A comparison between different volatility models

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2011-06-14

Abstract

The main purpose of this master thesis is to evaluate and compare different volatility models. The evaluation is based on how well the models are imitating the implied volatility of different stock options. Three different times to maturity will be studied. The volatility models are evaluated based on daily deviations from the implied volatility and on daily changes of the modelled volatility. Statistical measurements investigated are Mean Absolute Deviation and R^2. The models investigated are historical volatility models, a GARCH model and a model where the implied volatility of an index is scaled with a scaling factor based on historical returns of the asset and the index.

The investigation shows that the scaling factor model has a better performance than the other models, but it also shows that it may be better to use the implied volatility of an index, without scaling, instead of the volatility models. For the historical models it is shown that 50 to 75 observations is most appropriate to use to imitate the implied volatility. It is more difficult to evaluate the performance of the GARCH model, since the result of the model is varying. It is also concluded that a few single observations of high absolute returns can result in an overestimation of the modelled GARCH volatility.

Acknowledgement

I want to thank Harald Lang at the Mathematics Department of the Royal Institute of Technology in Stockholm for insightful comments and guidance. I also want to thank my supervisors Magnus Lundin and Erik Svensson at Handelsbanken for giving me this opportunity and for sharing their knowledge of financial risks.

Stockholm, June 2011

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1 Background

In recent years risk has been a hot topic in the financial sector. The set of regulations for banks, both on international and national level are today broader and more detailed than they were a couple of years ago. The number of tasks for the risk departments at banks has increased and the requirements are now more distinct. The general responsibility of the financial risks is located at the highest level of the bank, which has led to an increased focus of risk from the board of directors and the risk departments have been allocated with more resources.

New requirements from the authorities aim to unfold financial problems in banks at an earlier stage and to give the supervisory authorities a more powerful way of acting towards a mismanaged bank. Repeated infractions will be followed by fines, or even a revoked bank licence in some cases. In one way these new requirements are appreciated by banks, the risk of failure is reduced and the management can easier motivate the importance of having a comprehensive and fully working risk system. On the other hand, new requirements are increasing the costs for the banks and the time to adapting the new requirements is short. A well managed bank might have to change its way of handling risks because of other banks previous failures. Further, it might be related to high expenses for a bank to invest in new systems and there is a fear that some regulations are costing the bank more than the benefits they are providing.

Today, the risk department of a bank has the responsibility to validate valuations and perform risk calculations and as a consequence the distance between valuations and risk calculations of derivatives has decreased. From a risk perspective there is a huge difference for a bank to actively take positions and to trade back to back, but in practice, it is a question of interpretation of what an own position is. It can sometimes be very difficult for a bank to decrease their positions and slowly phase out their trading department. It is rather reverse; a bank in a financial distress may increase their positions in order to hedge their already existing positions, with an extended market exposure and maybe even a larger loss as a result.

Are some banks having an active trading department more exposed than other banks? Can a trading portfolio eventually adventure the existence of a bank? Apparently this is the case in the view of what has happened for a couple of banks around the world in recent years. For derivative trading, this might have been even more obvious last year when HQ-bank had their bank license revoked. The Swedish Financial Supervisory Authority claimed that HQ-bank did not use market information when pricing derivatives in a satisfied way, particularly the managing of volatility.

In this thesis, the focus will be placed on option trading and particularly the volatility of an option. Volatility has a large impact on the price of an option and most traders are pricing the options in terms of volatility; they are buying and selling volatility. Generally, traders want to buy an option when the volatility is low and sell when it is high. Future volatility can be regarded as an unknown parameter and is therefore a scope for assumptions. Historical volatility is a method of measuring the variation in the price of the underlying assets, but since that measurement is just historical and the volatility is varying over time it might not be a good way of measure future volatility. A volatility measurement that takes the market's expectation of the future volatility into account is the implied volatility. The implied volatility of an option can be derived from the option pricing formula given that all other parameters in the pricing formula are known. In a liquid market there is always a current update of the price of the option and as a result the implied volatility of the option can be calculated. But when the market is illiquid the implied volatility is unknown. The risk department does not know how the market is pricing the option which contributes to an increased uncertainty and risk of having a position. There are number of ways how to handle this problem and how to estimate the volatility. In this report three different volatility models are compared and evaluated based on how well they are imitating the implied volatility.

2 Problem formulation

The volatility is a fundamental variable in valuations and risk calculations of derivatives. The implied volatility is a measurement that takes the market expectations about the volatility into account. For a liquid option, it is possible to calculate the implied volatility and an interested person can get a good estimation of the market expectations of the variation in the price of the underlying asset in the future. But for an illiquid option, in the case an option is not frequently bought nor sold in a couple of days, there will be a problem to assess the current price of the option. Neither the price nor the calculation of the implied volatility will be up to date and an alternative method must be used in order to calculate the volatility. Therefore, the main purpose of this master thesis is to investigate how well different volatility models are imitating the implied volatilities, focusing on European at the money call options with three different times to maturity.

3 Methodology

The volatility models have been investigated and evaluated based on how well they are fitting the implied volatility. The models studied are historical volatility, GARCH volatility and a model where the implied volatility for an index is scaled with a scaling factor connected to the historical return in a stock and an index. To find out how well these three models are imitating the implied volatility, the models will be evaluated both visually and with statistical measurements. In order to compare and evaluate the models all options in this investigation are liquid, making the implied volatility always up to date. Options with strikes equal to the spot price, so called at the money options, are often the most liquid ones. The fact that implied volatility has different term structure makes the time series investigated in this thesis are all at the money with three different times to maturity; 30 days, 3 months and 6 months. In the first section of this thesis the theories behind these models are described. The theory section is followed by a presentation and analysis of the results from the investigation.

4 Options

An option contract gives the holder the right to trade in future at previously agreed price but there is no obligation to trade.[1] This implies that that the holder of a contact only will make the trade in the future if it is to his advantage.

There is a difference between European and American options. An American option gives the holder an opportunity to exercise at any time before maturity, while a European option can be exercised only at maturity day. An American option will never be less valuable than a European option, since an American option gives the holder more opportunities than a European option. In this thesis, only European options will be investigated.

A call option is the right to buy a particular asset for an agreed amount at a specified time in the future.	A put option is the right to sell a particular asset for an agreed amount at a specified time in the future.

The value of a European option at maturity day

<u>**Call option**</u>

$\max\{S_T - K, 0\}$

<u>**Put option**</u>

$\max\{K - S_T, 0\}$

Where

S_T = Spot price of the underlying asset

K = Strike price

[1] Paul Wilmott, 1998

One way of pricing a European option, is to use the well known Black & Scholes pricing formula.

The Black & Scholes pricing formula for a European call option:

$$C(S,t) = N(d_1) * Se^{-q(T-t)} - N(d_2) * Ke^{-r(T-t)}$$

The Black & Scholes pricing formula for a European put option:

$$P(S,t) = N(-d_2) * Ke^{-r(T-t)} - N(-d_1) * Se^{-q(T-t)}$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right) * (T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(r - q - \frac{\sigma^2}{2}\right) * (T-t)}{\sigma\sqrt{T-t}}$$

S=Price of the underlying asset

K= Strike price

R=Risk free rate

T-t=Time to maturity

σ =volatility of the underlying asset

q=The yearly dividend yield expressed with continuous compounding

Only one of the parameters in the Black & Scholes formula, the volatility, cannot be directly observed in the market and it is therefore a scope for speculation and uncertainty. For further reading and more assumptions behind the formula see Black & Scholes[2].

