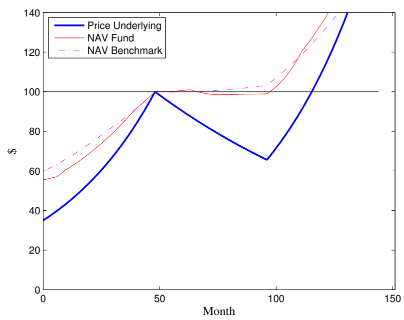
(d) Scenario B, −30% market crash.



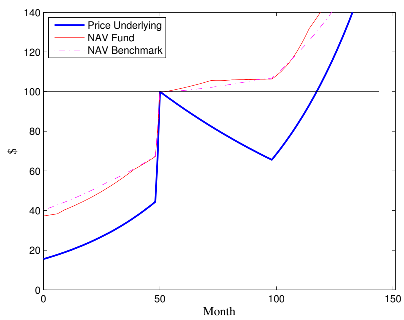
(e) Scenario A, −15% market crash.



(f) Scenario B, −15% market crash.



(g) Scenario A, −10% market crash.



(h) Scenario B, −10% market crash.

Figure 4.14: Illustrative example of the scenarios and movements of the SPF 1 (Portfolio 7) and the benchmark fund for $\varepsilon^{\text{SPF}} = 0.15$, $\varepsilon^{\text{bench}} = 0.3$, $\delta^{\text{SPF}} = 0.1$, $\delta^{\text{bench}} = 0.1$.

for a -15% market crash as depicted in Figure 4.14). Examples of outcomes for Portfolio 7 using different ϵ^{SPF} and δ^{SPF} with different scenarios are disclosed in Figure 4.14.

Thus a SPF should have the parameters $\varepsilon^{\text{SPF}} = 0.15$, $\delta^{\text{SPF}} = 0.1$ to provide better capital protection than a benchmark fund that has $\varepsilon^{\text{bench}} = 0.3$ and $\delta^{\text{bench}} = 0.1$.

These are just some of the possible boundaries; by increasing ε^{SPF} to 0.2 or 0.25 the fund is more aggressive and can take advantage of the momentum gained during bull markets (as well as increasing the possibility of a higher return after bear markets, Figure 4.14). Thus the level of ε^{SPF} needs to be decided in comparison with the competition and how the SPF is managed, the fund's goals etc. It is important to contemplate regarding what the fund is trying to accomplish and what tradeoffs different portfolio choices imply.

To summarize the previous sections: it is important to diversify the SPF over a numerous set of assets (Section 4.3). The fund should avoid investing in structured products that have maturity in the nearest future, since these are very volatile and in practice almost only a bet with a high theta (cost of making the bet). Also the fund should during a bear market avoid overweighting newly issued products (Section 4.3). In a downturn market investing in newly issued products increases the value held in options in the portfolio, thus increasing the risk. Thus the fund manager should, when trying to minimize the downside risk, allocate broadly over the spectrum of available products, except in the products that are about to mature (especially during bull markets) and newly issued products during bear markets.

It is important that the fund manager reallocates some of the return to either cash or short-term debt, as the SPF value increases, hence maintaining the ratio between the capital allocated in options and bonds at a reasonable level (Section 4.2 and 4.4). This implies that the fund locks in some of its previous return as it appreciates, thus protecting the previous profits. It also protects new investors from investing in a (too) high proportion of options in relation to the holdings in bonds. These effects are really desirable and in line with human behavior to rebalance due to loss-aversion (Kahneman and Tversky, 1979, [26]). When the manager does not rebalance the portfolio investors may invest in a fund with a high risk, due to the volatility of the option payoffs. It is, as shown in this study, imperative that the fund manager understands the risk of investing in structured products in a sophisticated portfolio, with different maturities, and how the capital protection is affected by the market returns and the reallocations.

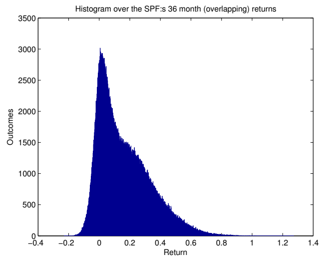
It is not only important to rebalance the portfolio after a series of appreciations to decrease the risk of the fund. The fund manager should after periods of deprecation rebalance (if any existing) holdings from short-term debt to the structured products to increase the optionality (in this case the proportion of options) in the portfolio to avoid that new investors will suffer from the fund's previous decline (Section 4.4). It is also important to rebalance the portfolio after a decline to give the previous investors the possibility to "win the losses back" as people prefer this re-investment strategy due to get-evenitis (Shefrin, 2002, [35]).

Thus when formulating an optimization algorithm, that allocates amongst different structured products, there are several constraints that should be made such that the risk of the portfolio is held under control. The proportion held in options should be restricted as done in this section, the fund manager decides the appropriate limits depending on what the SPF is trying to attain in capital protection sense and in relation to competition. It is also wise to limit the proportion invested in each structured product (to diversify adequately), and when minimizing the absolute downside it is recommended to avoid overweighting the newly issued products during bear markets and products maturing the upcoming period.

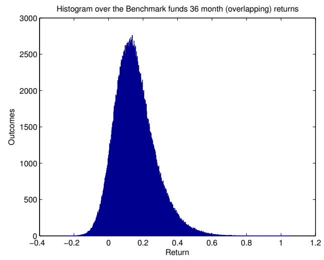
The SPF 1 has been stressed for the predefined scenarios, and the appropriate levels of ε^{SPF} and δ^{SPF} have been attained. Next follows an additional stress test to assure that these levels also provide better outcomes than the benchmark fund under simulated scenarios.

4.4.7 Additional stress test

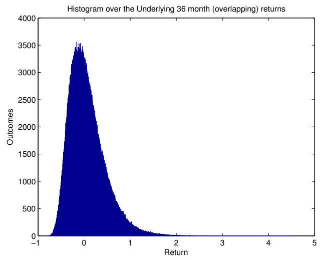
The stress test is conducted to ensure that the SPF 1 yields a higher degree of capital protection than a benchmark fund given the attained levels of ε^{SPF} and δ^{SPF} also for other scenarios than



(a) SPF 1: min: -23.05%, mean: 16.19%, max: 127.19%.



(b) Benchmark: min: -24.01%, mean: 15.35%, max: 105.45%.



(c) Underlying: min: -80.59%, mean: 6.3%, max: 455.26%.

Figure 4.15: Histograms over 36 months returns for the competing funds and the stress test.

the ones specified above. Thus SPF 1 has $\varepsilon^{\mathrm{SPF}} = 0.15$ and $\delta^{\mathrm{SPF}} = 0.1$, which should yield better worst-case outcomes than the benchmark fund according to the study conducted above.

The same type of Monte Carlo simulation as performed in Section 4.3 is used, thus the daily log returns for the index are modeled with a student's t-distribution with $\nu = 5.69$, $\sigma = 0.0105$ and $\mu = 0$ on a days basis. The following parameters are also used: $r_f = 6\%$ p.a. and the volatility used when pricing the options is 20.5% p.a.

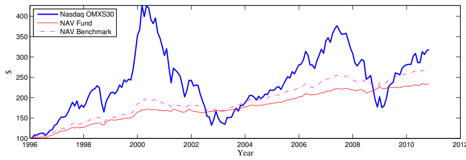
The drift equals to zero, since the model should be stressed towards negative outcomes, notable is that it is not important exactly what the parameters are since the study searches for patterns. The study is conducted through investigating 500,000 36-month periods and the 36 months return. The result is disclosed in Figure 4.15, which indicates that SPF 1 has the most desirable distribution of the three and the most limited downside. Thus the result (Figure 4.15 and the specified outcomes) shows that SPF 1 is a better investment alternative than the benchmark fund, and certainly the index when considering downside risk. Notable is also that SPF 1 has a higher upside than the benchmark fund (comparing Figure 4.15a and 4.15b), thus it may aspire to investors who are looking for a decent upside and a limited downside. Figure 4.15a indicates that SPF 1 has a positive skewed distribution, which is optimal for risk-averse investors, kurtosis is also displayed on the positive side and not the negative side. The results for a stress test where μ equals -30% p.a. is discloses in Appendix C (Figure C.1), notable is that SPF 1 is superior in extreme worst-case scenarios as well.

Now that it is concluded how a SPF should be constructed w.r.t. $\varepsilon^{\mathrm{SPF}}$ and δ^{SPF} it is interesting to investigate how the fund would have performed in the past through a backtest. Thus the next subsection focuses on a backtest for the SPF and how it would have performed historically.

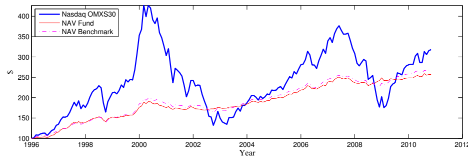
4.4.8 Backtesting

A backtest means that a modeled is tested on historical data and evaluated on that basis. Thus the strategy for the SPF described above (using Portfolio 7) is applied with the Nasdaq OMXS30 Index as underlying between 1996-01-02 − 2010-09-30. The volatility used is the realized volatility over every last three-month period. The interest rates for the zero-coupon bonds are given by the STIBOR (Stockholm Interbank Offered Rate). This is a good proxy for the interest rates for ELNs in the Nordic market where the bonds are issued by local investment banks, thus the STIBOR rate reflects the rate of the zero-coupon bonds for ELNs issued in Sweden in SEK. The risk-free rate is assumed to equal the STIBOR, since this is a quite conservative choice and a common rate to use as the risk-free rate (a higher r_f will yield a lower return for the SPF). For tenors over one year quoted SEK swap rates are used, available on Bloomberg. The backtest-results for different portfolios are disclosed below, for different $\varepsilon^{\mathrm{SPF}}$ and δ^{SPF}. Higher $\varepsilon^{\mathrm{SPF}}$ and δ^{SPF} implies a higher downside risk and the historical worst-case scenario is best for the most restrictive choice of $\varepsilon^{\mathrm{SPF}}$ and δ^{SPF}. Thus as mentioned earlier, the choice of these parameters should depend on the goals of the fund and thus determine the risk-level.

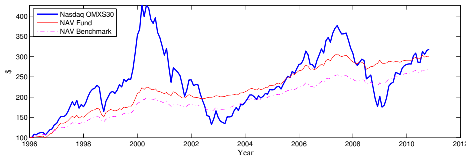
The time horizon considering capital protection is of three years as mentioned earlier. The worst-case outcome over all overlapping three years periods for the benchmark is −14.17% and for the index −67.79%, SPF 4.16a: −2.13%; SPF 4.16b: −8.15%; SPF 4.16c: −9.95%; SPF 4.16d: −16.30%. Thus most of the portfolios had a better worst-case return than the benchmark fund for the OMXS30 index between 1996 to the third quarter of 2010.



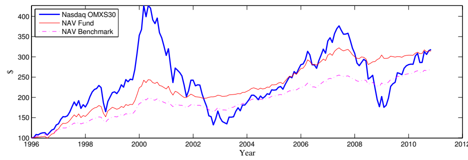
(a) $\varepsilon^{\text{SPF}} = 0.10$, $\delta^{\text{SPF}} = 0.05$.



(b) $\varepsilon^{\text{SPF}} = 0.15$, $\delta^{\text{SPF}} = 0.1$.

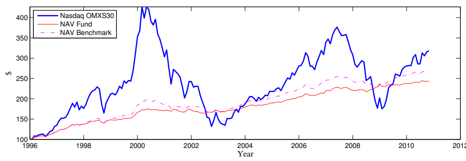


(c) $\varepsilon^{\text{SPF}} = 0.2$, $\delta^{\text{SPF}} = 0.1$.

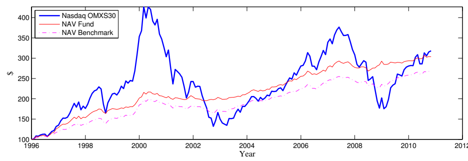


(d) $\varepsilon^{\text{SPF}} = 0.25$, $\delta^{\text{SPF}} = 0.1$.

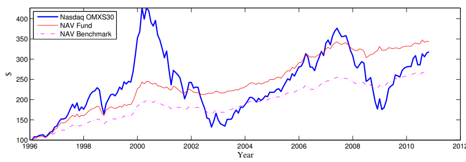
Figure 4.16: Backtest results: OMXS30 vs SPF 1 (Portfolio 7) and Benchmark.



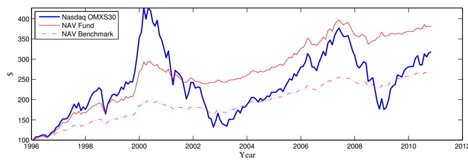
(a) $\varepsilon^{\text{SPF}} = 0.10$, $\delta^{\text{SPF}} = 0.05$.



(b) $\varepsilon^{\text{SPF}} = 0.15$, $\delta^{\text{SPF}} = 0.1$.



(c) $\varepsilon^{\text{SPF}} = 0.2$, $\delta^{\text{SPF}} = 0.1$.



(d) $\varepsilon^{\text{SPF}} = 0.25$, $\delta^{\text{SPF}} = 0.1$.

Figure 4.17: Backtest results: OMXS30 vs SPF 1 (Portfolio 3) and Benchmark.

The results indicate that the SPFs have the desired properties. Notable is that the comparison is done with the OMXS30, which is not including dividends and a benchmark fund, based on this index. Thus it is not a perfect benchmark and a more adequate benchmark is disclosed in Chapter 6. This standard index version is used in this chapter to be able to cover two market crashes; the total return version only exists since 2002.

The next subsection is regarding an alternative to the SPF 1 described above. The big difference lies in their allocation schemes for the structured products, but it will not be covered as extensively as SPF 1 since SPF 2 is mostly of theoretical interest.

4.4.9 Structured products fund 2

The structured products in SPF 1 were allocated using a fixed weights scheme, which indicated the weights allocated in the certain structured products. Thus it is impossible to guarantee that the fund will hold the desired amount of options from the different tenors in comparison to each other using SPF 1. This means that the portfolio may overweight certain options significantly, even though the total value held in options are under control, through the limits of ε^{SPF} and δ^{SPF}. The optimal portfolio weights that minimized the risk of movements in the underlying were discovered in Section 4.3. Thus to minimize the risk of movements in the underlying the fund should allocate the portfolio based rather on the options itself than based on the structured products. This means that the allocation algorithm calculates the weights for the structured products based on the proportion of options:bonds in the products and the desired relation of concentration amongst the options.

Now consider the same setting as for the SPF 1, the only thing that is different is that the weights are attained through a different scheme. The downside risk should be minimized for movements in the underlying. Thus option Portfolio 3 will be used as a base to calculate the adequate portfolio weights for the structured products. Important to notice is that this section is very theoretical and has less practical applications than SPF 1, which is quite realistic. SPF 2 is quite theoretical since the ratio options:bonds may decline significantly for some products if the market turns bust. Thus infinitesimal positions in some products and huge positions in others may be necessary to attain the desired level of options held in each tenor. This implies that the scheme may be adequate in times of normal market movements, but is disrupted in stressed scenarios. This scheme would also generate massive transaction costs, since the portfolio weights between two time periods often would change significantly, since the value of options is more volatile than the whole structured product.

The same products are available as for SPF 1. The fund reallocates its portfolio in the same manner as for SPF 1 (using the short-term debt etc.), the only difference is that $\tilde{\mathbf{w}}$ differs. The weight vector amongst the options is denoted $\mathbf{w}^{\text{options}}$ and is in this case given as follows,

$$\mathbf{w}^{\text{options}} = \begin{pmatrix} 0 & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} & \frac{1}{11} \end{pmatrix}^{\mathrm{T}},$$

let ξ be the first non-zero element of $\mathbf{w}^{\text{options}}$, j its index and $o_t^i = \frac{O_t^i}{\beta_t^i + O_t^i}$. A temporary weight vector $\mathbf{w}^{\text{temp}}(t)$ is created, given as,

$$w_i^{\text{temp}}(t) = \begin{cases} 0, & \text{if } i < j \\ \xi, & \text{if } i = j \\ w_i^{\text{options}} \dfrac{o_t^j}{o_t^i}, & \text{if } i > j \end{cases}$$

thus $\tilde{\mathbf{w}}$ is given as,

$$\tilde{\mathbf{w}}_t = \frac{\mathbf{w}^{\text{temp}}(t)}{\mathbf{1}^{\mathrm{T}}\mathbf{w}^{\text{temp}}(t)}.$$

The rest of the allocation scheme is exactly the same as in Section 4.4.3 for the SPF 1.

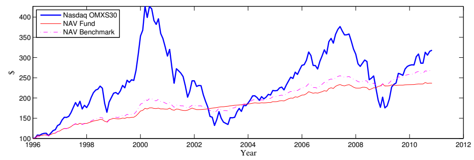
The same analysis is conducted for this fund construction as for SPF 1. The results for Scenario A and B are given in Table 4.3. By comparing the results in Table 4.1 and 4.3 it is easy to see that the SPF 2 has much better worst-case outcomes including every scenario and every selection of ε^{SPF} and δ^{SPF} than SPF 1. The Portfolio 3 was the best choice to minimize downside risk with in Section 4.3, thus it is reasonable that SPF 2 should beat SPF 1 in terms of worst-case outcome w.r.t. the underlying.

Figure 4.14 and 4.19 indicate that SPF 2 does not only provide a better protection during the market crash but also a better future potential when the market starts to appreciate and turns into a bull market.

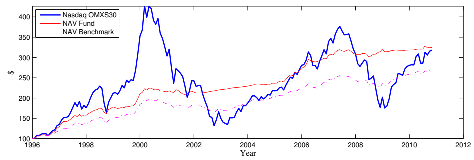
In the SPF 2 case it is possible to choose $\varepsilon^{\text{SPF}^2} = 0.20$ and still have a lower downside risk than the benchmark fund.

The same analysis, implications etc. are applicable to SPF 2 in the exact same way as SPF 1. Hence only the results are disclosed for the interested reader.

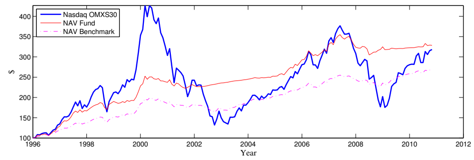
The following section discusses the terminology capital protection and how it should be used further in this thesis.



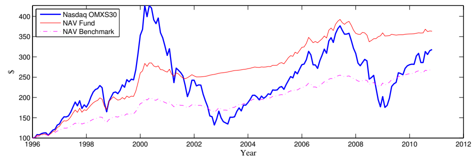
(a) $\varepsilon^{\text{SPF}} = 0.10$, $\delta^{\text{SPF}} = 0.05$



(b) $\varepsilon^{\text{SPF}} = 0.15$, $\delta^{\text{SPF}} = 0.1$

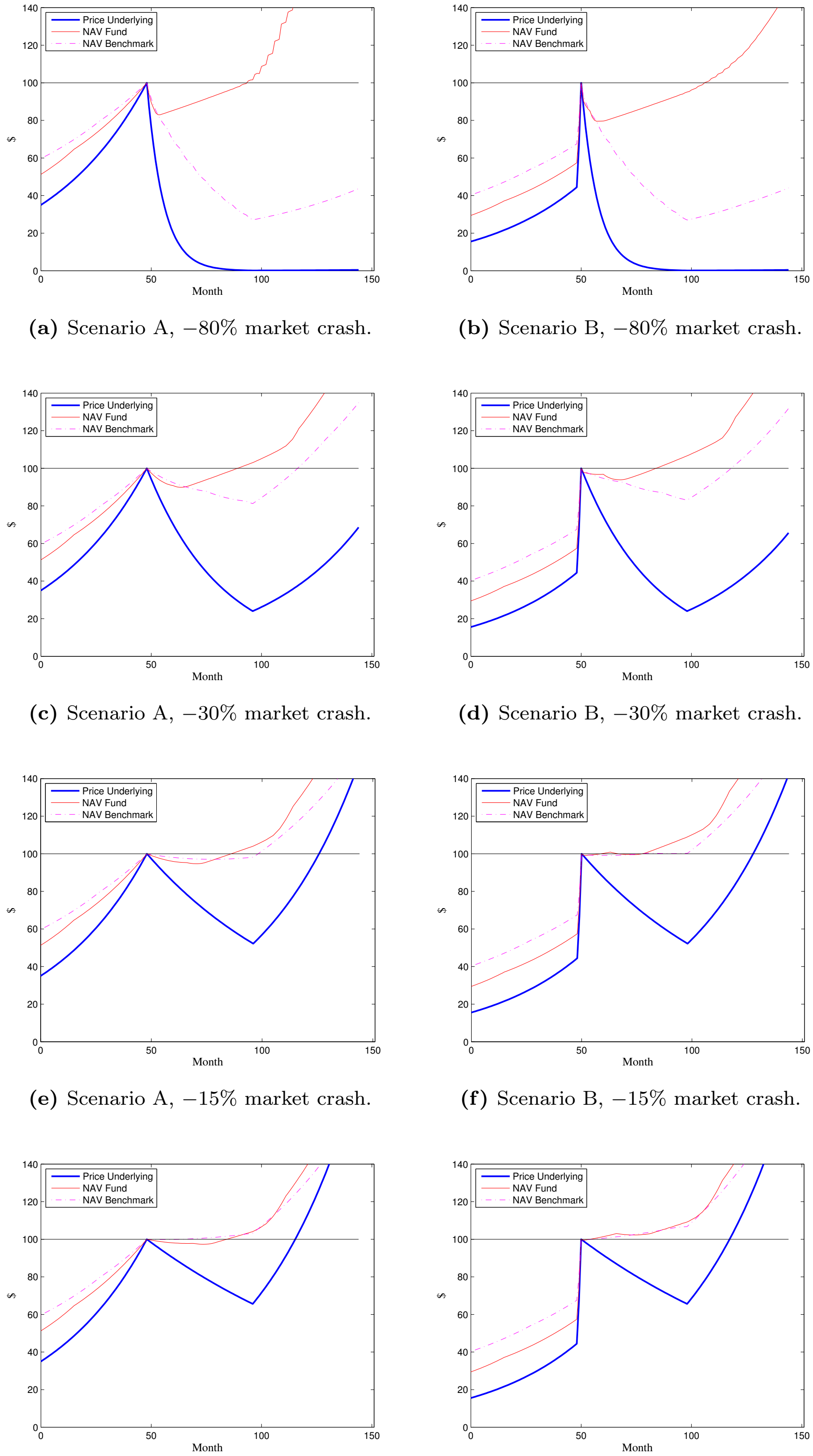


(c) $\varepsilon^{\text{SPF}} = 0.2$, $\delta^{\text{SPF}} = 0.1$



(d) $\varepsilon^{\text{SPF}} = 0.25$, $\delta^{\text{SPF}} = 0.1$

Figure 4.18: Backtest results: OMXS30 vs SPF 2 (Portfolio 3) and Benchmark.



(a) Scenario A, −80% market crash.

(b) Scenario B, −80% market crash.

(c) Scenario A, −30% market crash.

(d) Scenario B, −30% market crash.

(e) Scenario A, −15% market crash.

(f) Scenario B, −15% market crash.

(g) Scenario A, −10% market crash.

(h) Scenario B, −10% market crash.

Figure 4.19: Illustrative example of scenarios and movements of the SPF 2 (Portfolio 3) and benchmark fund for $\varepsilon^{\text{SPF}} = 0.15$, $\varepsilon^{\text{bench}} = 0.3$, $\delta^{\text{SPF}} = 0.1$, $\delta^{\text{bench}} = 0.1$.

Return Index Months 48-96 / 50-98 p.a.	$\varepsilon^{\mathrm{SPF}}$	δ^{SPF}	Scenario A Worst-Case Return	Scenario B Worst-Case Return
-80%	1	1	-11.79%	-30.67%
-80%	0.3	0.1	-11.79%	-20.76%
-80%	0.25	0.1	-11.79%	-18.51%
-80%	0.20	0.1	-4.74%	-14.24%
-80%	0.15	0.1	-4.58%	-10.11%
-80%	0.15	0.05	0.00%	-8.22%
-80%	0.10	0.05	1.20%	-2.61%
-50%	1	1	-10.89%	-21.42%
-50%	0.3	0.1	-10.89%	-14.66%
-50%	0.25	0.1	-10.89%	-12.07%
-50%	0.20	0.1	-4.30%	-6.10%
-50%	0.15	0.1	-2.47%	-2.51%
-50%	0.15	0.05	-3.26%	-3.90%
-50%	0.10	0.05	1.24%	2.25%
-30%	1	1	-9.48%	-15.39%
-30%	0.3	0.1	-9.48%	-9.08%
-30%	0.25	0.1	-9.48%	-6.76%
-30%	0.20	0.1	-3.70%	-3.11%
-30%	0.15	0.1	-2.24%	0.93%
-30%	0.15	0.05	-2.49%	-0.74%
-30%	0.10	0.05	-1.16%	4.08%
-20%	1	1	-8.17%	-11.80%
-20%	0.3	0.1	-8.17%	-6.90%
-20%	0.25	0.1	-8.17%	-4.69%
-20%	0.20	0.1	-3.98%	-1.20%
-20%	0.15	0.1	-1.61%	2.82%
-20%	0.15	0.05	-1.72%	0.88%
-20%	0.10	0.05	1.76%	5.54%
-10%	1	1	-5.88%	-7.25%
-10%	0.3	0.1	-5.88%	-3.77%
-10%	0.25	0.1	-5.88%	-1.60%
-10%	0.20	0.1	-1.67%	0.80%
-10%	0.15	0.1	0.00%	4.69%
-10%	0.15	0.05	-0.60%	2.36%
-10%	0.10	0.05	3.29%	7.16%

Table 4.3: SPF 2: Worst-case outcomes for the SPF 2 based on Portfolio 3 option weights given scenarios A and B, $\varepsilon^{\mathrm{SPF}}$ and δ^{SPF}.

4.5 Alternative definitions of capital protection

Since it is not possible to construct a capital guaranteed SPF it is imperative to discuss the usage of the terminology capital protection. The terminology capital guarantee is something absolute, which indicates that the portfolio will not have a negative return. It is absolute in the sense that it cannot be distorted and it is not possible to argue that a placement is capital guaranteed if it may have a negative return (still not taking the credit risk into consideration). The terminology capital protection on the other hand is more diffuse and flexible.

Still, it is quite easy to have the same associations to the word protection as the word guarantee, and many investors mix up the terminologies (Chuan, 2008, [12]). Thus while it is not possible to call the fund capital guaranteed it is essential that the fund uses the terminology capital protected in its communication with the investors to appeal to their loss-aversion (Kahneman and Tversky, 1979, [26]). What does it mean that an investment is capital protected? Capital protection means, according to UBS (2010, [37]), that the investor is guaranteed a certain minimum repayment of the invested amount in the end of its investment term (time horizon). Thus when claiming that a fund is capital protected it should have a limited downside risk, i.e. a certain percentage of the investment should be guaranteed.

This leads us to the purpose of this section, alternative definitions of capital protection. By using the original definition and modify it slightly it may be possible to construct a fund which fulfills the requirements for an alternative definition of capital protection.

The focus lies on two alternative definitions of capital protection. These definitions are evaluated regarding their pros and cons and which definition is best to use. Note that these alternative definitions are for capital protection, not guarantee. The capital protection property is regarding a certain time horizon, i.e. if the investors hold their position during this time horizon they will receive a certain level of capital protection.

4.5.1 Definition 1 - Absolute lower bound

This definition is a worst-case scenario based definition, which means that the portfolio's performance should be Monte Carlo simulated with a certain number of scenarios, N, which generates a worst-case outcome for the portfolio. The aim of the portfolio allocation is that the worst-case scenario should exceed the invested amount X_0 times the degree of capital protection κ (determines the degree of capital protection, can be any positive number). Hence the portfolio is considered as capital protected if,

$$\min_i X_T^i > \kappa X_0 \qquad\qquad i = 1, ..., N,$$

where X_T^i is the portfolio value at time T (end of the investment horizon) for the ith scenario.

Pros

This is probably the most concrete definition of capital protection since the worst outcome simulated actually would exceed the minimum acceptable level of return. The investor can be quite confident that it will not lose more capital than the given level over the time horizon. It is also possible to use a sophisticated algorithm to solve the linear optimization problem as given in Section 2.3.2, ensuring a well-defined solution to the problem.

Cons

The point of using this definition is to quantify the worst-case outcome, but the worst-case outcome will always be based on the scenarios, the assumptions regarding the market model and how the underlying is modeled, the options, the yield curve etc. Also, how many scenarios are needed? Choosing between N = 10,000, 50,000 or 100,000 scenarios, which one is adequate? The problem is that when increasing the number of scenarios, the number of extreme outliers are also increased, so how is it possible to determine the adequate number of scenarios to use, since the worst-case scenario would for most portfolios converge to −100% in return given that $N \to \infty$? Another problem with this definition is that it can be extremely hard to find a portfolio that actually fulfills this requirement (depending on the choice of κ, e.g. if $\kappa = 1$ the portfolio may end up in only investing in bonds and almost nothing in options, thus eliminating the possibility to gain when the market goes up). A negative side of this definition is also that the risk measure is only saying something about the worst-case outcome, not anything about the remaining unfavorable outcomes.

4.5.2 Definition 2 - CVaR

This definition is based on CVaR, which means that the CVaR for the portfolio at some quantile α must be less than $(1-\kappa)\,X_0$, for a determined level of capital protection κ. For more information regarding the definition of CVaR please consult Section 2.1.3. Thus the portfolio is considered to be capital protected if,

$$\mathrm{CVaR}_\alpha\left(L^w\right) = \mathbb{E}\left[L^w \mid L^w \geq \mathrm{VaR}_\alpha\left(L^w\right)\right] \leq (1-\kappa)\,X_0$$

Pros

A positive side of this definition is that it is more realistic (in sense that a portfolio may actually fulfill the requirement even for high values of κ) than the worst-case definition. It is more robust, it is a coherent risk measure and the result converges as the number of scenarios grows. Also it may be a more adequate definition than the worst-case definition, since it discloses, "if it goes bad, how bad does it get?" and not only for the worst-case scenario. For example by using the quantile $\alpha = 0.95$ CVaR is the expected value of the loss for the 5% worst outcomes. It is also a risk measure that can capture the risk of heavy tails and skewness. Thus it is possible to capture a lot of the risk using CVaR, much more than using a worst-case outcome definition. Another advantage is that the linear optimization problem for CVaR is well defined and easy to solve using the simplex method.

Cons

The negative aspect is that it does not provide an absolute definition of capital protection, if the CVaR is positive it does not guarantee that there are no negative outcomes.

CVaR is the risk measure that will be used further on in this thesis, due to its major benefits. Note that it is possible to use the alternative definition 1 in the same manner as with alternative definition 2 in the upcoming scenario optimization, by solving the minimum regret optimization problem instead. The portfolio weights will in general differ for the two different risk measures, since they measure different things. The focus on minimizing the worst-case outcome may impose a high risk for the investor, leading to a suboptimal portfolio.

CHAPTER 5

Modeling Financial Assets

Modeling the financial assets is one of the most important issues in portfolio optimization and allocation problems. It is of great importance that the risk factors are modeled in a way such that they reflect the underlying risk factors. This thesis studies Equity-Linked Notes, thus the components that must be modeled are: the yield curve, the underlying and the volatility. These risk factors determine the fair price of the fund. The reason why the previous study has been so extensive is to avoid, as far as possible, dependence on the chosen market model in this chapter.

5.1 Volatility and option pricing

It is important that the options in this study can be priced through a closed-form expression, since the study involves the usage of Monte Carlo simulation and a simulation based pricing model would thus be too computer intensive. The Black-Scholes formula for option pricing will therefor be used for pricing the plain vanilla options, since it provides a fast and adequate pricing model. The problem with Black-Scholes formula is that it does not always price the options correctly due to the misspecification that the price of an asset follows a geometric brownian motion. The implied volatility is the volatility that should be used in Black-Scholes formula so that it gives the fair price of the option. Thus by using Black-Scholes formula and implied volatility the correct price is attained. The thesis is regarding ELNs and as discussed earlier, in Section 3.3.1, there are no systematic differences for the prices of ELNs when using either implied volatility or realized volatility (Wasserfallen and Schenk, 1996, [39]). Thus the volatility will be modeled as the realized volatility during one year, thus it is based on 52 weekly observations of the index log returns.

It is important to notice that the liquidity of options on OMXS30 is quite poor, which implies that it is hard to find appropriate historical data such that the price of the options can be modeled through implied volatility (surfaces). Thus it may reduce the misspecification and error in pricing on the Nordic market to use realized volatility instead of implied volatility.

The dividend rate of the index must be estimated, since the OMXS30 index is not including dividends. There exists a total return version of OMXS30, where dividends are reinvested in the index, which is available on Bloomberg. When comparing the total return index with the standardized version it is easy to see that the total return index yields a higher return of approximately 3.0% p.a. using continuous compounding during the observed time period (since 2002 when it first was started to 15th of November 2010), which implies that the expected dividend yield is 3.0% p.a. in the Black-Scholes model.

5.2 Index

There exists countless of models describing the movements of a stock or an index. Probably the most popular model is to describe the movements of the underlying as a geometric brownian motion as done by Black and Scholes (1973, [9]). It is also common to model the returns by using for example normal variance mixtures (Aas, 2006, [1]; Hult and Lindskog, 2009, [23]).

Another popular model is proposed by Heston (1993) in [20] where he proposes a model including stochastic volatility that results in a closed-form expression for option pricing.

Since the aim of this chapter is to model scenarios, for the risk factors so that the scenarios can be evaluated in an optimization problem, it is desirable to have a model that is easy to sample from, also it is important to maintain the correlation in the data. Not only the underlying should be modeled but also the yield curve, thus it is necessary to consider the dependence structure between the index and the yield curve. An adequate method to use is principal component analysis (PCA) as described in Section 2.2. There exists significant dependence between a yield curve and its market's corresponding index, e.g. in the Swedish market the STIBOR is not independent of the stock index OMXS30. The correlation between OMXS30 and the Swedish yield curve is negative for shorter tenors and positive for tenors above one year (also the absolute value of the correlation is much higher for longer tenors). The correlation is in general higher for weekly returns than for daily returns during the observed time period. The correlation matrices for daily, weekly and yearly log returns between OMXS30 and the SEK rates between 1M to 3Y are disclosed in Appendix D. Thus it is better to use weekly log returns for the simulation since the dependence structure amongst the data is not contaminated as much as the daily log returns by noise.

Since one of the most appropriate ways to model the yield curve is by PCA and since PCA also is a good method to describe stock/index returns it is a natural choice to use PCA for the two combined.

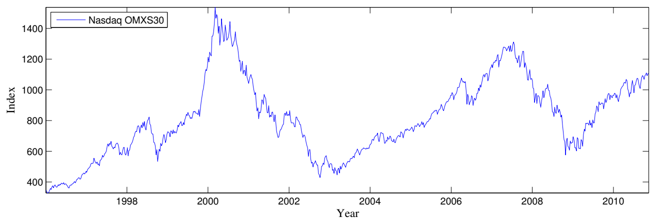


Figure 5.1: The closing index levels of OMXS30 between January 1996 and November 2010.

5.3 Yield curve

It is as mentioned earlier a good choice to use PCA to model the yield curve changes since a yield curve can, in general, be described by three dominant factors in a three-factor model, these are the level, steepness and curvature (Litterman and Scheinkman, 1991, [29]). Therefor the yield curve will be modeled as described in Section 2.2 together with the index to maintain the dependence structure. Thus the PCA should result in four significant factors, i.e. principal components, which will be enough to explain most of the dependence structure in the data. This choice of model guarantees that the sampling of scenarios will be parsimonious.

In the described market model the STIBOR will be used as the risk-free rate, since this is a common choice as the risk-free rate when pricing in the Swedish market. STIBOR is the daily interbank offered rate in Stockholm, thus it also reflects the yield on the zero-coupon bonds that are issued by the local investment banks. Hence the STIBOR is an appropriate rate both for the risk-free rate and the zero-coupon rate. The Swedish Riksbank publishes daily quotes of the STIBOR on their homepage http://www.riksbank.se.

5.4 A PCA model for the yield curve and index

A four-factor model is, as mentioned earlier, enough to describe the market movements, since the yield curve is described by three factors and the index (mainly) by the remaining factor.

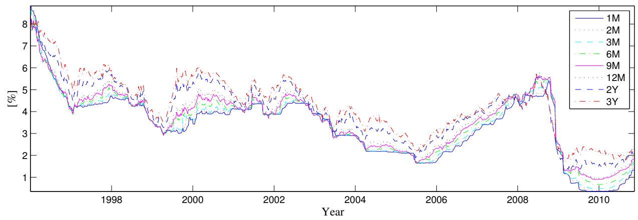


Figure 5.2: SEK STIBOR and Swap rates between January 1996 and November 2010.

The index and the yield curve must be rewritten on a stationary form since they are not stationary. Thus the index and the yield curve nodes are modeled with log returns, which are weakly stationary. Even though it may seem strange to use log returns for the yield curve, since these are in some sense already log returns, it is possible to prevent negative interest rates by this procedure, which is very important. Also it seems more adequate for the observed data sample to model with log returns than absolute changes.

5.4.1 Data set

The OMXS30 from the beginning of 1996 until the middle of November 2010 is used for the simulation. The STIBOR is used as the risk-free rate up to one year and above one year the SEK swap rates are used, the data set ranges from the beginning of 1996 until the middle of November 2010. The volatility is modeled by using the one year realized volatility. STIBOR has the tenors 1M, 2M, 3M, 6M, 9M, 12M and the swap rates the tenors 2Y and 3Y, thus there is a total of nine PCs. A linear interpolation is performed for products with a time to maturity that does not coincide with these tenors.

5.4.2 PCA results

A PCA is performed on the whole data set; the first four PCs explain 97.94% of the variation in the data. This is in line with the results found by Litterman and Scheinkman (1991, [29]) as well as by Barber and Copper (2010, [6]) that most PCAs for yield curves can with only three PCs describe more than 95% of the variation (notable is that by removing the index the first three factors explains 97.68% of the variation). Notable is that the four PCs describe more than 95% of the variation, which is more than enough.

PC	Explained variation [%]	Cumulative explained variation [%]
1	68.00	68.00
2	16.81	84.82
3	9.62	94.44
4	3.50	97.94
5	0.72	98.65
6	0.57	99.22
7	0.37	99.59
8	0.30	99.89
9	0.11	100.00

Table 5.1: PCA results: Variation explained by each principal component and total explained variation (cumulative explained variation).

By inspecting Figure 5.3 it is easy to see that the first PC's factor loadings are all positive and can be interpreted as the level change, also the factor loading for the index is almost zero. The

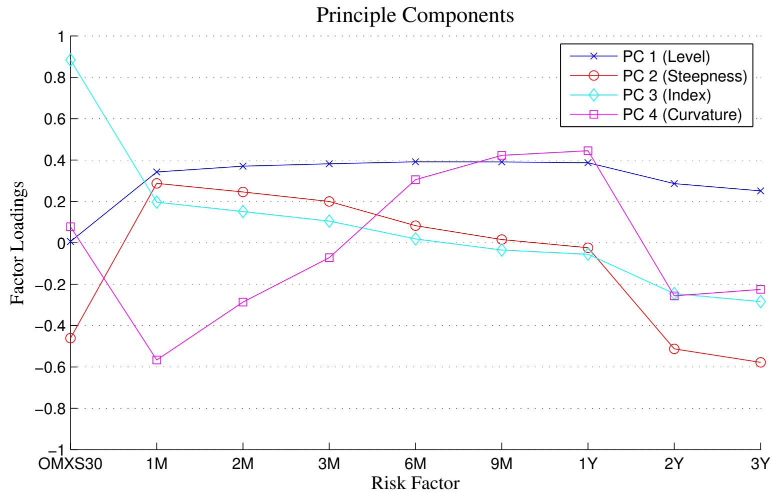


Figure 5.3: Factor loadings for the first four principle components, explaining changes in OMXS30 and the yield curve up to three years.

second PC corresponds to steepness since its factor loadings change sign moving to longer tenors. The third PC corresponds to the index since the factor loading is close to one for OMXS30 and small for the rest of the factors. The fourth PC corresponds to curvature since products in the middle tenors have positive factor loadings when the others have negative.

The factor loadings have been estimated during a period of approximately fifteen years. It is important that the factor loadings are stable through time if the distribution is used for sampling (since the estimation is used as a proxy for the future). To investigate if the model is adequate the factor loadings and explanatory power of overlapping PCAs must be studied through time and investigated for stability.