5 Implied volatility

There is only one value of the volatility(σ) in the Black & Scholes formula that gives a theoretical price equal to the market price of an option. This value is called the implied volatility. When all parameters in the Black & Scholes formula are known, the contract value from quoted prices on the market included, it is possible to calculate the implied volatility. Since the value is derived from the

[2] Black & Scholes, 1973

formula it is a measurement that takes the market's expectations about the future volatility for an option into account. By using the implied volatility, the market valuation and risk calculation of the option will be more correct in a market risk perspective. Another important reason to calculate the implied volatility of a vanilla option is that the implied volatility also can be used to evaluate and calculate risk for more exotic derivatives.

Only call options have been studied in this thesis, but the implied volatility is the same for both call and put options with the same time to maturity and the same strike price due to the put-call parity.[3]

The put-call parity

$$C(t) - P(t) = S - Ke^{-r(T-t)}$$

The Black & Scholes formula assumption of volatility being constant can be shown to be incorrect. By backing out the implied volatility from the Black & Scholes formula, it can be shown that the volatility changes due to variations in time to maturity and in strike price.

5.1 Volatility smile and skew

Options having the same underlying asset and term structure, but different strike prices are having different implied volatilities. A plot of the implied volatilities of an option as a function of theirs strike prices is known as a volatility smile. A volatility smile as presented in figure 1 is typical for FX-options. An FX-option contract gives the holder an option to exchange money in one currency into another currency at a previously agreed exchange rate. The implied volatility of at the money options is lower than for other strike prices.

[3] Poon, 2005

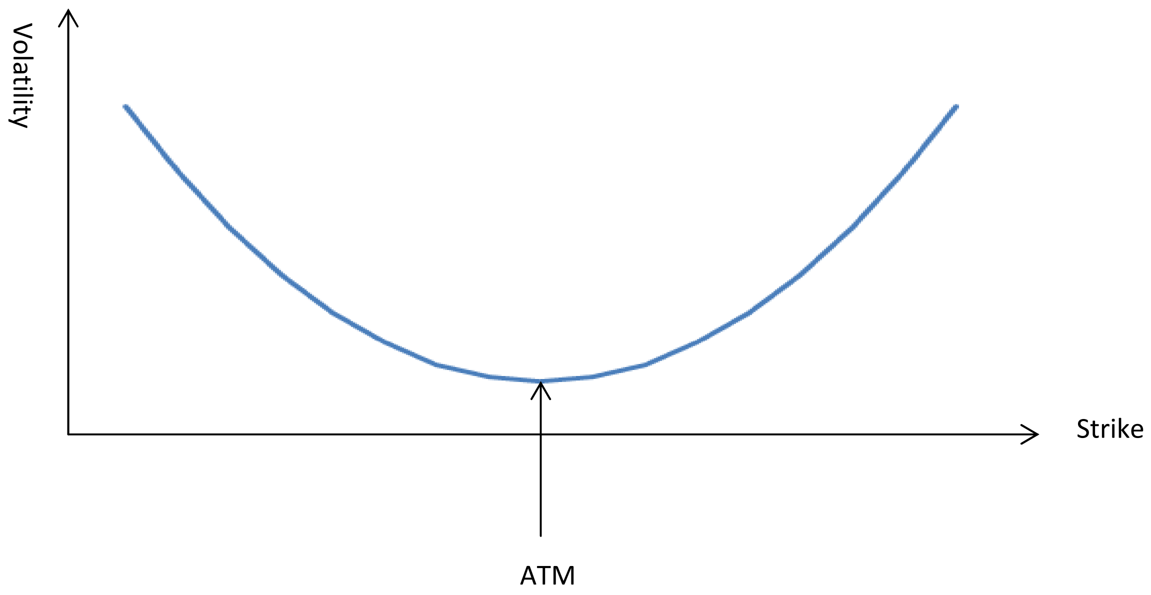

Figure 1 Volatility smile

For stock options, a downward slope, volatility skew, as shown in figure 2 is more common. The implied volatilities of stock options with high strike prices are usually lower than those of at the money stock options. In this thesis only stock options will be studied.

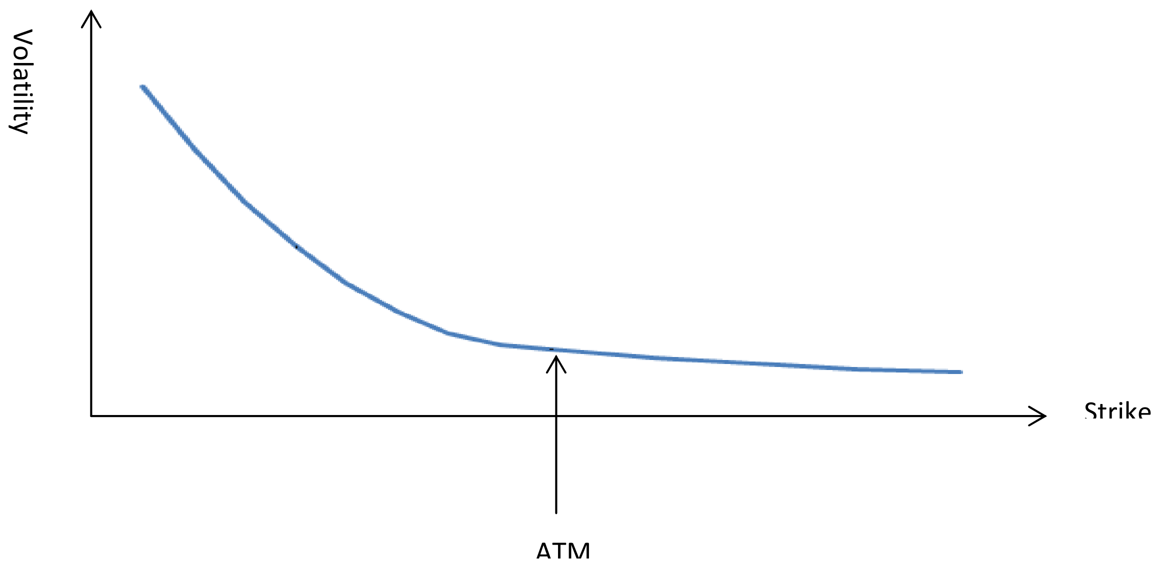

Figure 2 Volatility skew

5.2 Volatility term structure

The term structure of implied volatility describes the relationship between time to maturity and implied volatility. Options having the same underlying asset and the same strike price may have different implied volatilities due to different term structure. The term structure is for example depending on upcoming market events. The market knows that an important report will be released at a predetermined point in time and that the result of the report will have a large impact on the company's stock price and therefore also on the volatility of the option.

5.3 Volatility surface

A volatility surface of the implied volatility is shown in figure 3. The implied volatility is plotted in relation to both the time to maturity and the strike price. The surface represents the value of volatility giving each traded option a theoretical volatility value equal to the market value. The surface may vary a lot over time and between different underlying assets. When the implied volatility of all known options is marked in a graph, an interpolation method is used to get the whole volatility surface.