To conduct the study the data between January 1996 and June 2010 is divided into overlapping three years intervals with a distance of six months. Thus this gives approximately 150 observations, which is enough to retrieve a good estimation of the factor loadings.

The results are disclosed in Table 5.2, the first PC explains between 59% and 74% of the variation and the other PCs exhibit similar patterns. On the other hand the total explanatory power of the four PCs is always above 93.48%. Thus the four PCs explain most of the variation in the data sample regardless of which sample period selected. Notable is that the explanatory power for the whole period was 97.94% and that the explanatory power in the model is higher during the latter part of the time period. The PCA is computed on weekly returns in this section, performing the PCA on daily returns generates less stable results, with lower explanatory power.

The PCs' factor loadings for the overlapping periods are disclosed in Figure 5.4. PC 1 has the smallest variation, while the other three have higher levels of variation; the factor loadings have the same structure for all samples. Concluded: the factor loadings are relative stable through time and stable enough to serve for simulation, even though the factor loadings exhibit some sample dependence.

5.4.3 Historical simulation

Historical simulation is a non-parametric model that draws samples from the empirical distribution with replacement. Thus to simulate new PCs; samples of factor scores should be drawn from the empirical distribution. This means sampling random dates (with equal probabilities) that are used to extract the corresponding factor scores, and replace the sample before the next draw. The advantage of using historical simulation is that the scenarios have happened in the

Start	End	PC 1	PC 2	PC3	PC 4	Cumulative [%]
Jan 1996	Dec 1998	73.51	11.18	9.74	3.09	97.52
Jul 1996	Jun 1999	74.35	11.10	9.54	2.88	97.87
Jan 1997	Dec 1999	67.34	11.71	10.47	3.99	93.51
Jul 1997	Jun 2000	65.98	12.96	10.64	3.90	93.48
Jan 1998	Dec 2000	65.80	12.96	10.74	4.08	93.59
Jul 1998	Jun 2001	66.19	12.84	10.91	4.01	93.95
Jan 1999	Dec 2001	64.59	14.88	11.30	3.69	94.46
Jul 1999	Jun 2002	62.84	16.05	11.21	3.85	93.94
Jan 2000	Dec 2002	67.95	17.38	9.75	2.91	98.00
Jul 2000	Jun 2003	66.26	20.42	8.81	2.89	98.38
Jan 2001	Dec 2003	65.79	21.01	8.64	2.99	98.43
Jul 2001	Jun 2004	61.88	23.59	9.15	3.68	98.30
Jan 2002	Dec 2004	58.80	24.77	9.00	4.53	97.10
Jul 2002	Jun 2005	60.27	23.34	8.85	4.66	97.12
Jan 2003	Dec 2005	61.38	21.96	9.17	4.59	97.09
Jul 2003	Jun 2006	61.77	20.58	9.93	4.62	96.90
Jan 2004	Dec 2006	62.62	19.84	10.16	4.44	97.07
Jul 2004	Jun 2007	64.24	17.53	10.55	4.42	96.73
Jan 2005	Dec 2007	63.68	18.51	10.66	4.52	97.37
Jul 2005	Jun 2008	59.18	20.21	11.24	5.48	96.11
Jan 2006	Dec 2008	69.61	20.96	7.58	1.20	99.35
Jul 2006	Jun 2009	72.57	16.85	7.28	2.52	99.22
Jan 2007	Dec 2009	71.42	17.58	7.09	2.70	98.79
Jul 2007	Jun 2010	69.89	18.76	7.17	2.92	98.74
Jan 1996	October 2010	68.00	16.81	9.62	3.50	97.94

Table 5.2: Variation explained by each principal component and total explained variation (cumulative explained variation) for 24 overlapping time periods between January 1996 and June 2010.

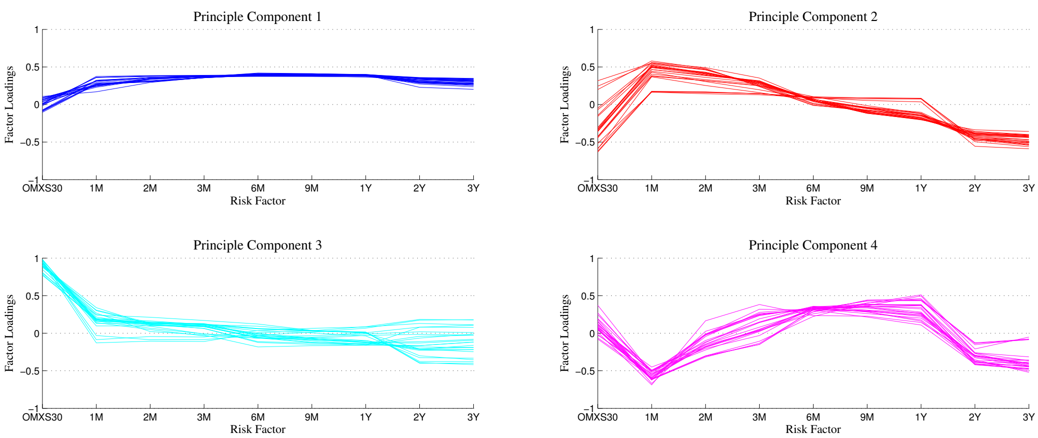


Figure 5.4: Factor loadings for the first four PCs for 24 overlapping time periods between January 1996 and June 2010, each line corresponds to one sample.

past, they are realistic and reflect the empirical distribution. Also it is really simple to implement and use historical simulation. On the other hand the problem is that there is no possibility of implementing new scenarios, thus the model is restricted to the outcomes of the past. This means that if no extreme events have happened in the past the probability of generating an extreme scenario is zero. But the model is not restricted in this setup since the return over a three-year period is generated from weekly returns. The model would be limited when studying only weekly returns, not yearly or monthly returns, thus simulating three-years returns using weekly data does not put any serious restrictions on the simulation.

The ith sample of the jth variable's return using the first four PCs in a historical simulation is given as,

$$r_{i,j} = \sum_{k=1}^{4} \mathbf{Q}^{\mathrm{T}}_{k,j}\mathbf{F}_{i,k} + \varepsilon_{i,j},$$

where,

$$\varepsilon_{i,j} \sim N\left(0, \mathrm{Var}\left[\sum_{k=5}^{9} \mathbf{Q}^{\mathrm{T}}_{k,j}\mathbf{F}_{\bullet,k}\right]\right).$$

The sample should also be rescaled with its historical mean and standard deviation to attain the final sampled data.

5.4.4 Monte Carlo simulation

The idea of a Monte Carlo simulation is to fit a parametric distribution to the observed data; random samples are drawn from the parametric distribution to generate new data. Thus in the case of the PCA a parametric distribution should be fitted to the observed factor scores.

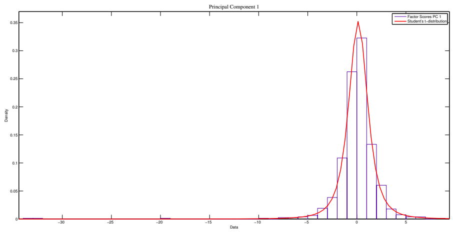
By using QQ-plots, or other graphical illustrations such as histogram-fits and scatter plots, it is possible to illustrate and identify appropriate parametric distributions. It is easy to see that the PCs can be described by t-distributions with different levels of freedom, location and scale, as shown in Figure 5.5. Scatter plots indicate that the four PCs come from a spherical distribution and that there exists dependence between the samples (scatter plots have elliptical shapes) as seen in Figure 5.6.

	t-location scale		
PC	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\nu}$
1	0.00	1.0296	2.4189
2	0.00	0.8880	4.1763
3	0.00	0.8189	8.8734
4	0.00	0.2976	2.7824

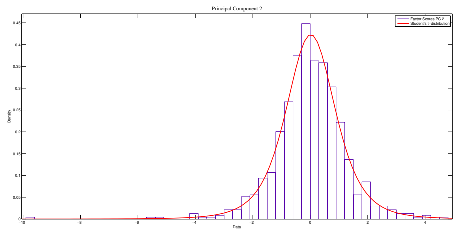
Table 5.3: The fit for a student's t-distribution to each marginal distribution (factor scores) for the first four PCs attained through a maximum likelihood estimation.

Multivariate t-distribution

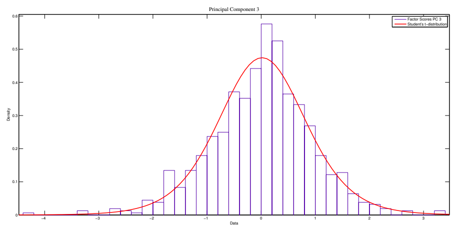
If ν (degrees of freedom) for the marginal distributions (in this case the individual PCs) are close to each other it is possible to use a multivariate t-distribution, instead of using e.g. copulas. A relative good fit is actually attained for this sample with a multivariate t-distribution, but it is not as good as the t-copula model as disclosed further down. The following parameters for the



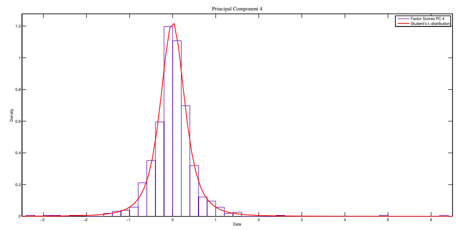
(a) PC 1



(b) PC 2



(c) PC 3



(d) PC 4

Figure 5.5: Histograms for the distribution of factor scores (PC 1-4), the red line illustrates the pdf of a t-distribution, the t-distribution seems to be an adequate parametric distribution for the PCs' marginal distributions.

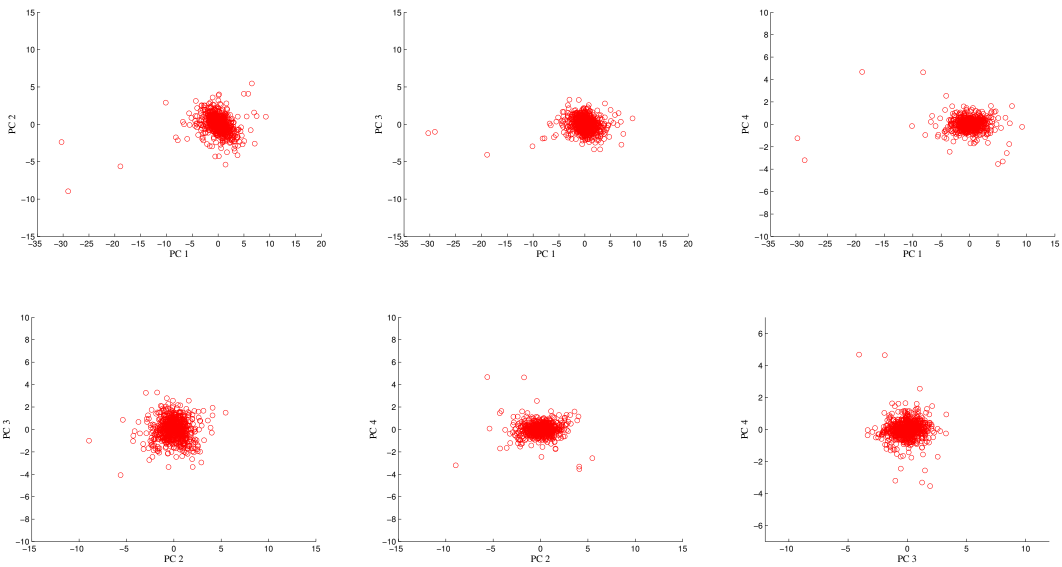


Figure 5.6: Scatter plots of pairwise PCs and corresponding factor scores for the empirical (observed) samples.

multivariate t-distribution are attained using a maximum likelihood estimation,

$$\hat{\mu} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad \hat{\mathbf{\Sigma}} = \begin{pmatrix} 2.337 & 0 & 0 & 0 \\ 0 & 0.5770 & 0 & 0 \\ 0 & 0 & 0.3301 & 0 \\ 0 & 0 & 0 & 0.1201 \end{pmatrix}, \qquad \hat{\nu} = 3.2326.$$

It is common to assume that the residual term is multivariate normal distributed. This is not a necessary condition and the residual term ε can be rewritten as a matrix (factor loadings) times a stochastic vector (vector of the remaining PCs). For the observed data set the stochastic vector is approximately multivariate t-distributed. Thus,

$$r_{i,j} = \sum_{k=1}^{4} \mathbf{Q}_{k,j}^{\mathrm{T}} \mathbf{F}_{i,k} + \sum_{k=5}^{9} \mathbf{Q}_{k,j}^{\mathrm{T}} \mathbf{F}_{i,k}^{\varepsilon}, \tag{5.1}$$

where,

$$\mathbf{F} \sim t_{\nu}\left(\mathbf{0}, \mathbf{\Sigma}\right), \quad \mathbf{F}^{\varepsilon} \sim t_{\nu_{\varepsilon}}\left(\mathbf{0}, \mathbf{\Sigma}_{\varepsilon}\right),$$

and,

$$\hat{\mathbf{\Sigma}}_{\varepsilon} = \begin{pmatrix} 0.0195 & 0 & 0 & 0 & 0 \\ 0 & 0.0154 & 0 & 0 & 0 \\ 0 & 0 & 0.0099 & 0 & 0 \\ 0 & 0 & 0 & 0.0082 & 0 \\ 0 & 0 & 0 & 0 & 0.0030 \end{pmatrix}, \qquad \hat{\nu}_{\varepsilon} = 2.8622.$$

Hence to perform a Monte Carlo simulation, using a multivariate t-distribution (in this setting), draw samples of $\mathbf{F}$ and $\mathbf{F}^{\varepsilon}$ and plug in the sampled values into Equation 5.1 above. The multivariate t-distribution is a normal variance mixture and hence easy to sample from e.g. MATLAB has an inbuilt function to perform the operation.

Copula

The marginal distributions for the factor scores are approximately t-distributed as seen above. Thus by using a Copula it is possible to maintain the marginal distributions intact and use the dependence structure from another distribution. Thus an adequate choice is to use t-marginals and a t-copula since the dependence structure is spherical and similar to the one sampled from a t-copula. If all the marginals had the same levels of freedom it would be the exact same model as with a multivariate t-distribution, but as seen in Table 5.3 this is not the case. The marginals described in Table 5.3 will be used for the first four PCs.

The best fit (using MATLAB's copulafit) is a t-copula with $\hat{\nu} = 3.0617$, which has the following dependence structure,

$$\hat{\rho} = \begin{pmatrix} 1.0000 & -0.4985 & -0.2170 & 0.1680 \\ -0.4985 & 1.0000 & -0.0841 & 0.0733 \\ -0.2170 & -0.0841 & 1.0000 & 0.1202 \\ 0.1680 & 0.0733 & 0.1202 & 1.0000 \end{pmatrix}.$$

To sample from the copula with t-marginals sample from a multivariate t-distribution with $\hat{\nu} = 3.0617$ as above, use the cumulative distribution for a univariate $t_{\hat{\nu}}$ variable on the sample, the samples are now uniformed samples from the $t_{\hat{\nu}}$ copula. Take the inverse of each cumulative marginal distribution on the samples to attain the new centered and rescaled samples, multiply these samples with each dispersion coefficient and add the mean to attain the final factor score sample.

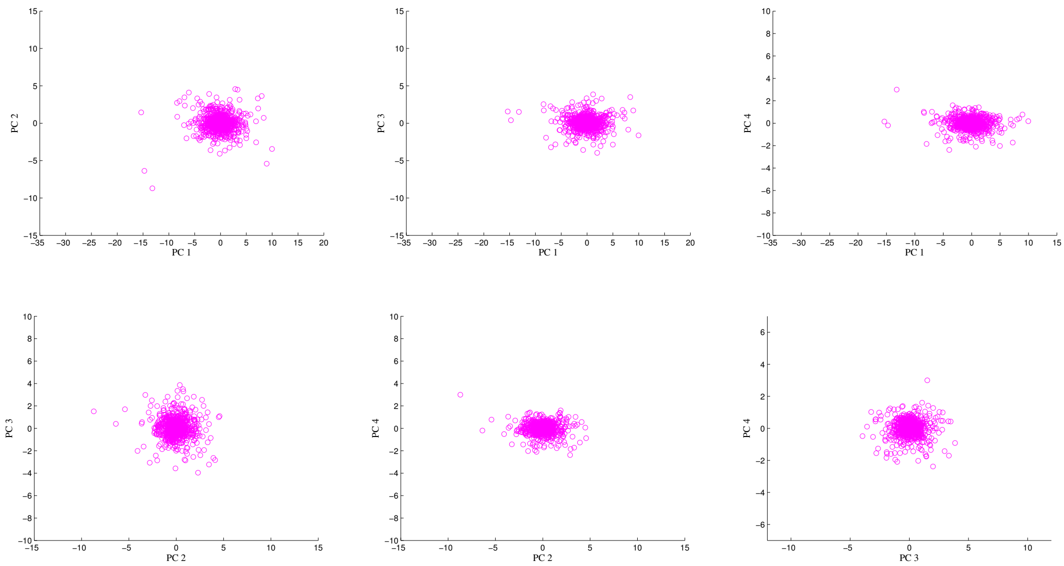


Figure 5.7: Scatter plots of pairwise PCs and corresponding factor scores sampled from the multivariate t-distribution.

The error term can be modeled either through a multivariate t-distribution or any other parametric distribution such as a copula model. It does not matter a lot which distribution is chosen since the error explains the remaining noise in the data (which is small, less than 2.5% of the total variance). By fitting a t-copula with t-marginals the following result is retrieved,

	t-location scale		
PC	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\nu}$
5	0.00	0.1405	2.8771
6	0.00	0.1080	2.5897
7	0.00	0.0808	2.4963
8	0.00	0.0832	2.6872
9	0.00	0.0692	3.8576

Table 5.4: Parameters for the residual factor scores attained through a maximum likelihood estimation of t-distributions.

$$\hat{\rho}^{\varepsilon} = \begin{pmatrix} 1.0000 & 0.0658 & 0.1179 & 0.1837 & 0.1133 \\ 0.0658 & 1.0000 & 0.1380 & 0.1260 & -0.0183 \\ 0.1179 & 0.1380 & 1.0000 & 0.2901 & 0.1661 \\ 0.1837 & 0.1260 & 0.2901 & 1.0000 & 0.1338 \\ 0.1133 & -0.0183 & 0.1661 & 0.1338 & 1.0000 \end{pmatrix}, \qquad \hat{\nu}^{\varepsilon} = 3.4124.$$

Figure 5.9 shows that the copula provides a much better fit for the marginals than the multivariate t-distribution. The copula provides also a better dependence structure, when comparing Figure 5.7 and 5.8 with Figure 5.6. Thus it is easy to conclude from the plots that the copula provides a better fit, notable is that the multivariate t-distribution does not provide a bad fit at all, just not as good as the copula model.

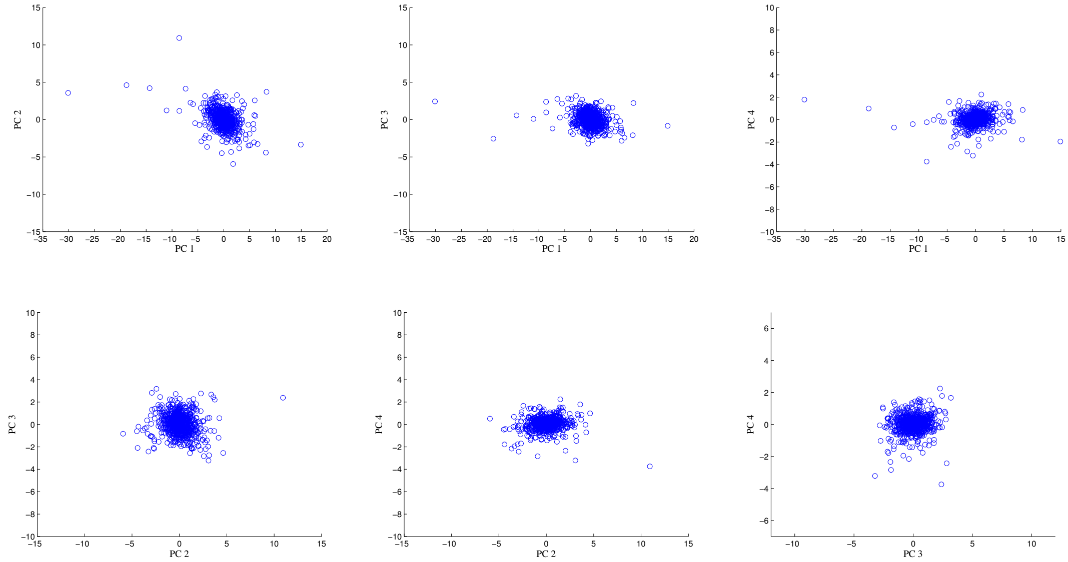


Figure 5.8: Scatter plots of pairwise PCs and corresponding factor scores sampled from the t-copula with t-marginals.

Hence an appropriate choice is to use either the copula model with Monte Carlo simulation or historical simulation, the result does not differ a lot in practice. This is particularly the case since weekly return data is used to simulate three years trajectories. The two models would differ much more if the return for just one week should be simulated. When sampling one week's data only one draw is required, thus limiting the outcomes to the empirical distribution. When sampling three-years returns from weekly returns it is still possible to generate almost an infinite number of combinations using the empirical distribution, 156 return samples with replacement out of 781 observed weeks provides 781^{156} different combinations, which converges towards infinity.

The advantage of using historical simulation instead of the parametric simulation is that it is much faster to use, reducing the sample time in many cases from several hours to minutes. Thus a historical simulation is used further in this thesis since a sensitivity analysis shows that the results are in practice identical using both.

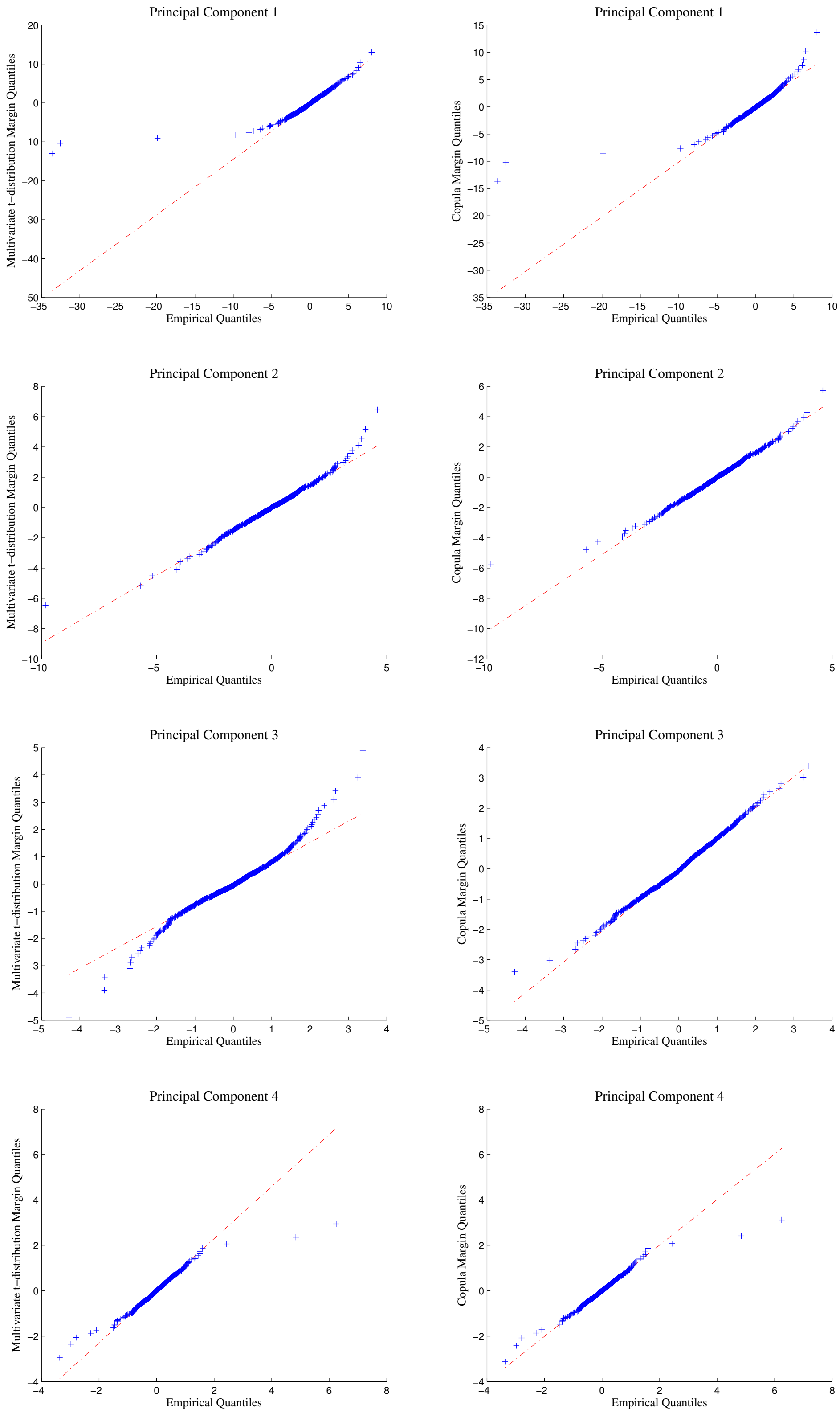


Figure 5.9: QQ-plots of the empirical marginal distributions against the multivariate t-distribution's marginals (to the left) and the copula's marginals (to the right).

CHAPTER 6

Portfolio Optimization

The fund should yield a high return as well as limit the risk under CVaR. CVaR is used as risk measure since the distribution of the structured products is skewed and exhibits kurtosis, thus risk measures such as standard deviation are inappropriate (as described earlier). In this chapter, the fund is allowed to rebalance the portfolio every quarter, the investment horizon is three years, the fund is allowed to allocate a maximum of 30% in each asset, CVaR will be considered for $\alpha = 0.95$. All structured products in this chapter are 100% capital guaranteed and follows the dynamics given in Chapter 3, thus the structured products have a time to maturity of three years at issuance, the bond's notional amount equals the price of the underlying, the strike as well as the price of the product itself at issuance.

Only proportional transaction costs are considered since fixed transaction costs transforms the linear CVaR optimization problem into a mixed integer linear problem, which increases the complexity of the problem a lot.

The chapter covers three different types of portfolio choices: fixed portfolio weights, rolling portfolio weights and dynamic portfolio weights. The different alternatives are evaluated in relation to a benchmark fund and the most beneficial fund construction is investigated. The results indicate that the most beneficial setup is the dynamic portfolio weights scheme, where a modified Korn and Zeytun framework is proposed. The study indicates that the most imperative factor that affects the return is in fact not the portfolio selection itself but instead the transaction costs.

The first portfolio allocation strategy/scheme that is described is the fixed portfolio weight strategy.

6.1 Fixed portfolio weights

Fixed portfolio weights is the setup where product one always is the product closest to maturity and product twelve the product with the longest time to maturity (since it exists only twelve products), thus it is similar to the portfolios used in Section 4.3, where there is always twelve capital guaranteed structured products available. The portfolio will be allocated using the CVaR algorithm described in Chapter 2 with the modification that no binary variables will be used (no fixed transaction costs), since they turn the problem into a mixed integer linear problem, which is not desirable. The return is used as target function since this problem is better defined than having the return as a constraint (needs less iterations via the simplex method). Since the fixed weights concept is used, which is a relative theoretical concept, it is interesting to investigate which products are more optimal than others to allocate in, when considering tenors. Thus the study disregards transaction costs initially to investigate the optimal choice in a perfect market setting. The result should be seen as a guideline for optimal portfolio choice, i.e. which types of products that are better to invest in than others, regarding the tenor.

The results presented in Section 4.3 are in line with the results found in this section. The products closest to maturity are associated with the highest risk, i.e. CVaR, where it was previously found that the products closest to maturity have the worst outcomes of equivalent option portfolios. Investing in the products closest to maturity imposes an "all or nothing situation" for the options, while retrieving a more smooth return distribution for newly issued products as proven earlier (even though the interest rate was ignored in the previous chapters). Since similar results are retrieved when including the effect of interest rate changes (as seen in Table 6.1) the assumption that the risk of movements in the underlying is the most prevalent risk factor can be confirmed. Notable is that the expected return is higher for products with higher risk, thus it seems to be adequate to use a CVaR optimization scheme to attain a smooth efficient frontier. The analysis is performed via 100,000 six-year trajectory samples based on the model described in Chapter 5. The return over the last three years of each sample trajectory is considered, since the first three years are needed to generate relevant price data as the price process is path dependent. The last three years observed price data on the market is not used as start data since the analysis should be conducted independent of the current market. Instead different start levels for the yield curve will be considered, i.e. the yield curve that is in the beginning of the six-year period. In this section, the yield curve as of 15th November 2010, representing a low yield case will be used. In the appendix the case with an arbitrary yield curve is disclosed. The results are in general the same, mostly imposing just higher expected return and risk.

It is important to understand that the result may not look exactly the same for all markets. It is also important to notice that a more general concept is investigated in this section since the previous three-year trajectories are simulated. Thus the efficient frontier would look different if an investor would want to invest in structured products today since the current yield curve, index level, strikes and participation rates are known. Hence the results in this chapter are mainly general results, i.e. in the long run the efficient frontiers etc. would look like this, not necessary all the time.

Thus the hypothesis that was presented in Section 4.3 was correct, the highest risk is associated with products that are closest to maturity and the newly issued products have a lower risk as disclosed in Table 6.1. Thus it is reasonable to assume that the investment strategy proposed earlier in Chapter 4 is still adequate.

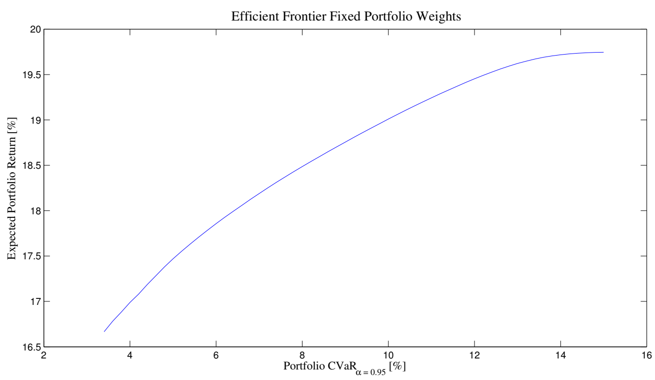


Figure 6.1: Efficient frontier for the optimization choice using the fixed portfolio weights setting, $\alpha = 0.95$.

The CVaR algorithm described in Chapter 2 is applied to produce a robust efficient frontier; the result is disclosed in Figure 6.1. Transaction costs have been disregarded and the restriction that it is only allowed to invest a maximum of 30% in each asset is in place. Since the optimization is computer intensive, it is not possible to use a lot more than approximately 1,000 scenarios for the efficient frontier (in a timely manner on a regular PC). Thus to attain robust results 100 efficient frontiers are simulated for different risk levels (CVaR) as constraint and the average of these weights are calculated to attain robust portfolio weights. Thus yielding a more robust efficient frontier, which reflects the intrinsic frontier in a better manner.

Figure 6.1 indicates that the expected return increases as the risk increases, and the frontier is quite similar the frontier for a classic Markowitz optimization problem with standard deviation as risk measure.

Product Number, $CVaR_{0.95}$ [%]											
1	2	3	4	5	6	7	8	9	10	11	12
18.46	14.88	12.41	10.52	9.33	8.04	7.08	6.40	4.68	4.23	3.92	3.74
Product Number, Expected return [%]											
1	2	3	4	5	6	7	8	9	10	11	12
20.14	19.71	19.63	19.07	18.12	17.71	17.30	16.93	16.48	16.14	15.81	15.51

Table 6.1: The table describes the CVaR at the 95% confidence level and expected return for the different products over the three-year period using the fixed portfolio weights setting.

Next the sensitivity of each scenario's portfolio weights is analyzed. It is important to analyze the standard deviation of the portfolio weights, since it discloses how robust the weights are, and if it is necessary to use multiple samples to attain a representative solution to the optimization problem. 100 samples of 1,000 scenarios for three different levels of CVaR are used, one low risk, one medium risk and one high-risk level. The portfolio weights are approximately equally robust for the low risk scenarios and for high-risk scenarios, with slightly more robust portfolio weights for the high-risk level, as Table 6.2 indicates.

	Weights, standard deviation [%]											
$CVaR_{0.95}$	1	2	3	4	5	6	7	8	9	10	11	12
4%	0.80	0.95	8.69	9.7	0.04	5.25	8.82	10.67	9.07	11.03	11.94	10.06
8%	6.42	7.37	5.50	6.91	3.73	8.01	9.00	11.05	8.01	7.69	6.37	4.95
12%	9.75	9.94	1.80	5.73	2.62	3.72	2.47	3.92	1.57	1.11	0.39	0.50

Table 6.2: The table describes the robustness (standard deviation) of the weights for a portfolio allocation using 1,000 scenarios and for three different CVaR levels using the fixed portfolio weights setting.

The sensitivity of the portfolio weights is higher with less sample scenarios. Also it is more adequate to use CVaR as a constraint and maximizing the return as the target, than the other way around, since it does not only require less iterations but also generates more stable portfolio weights. Table 6.2 shows that there exists significant standard deviation for the portfolio weights. So what does this imply? Since each sample has different paths up till the start of investment a high standard deviation implies that it is important to not use a generalized allocation scheme, since the weights shift much depending on the scenarios.

From the distribution and CVaR results for the individual products it is reasonable to assume that lower levels of CVaR allocate most of the capital to the products with the longest time to maturity (which is the case). But as the maximum CVaR allowed increases the portfolio will allocate in shorter products to yield a higher expected return (since the expected return increases almost linear over the twelve assets). Thus the frontier portfolios (weights), disclosed in Figure 6.2, are attained.

Hence depending on the desired maximum risk level the portfolio weights should be chosen according to Figure 6.2 and the portfolio would receive the expected return according to Figure 6.1.

Thus following the path of capital guarantee; a fund should allocate mainly in the last four - five products to minimize the CVaR for the portfolio. It should also, depending on its risk tolerance (and transaction costs), sell of assets after a number of quarters elapsed. The risk level and expected return will be very dependent on the transaction costs, which is important to notice. Consider transaction costs of 2% (2% in proportional transaction costs each time the fund sells or buys an asset) and a portfolio which holds equal weights in the longest tenors $w_9 = w_{10} = w_{11} = w_{12}$. The portfolio has without transaction costs a $CVaR_{0.95}$ of 3.62% and

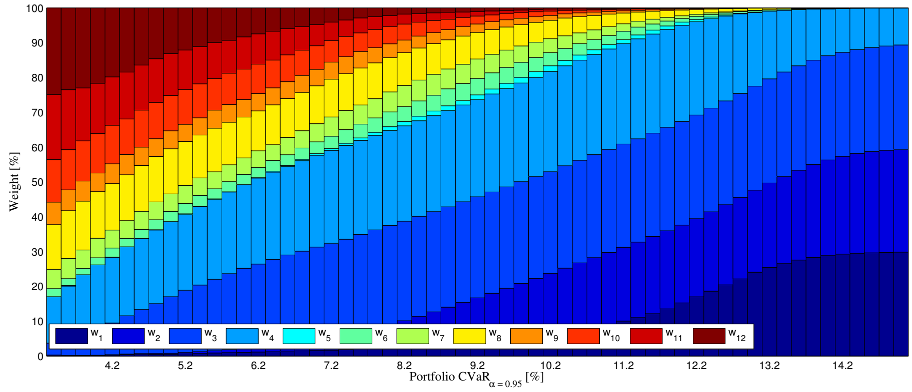


Figure 6.2: Portfolio weights for the efficient frontier using the fixed portfolio weights setting, $\alpha = 0.95$.

an expected return of 15.98% over three years. Including the transaction costs it will have a $\mathrm{CVaR}_{0.95}$ of 9.29% and an expected return of 9.16%. On the other hand a portfolio holding equal amounts invested in products twelve throughout seven has a $\mathrm{CVaR}_{0.95}$ of 7.52% and an expected return of 11.75% including the transaction cost. Thus the problem must be modified to find a solution including transaction costs.

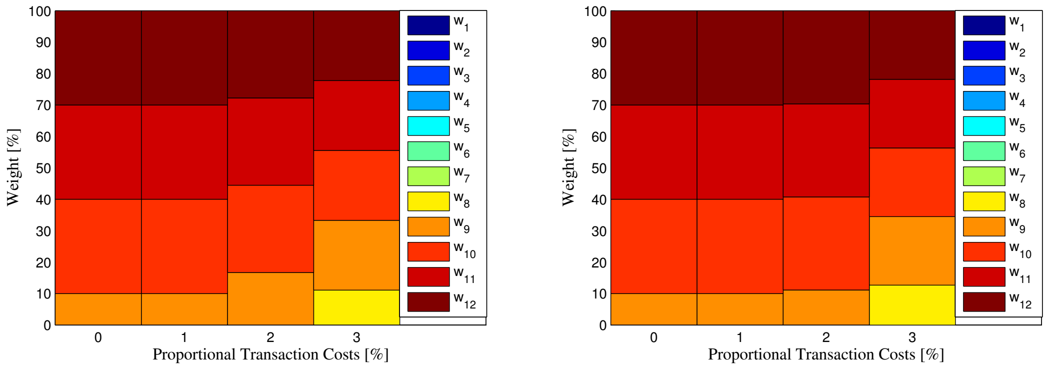
To solve how to allocate in an optimal way, including transaction costs, an alternative CVaR algorithm with proportional transaction costs is introduced, which can approximate the induced transaction costs of a portfolio with fixed weights. The weight held in asset 12 one quarter is approximately held in asset 11 the next quarter and so on. Thus the fund can, by solving the optimization problem below, attain portfolio weights that are desirable when taking in consideration transaction costs.

$$\max_{\mathbf{w},\mathbf{w}^+,\mathbf{w}^-,\mathbf{z},\beta} \frac{1}{S}\sum_{k=1}^{S}\left(1+R_{T,k}^{w}\right),$$

such that:

$$\begin{aligned}
&\beta + \frac{1}{S(1-\alpha)}\sum_{k=1}^{S} z_k \leq C, && \\
&1+R_{T,k}^{w} = \left(1+R_{T,k}^{1}\right)w_1 + \ldots + \left(1+R_{T,k}^{N}\right)w_N, && k=1,\ldots,S \\
&R_{T,k}^{w} + \beta + z_k \geq 0, && k=1,\ldots,S \\
&z_k \geq 0, && k=1,\ldots,S \\
&\sum_{i=1}^{N} w_i + \sum_{i=1}^{N-1}\left(TC_i^+ w_i^+ + TC_i^- w_i^-\right) + TC_N^+ w_N = 1, && \\
&w_i = w_{i+1} + w_i^+ - w_i^-, && i=1,\ldots,N-1 \\
&w_N = w_1 + w_N^+ - w_N^- && \\
&w_i \geq 0, \;\; w_i^{\pm} \geq 0, && i=1,\ldots,N
\end{aligned}$$

The optimal portfolio weights are investigated for the cases of no transaction costs and three different levels of transaction costs, when minimizing the CVaR, given a positive return. The optimal weights are investigated in a setting of a low initial yield curve and the setting of an arbitrary, random start yield curve, the results are disclosed in Figure 6.3. It is in general most optimal to allocate in the first four products, and as the transaction costs rises it is less optimal to rebalance the portfolio often, thus leading to that the fund allocates to more products.



(a) Low yield **(b)** Generalized yield

Figure 6.3: Optimal portfolio weights for the fixed portfolio weight scheme for different proportional transaction costs, minimizing $\mathrm{CVaR}_{0.95}$ given the requirement of a positive return, according to the algorithm described above.

6.2 Rolling portfolio weights

Rolling portfolio weights is the setup where product one is a product that has one quarter left to maturity and is rolled over at maturity to a new equivalent capital guaranteed structured product with three years to maturity. Product two has two quarters to maturity before it is rolled over to a new capital guaranteed structured product, and so on. Thus this rolling product is a perpetual product (similar to the bond roll) where transaction costs occur when the investor rolls over to the new product (thus each product will over a three-year period have equal transaction costs in percentage). A CVaR optimization algorithm, the same as in the beginning of the previous section, is performed for the fixed portfolio weights. The transaction costs will not impact the portfolio choice itself since all products are associated with the same degree of transaction costs (when the roll is performed), thus transaction costs are not considered in this section.