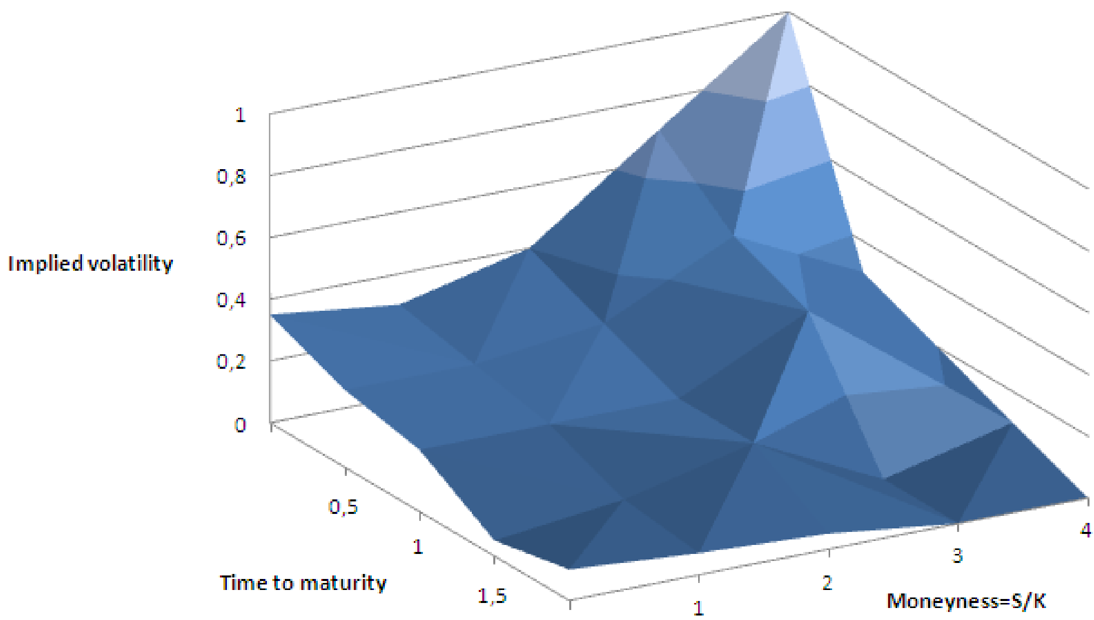

Figure 3 Volatility surface

The implied volatility time series studied in this thesis have a fixed time to maturity. The implied volatility is calculated every day based on a weighted average of the two call options closest to the at-the-money strike. Due to this fact, an interpolation between the two closest options has to be made to receive the searched value with correct time to maturity and strike price. The option market is most liquid when the spot price is equal to the strike price. Therefore, the time series of the implied volatility will be calculated from at the money options in this investigation.

6 Historical volatility

The historical volatility σ_n is estimated from historical spot prices of the underlying asset.[4]

The calculation method of the historical volatility is described below. At first the return for each day is calculated as

$$r_i = \frac{S_i - S_{i-1}}{S_{i-1}}$$

where S_i is the spot price of the underlying asset at day i.

Then, the variance is calculated as

$$\sigma_n^2 = \frac{1}{m-1}\sum_{i=0}^{m-1}(r_{n-i} - \bar{r})^2$$

Where m is the number of observations and $\bar{r}$ is the average return of the m observations at the time n.

$$\bar{r} = \frac{1}{m}\sum_{i=0}^{m-1} r_{n-i}$$

σ_n^2 is given as a daily measure of the variance, but since the comparison is based on yearly volatilities(σ_n) the daily variance needs to be scaled as below to get the yearly volatility

$$\sigma_{yearly} = \sqrt{252\sigma_n^2}$$

In this thesis 30, 50, 75 and 100 daily observations are used to calculate the historical volatility.

7 Scaling index volatility model

The implied volatility is almost always known for large indices, since they have more liquid options. By using the CAPM formula and other assumptions and well known calculations, an estimation of the volatility for other options with other underlying assets can be calculated. The model is described below.

This model uses historical prices to estimate the CAPM beta (β)[5] for an asset. The β is describing the return between two assets or portfolios, in this thesis the return between a stock and an index.

[4] Hull, 2006

[5] Sharpe,1964

The CAPM β is calculated as:

$$\beta_i = \frac{cov(r_i, r_M)}{var(r_M)} = \frac{\sigma_{iM}}{\sigma_M^2}$$

where

r_i= rate of return of the stock

r_M= Rate of return of the index

The variance (σ_M^2) of the index is calculated as described earlier, section 6, and the covariance (σ_{iM}) as:

$$\sigma_{iM} = cov(r_i, r_M) = \frac{1}{m-1}\sum_{t=0}^{m-1}(r_{it} - \bar{r}_i)\,(r_{Mt} - \bar{r}_M)$$

The linear correlation ρ of two different assets can be calculated as Pearson product-moment correlation coefficient.

$$\rho_{i,M} = corr(r_i, r_M) = \frac{cov(r_i, r_M)}{Stdev(r_i)Stdev(r_M)} = \frac{\sigma_{iM}}{\sigma_i \sigma_M} \qquad \Leftrightarrow \qquad \sigma_{iM} = \rho_{i,M}\sigma_i\sigma_M$$

Inserting this in the formula above gives

$$\beta_i = \frac{\rho_{i,M}\sigma_i\sigma_M}{\sigma_M^2}$$

The parameter that is searched is the volatility of the stock (σ_i) and a rewriting of the formula gives

$$\sigma_i = \frac{\beta_i \sigma_M}{\rho_{i,M}} \qquad \Rightarrow \qquad \hat{\sigma}_i^{implied} = \frac{\hat{\beta}_i \sigma_M^{implied}}{\hat{\rho}_{i,M}}$$

In the formula above $\hat{\beta}$ and $\hat{\rho}$ are based on historical values and while σ_M is the implied volatility of the index. 100 respective 1000 historical returns are used to calculate β and ρ. A new calculation of the measurements is made every day.

8 Garch(1,1) volatility

The GARCH (Generalized Autoregressive Conditional Heteroskedasticity)[6] process was introduced by Tim Bollerslev in 1986 and was a development from the ARCH process introduced by Engle[7]. Below a Garch(1,1) process is described, with the volatility of today expressed by the volatility and the disturbance from yesterday.

$\varepsilon_t = \sigma_t z_t$

where

$z_t \sim N(0,1)$

$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

where

α_0, α_1 and β_1 are estimated coefficients.

In the investigation the GARCH volatility of tomorrow is compared with the implied volatility of today.

8.1 Estimation of parameters

A new parameter estimation is calculated every day. To estimate the parameters in the GARCH-process Matlab is used. Matlab uses a default formula to estimate the parameters in the GARCH process:

$Y_t = \mathrm{C} + \varepsilon_t$

where

Y_t is the observed return for day t and C is a constant

To estimate the parameters (θ) =(C, $\alpha_0, \alpha_1, \beta_1$) a maximum likelihood function is used

$$L(\theta) = \prod_{t=1}^{T} f(\theta)$$

where

$$f(\theta) = \frac{1}{\sqrt{2\pi\sigma_t^2}} exp\left(-\frac{\varepsilon_t^2}{2\sigma_t^2}\right)$$

Taking the logarithms of both sides gives

$$\ln L(\theta) = \sum_{t=1}^{T} \ln f(\theta)$$

This gives the log likelihood function that will be maximized

[6] Bollerslev, 1986

[7] Engle,1982

$$\tilde{L}(\theta) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T} log\sigma_t^2 - \frac{1}{2}\sum_{t=1}^{T}\frac{\varepsilon_t^2}{\sigma_t^2}$$

9 Methods to evaluate the models

The models can be evaluated in several different ways. By investigating the graphs, a first indication of obvious errors in the models might be received. A more quantitative way of evaluating the models is to investigate statistical measurements. It is difficult to evaluate how well a model fits another model with one single measurement. The measurements may give different information and the choice of what measurement to study depends on what the investigation aims to illustrate.