The CVaR algorithm with CVaR as constraint and maximizing the expected return as the target function is performed 100 times (for robustness) to produce a robust efficient frontier; the result is disclosed in Figure 6.4.

Figure 6.4 shows that the expected return increases as the risk increases, but only in a very small extent. The expected return is almost similar for all risk levels. It is preferable to invest in product twelve since its time to maturity for the first roll equals the investment horizon, thus providing intrinsic capital guarantee. The risk associated with the products is less for products that are close to roll over than products in the middle spectrum. The reason why the return of the efficient frontier is almost equal for all risk levels is that almost all products have the same expected return, since they are over a three-year horizon almost equivalent products in return sense.

Table 6.3 indicates that the products that are closer to their first roll over are more risky than products that have a long time to their first roll. The different risk levels for the products are derived from the participation rate. A higher participation rate (due to often a higher yield) imposes a higher risk. The probability of attaining a higher participation rate is higher for products that roll over later. On the other hand the risk associated with options is higher when the time to maturity is smaller. Thus the products that have the longest time until their roll will

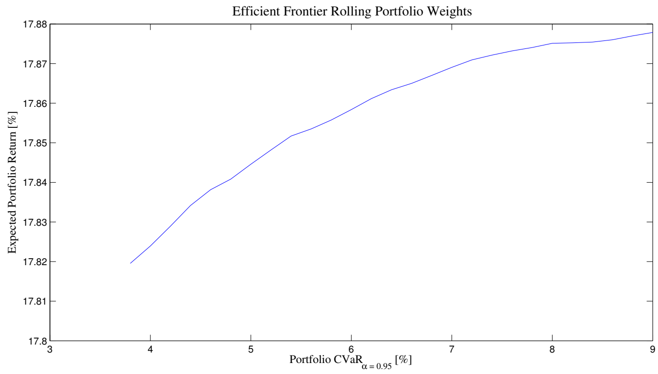


Figure 6.4: Efficient frontier for the optimization choice using the rolling portfolio weights setting, $\alpha = 0.95$.

risk receiving higher participation rates in the end, but they will only go through a few quarters (the ones with the least risk) until the investment horizon ends with this high participation rate. Also these products will benefit of the previous roll before the high participation rate that with a high probability had a low participation rate. Thus the average participation rate over the investment horizon is higher for products with 1-9 quarters to the next roll, imposing a higher risk. The risk levels and expected returns for the twelve different rolls are given in Table 6.3.

Product (months until next roll), $CVaR_{0.95}$ [%]											
1	2	3	4	5	6	7	8	9	10	11	12
9.28	10.57	11.43	10.63	10.68	10.31	9.73	9.12	8.15	6.86	5.06	0.00
Product (months until next roll), Expected return [%]											
1	2	3	4	5	6	7	8	9	10	11	12
18.04	18.01	17.98	17.57	17.57	17.54	17.58	17.60	17.64	17.69	17.72	17.81

Table 6.3: The table describes the CVaR at the 95% confidence level and expected return for the different products over the three-year period for the different rolls.

The robustness of the weights is also studied through investigating the standard deviations of the weights as previously, the result is presented in Table 6.4.

	Weights, standard deviation [%]											
$CVaR_{0.95}$	1	2	3	4	5	6	7	8	9	10	11	12
4%	9.35	11.65	7.86	5.89	4.32	4.83	2.98	2.78	6.59	8.34	10.44	4.72
6%	12.76	12.19	10.43	7.94	7.89	7.22	5.86	5.05	12.66	13.40	11.89	12.30
8%	12.25	11.83	9.89	7.95	8.71	7.77	8.69	5.62	13.87	13.73	12.03	12.34

Table 6.4: The table describes the robustness (standard deviation) of the weights for a portfolio allocation using 1,000 scenarios and for three different CVaR levels using the rolling portfolio weights setting.

The standard deviation result is similar to the result in the previous section, which indicates that it is important as a portfolio manager to not use a generalized scheme for the allocation of the portfolio since the portfolio weights depends a lot on the previous path.

Investigating the portfolio weights for the robust efficient frontier shows that the portfolio should, to attain lower levels of CVaR, allocate to the products that have a lower individual CVaR, which is reasonable as disclosed in Figure 6.5.

The expected return will in the long run be very similar for all rolls, while the risk profile of

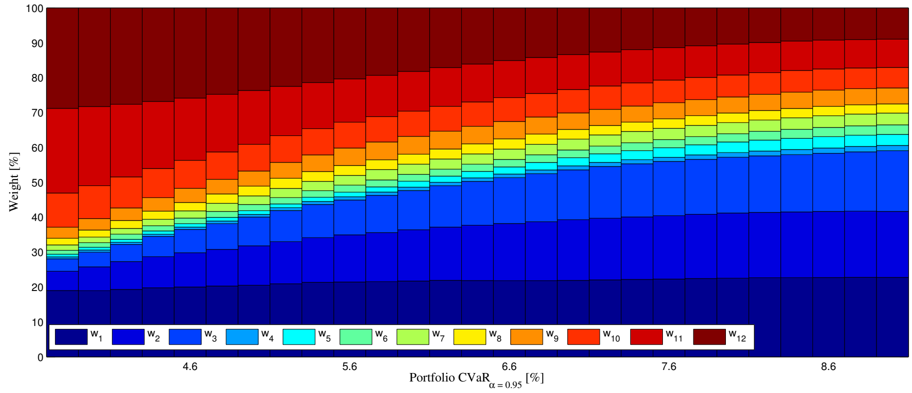


Figure 6.5: Portfolio weights for the efficient frontier using the rolling portfolio weights setting, $\alpha = 0.95$.

the products differs. Thus it is better to invest in products with a long time to the next roll. The high standard deviation of the portfolio weights implies that it is very important to evaluate the portfolio depending on the observed paths (yield curve, participation rate, index level etc) since they will impact both the expected return and the risk profile.

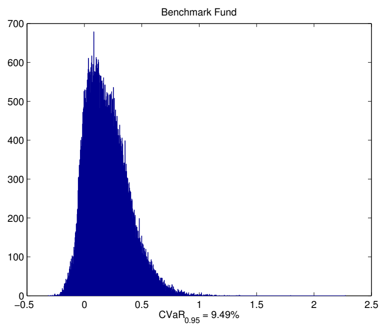
Thus when investing with the rolling weight scheme, it is better to allocate to the newly issued products (as stated in the previous chapters and sections). It is better to invest with rolling portfolio weights than the fixed weight scheme if the expected return it yields is satisfactory. Thus by requiring a maximum of 3.80% in $\text{CVaR}_{0.95}$ the portfolio will actually have an expected return of 17.82% over three years (excluding transaction costs, which would increase the CVaR and decrease the expected return), compared to the fixed weights which only yields an expected return of 16.88% for the same risk level, which also induces much higher transaction costs.

As described earlier in this paper, a portfolio's risk and return is only relevant in a certain context, thus it is imperative that the different portfolio constructions are compared with a benchmark fund.

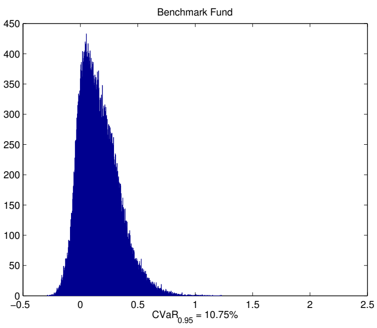
6.3 Benchmark fund

The analysis would be less relevant without a comparison with a relevant benchmark. Observed market data for the Swedish market has been used throughout the analysis in this chapter, thus the SPF must be benchmarked against alternatives at the Swedish market. Consider the benchmark fund described earlier, which places 30% of its capital in the index, plus minus 10%, and the rest of the capital in a basket of bonds with time to maturity between three months to three years (approximately equally weighted, rolling). This yields an expected return, over the three years investment horizon, of 19.81% including the expectation of 3% annual dividends in the index and a $\text{CVaR}_{0.95}$ of 9.49%. The distribution of the fund returns is given in Figure 6.6. When disregarding the expectation of the dividends the portfolio instead receives an expected return of 16.12% and a $\text{CVaR}_{0.95}$ of 10.75%.

Thus the distributions of the two, with or without dividend, are of course almost identical, the only thing that differs is approximately a parallel shift. The results indicate that it is hard for the SPF to compete with the benchmark fund including dividends (in terms of return) since the benchmark has a much higher expected return given the same risk level. Thus the SPF must compete with the benchmark portfolio given low risk. In Table 6.5 results for different benchmark funds are disclosed. Notable is that transaction costs have been disregarded, which would decrease the expected return of these products.



(a) Incl. dividend



(b) Excl. dividend

Figure 6.6: Histograms over the sampled three-year returns for the benchmark fund, $\varepsilon^{\text{bench}} = 0.3$.

	Incl. Dividends.		Excl. Dividends.	
Index allocation	Expected Return	$\text{CVaR}_{0.95}$	Expected Return	$\text{CVaR}_{0.95}$
10%	11.40	-1.66	10.16	-1.20
20%	15.43	3.25	13.05	4.38
30%	19.50	8.72	16.12	10.75
40%	23.88	14.35	19.16	17.13

Table 6.5: The table discloses the expected return and CVaR for different levels of index weights over the three-year period for a benchmark fund.

Table 6.1, 6.3 and 6.5 indicate that the SPF can compete initially (without transaction costs) in the low risk spectrum, where the rolling weights portfolio generates higher expected returns relative to the risk, which is also the case for the fixed portfolio weights. Thus these types of funds would be able to compete with the benchmark fund in the low risk spectrum. This is also in line with the idea of the SPF, which is that it should be a low risk fund, and the results show that it is only possible for the SPF to compete against other low risk funds to yield a higher expected return.

The next section will investigate the last allocation scheme, the dynamic portfolio weights allocation scheme.

6.4 Dynamic portfolio weights

The allocation problem is quite complex for the SPF. It is assumed that product one is the product with one quarter until maturity, product two is the product with two quarters until maturity and so on as previously, hence there exists twelve products (the products are also standard). The fund manager has the possibility today to invest in twelve structured products, where only one of the products does not mature before the end of the investment horizon. Thus when considering to allocate the portfolio over a three-year period the fund manager needs to take in count how the payoff of the products that mature before the end of the investment horizon should be reinvested. It is necessary to consider the full three-year time horizon since a portfolio allocated on a three-month basis would be suboptimal and induce high transaction costs. Thus it is desirable to investigate the portfolio choice for the whole three-year time period to avoid excessive transaction costs (since this is the time to maturity of the longest product that exists). Korn and Zeytun (2009, [27]) propose a method to solve optimal investment problems with structured products under CVaR constraints, where the investor's investment horizon is longer than the time to maturity. Korn and Zeytun proposes the following modified multiple period CVaR allocation algorithm,

$$\max_{\mathbf{w},\nu,\mathbf{z},\beta} \frac{1}{S} \sum_{k=1}^{S} R_{T,k}^{w,\nu},$$

such that:

$$\begin{aligned}
&R_{T,k}^{w,\nu} = w_1 R_{T,k}^{1,\nu} + ... + w_{N-1} R_{T,k}^{N-1,\nu} + w_N R_{T,k}^{N}, && k = 1, ..., S \\
&R_{T,k}^{j,\nu} = \left(1 + \Pi_k^j\right) \left[\nu_{j+1}^j \left(1 + r_{T-j,k}^{j+1}\right) + ... + \nu_N^j \left(1 + r_{T-j,k}^{N}\right)\right] - 1, && j = 1, ..., N-1 \\
&R_{T,k}^{w,\nu} + \beta + z_k \geq 0, && k = 1, ..., S \\
&\beta + \frac{1}{S(1-\alpha)} \sum_{k=1}^{S} z_k \leq C, && \\
&z_k \geq 0, && k = 1, ..., S \\
&w_1 + ... w_N = 1, && \\
&w_i \geq 0, && i = 1, ..., N \\
&\nu^j \geq 0, && j = 1, ..., N-1
\end{aligned}$$

where ν_l^j is the percentage of the product j's payoff that is allocated to asset l after maturity, $r_{T-j,k}^l$ is the return of asset l for the remaining time period (time until the end of the investment horizon from time j), Π_k^j is the return of asset j from the start of the investment period until its maturity and k corresponds to the scenario. Thus each product that matures is reinvested using the weights ν_l^j. Note that the last product does not mature prior to the investment horizon thus $R_{T,k}^N = \Pi_k^N$. The optimization problem above is without transaction costs, the proportional transaction costs constraints and budget constraints can be added in the exact same way as for the regular CVaR optimization problem.

The problem with this setting is that it is no longer a linear optimization problem. Thus it cannot be solved with only a simplex procedure. It must instead be solved by creating optimization loops, where each intermediate rebalancing point creates a new loop. Eleven outer optimization loops, with twelve possible products to allocate within, are needed since the portfolio has eleven intermediate rebalancing points. It is not feasible to allocate with this sophisticated optimization algorithm in this setting since the time to attain a solution would be months, not minutes or hours, and for robust weights it would take years to solve the optimization problem. Also, the optimization problem is not always convex, thus resulting in different solutions, given different start values in the optimization.

6.4.1 Modifying the Korn and Zeytun framework

The algorithm described by Korn and Zeytun is adequate for a limited number of intermediate time periods, thus relative few assets, since the iterations and as a consequence the computing time grows exponentially. Concluded: it is not possible to use this algorithm to solve the allocation problem in this setting.

The optimization problem is modified by setting constraints on ν_l^j, such that the problem is easier to solve. Let ν_l^j be the weights that minimize the CVaR for the particular product j including the reinvestment, thus reducing the total problem to only solving twelve linear optimization problems.

Thus the modified algorithm is as follows,

- **i.** Sample N trajectories from the model described in Chapter 5 to attain returns for the structured products.
- **ii.** Solve the CVaR optimization problem with minimizing CVaR as target and the constraint of a positive return for the period between $N-1$ to N to attain ν^{N-1}, $r^{N-1}_{T-(N-2),k}$ and complete the full return trajectory for product $N-1$.
- **iii.** Repeat **ii** until the full price trajectories for all products are known, thus between $N-2$ to N, $N-3$ to N and so on (in this case eleven optimization problems).
- **iv.** Solve the well-defined CVaR optimization problem between 1 and N to attain initial weights $\mathbf{w}$ (since all scenario trajectories are known).
- **v.** Repeat steps **i - iv** 100 times to attain robust portfolio weights.

The results attained through this allocation scheme are much more beneficial than the previous two allocation schemes. Thus it is this allocation scheme that should be used in relation to the benchmark portfolio, since it can generate higher expected returns to the same risk level. Without taking in count transaction costs the robust efficient frontier in Figure 6.7 is attained.

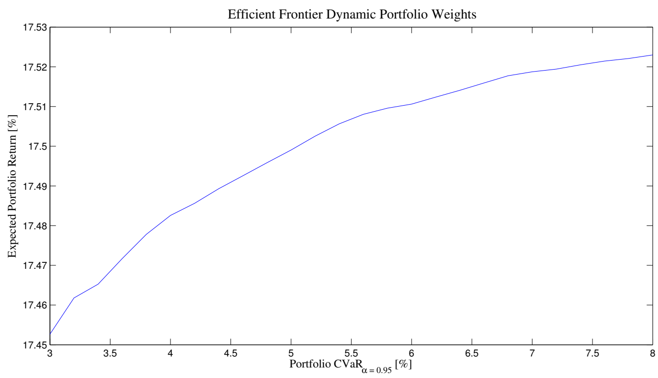


Figure 6.7: Efficient frontier for the optimization choice using the dynamic portfolio weights and the modified Korn and Zeytun framework, $\alpha = 0.95$.

The expected return does not increase a lot as the portfolio increases its risk tolerance, most structured products in this setting have similar expected returns. Most of the reinvested capital will be placed in the newly issued product, since the products with the longest time to maturity have the lowest risk. Thus the solution is in practice very similar to the one of rolling portfolio weights. Notable is that this is for the low yield setting and the solution to the problem is not always the same, thus the solution will in many cases be similar to the one of rolling portfolio weights, but since the returns are path dependent, the risk profile will be different, depending on the previous price data. Thus the big difference between these two models is that the modified Korn and Zeytun framework is dynamic and minimizes the risk of the reinvested capital and allocates based on that knowledge. Hence even though the solution in this case is extremely similar to the rolling weights problem, it is not the same (it generates lower risk, but also lower return). The modified Korn and Zeytun algorithm allocates even more in the newly issued products, especially for the lower risk segments. The expected return and CVaR for this setup are disclosed in Table 6.6.

The standard deviation of the weights are analyzed in the same way as in previous sections and disclosed in Table 6.7.

The standard deviation result indicates that it is important to use the scheme actively, and not just use a static allocation scheme, to choose the portfolio weights depending on the previous

Product, months until maturity, $CVaR_{0.95}$ [%]											
1	2	3	4	5	6	7	8	9	10	11	12
8.80	10.04	10.96	10.26	10.24	9.87	9.48	8.74	7.74	6.38	4.68	0.00
Product, months until maturity, Expected return [%]											
1	2	3	4	5	6	7	8	9	10	11	12
17.64	17.62	17.66	17.27	17.24	17.24	17.20	17.25	17.31	17.36	17.39	17.45

Table 6.6: The table describes the CVaR at the 95% confidence level and expected return for the different products over the three-year period using the dynamic portfolio weights and the modified Korn and Zeytun framework .

	Weights, standard deviation [%]											
$CVaR_{0.95}$	1	2	3	4	5	6	7	8	9	10	11	12
4%	11.24	9.20	9.34	1.33	3.28	3.98	0.68	4.60	8.46	12.36	10.05	4.31
6%	12.81	13.06	12.97	2.37	6.22	7.44	4.06	6.66	9.67	12.49	13.74	12.52
8%	12.23	13.54	13.87	6.42	7.31	8.22	4.71	7.28	9.88	12.42	12.61	12.85

Table 6.7: The table describes the robustness (standard deviation) of the weights for a portfolio allocation using 1,000 scenarios and for three different CVaR levels using the dynamic portfolio weights and the modified Korn and Zeytun framework.

paths.

When analyzing the portfolio weights for the different risk levels it is easy to notice that the products with a long time to maturity are very attractive, since they both have high expected return and low risk. Also the products that are close to maturity have a higher expected return and a lower risk than the products in the middle spectrum. Therefor the algorithm allocates mainly in products 1-3 and 10-12, as disclosed in Figure 6.8. Notable is that the algorithm is best for low risk strategies, since it minimizes the risk of the reinvested capital, which is the goal of this thesis.

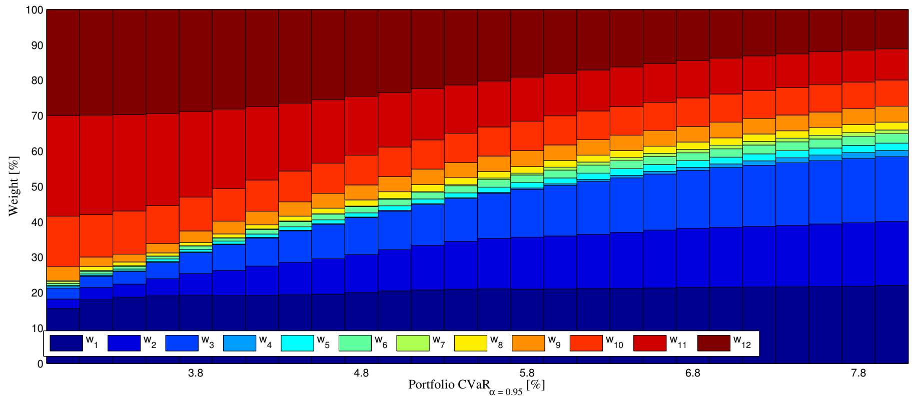


Figure 6.8: Portfolio weights for the efficient frontier using the dynamic portfolio weights and the modified Korn and Zeytun framework, $\alpha = 0.95$.

6.4.2 Dependence of the initial yield curve

It is important to note that the risk level and level of expected return are significantly dependent on the initial yield curve. In this chapter the yield curve from middle of November 2010 on the Swedish market is used as an initial yield curve of the six-year period. Thus the results are dependent on this initial yield curve. In Appendix E the corresponding figures and tables are disclosed where an arbitrary start yield curve is used instead to attain unbiased results. Notable is that the results are very similar to each other, the thing that differs the most is the level of expected return and CVaR. The optimal portfolio weights are very similar to each other for both low-risk and high-risk strategies.