This report is written from a risk perspective, which implies that the investigation and the evaluation must be based on how well the models are corresponding to the implied volatility at every point in time, but also on how well and how fast the models are adapting to sudden changes.

To evaluate the models, both visual graphs and statistical measurements will be studied. At first a graph and a summary of statistical measurements will be presented for each time series. This section will be followed by some measurements on how well the models are fitting the implied volatility at each day. Finally the daily changes of the volatility will be studied for each model.

The measurements used in the evaluation

Max= *the highest value in a data set*

Min= *the lowest value in a data set*

Stdev= *the standard deviation of a data set*

$$\boldsymbol{MAD} = mean\ absolute\ deviation = \frac{\sum_{i=1}^{k} |m_i - d_i|}{k}$$

Where

m_i =the model value at time point i.

d_i =the implied volatility value at time point i.

K=the number of observations.

The MAD measurement is easy to understand, to interpret and the calculation is straight forward. The MAD is a measurement on a models average deviation from the data. The MAD measurement gives all deviations equal weights in the calculation, no matter if they are big or small, positive or negative.

$$\boldsymbol{R^2} = 1 - \frac{SS_{err}}{SS_{tot}}$$

where

$$SS_{tot} = \sum_i (d_i - \bar{d})^2$$

$$SS_{err} = \sum_i (d_i - m_i)^2$$

R^2 is often called the coefficient of determination. The measurement is taking into account the variation in the data set that the model is trying to fit, in this thesis the variation in the implied volatility. R^2 is penalising large deviations severely, since the deviations are squared in the calculation. An R^2 value equal to 1 implies that the model is fitting the data perfectly.

10 Presentation of the data

The Data used in the analysis are collected from Bloomberg, where daily implied volatilities are stored since 2005-05-06. Hence, the time series are from 2005-05-06 until 2010-11-30 which gives 1404 daily observations. Further 800 observations of historical closing prices of the underlying asset have been used to estimate the GARCH-parameters. Below some raw data and key ratios for the time series are presented to the reader:

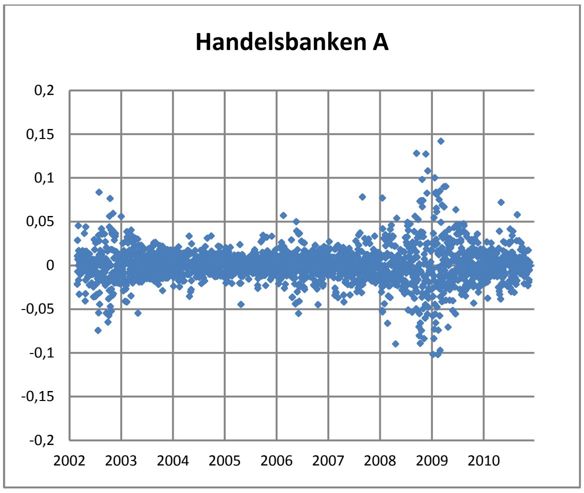

Figure 4 ***Handelsbanken*** *daily return.*

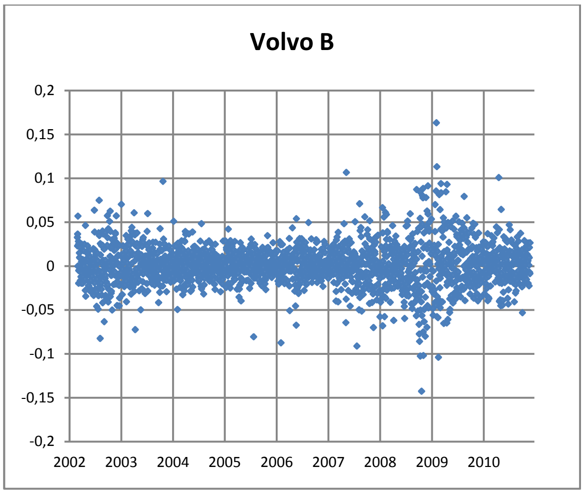

Figure 5 ***Volvo*** *daily return.*

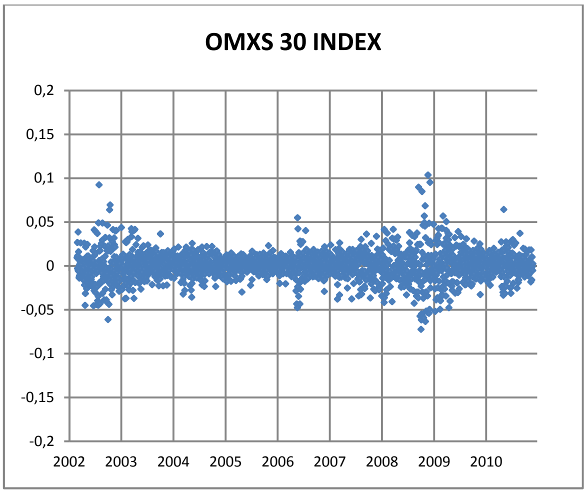

Figure 6 ***OMXS 30 Index*** *daily return.*

In figures 4-6 the daily return for Handelsbanken A, Volvo B and the OMXS 30 Index are plotted. The first 800 observations are only used in order to calculate the GARCH parameters of the first day (2005-05-06) in the investigation. To estimate the parameters and the volatility of each day a rolling window is used, moving the estimation period one step forward per day ahead in time. As shown in figures 4-6 the variations in the return of all assets are increased in 2008-2009 during the global financial crisis (Late-2000s financial crisis). Another conclusion is that Volvo has the greatest variations in the return, followed by Handelsbanken and the OMXS 30 INDEX.

Further, in this section some general statistics and graphs of the implied volatility and the investigated models are presented. All volatility measurements are expressed as percentage and the changes as percentage points. First a description of the time series is presented.

Implied **30d**=implied volatility **30 days** to maturity of the investigated asset.

OMX **3m**= implied volatility **3 months** to maturity of the investigated index.

100 Scaling **6m**= the scaling factor model using **100** observations to calculate the scaling factor. **6m** means that the time to maturity is **6 months**.

30Historical= the historical model using **30** observations to calculate the volatility.

GARCH= the GARCH model.

10.1 30 days to maturity

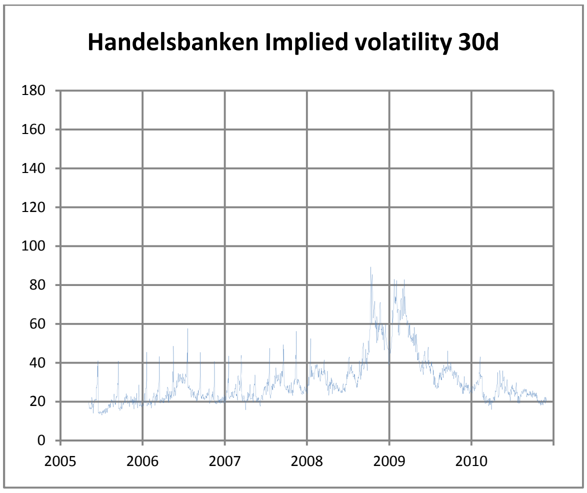

Figure 7 *An overview of **Handelsbanken** implied volatility 30 days*

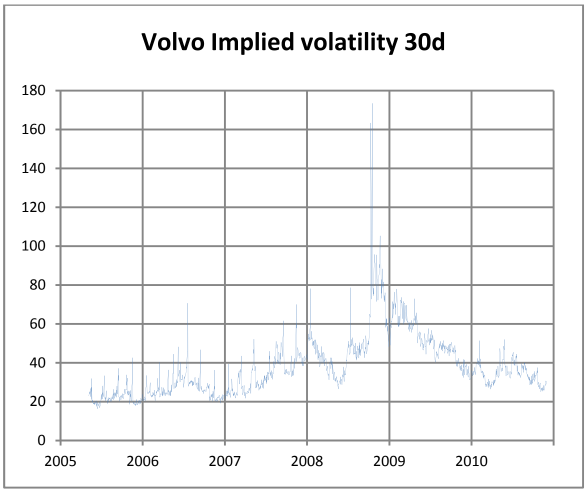

Figure 8 *An overview of **Volvo** implied volatility 30 days*

As shown in figures 7 and 8 the time series of the *implied volatility 30 days* had heavy periodic changes in the first two years. This was most likely caused by the option market being less extensive by that time, with less number of liquid options. The time series of the implied volatility are created by interpolating the implied volatility of the two options having strike prices closest to the spot prices (at the money) and that are maturing in 30 days. If, for example new options are having longer term to maturity and are issued less often, it will sometimes be impossible to interpolate. Instead a much more unsteady extrapolating procedure will be used. Investigations of time series having even shorter time to maturity (<30 days) are showing that the problem is even more extended for those time series and therefore evaluations are difficult to perform on these short term to maturities. During the last couple of years there have been more options available and better liquidity in the market, making the interpolation processes working better and the time series smoother. Only data from 2008-01-02 will be used in the comparison of 30 days to maturity due to the data errors before year 2008. The market was unsteady during the financial crisis with some odd market notations. In figures 7 and 8 it can be noted that the implied volatility had a couple of extreme values in late 2008, especially for Volvo.

	Average	Max	Min	Stdev
Implied 30d	**36,06**	**89,31**	**15,87**	**14,28**
OMX 30d	30,74	80,82	15,25	11,79
100Scaling 30d	42,64	102,71	19,93	16,87
1000Scaling 30d	40,64	100,76	21,44	14,36
30Historical	40,54	96,35	14,38	20,90
50Historical	40,94	90,92	15,24	19,86
75Historical	41,25	84,13	18,64	19,11
100Historical	41,50	82,87	20,42	18,45
GARCH	40,40	119,85	16,51	20,67

Table 1 *Presentation of the time series.* ***Handelsbanken*** *implied volatility 30 days and the investigated models*

	Average	Max	Min	Stdev
Implied 30d	**46,47**	**173,46**	**25,23**	**15,87**
OMX 30d	30,74	80,82	15,25	11,79
100Scaling 30d	48,47	111,67	26,13	15,46
1000Scaling 30d	47,46	121,12	23,72	17,34
30Historical	45,17	100,11	21,10	18,90
50Historical	45,77	92,35	24,67	17,61
75Historical	46,25	84,65	25,87	16,67
100Historical	46,64	84,52	26,20	15,89
GARCH	46,20	132,08	22,86	18,47

Table 2 *Presentation of the time series.* ***Volvo*** *implied volatility 30 days and the investigated models*

As shown in table 1, all models are related to higher average values than that of Handelsbanken implied volatility 30 day. The estimated time series are also having higher values of the standard deviation compared to the implied volatility. The OMX index has a lower average value and standard deviation than Handelsbanken. In table 2, Volvo 30 days to maturity, it is shown that the models are having similar average values and standard deviations as the implied volatility. Notable for the investigation is that OMX is having a much lower average value and standard deviation than the implied volatility of Volvo.

10.2 3 months to maturity

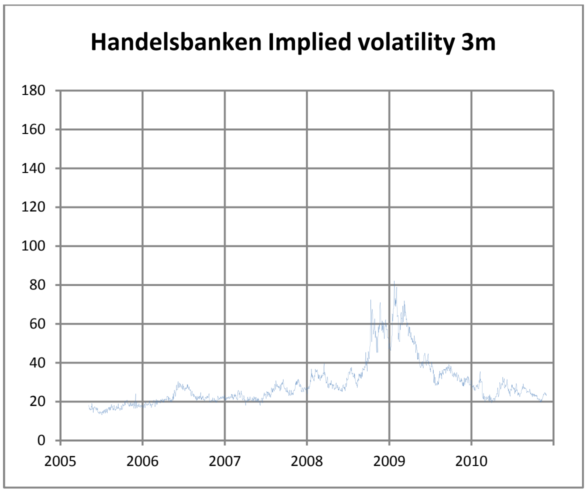

Figure 9 *An overview of* ***Handelsbanken*** *implied volatility 3 months*

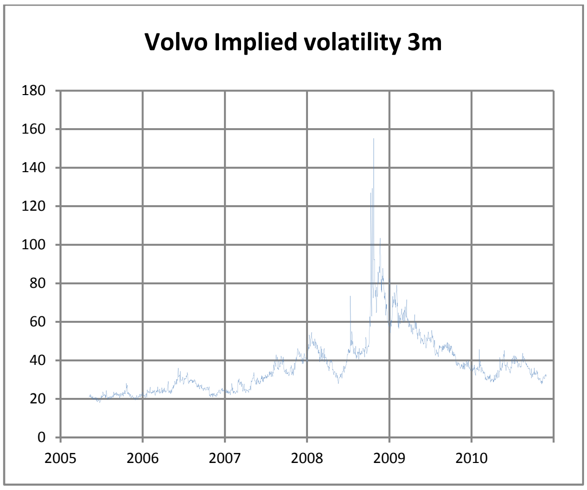

Figure 10 *An overview of* ***Volvo*** *implied volatility 3 months*

The interpolation problem from implied volatility 30 days is eliminated with this term to maturity, figure 9 and 10. This is probably due to options always being liquid and having term to maturity at both sides of 3 months to interpolate between.

	Average	Max	Min	Stdev
Implied 3m	**28,90**	**82,14**	**13,17**	**11,92**
OMX 3m	25,12	65,49	11,44	9,24
100Scaling 3m	33,79	83,23	12,95	13,86
1000Scaling 3m	31,35	81,65	11,47	13,10
30Historical	31,08	96,35	7,95	18,69
50Historical	31,36	90,92	9,11	18,00
75Historical	31,56	84,13	10,37	17,52
100Historical	31,65	82,87	12,58	17,20
GARCH	30,63	119,85	11,11	18,56

Table 3 *Presentation of the time series.* ***Handelsbanken*** *implied volatility 3 months and the investigated models*

	Average	Max	Min	Stdev
Implied 3m	**36,87**	**155,23**	**18,08**	**14,79**
OMX 3m	25,12	65,49	11,44	9,24
100Scaling 3m	40,53	92,03	18,02	13,39
1000Scaling 3m	37,77	98,91	14,20	14,95
30Historical	36,62	100,11	13,27	17,66
50Historical	36,97	92,35	13,87	16,73
75Historical	37,18	84,65	16,33	16,22
100Historical	37,29	84,52	17,21	15,87
GARCH	37,00	132,08	18,41	17,16

Table 4 *Presentation of the time series.* ***Volvo*** *implied volatility 3 months and the investigated models*

As shown in table 3, the average value of Handelsbanken implied volatility 3 month is lower than those of the models. The standard deviation is following the same pattern by having considerably higher values than the implied volatility. The OMX index has a lower average value and standard deviation than both Handelsbanken and Volvo. Most of the models in table 4 have slightly higher average values and standard deviation than those received from the implied volatility.