The fixed portfolio weights scheme gains a much higher expected return given the same risk levels for the arbitrary yield curve case. The same patterns of weights for different risk levels are shown for the arbitrary yield setting, thus a low risk strategy in one setting can be expected to also be a low risk strategy in the other setting. Notable is also that by inspecting the efficient frontier for the fixed portfolio weights, it is possible to see that the low risk strategy of fixed portfolio weights cannot compete with the low risk strategy of the rolling portfolio weights, but on the other hand there are portfolios which impose higher expected returns than for the rolling and dynamic weights in the fixed portfolio weight scheme. Thus using a strategy where the fund always buys structured products close to maturity is the only way to trigger a higher expected return in the SPF, which on the other hand increases the risk dramatically. Notable is also that the fixed portfolio weights strategy will in most cases impose a lot more transaction costs than the other strategies, thus reducing its attractiveness.

Almost the exact same risk levels are attained in the rolling portfolio weights setting to a much higher expected return in the arbitrary yield case (compared to the low yield case). This can be derived from the fact that increasing the limit for CVaR allowed does not necessary increase the expected return. Since most products have approximately equal expected return the only difference is that the arbitrary yield curve case has a higher expected return and a slightly different risk level.

For the benchmark fund the expectations on the dividends in the future are really important, and it is important for the portfolio manager to understand the impact of having different expectations for the dividends. The results are a little bit different when comparing the benchmark fund with a low yield curve against an arbitrary yield curve, than for the SPF. The expected return increases (due to the higher yield) but the risk of the index does not increase, thus actually reducing the risk and increasing the expected return.

Concluded: the SPF is more beneficial for investors in a market state of a low yield, since it imposes less risk for the investor in the SPF (lower participation rate) at the same time as competing mixed funds actually have increased risk.

By comparing the alternatives of rolling and dynamic portfolio weights against investing in a competing benchmark portfolio it is possible to notice that there are levels where it is more beneficial to invest in the SPF. This is what the thesis is looking for, if the SPF can generate a lower risk for the same level of expected return as existing funds, it is definitely a good choice to invest in the SPF.

6.4.3 Observed paths

There exists in general an optimal choice for the portfolio weights given that the portfolio is resampled. On the other hand the standard deviation of the portfolio weights was quite high which indicates that it is important to choose the portfolio weights actively. A high standard deviation for the portfolio weights indicates that the optimal portfolio weights are depending on the previous paths. Therefor the optimal portfolio choice with an investment horizon that starts the 16th of November 2010 in the Swedish market will now be studied. Figure 6.9 discloses the closing prices of OMXS30 during the three-year time period prior to the day of investment and the robust efficient frontier for the investment choice according to the modified Korn and Zeytun algorithm.

The expected return is significantly higher for this scenario than for the generalized case described above. Also this scenario has a much lower minimum risk level for the CVaR.

It is interesting to investigate the standard deviation for the portfolio weights given the previous price paths. By analyzing the standard deviation of the portfolio weights it is possible

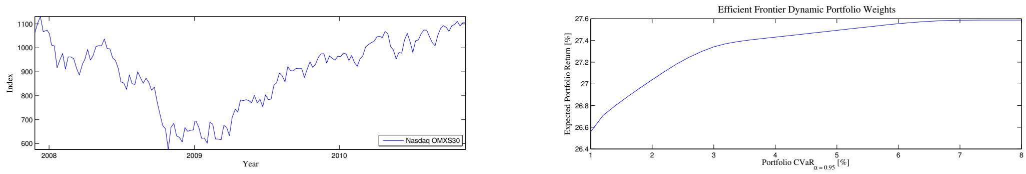


Figure 6.9: Observed price paths of OMXS30 during the three years leading up to 16th of November 2010 and the efficient frontier using the dynamic portfolio weights and the modified Korn and Zeytun framework for 16th of November 2010.

to assess if 1,000 scenarios is enough to determine the optimal portfolio weights, the result is disclosed in Table 6.8.

	Weights, standard deviation [%]											
$\text{CVaR}_{0.95}$	1	2	3	4	5	6	7	8	9	10	11	12
1%	1.82	1.25	0.10	0.00	0.00	0.00	0.00	0.52	1.60	0.34	0.00	0.00
4%	0.00	8.96	5.70	0.00	0.00	0.00	0.00	0.00	0.00	3.94	0.00	0.00
7%	0.00	4.57	2.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00	5.82	0.00

Table 6.8: The table describes the robustness (standard deviation) of the weights for the portfolio allocation using 1,000 scenarios and for three different CVaR levels using the dynamic portfolio weights and the modified Korn and Zeytun framework for 16th of November 2010.

The result indicates that the specified setting (known paths) has a much lower standard deviation for the portfolio weights than the generalized investment scheme. The standard deviation is higher for the scenarios with higher risk, but the products that have standard deviation are the products that the portfolio allocates in. Thus it is the same assets that are more or less most optimal in each optimization, but the amount allocated in each asset differs slightly. Thus the result indicates that it is reasonable to use 1,000 simulations to attain relative stable portfolio weights. On the other hand the quality of the result is still enhanced when resampling the portfolio weights (as most often).

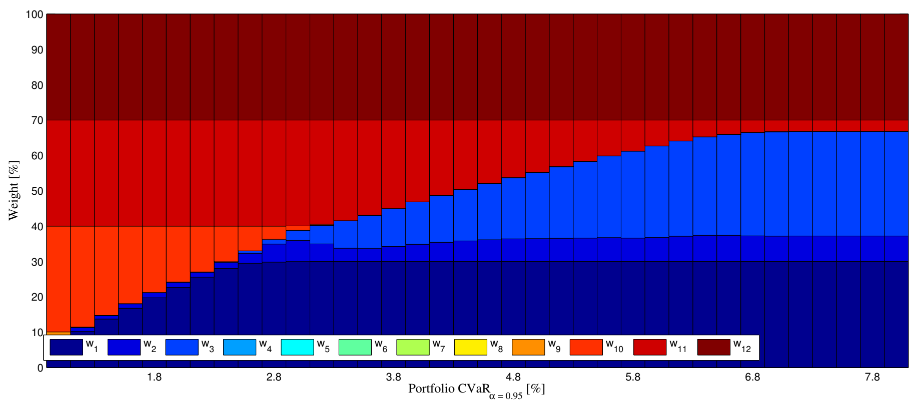


Figure 6.10: Portfolio weights for the efficient frontier using the dynamic portfolio weights and the modified Korn and Zeytun framework for 16th of November 2010, $\alpha = 0.95$.

The portfolio weights for the efficient frontier for the optimal investment choice as of the 16th November 2010 are disclosed in Figure 6.10. The weights are quite similar as in the generalized case, but they differ slightly (adjusted for the particular scenario). Thus it is imperative to adjust the portfolio optimization depending on the previous observed paths.

6.5 Transaction costs

This section focuses on the dynamic portfolio weights setting and how transaction costs impact the risk and return levels in comparison to the benchmark fund. The SPF is compared with a benchmark fund, which includes dividends from the index (the most realistic benchmark), for different participation levels as well as for different levels of proportional transaction costs. The transaction costs occur when an asset is bought or sold, all assets have the same level of transaction costs, for buying as well as selling. Thus rebalancing a portfolio induces transaction costs, both when the asset is sold and when a new asset is bought. As seen in the previous section; the most desirable setting for the dynamic portfolio weights is to minimize the portfolio's risk, thus this will be conducted given the constraint of a positive return. Both the setting with a low start yield curve (three years prior to the initial investment horizon) and the setting with an arbitrary start yield curve are investigated. Fixed transaction costs are disregarded to avoid transforming the problem into a mixed integer linear problem. In Table 6.9 the expected return and $\mathrm{CVaR}_{0.95}$ for different benchmark funds and SPFs are disclosed.

Table 6.9 indicates that the SPF can compete against different benchmark funds during different market settings. In the low yield case the SPF provides, in general, lower risk to a higher expected return than the benchmark fund with an aim of 20% index investments (when assuming the same transaction costs level). Also the SPF is a lot less risky than the benchmark funds at the 30% and 40% levels.

In the arbitrary yield setting: the SPF has higher risk associated with it than the benchmark fund at 20% level (assuming equal transaction costs for the two funds). On the other hand the SPF has a higher expected return and lower risk than the 30% benchmark fund.

The risk of the SPF increases as the yield increases, since the participation rate increases. Notable is that the risk of the bond part decreases (a bond's delta is increasing in yield (higher yield, less sensitive to yield changes)), which implies that the benchmark portfolio's risk decreases as the yield increases. Thus a fund manager must be aware of these characteristics and how they impact the portfolio's expected return and risk. With a higher yield the bond portfolio gains a higher return, which explains the higher expected return for both the SPF and benchmark portfolio in the arbitrary yield setting.

The results indicate that transaction costs is one of the most important concepts for the fund manager to consider in its portfolio. Assume that the proportional transaction costs for structured products are approximately 3% and for the benchmark portfolio 1%, when studying the arbitrary yield case. Thus the SPF's expected return is 21.83%, has a $\mathrm{CVaR}_{0.95}$ of 7.36% and the benchmark fund which allocates 30% of its assets in the index has an expected return of 23.08%, a $\mathrm{CVaR}_{0.95}$ of 6.32% (as shown in Table 6.9). Thus the benchmark fund has both a higher expected return and a lower risk than the SPF, thus it would be an irrational decision to invest in the SPF.

It is not unreasonable that big funds (or investment banks) have the possibility to invest to as low transaction costs as 0.5% or even 0.25%. Thus the SPF must in the arbitrary yield case have as low as 1.5% in proportional transaction costs to be more attractive than the benchmark fund investing 30% of its capital in the index. It is important that the reader takes care and understands that these levels are dependent on the data that is used in the modeling, as well as the underlying assumptions. The same patterns are exhibited for a huge variety of data sets, but the expected returns, CVaR at different levels varies of course. The low yield setting reflects the setting of having a yield of approximately 1.5% p.a. in the three-month tenor and 2.3% p.a. for the three-year tenor.

Concluded: the whole idea of a structured products fund is very dependent on the transaction costs and it is only possible for the fund to compete with other funds, such as mixed funds, if

	SPF Dynamic Weights Low yield		SPF Dynamic Weights Arbitrary yield	
TC [%]	Expected Return [%]	$CVaR_{0.95}$ [%]	Expected Return [%]	$CVaR_{0.95}$ [%]
0.00	17.69	2.79	27.19	3.99
0.25	17.26	3.10	26.73	4.29
0.50	16.82	3.41	26.25	4.59
1.00	16.00	3.98	25.32	5.16
1.50	15.21	4.54	24.41	5.73
2.00	14.44	5.08	23.53	6.28
2.50	13.70	5.61	22.67	6.82
3.00	12.97	6.13	21.83	7.36
	Benchmark 20% Low yield		Benchmark 20% Arbitrary yield	
TC	Expected Return [%]	$CVaR_{0.95}$ [%]	Expected Return [%]	$CVaR_{0.95}$ [%]
0.00	15.43	3.25	20.81	0.13
0.25	15.11	3.39	20.51	0.23
0.50	14.78	3.70	20.18	0.58
1.00	14.22	4.18	19.58	1.14
1.50	13.65	4.70	18.99	1.69
2.00	13.05	5.09	18.36	2.09
2.50	12.56	5.56	17.82	2.62
3.00	11.94	6.10	17.26	3.07
	Benchmark 30% Low yield		Benchmark 30% Arbitrary yield	
TC	Expected Return [%]	$CVaR_{0.95}$ [%]	Expected Return [%]	$CVaR_{0.95}$ [%]
0.00	19.50	8.72	24.37	5.43
0.25	19.26	8.97	24.04	5.79
0.50	18.93	9.09	23.75	6.04
1.00	18.30	9.66	23.08	6.32
1.50	17.69	10.13	22.59	6.90
2.00	17.08	10.51	21.97	7.30
2.50	16.48	10.93	21.28	7.89
3.00	15.84	11.44	20.68	8.20
	Benchmark 40% Low yield		Benchmark 40% Arbitrary yield	
TC	Expected Return [%]	$CVaR_{0.95}$ [%]	Expected Return [%]	$CVaR_{0.95}$ [%]
0.00	23.88	14.35	28.18	11.46
0.25	23.56	14.41	27.85	11.64
0.50	23.20	14.57	27.40	12.00
1.00	22.58	15.25	26.78	12.41
1.50	21.87	15.59	26.16	12.75
2.00	21.32	16.02	25.56	13.17
2.50	20.64	16.44	24.91	13.61
3.00	20.07	16.78	24.19	14.15

Table 6.9: The table discloses the expected return and $CVaR_{0.95}$ for different levels of proportional transaction costs for the SPF and benchmark funds over a three-year investment horizon for both the low yield setting and the arbitrary yield setting.

the fund keeps down the transaction costs. If the SPF can have the same level of transaction costs as the benchmark funds it will be able to generate a higher expected return to a lower risk. Notable is that as the interest increases the ability for the SPF to compete with the benchmark portfolios decreases (in terms of low risk). Thus it is very important that the fund manager understands how its choices affect the return and risk of the fund.

The most beneficial investment setup to use is the modified Korn and Zeytun framework. The scheme usually implies that the fund allocates most of its capital in the three newest products (the products with the longest time to maturity) and almost all of the remaining capital in the product closest to maturity, since these can soon be reallocated in the newly issued products.

6.6 Summary

How can the results found in Chapter 4 and 6 be combined into an investment strategy? It has been established that the products with the longest time to maturity have, in general, the lowest CVaR (and expected return). Chapter 4 showed that it is important (if the fund manager wants to minimize the risk) to limit the level of options held in the portfolio. Thus when competing with a benchmark fund the fund manager could by allowing between 5-25 percent of the portfolio value to be allocated in options provide a better protection against downside risk. The investors are more or less guaranteed, by limiting the option portfolio in this manner, that a market crash will not erase more than 25% of the fund's portfolio value. Rebalancing in this manner implies that the fund gains capital protection on its previous gains, since it locks them in by investing in bonds. Obviously by minimizing the risk in this manner the potential return is also reduced.

Section 4.4 indicated that the fund should not overweight products that have been issued the last quarter during downturn markets and also avoid products close to maturity to attain protection to the worst-case outcomes. Minimizing the worst-case outcome imposes a slightly different portfolio than minimizing CVaR. Hence it is very important for the investor/portfolio manager to understand how the different decisions affect the risk profile. Investing only in products close to maturity would provide a higher expected return and impose a higher risk. This paper is not stating that any option is better than the other; it is only presenting the impact of different portfolio options and goals.

The other main investment alternative is to minimize CVaR with the whole investment universe available as done in this chapter, thus allocating most of the capital in the newly issued products since they have the best risk profile in CVaR sense. When comparing the results of the three constructions (fixed, rolling and dynamic) it is concluded that dynamic portfolio weights is the best choice in risk adjusted return sense (generates lower risk to the same return levels). The modified Korn and Zeytun framework will not overweight newly issued assets during downturn markets since it is minimizing the risk, thus rather investing in the products with a low ratio options:bonds. The downside with this strategy is that it may induce relative high transaction costs.

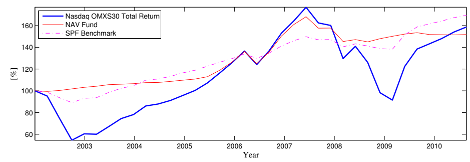
The constraint of holding a maximum of 25% value in options can be added, thus forcing the portfolio manager to reduce the worst-case outcome risk at the same time as the CVaR (but notable minimizing the CVaR will in most cases avoid allocating too much capital in the options).

Different portfolio managers will make different choices regarding which alternative is best, none is better than the other since they are constructed to restrict different events.

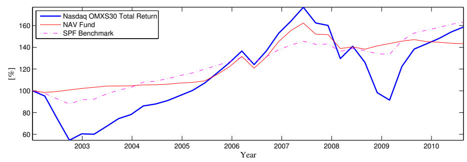
Transaction costs will be a central question no matter which investment strategy is chosen. The results are only relevant in relation to the investment environment, thus it is necessary that the SPF has the possibility of being more attractive than the benchmark fund, either at risk or return. Notable is that transaction costs are more important for the expected return and risk level than the portfolio choice itself. Thus given high transaction costs it is impossible for a SPF to compete with a benchmark fund with low transaction costs. Hence if the reader is considering to start a SPF it should focus on the transaction costs, it is impossible to outrun the mixed funds available at the market in the long run without limiting the transaction costs.

6.7 Backtesting

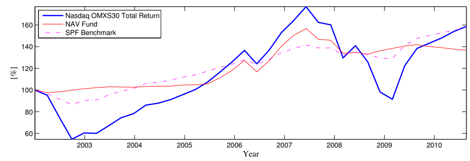
This section investigates how the modified Korn and Zeytun algorithm would have performed historically. The model is backtested from the beginning of 2002 to the middle of 2010 (data for OMXS30 total return is only available since 2002 from Bloomberg). In each time step the simulation is based on the last four year's data (log returns), to capture approximately a market cycle. Proportional transaction costs at the levels 0%, 1%, 2% and 3% are considered. The CVaR is minimized given the constraint of a positive return. Notable is that the scheme is only rebalancing matured capital, to avoid as much unnecessary transaction costs as possible. The results are compared with the benchmark fund described earlier (with 30% market participation), which is based on the total return version of OMXS30, and with the actual OMXS30 total return version. Figure 6.11 shows that the SPF had the desired characteristics during the financial turmoil during 2002 and during 2008-2009, imposing a high level of capital protection. Also the SPF did not actually yield any return during the latter part of 2009 and 2010, rather negative return. This is due to that the portfolio is rebalanced quite significantly during the last quarters (imposing negative returns due to transaction costs) and the volatility drops significantly, which implies that the value of the options decreases. The interest rate movements also impact the return during these quarters negative, the increase in the yield results in lower bond prices.



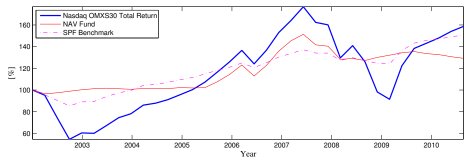
(a) TC = 0%, SPF total return 51.73%.



(b) TC = 1%, SPF total return 43.35%.



(c) TC = 2%, SPF total return 36.73%.



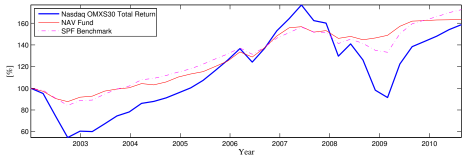
(d) TC = 3%, SPF total return 29.25%.

Figure 6.11: Backtesting results for the dynamic portfolio weights SPF against a benchmark mixed fund with 30% participation rate and OMXS30.

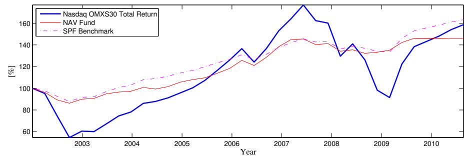
The transaction costs impact the return significantly (notable is that the benchmark fund suffers in approximately the same extent). The problem with investing using a structured products fund is that the structured products mature and the capital must be reinvested in new products, imposing new transaction costs. Thus if the fund instead invests in products that have a longer time to maturity it is possible to both reduce the risk (since products with a longer time to maturity have lower risk) and the transaction costs.

A backtest is also conducted for the fixed portfolio weights strategy using the algorithm from Section 6.1 and in Figure 6.3, which takes in count the transaction costs, thus allocating mostly in the newly issued products, the result is disclosed in Figure 6.12. This strategy generates a higher return than the dynamic portfolio weights strategy during the time period, which is more due to a coincidence than an optimal portfolio choice, since this is just one observation. The fixed portfolio weights setting generates higher transaction costs than the dynamic portfolio weights setting.