10.3 6 months to maturity

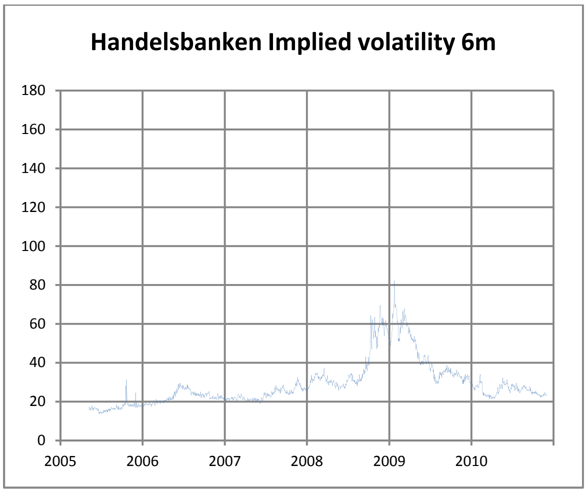

Figure 11 *An overview of* ***Handelsbanken*** *implied volatility 6 months*

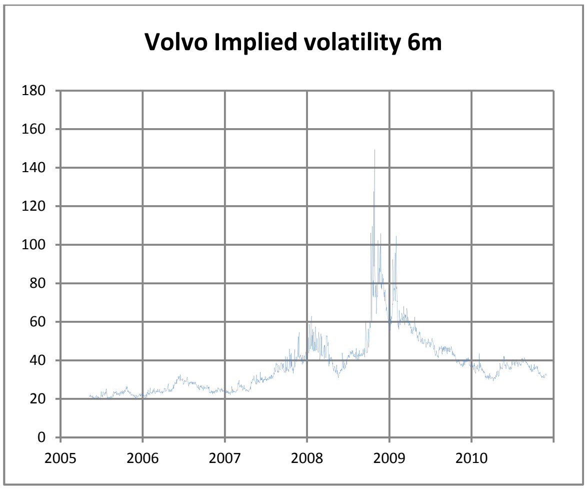

Figure 12 *An overview of* ***Volvo*** *implied volatility 6 months*

The implied volatility 6 months, figure 11 and 12, does not have the interpolation problem either that implied volatility 30 days had before year 2008. The extreme values in the end of 2008 exist also for this time to maturity.

	Average	Max	Min	Stdev
Implied 6m	**28,95**	**82,24**	**13,59**	**11,55**
OMX 6m	24,77	59,75	10,29	8,18
100Scaling 6m	33,36	76,80	11,97	12,70
1000Scaling 6m	30,95	74,77	10,30	12,01
30Historical	31,08	96,35	7,95	18,69
50Historical	31,36	90,92	9,11	18,00
75Historical	31,56	84,13	10,37	17,52
100Historical	31,65	82,87	12,58	17,20
GARCH	30,63	119,85	11,11	18,56

Table 5 *Presentation of the time series.* ***Handelsbanken*** *Implied volatility 6 months and the investigated models*

	Average	Max	Min	Stdev
Implied 6m	**37,18**	**149,45**	**19,65**	**14,85**
OMX 6m	24,77	59,75	10,29	8,18
100Scaling 6m	40,09	83,54	18,22	12,11
1000scaling 6m	37,25	90,63	12,78	13,48
30Historical	36,62	100,11	13,27	17,66
50Historical	36,97	92,35	13,87	16,73
75Historical	37,18	84,65	16,33	16,22
100Historical	37,29	84,52	17,21	15,87
GARCH	37,00	132,08	18,41	17,16

Table 6 *Presentation of the time series.* ***Volvo*** *Implied volatility 6 months and the investigated models*

The models compared to the implied volatility 6 months are showing similar results as 30 days and 3 months implied volatility. The implied volatility of Handelsbanken, table 5, is having lower average value and standard deviation than the values received by the investigated models. In table 6, the models values are more similar to the values of implied volatility. The models are showing just slightly higher or lower values than those of the implied volatility. The OMX index has lower average value and standard deviation than Handelsbanken and Volvo.

11 Daily deviations from the implied volatility

In this section the daily difference between the models and the implied volatilities will be in focus. Below, in the tables some measurements on how well the models fits the implied volatilities are presented.

11.1 30 days to maturity

	MAD	MAX	MIN	R2
OMX 30d	6,15	19,64	-34,83	0,63
100Scaling 30d	7,61	38,60	-17,83	0,50
1000Scaling 30d	6,24	37,39	-20,64	0,69
30Historical	8,88	35,85	-34,23	0,32
50Historical	8,18	35,85	-33,02	0,33
75Historical	8,48	37,93	-37,93	0,28
100Historical	9,05	43,45	-43,45	0,20
GARCH	7,92	54,74	-30,84	0,34

Table 7 *The comparison between the models and the implied volatility of* ***Handelsbanken*** *30 days*

	MAD	Max	Min	R2
OMX 30d	15,73	2,68	-92,64	-0,17
100Scaling 30d	4,97	30,76	-65,67	0,80
1000Scaling 30d	4,69	29,65	-52,33	0,81
30Historical	7,57	34,28	-104,37	0,52
50Historical	7,08	34,52	-110,86	0,52
75Historical	7,58	33,25	-115,43	0,46
100Historical	8,20	25,02	-80,60	0,39
GARCH	6,65	73,36	-80,60	0,56

Table 8 *The comparison between the models and the implied volatility of* ***Volvo*** *30 day*

In table 7, it is shown that OMX has a lower MAD value than those of the models. The fact that the models are not performing better than an index shall be regarded as a failure, particularly for the scaling factor model since it is given by the index value multiplied by a factor. The scaling factor model using 1000 historical observations is having the highest R^2 value and is giving a better fit than using 100 observations.

In table 8, it is shown that the index is not matching Volvo well at all. The OMX is generally having a much lower value than Volvo and as a result the MAD value is very high and the R^2 even negative. The scaling factor model is performing well and there is not a large difference between using 100 or 1000 historical observations when calculating the scaling factors.

The historical volatility models are consistently having lower R^2 and higher MAD values than the scaling factor models. A comparison between the different historical models shows that 50 observations is the best performing model. The GARCH model for both Handelsbanken and Volvo is by a small margin the better performing model compared to the historical models.

11.2 3 months to maturity

	MAD	Max	Min	R2
OMX 3m	4,18	7,50	-32,12	0,74
100Scaling 3m	5,34	31,15	-16,46	0,65
1000scaling 3m	4,31	23,64	-18,28	0,79
30Historical	6,68	37,62	-15,88	0,38
50Historical	5,65	33,26	-16,14	0,48
75Historical	5,30	36,98	-21,05	0,49
100Historical	5,31	35,93	-26,57	0,45
GARCH	6,22	56,09	-13,68	0,37

Table 9 *The comparison between the models and the implied volatility of* ***Handelsbanken*** *3 months*

	MAD	Max	Min	R2
OMX 3m	11,75	2,20	-97,83	0,17
100Scaling 3m	5,67	25,52	-79,25	0,74
1000Scaling 3m	3,60	16,80	-69,46	0,89
30Historical	6,48	34,89	-79,47	0,65
50Historical	5,50	28,50	-92,00	0,71
75Historical	5,15	27,87	-96,87	0,71
100Historical	5,13	26,87	-101,03	0,68
GARCH	5,32	74,76	-76,80	0,66

Table 10 *The comparison between the models and the implied volatility of* ***Volvo*** *3 months*

In table 9 and 10, it is shown that the scaling factor model using 1000 historical observation is having the highest R^2 value. For Volvo it is also having considerably lower MAD than the other models. Also at this term to maturity, the OMX index is having a lower MAD value compared to the scaling factor models for Handelsbanken.

Evaluating 3 months to maturity by MAD and R^2, shows that the usage of 50 or 75 observations is the most appropriate number of observations when using the historical model. The GARCH model is not matching the implied volatility better than historical volatility models.

11.3 6 months to maturity

	MAD	Max	Min	R2
OMX 6m	4,41	5,83	-34,63	0,68
100Scaling 6m	5,23	27,73	-21,21	0,66
1000Scaling 6m	4,21	16,42	-22,19	0,79
30Historical	6,82	36,10	-15,98	0,33
50Historical	5,70	32,21	-12,92	0,45
75Historical	5,12	36,04	-14,33	0,48
100Historical	4,95	34,11	-18,45	0,44
GARCH	6,45	58,46	-16,13	0,28

Table 11 *The comparison between the models and the implied volatility of* ***Handelsbanken*** *6 months*

	MAD	Max	Min	R2
OMX 6m	12,41	1,55	-89,70	0,04
100Scaling 6m	6,02	27,71	-65,91	0,69
1000scaling 6m	3,60	12,51	-58,98	0,86
30Historical	6,87	36,54	-64,69	0,62
50Historical	5,80	27,28	-79,38	0,70
75Historical	5,30	28,17	-85,57	0,70
100Historical	5,24	27,43	-90,68	0,67
GARCH	5,49	74,95	-49,21	0,64

Table 12 *The comparison between the models and the implied volatility of* ***Volvo*** *6 months*

The scaling factor model using 1000 observations has at 6 months to maturity the highest R^2 and the lowest MAD value for both Handelsbanken and Volvo. The scaling factor models matching with Handelsbanken are just slightly better than the OMX index. Of the historical models, 75 observations is giving the highest R^2 and GARCH is not showing a better overall performance than the other models except the 30 days historical volatility model.

12 Daily changes in the volatility

In this section, the daily changes in the time series are studied. Remember that only observations from 2008 and after are used in the investigation of 30 days to maturity due to the interpolation problem. In the end of this chapter histograms of the daily changes can be found.

	Max	Min	Stdev
Implied 30d	**17,94**	**-19,90**	**3,82**
OMX 30d	19,75	-12,63	2,95
100Scaling 30d	26,23	-16,96	3,98
1000Scaling 30d	24,99	-15,95	3,80
30Historical	15,22	-11,55	1,93
50Historical	8,45	-8,01	1,17
75Historical	6,84	-5,87	0,80
100Historical	5,61	-4,33	0,63
GARCH	49,69	-9,37	5,20

Table 13 *Presentation of the daily changes in the implied volatility 30 days and the investigated models for* ***Handelsbanken***

	Max	Min	Stdev
Implied 30d	**84,47**	**-84,93**	**7,41**
OMX 30d	19,75	-12,63	2,95
100Scaling 30d	26,57	-27,83	4,59
1000Scaling 30d	29,66	-19,65	4,54
30Historical	17,49	-11,51	1,90
50Historical	8,65	-8,08	1,16
75Historical	5,32	-6,15	0,80
100Historical	4,49	-5,04	0,63
GARCH	50,32	-10,48	4,20

Table 14 *Presentation of the daily changes in the implied volatility 30 days and the investigated models for* ***Volvo***

The focus will be put on the standard deviations of the daily changes. Looking at the 30 days to maturity, a large difference between Handelsbanken and Volvo can be noted. Volvo has a considerably higher standard deviation. The OMX index has a lower standard deviation than both Handelsbanken and Volvo. For Handelsbanken, the scaling factor model almost has equal standard deviation as the implied volatility and even if the scaling factor model is having less fit with VOLVO, it is clearly better than the historical models. The values of the standard deviation of the GARCH volatilities are higher than the one of implied volatility for Handelsbanken and lower than the implied volatility for Volvo.

	Max	Min	Stdev
Implied 3m	**12,93**	**-12,70**	**2,02**
OMX 3m	13,79	-14,87	1,84
100Scaling 3m	24,89	-19,09	2,65
1000Scaling 3m	17,51	-18,64	2,27
30Historical	15,22	-11,55	1,57
50Historical	8,45	-8,01	0,97
75Historical	6,84	-5,87	0,66
100Historical	5,61	-4,33	0,50
GARCH	49,69	-9,37	4,10

Table 15 *Presentation of the daily changes in the implied volatility 3 months and the investigated models for* ***Handelsbanken***

	Max	Min	Stdev
Implied 3m	**80,84**	**-63,10**	**4,81**
OMX 3m	13,79	-14,87	1,84
100Scaling 3m	18,59	-20,50	2,91
1000Scaling 3m	20,71	-22,45	2,72
30Historical	17,49	-14,06	1,78
50Historical	9,10	-8,08	1,05
75Historical	5,75	-6,15	0,73
100Historical	4,65	-5,04	0,57
GARCH	50,32	-10,48	3,70

Table 16 *Presentation of the daily changes in the implied volatility 3 months and the different models for* ***Volvo***

A comparison of the standard deviation for implied volatility 3 months to maturity shows that the scaling factor models have higher values and also closer values to the implied volatility than the historical models. For Handelsbanken the standard deviation in absolute terms of the OMX index is closer to the implied volatility than the scaling factor models. All models are also having too low values of the standard deviations compared to the implied volatility of Volvo.

	Max	Min	Stdev
Implied 6m	**11,08**	**-13,71**	**1,72**
OMX 6m	22,74	-9,25	1,65
100Scaling 6m	28,79	-11,34	2,21
1000Scaling 6m	28,34	-11,38	1,96
30Historical	15,22	-11,55	1,57
50Historical	8,45	-8,01	0,97
75Historical	6,84	-5,87	0,66
100Historical	5,61	-4,33	0,50
GARCH	49,69	-9,37	4,10

Table 17 *Presentation of the daily changes in the implied volatility 6 month and the investigated models for* ***Handelsbanken***

	Max	Min	Stdev
Implied 6m	**60,68**	**-56,77**	**4,81**
OMX 6m	22,74	-9,25	1,65
100Scaling 6m	30,29	-14,61	2,67
1000scaling 6m	34,07	-13,63	2,39
30Historical	17,49	-14,06	1,78
50Historical	9,10	-8,08	1,05
75Historical	5,75	-6,15	0,73
100Historical	4,65	-5,04	0,57
GARCH	50,32	-10,48	3,70

Table 18 *Presentation of the daily changes in the implied volatility 6 month and the investigated models for* ***Volvo***

The investigation of 6 months to maturity is showing similar results as 3 months. For Volvo the scaling factors are having values of the standard deviation closer to the implied volatility than the ones of the historical models. The values of the standard deviation are too low compared to the ones of the implied volatility. For Volvo GARCH is having the closest value of the standard deviation and the farthest for Handelsbanken.

It is important to study the daily changes in the volatility to get to knowledge in how fast the volatility models are adjusting with sudden changes in the market. Another important reason to evaluate the daily changes is to get knowledge about the risks of holding an option. A very common method of measuring risk is Value at Risk. Value at Risk is the expected maximum loss over a target horizon within a given confidence interval[8]. Interval commonly used is 95%, 99% and 99,9%. The volatility is one risk factor when computing Value at Risk. If an empirical distribution will be used in the calculation, the daily changes in the volatility will be very important. In figure 13-29, the histograms of the daily changes for each volatility model are compiled. The histograms are showing that the variation in the daily changes are too low for the historical volatility models, which implies that the risk measurement will not take into account that bigger changes in the volatility can occur from day to day. Among the historical models, a shorter estimation period seems to be the most appropriate since the variation is larger for shorter periods. The scaling factor models are having a similar approach as the implied volatility in the histograms and this model also seems to be the most proper model to use in this aspect.