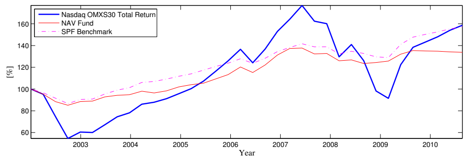
It is really important to not rebalance the whole portfolio every quarter, instead the fund should mainly rebalance capital that has matured, in a setting with high transaction costs. Rebalancing the whole portfolio every quarter would in the dynamic portfolio weights setting,



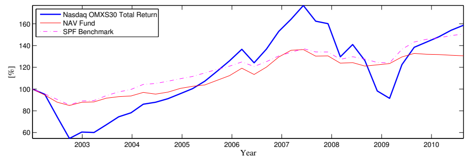
(a) TC = 0%, SPF total return 63.64%.



(b) TC = 1%, SPF total return 45.86%.



(c) TC = 2%, SPF total return 33.81%.



(d) TC = 3%, SPF total return 30.60%.

Figure 6.12: Backtesting results for the fixed portfolio weights SPF including proportional transaction costs against a benchmark mixed fund with 30% participation rate and OMXS30.

with 3% in transaction costs, yield 15% in return instead of approximately 29%. Thus the portfolio manager should assume a type of buy and hold strategy and mainly reallocate matured capital. If the fund assumes a buy and hold strategy the portfolio manager should be aware of that it still needs to rebalance the whole portfolio sometimes, e.g. when the option portfolio value exceeds its allowed boundary.

Thus it is recommended that, in a setting of high transaction costs, the portfolio manager uses the modified Korn and Zeytun framework to initially allocate its capital, and to reallocate the matured capital. The importance of using a more active portfolio strategy is higher when the possibility of diversifying in several structured products with different underlying exists.

CHAPTER 7

Conclusions

The thesis has studied the concept of a structured products fund (SPF) and how it should be constructed to be competitive in risk sense. The study shows that the concept should not be disregarded and that it is possible to construct a SPF that is competitive.

Transaction costs affect the expected return and risk level for the SPF in a high extent. Thus the study indicates that the most important factor for a portfolio manager to consider is the transaction costs. A SPF with high transaction costs will not have the possibility of beating the competing mixed funds at either risk or return. Thus the fund needs low transaction costs (in level with the competing funds) in order to not increase the risk and decrease the return in a too high extent.

The study focuses a lot on the issue of capital guarantee since it is desirable that the SPF has the same characteristics as a capital guaranteed product. The study indicates that it is not possible to attain a portfolio based on structured products that is capital guaranteed. The thesis instead studies the concept of alternative capital protection and finds that a fund can be capital protected at a certain level for a given risk measure, the thesis uses the risk measure CVaR. Thus a fund that has a $\text{CVaR}_{0.90}$ less than for example 0% can claim that the portfolio is capital protected on a 100% level with a confidence level of 90%. CVaR is used as a risk measure since it captures both skewness and kurtosis and is adequate to use in scenario based optimization. It is also common to describe a fund's risk characteristics via CVaR, thus making it an adequate risk measure to benchmark through.

The thesis finds that a portfolio manager has mainly two investment schemes to allocate the portfolio with, to minimize the downside risk, thus attaining as much capital protection as possible. The first investment scheme is a type of rolling investment scheme where the portfolio manager invests broadly in almost all the available products (preferably equal weights). The fund should sell of the products when they are close to maturity (a quarter or two left), since these assets exhibit a lot of risk especially if they are in the money. The capital that is gained when assets are sold should be reinvested in the newly issued products since they have in general the lowest risk associated with them. On the other hand as the index starts to decrease (during bear markets) the portfolio should underweight newly issued products (products with the longest time to maturity) since they are risky products in downturn markets. This scheme is used to both restrict the risk and the induced transaction costs. The benefit with this scheme is that it is easy to implement and that it serves as a reasonable investment choice during most time periods. On the other hand the scheme can induce a high-risk portfolio during unwanted market states, since it does not adjust to changes in the market. Thus it is impossible to control the expected return of the investment as well as the risk level. Hence the thesis instead proposes a more sophisticated allocation algorithm, which is referred to as the modified Korn and Zeytun framework.

The modified Korn and Zeytun framework is used to minimize the risk (CVaR) given the constraint of the expected return. This framework is very beneficial and creates low risk portfolios with high expected return. The framework that is presented in detail in Section 6.4 shows that it is possible to generate portfolios that have a higher expected return than competing funds to a lower risk, given that the transaction costs are low. The study is conducted with both low yield and arbitrary yield start trajectories. The results show that a SPF is most beneficial during times of low yield since the risk of the SPF increases a lot as the yield goes up.

The results indicate that a SPF can be a superb investment vehicle for investors searching for low risk alternatives with a limited downside. They also show that it is imperative that a portfolio manager understands how the portfolio allocation affects the characteristics of the fund, since it is a rather complex issue.

It is strongly recommended to use the modified Korn and Zeytun framework to allocate the portfolio with, for a SPF, since it generates well-balanced portfolios in risk and return sense. Allocating according to the modified Korn and Zeytun framework will in most cases lead to allocating in the best available portfolio (given the constraints) in every time step since it can be used for specific paths (adjust for the current market data).

The thesis finds that it is important, no matter which scheme is used, that the portfolio is not allowed to hold more than a certain degree of value attained in options (a limit), to avoid downside risk for its investors. A recommended level is that the SPF is allowed to allocate a maximum of 25% of its capital in options (within the structured products) with a target level of 15%. The portfolio should be rebalanced if this limit is exceeded, hence reducing the downside risk.

The thesis studies ELNs with a time to maturity of three years and the results can be generalized to products with a longer time to maturity at issuance (newly issued products, less risk). The thing that differs for a capital guaranteed ELN with a time to maturity of three years at issuance and one with a longer time to maturity (or shorter) is the ratio options:bonds. Thus a newly issued ELN with five years to maturity has a higher risk than a newly issued ELN with three years to maturity since it has a higher proportion options. Thus it is very important to use the boundaries of allowed proportion options:bonds especially when combining different types of ELNs. The big advantage by using the modified Korn and Zeytun framework is that it adjusts for these discrepancies and that the portfolio manager does not need to modify the allocation model if it modifies its investment universe.

Thus the main result is: yes it is possible to construct a fund based on structured products that is competitive and exhibits a type of capital protection (i.e. in CVaR sense). Only one underlying for the ELNs has been used and the results can be generalized without serious loss of generality to the multi-dimensional case. A Black-Litterman approach to the optimization problem is left for future studies where it, in combination with the multi-underlying case, can provide superior return due to subjective views on the market.

Bibliography

[1] Aas, K. (2006). The Generalised Hyperbolic Skew Student's t-distribution. *Journal of Financial Econometrics*, 4 (2), 275-309.

[2] Abdi, H., & Williams, L. J. (2010). Principal Component Analysis. *Wiley Interdisciplinary Reviews: Computational Statistics*, 2, 1-47.

[3] Acerbi, C. (2002). Spectral measures of risk: A coherent representation of subjective risk aversion. *Journal of Banking & Finance*, 26 (7), 1505-1518.

[4] Alexander, S., Coleman, T.F., & Li, Y. (2006). Minimizing CVaR and VaR for a portfolio of derivatives. *Journal of Banking & Finance*, 30 (2), 583-605.

[5] Bank For International Settlements. (2006). *Basel II: International Convergence of Capital Measurement and Capital Standards: A Revised Framework*. Basel: BIS

[6] Barber, J. R., & Copper, M. L. (2010). Principal component analysis of yield curve movements. *Journal of Economics and Finance*, 1-16, doi:10.1007/s12197-010-9142-y.

[7] Björk, T. (2009). *Arbitrage Theory in Continuous Time* (3rd ed.). Oxford: Oxford Finance.

[8] Black, F., & Litterman, R. (1992). Global Portfolio Optimization. *Financial Analysts Journal*, 48 (5), 28-43.

[9] Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81 (3), 637-654.

[10] Brooks, C., & Persand, G. (2000). *Value at Risk and Market Crashes*. Discussion Paper in Finance 2000-01. University of Reading.

[11] Browning, E., S. (2007, October 15). Exorcising Ghosts of Octobers Past, *Wall Street Journal*, p. C1.

[12] Chuan, T. (2008, August, 5). *Capital Protected is not Capital Guaranteed*. Financial Planning Central:
http://www.ifa-sg.com/capital-protected-is-not-capital-guaranteed/

[13] Deng, G., Li, G., Liu, S., Mallett, J., & McCann, C. (2009). *Structured Products In the Aftermath of Lehman Brothers*. Working Papers Securities Litigation & Consulting Group.

[14] Fisher, A. (1999). *Multivariate Stock Returns Around Extreme Events: A Reassessment of Economic Fundamentals and the 1987 Market Crash*. NYU Working Paper, No. FIN-99-071.

[15] Frazzini, A. (2006). The Disposition Effect and Underreaction to News. *The Journal of Finance*, 61 (4), 2017-2046.

[16] Glot, P. (2005). Relationship Between Implied Volatility Indexes and Stock Index Returns. *The Journal of Portfolio Management*, 31 (3), 92-100.

[17] Grinold, R., C. (1999). Mean-Variance and Scenario-Based Approaches to Portfolio Selection. *Journal of Portfolio Management*, 25 (2), 10-22.

[18] Hagan, P. S., Kumar, D., Lesniewski, A. S., & Woodward, D. E. (2002). Managing Smile Risk. *Wilmott Magazine September 2002*, 84-108.

[19] Hansen, P., & Lärfars, M. (2010). *Evaluating Long-Term Performance of Structured Products.* Stockholm: Kungliga Tekniska Högskolan.

[20] Heston, S. L. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *The Review of Financial Studies*, 6 (2), 327-343.

[21] HSBC. (2010, September 14). *Structured Product Glossary.* HSBC: http://www.hsbcnet.com/treasury/structured-products/wms-global/struc_prod_glossary.html

[22] Hull, J.C. (2002). *Options, Futures, and Other, Derivatives* (5th ed.). New Jersey: Prentice Hall.

[23] Hult, H., & Lindskog, F. (2009). *Lecture notes in Risk management.* Stockholm: Royal Institute of Technology.

[24] Hult, H., Lindskog, F., Hammarlid, O., & Rehn, C.-J. (2010). *Risk and portfolio analysis: principles and methods, Part I.* Stockholm: Royal Institute of Technology.

[25] Johansson, T., & Lingnardz, H. (2010). *Evaluation of Equity-Linked Notes - an Empirical Study of SEB Equity-Linked Notes.* Lund: Lund University.

[26] Kahneman, D., & Tversky, A. (1979). Prospect Theory: An Analysis of Decision under Risk. *Econometrica*, 47 (2), 263-291.

[27] Korn, R., & Zeytun, S. (2009). Solving optimal investment problems with structured products under CVaR constraints. *Optimization*, 58 (3), 291-304.

[28] Krokhmal, P., Palmquist, J., & Uryasev, S. (2002). Portfolio optimization with conditional value-at-risk objective and constraints. *Journal of Risk*, 4 (2), 11-27.

[29] Litterman, R., & Scheinkman, J. (1991). Common Factors Affecting Bond Returns. *The Journal of Fixed Income*, 1 (1), 54-61.

[30] Markowitz, H. (1952). Portfolio selection. *Journal of Finance*, 7 (1), 77-91.

[31] Martellini, L., Simsek, K., & Goltz, K. (2005). *Structured forms of investment strategies in institutional investors portfolios.* Technical report, EDHEC Risk and Asset Management Research Centre.

[32] My News Desk. (2010, January 29). *Swedbank störst på indexobligationer.* My News Desk: http://www.mynewsdesk.com/se/pressroom/swedbank/pressrelease/view/swedbank-stoerst-paa-indexobligationer-366818

[33] Rockafellar, R., T., & Uryasev, P. (2001). *Conditional Value-at-Risk for General Loss Distributions.* ISE Dept. Working Paper No. 2001-5.

[34] Scherer, B. (2004) *Portfolio Construction and Risk Budgeting* (2nd ed.). London: Risk Books.

[35] Shefrin, H. (2002). *Beyond Greed and Fear* (2nd ed.). Oxford: Oxford University Press.

[36] Soto, G. (2004) *Using Principal Component Analysis to Explain Term Structure Movements: Performance and Stability.* Available online at: http://ssrn.com/abstract=985404.

[37] UBG AG. (2010, October, 12). *Banking terms.* UBS AG: http://www.ubs.com/1/e/gcc/bankingterms.html.

[38] Uryasev, P. (2000). Conditional Value-at-Risk: Optimization Algorithms and Applications. *Financial Engineering News*, 14, 1-5.

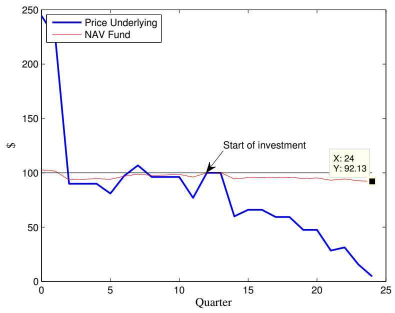
[39] Wasserfallen, W., & Schenk, C. (1996). Portfolio Insurance for the Small Investor in Switzerland, *The Journal of Derivatives*, 3 (3), 37-43.

Appendices

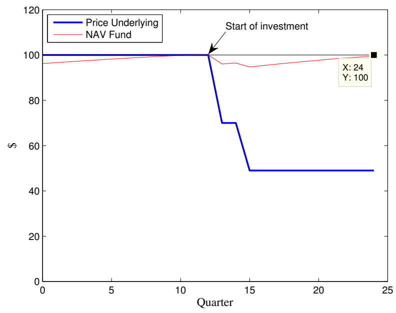
APPENDIX A

Naive fund constructions - Additional scenarios

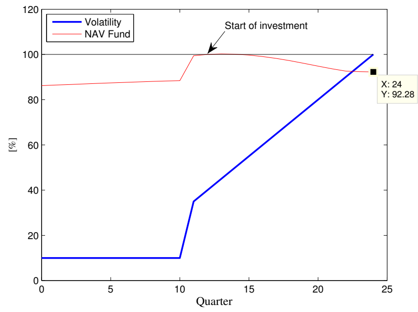
Naive fund construction 1



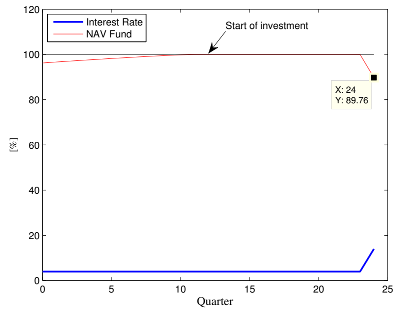
(a) Scenario 3, −7.87%.



(b) Scenario 4, 0%.



(c) Scenario 5, −7.72%.



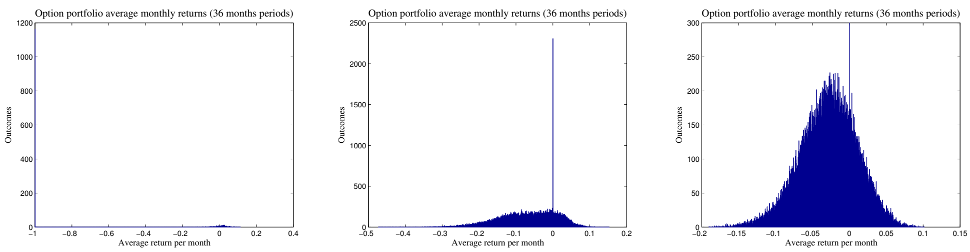
(d) Scenario 6, −10.24%.

Figure A.1: Scenarios 3-6, return between Q12-Q24.

APPENDIX B

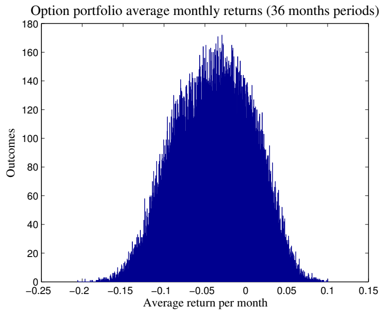
Option portfolios - Additional portfolios

$$
\begin{aligned}
\mathbf{w}^9 &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^{\mathrm{T}} \\
\mathbf{w}^{10} &= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^{\mathrm{T}} \\
\mathbf{w}^{11} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}^{\mathrm{T}} \\
\mathbf{w}^{12} &= \begin{pmatrix} 0 & 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \end{pmatrix}^{\mathrm{T}} \\
\mathbf{w}^{13} &= \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}^{\mathrm{T}} \\
\mathbf{w}^{14} &= \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^{\mathrm{T}} \\
\mathbf{w}^{15} &= \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \end{pmatrix}^{\mathrm{T}} \\
\mathbf{w}^{16} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}^{\mathrm{T}} \\
\mathbf{w}^{17} &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{10} & \frac{3}{10} & \frac{3}{10} & \frac{3}{10} \end{pmatrix}^{\mathrm{T}} \\
\mathbf{w}^{18} &= \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & 0 & 0 & 0 \end{pmatrix}^{\mathrm{T}} \\
\mathbf{w}^{19} &= \begin{pmatrix} \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & 0 & 0 & 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}^{\mathrm{T}}
\end{aligned}
$$

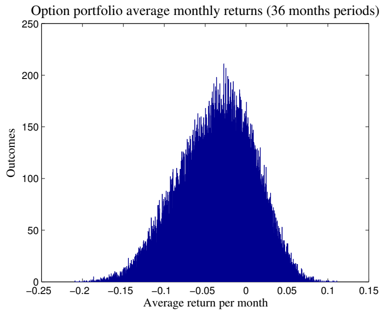


(a) Portfolio 9, −100%. **(b)** Portfolio 10, −47.37%. **(c)** Portfolio 11, −19.02%.

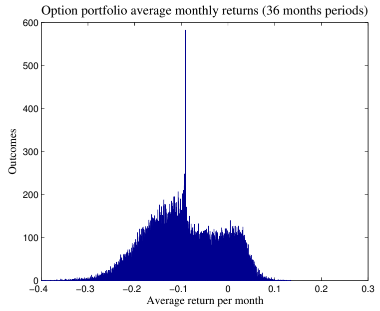
Figure B.1: Histograms over the outcomes for option portfolios 9-11, the worst-case average 36 month return is given in the caption.



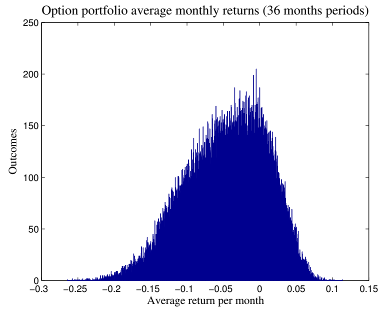
(a) Portfolio 12, −20.55%.



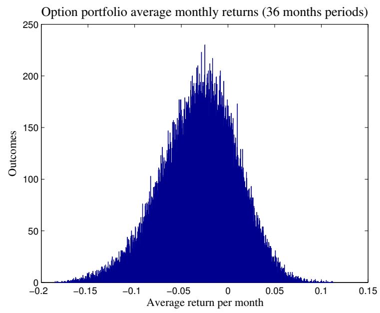
(b) Portfolio 13, −20.93%.



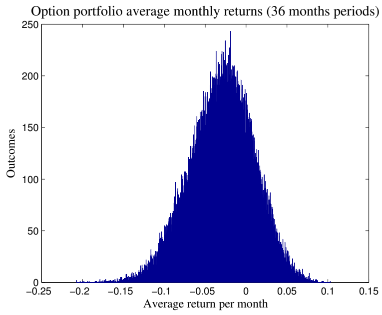
(c) Portfolio 14, −39.94%.



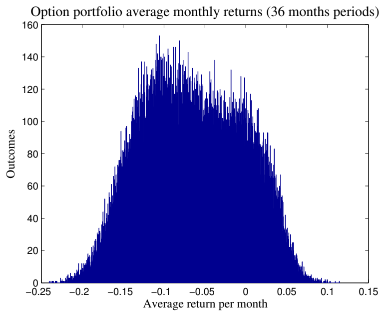
(d) Portfolio 15, −26.46%.



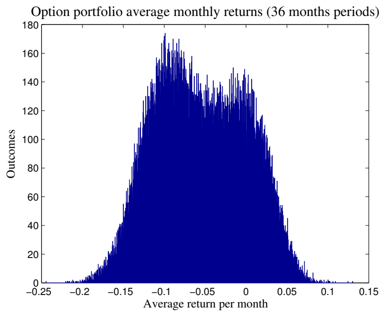
(e) Portfolio 16, −20.44%.



(f) Portfolio 17, −20.74%.



(g) Portfolio 18, −24.04%.



(h) Portfolio 19, −24.47%.

Figure B.2: Histograms over the outcomes for option portfolios 12-19, the worst-case average 36 month return is given in the caption.

APPENDIX C

SPF vs Benchmark - Additional scenarios

Scenario C & D

Return Index Months 49-96 / 51-98 p.a.	$\varepsilon^{\text{bench}}$	δ^{bench}	Scenario C Worst-Case Return	Scenario D Worst-Case Return
-80%	0.7	0.1	-97.36%	-98.12%
-80%	0.5	0.1	-91.23%	-94.01%
-80%	0.3	0.1	-71.41%	-78.26%
-80%	0.20	0.1	-51.70%	-60.40%
-80%	0.10	0.05	-23.56%	-32.10%
-50%	0.7	0.1	-87.92%	-91.10%
-50%	0.5	0.1	-71.47%	-79.34%
-50%	0.3	0.1	-51.05%	-62.53%
-50%	0.20	0.1	-29.10%	-40.95%
-50%	0.10	0.05	-8.77%	-18.58%
-30%	0.7	0.1	-77.85%	-83.52%
-30%	0.5	0.1	-57.83%	-69.09%
-30%	0.3	0.1	-37.75%	-52.21%
-30%	0.20	0.1	-19.82%	-32.96%
-30%	0.10	0.05	2.26%	-12.32%
-20%	0.7	0.1	-71.86%	-78.88%
-20%	0.5	0.1	-50.02%	-63.15%
-20%	0.3	0.1	-32.27%	-47.57%
-20%	0.20	0.1	-14.70%	-29.60%
-20%	0.10	0.05	0.76%	-10.23%
-10%	0.7	0.1	-64.91%	-73.57%
-10%	0.5	0.1	-42.02%	-57.13%
-10%	0.3	0.1	-26.33%	-43.11%
-10%	0.20	0.1	-11.22%	-25.90%
-10%	0.10	0.05	2.69%	-8.10%

Table C.1: Benchmark fund: Worst-case outcome given scenarios C and D, $\varepsilon^{\text{bench}}$ and δ^{bench}.

Scenario C	Scenario D
Months 0-48, return 30% p.a. Months 48-49, return -80% p.m. Months 49-96, return -X% p.a. Months 96-144, return 30% p.a.	Months 0-48, return 30% p.a. Months 48-50, return 50% p.a. Months 50-51, return -80% p.m. Months 51-98, return -X% p.a. Months 98-144, return 30% p.a.

Return Index Months 49-96 / 51-98 p.a.	ε^{SPF}	δ^{SPF}	Scenario C Worst-Case Return	Scenario D Worst-Case Return
-80%	1	1	-20.35%	-61.49%
-80%	0.3	0.1	-20.35%	-61.49%
-80%	0.25	0.1	-11.25%	-51.12%
-80%	0.20	0.1	-10.42%	-50.03%
-80%	0.15	0.1	1.32%	-30.87%
-80%	0.10	0.05	3.79%	-25.69%
-50%	1	1	-20.65%	-62.31%
-50%	0.3	0.1	-20.65%	-62.31%
-50%	0.25	0.1	-11.58%	-52.17%
-50%	0.20	0.1	-10.76%	-51.10%
-50%	0.15	0.1	0.94%	-32.25%
-50%	0.10	0.05	3.41%	-27.27%
-30%	1	1	-24.26%	-63.95%
-30%	0.3	0.1	-24.26%	-63.95%
-30%	0.25	0.1	-15.61%	-54.24%
-30%	0.20	0.1	-14.82%	-53.22%
-30%	0.15	0.1	-3.65%	-35.28%
-30%	0.10	0.05	-1.30%	-30.43%
-20%	1	1	-25.671%	-64.62%
-20%	0.3	0.1	-25.67%	-64.62%
-20%	0.25	0.1	-17.18%	-55.09%
-20%	0.20	0.1	-16.41%	-54.09%
-20%	0.15	0.1	-5.45%	-36.48%
-20%	0.10	0.05	-3.14%	-31.72%
-10%	1	1	-26.26%	-64.91%
-10%	0.3	0.1	-26.26%	-64.91%
-10%	0.25	0.1	-17.84%	-55.46%
-10%	0.20	0.1	-17.07%	-54.47%
-10%	0.15	0.1	-6.20%	-37.00%
-10%	0.10	0.05	-3.91%	-32.28%

Table C.2: SPF 1: Worst-case outcomes for the SPF 1 Portfolio 7 given the scenarios C and D, ε^{SPF} and δ^{SPF}.

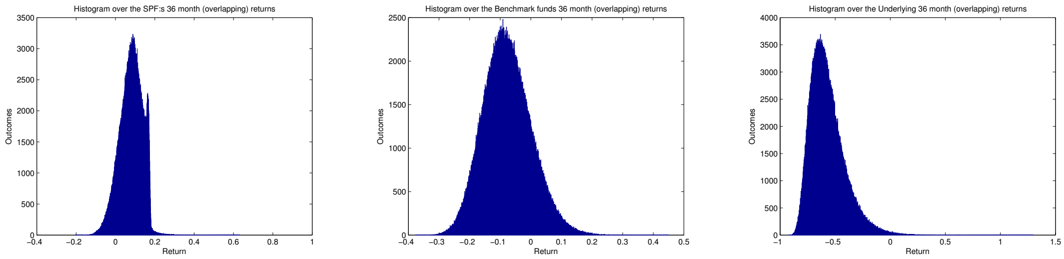
Portfolio 3, Scenario A & B

Return Index Months 48-96 / 50-98 p.a.	$\varepsilon^{\mathrm{SPF}}$	δ^{SPF}	Scenario A Worst-Case Return	Scenario B Worst-Case Return
-80%	1	1	-26.11%	-57.88%
-80%	0.25	0.1	-26.11%	-40.90%
-80%	0.20	0.1	-19.68%	-35.51%
-80%	0.15	0.1	-15.98%	-26.94%
-80%	0.15	0.05	-14.77%	-29.43%
-80%	0.10	0.05	-13.02%	-20.77%
-50%	1	1	-24.87%	-49.59%
-50%	0.25	0.1	-24.87%	-31.78%
-50%	0.20	0.1	-18.96%	-27.15%
-50%	0.15	0.1	-16.01%	-19.28%
-50%	0.15	0.05	-16.69%	-23.33%
-50%	0.10	0.05	-12.71%	-14.78%
-30%	1	1	-20.46%	-37.45%
-30%	0.25	0.1	-20.46%	-22.40%
-30%	0.20	0.1	-15.21%	-16.62%
-30%	0.15	0.1	-12.92%	-9.19%
-30%	0.15	0.05	-13.19%	-13.63%
-30%	0.10	0.05	-8.82%	-12.84%
-20%	1	1	-16.53%	-26.71%
-20%	0.25	0.1	-16.53%	-12.91%
-20%	0.20	0.1	-11.56%	-8.31%
-20%	0.15	0.1	-8.71%	-1.35%
-20%	0.15	0.05	-9.36%	-5.38%
-20%	0.10	0.05	-5.12%	-0.79%
-10%	1	1	-9.92%	-13.72%
-10%	0.25	0.1	-9.92%	-3.5%
-10%	0.20	0.1	-5.94%	-1.28%
-10%	0.15	0.1	-0.53%	5.44%
-10%	0.15	0.05	-4.49%	0.00%
-10%	0.10	0.05	0.00%	3.35%

Table C.3: SPF 1: Worst-case outcomes for the SPF 1 Portfolio 3 given scenarios A and B, $\varepsilon^{\mathrm{SPF}}$ and δ^{SPF}.

Stress testing

$\mu = -30\%$ p.a. for underlying instead of 0% p.a. as in Section 4.4.7



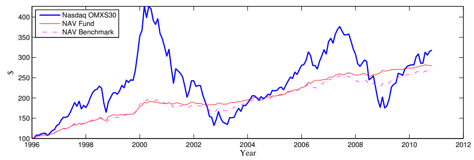
(a) SPF 1: $\varepsilon^{\text{SPF}} = 0.15$, $\delta^{\text{SPF}} = 0.1$, min: -20.59%, mean: 8.01%, max: 63.24%.

(b) Benchmark: $\varepsilon^{\text{bench}} = 0.3$, $\delta^{\text{bench}} = 0.1$, min: -37.83%, mean: -7.70%, max: 45.03%.

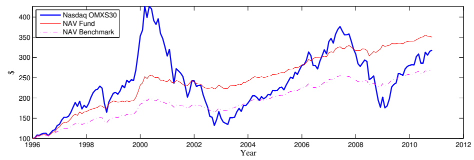
(c) Underlying: min: -92.60%, mean: -56.87%, max: 129.82%.

Figure C.1: Histograms over 36 months returns for the competing funds and the modified stress test

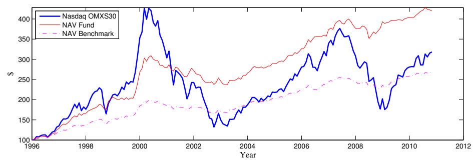
Backtesting



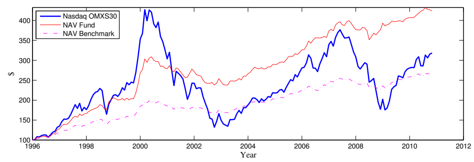
(a) $\varepsilon^{\text{SPF}} = 0.10$, $\delta^{\text{SPF}} = 0.05$.



(b) $\varepsilon^{\text{SPF}} = 0.15$, $\delta^{\text{SPF}} = 0.1$.



(c) $\varepsilon^{\text{SPF}} = 0.2$, $\delta^{\text{SPF}} = 0.1$.



(d) $\varepsilon^{\text{SPF}} = 0.25$, $\delta^{\text{SPF}} = 0.1$.

Figure C.2: Backtest results: OMXS30 vs SPF 1 (Portfolio 4) and Benchmark..

APPENDIX D

Correlations between the market index and the yield curve

Correlation matrix between OMXS30 and SEK yield curve, daily log returns (Jan 1996 - Nov 2010).									
	Index	1M	2M	3M	6M	9M	1Y	2Y	3Y
Index	1.0000	-0.0080	-0.0023	-0.0117	0.0012	0.0070	0.0193	0.1900	0.1891
1M	-0.0080	1.0000	0.8970	0.8559	0.7678	0.7081	0.6855	0.3166	0.2382
2M	-0.0023	0.8970	1.0000	0.9149	0.8422	0.7850	0.7628	0.3610	0.2802
3M	-0.0117	0.8559	0.9149	1.0000	0.8947	0.8429	0.8229	0.3914	0.3070
6M	0.0012	0.7678	0.8422	0.8947	1.0000	0.9362	0.9214	0.4487	0.3657
9M	0.0070	0.7081	0.7850	0.8429	0.9362	1.0000	0.9610	0.4783	0.4100
1Y	0.0193	0.6855	0.7628	0.8229	0.9214	0.9610	1.0000	0.4981	0.4317
2Y	0.1900	0.3166	0.3610	0.3914	0.4487	0.4783	0.4981	1.0000	0.8916
3Y	0.1891	0.2382	0.2802	0.3070	0.3657	0.4100	0.4317	0.8916	1.0000

Correlation matrix between OMXS30 and SEK yield curve, weekly log returns (Jan 1996 - Nov 2010).									
	Index	1M	2M	3M	6M	9M	1Y	2Y	3Y
Index	1.0000	-0.0504	-0.0484	-0.0458	-0.0211	-0.0120	-0.0000	0.1741	0.1899
1M	-0.0504	1.0000	0.9431	0.8971	0.8016	0.7497	0.7226	0.3765	0.2666
2M	-0.0484	0.9431	1.0000	0.9578	0.8897	0.8471	0.8191	0.4488	0.3378
3M	-0.0458	0.8971	0.9578	1.0000	0.9345	0.8965	0.8693	0.4975	0.3915
6M	-0.0211	0.8016	0.8897	0.9345	1.0000	0.9711	0.9513	0.5965	0.4993
9M	-0.0120	0.7497	0.8471	0.8965	0.9711	1.0000	0.9864	0.6438	0.5646
1Y	-0.0000	0.7226	0.8191	0.8693	0.9513	0.9864	1.0000	0.6684	0.5972
2Y	0.1741	0.3765	0.4488	0.4975	0.5965	0.6438	0.6684	1.0000	0.9412
3Y	0.1899	0.2666	0.3378	0.3915	0.4993	0.5646	0.5972	0.9412	1.0000

Correlation matrix between OMXS30 and SEK yield curve, monthly log returns (Jan 1996 - Nov 2010).									
	Index	1M	2M	3M	6M	9M	1Y	2Y	3Y
Index	1.0000	-0.1041	-0.1328	-0.1320	-0.1029	-0.0935	-0.0757	0.1121	0.1233
1M	-0.1041	1.0000	0.9608	0.9258	0.8415	0.7791	0.7473	0.5317	0.3837
2M	-0.1328	0.9608	1.0000	0.9777	0.9166	0.8634	0.8325	0.5900	0.4410
3M	-0.1320	0.9258	0.9777	1.0000	0.9553	0.9134	0.8849	0.6419	0.4912
6M	-0.1029	0.8415	0.9166	0.9553	1.0000	0.9800	0.9626	0.7399	0.5965
9M	-0.0935	0.7791	0.8634	0.9134	0.9800	1.0000	0.9932	0.7940	0.6677
1Y	-0.0757	0.7473	0.8325	0.8849	0.9626	0.9932	1.0000	0.8223	0.7048
2Y	0.1121	0.5317	0.5900	0.6419	0.7399	0.7940	0.8223	1.0000	0.9563
3Y	0.1233	0.3837	0.4410	0.4912	0.5965	0.6677	0.7048	0.9563	1.0000

APPENDIX E

Optimization problem - Arbitrary start yield curve

This appendix uses an arbitrary start yield curve, thus the initial yield curve is drawn randomly from the observed sample. The initial yield curve is drawn arbitrary between January 1996 to November 2010.

Fixed portfolio weights

Product Number, $\text{CVaR}_{0.95}$ [%]											
1	2	3	4	5	6	7	8	9	10	11	12
22.58	18.78	16.27	14.19	12.53	11.20	10.21	9.63	7.17	6.75	6.58	6.68

Product Number, Expected return [%]											
1	2	3	4	5	6	7	8	9	10	11	12
28.73	28.76	28.45	27.99	27.20	26.76	26.33	25.94	25.91	25.64	25.38	25.14

Table E.1: The table describes the CVaR at the 95% confidence level and expected return for the different products over the three-year period using the fixed portfolio weights setting.

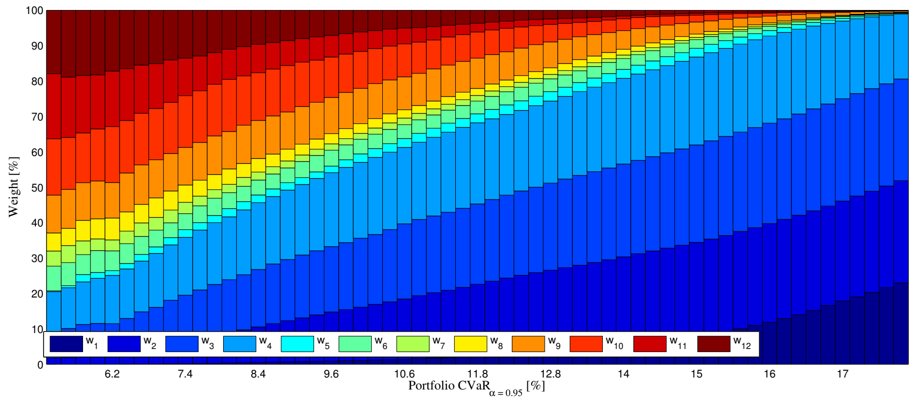


Figure E.1: Portfolio weights for the efficient frontier using the fixed portfolio weights setting, $\alpha = 0.95$.

Rolling portfolio weights

Product (months until next roll), $CVaR_{0.95}$ [%]											
1	2	3	4	5	6	7	8	9	10	11	12
7.68	10.27	11.95	13.18	13.96	14.41	14.72	14.49	13.89	12.41	9.99	0

Product (months until next roll), Expected return [%]											
1	2	3	4	5	6	7	8	9	10	11	12
26.95	26.87	26.83	26.73	26.67	26.64	26.63	26.64	26.76	26.89	26.97	27.13

Table E.2: The table describes the CVaR at the 95% confidence level and expected return for the different products over the three-year period using the rolling portfolio weights setting.

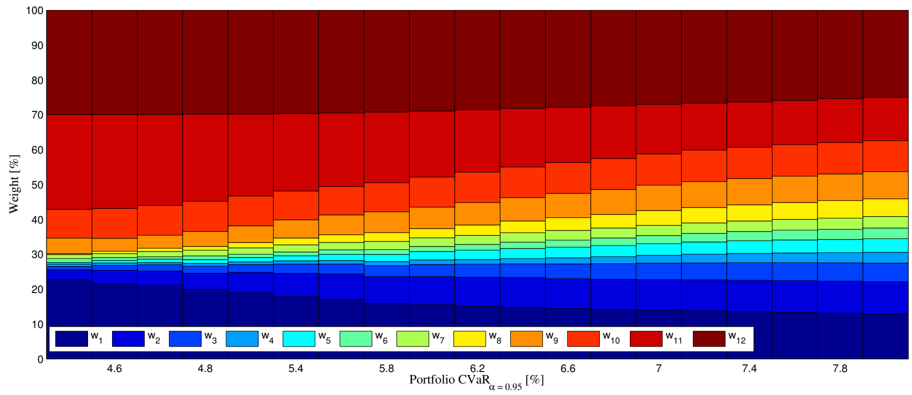


Figure E.2: Portfolio weights for the efficient frontier using the rolling portfolio weights setting, $\alpha = 0.95$.

Benchmark fund

	Incl. Dividends.		Excl. Dividends.	
Index allocation	Expected Return	$CVaR_{0.95}$	Expected Return	$CVaR_{0.95}$
10%	17.25	-3.76	16.09	-3.17
20%	20.81	0.13	18.46	1.31
30%	24.37	5.43	20.83	7.57
40%	28.18	11.46	23.33	14.41

Table E.3: The table discloses the expected return and CVaR for different levels of index weights over the three-year period for a benchmark fund.

Dynamic portfolio weights

Product, months until maturity, $CVaR_{0.95}$ [%]											
1	2	3	4	5	6	7	8	9	10	11	12
9.85	12.34	13.70	14.45	14.63	14.53	14.00	13.14	12.01	10.34	7.70	0.00

Product, months until maturity, Expected return [%]											
1	2	3	4	5	6	7	8	9	10	11	12
26.69	26.58	26.52	26.41	26.35	26.31	26.35	26.46	26.52	26.61	26.74	26.88

Table E.4: The table describes the CVaR at the 95% confidence level and expected return for the different products over the three-year period using the dynamic portfolio weights and the modified Korn and Zeytun framework .

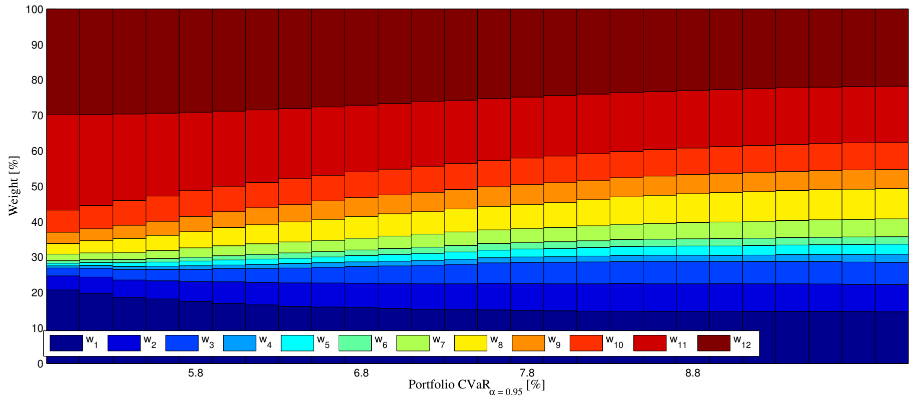


Figure E.3: Portfolio weights for the efficient frontier using the dynamic portfolio weights and the modified Korn and Zeytun framework, $\alpha = 0.95$.