[8] Jorion, 2001

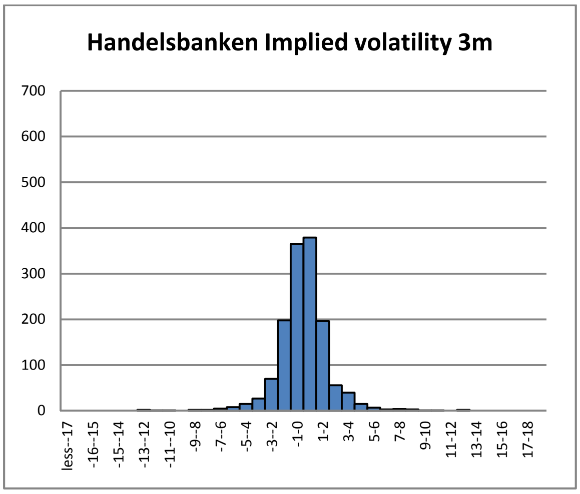

Figure 13 *Histogram over the daily changes in Handelsbanken implied volatility 3 months*

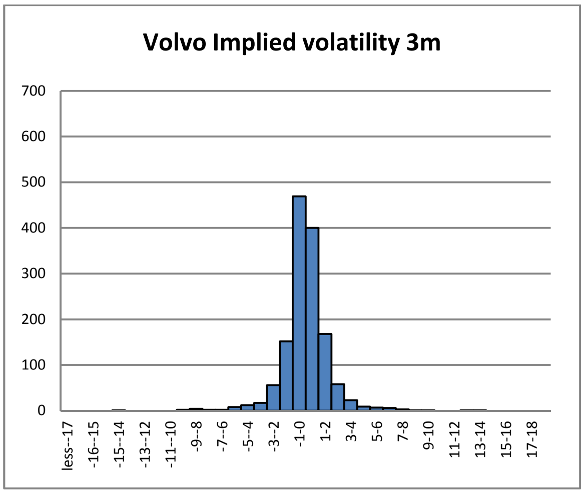

Figure 14 *Histogram over the daily changes in Volvo implied volatility 3 months*

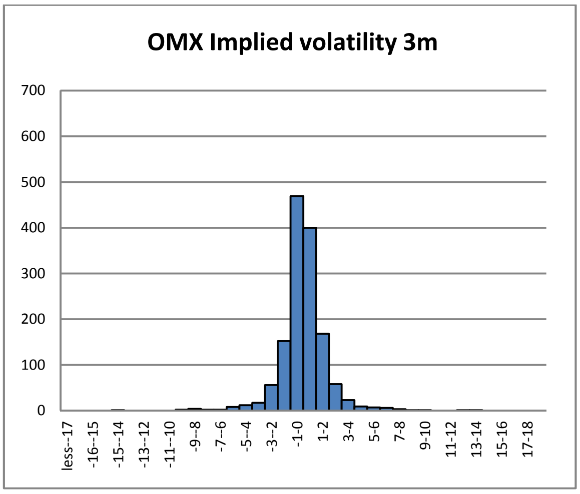

Figure 15 *Histogram over the daily changes in OMXS 30 implied volatility 3 months*

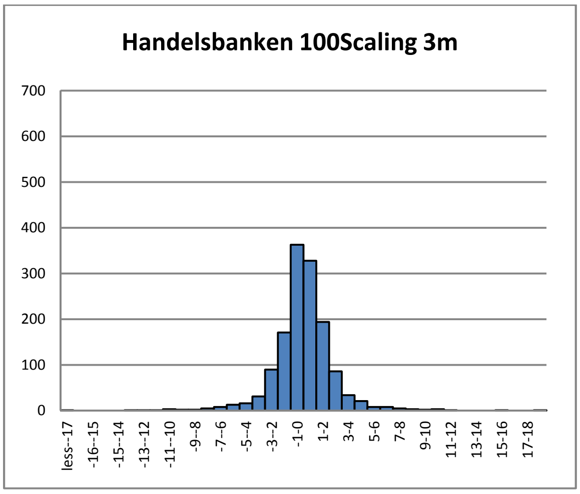

Figure 16 *Histogram over the daily changes in Handelsbanken 100 Scaling factor 3 months*

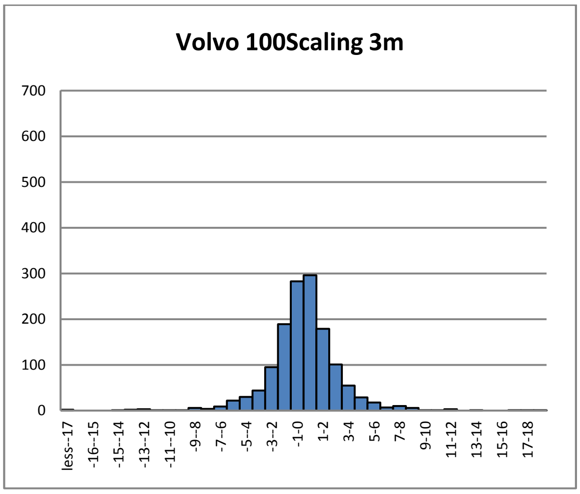

Figure 17 *Histogram over the daily changes in Volvo 100 Scaling factor 3 months*

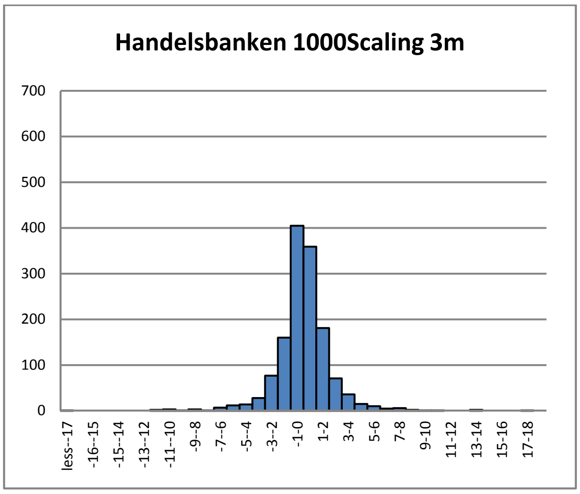

Figure 18 *Histogram over the daily changes in Handelsbanken 1000 Scaling factor 3 months*

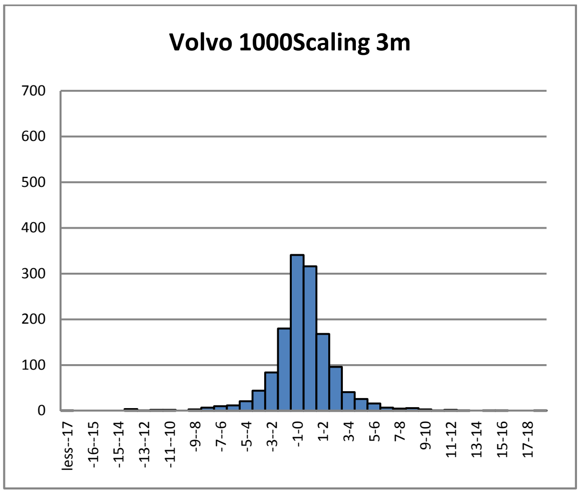

Figure 19 *Histogram over the daily changes in Volvo 1000 Scaling factor 3 months*

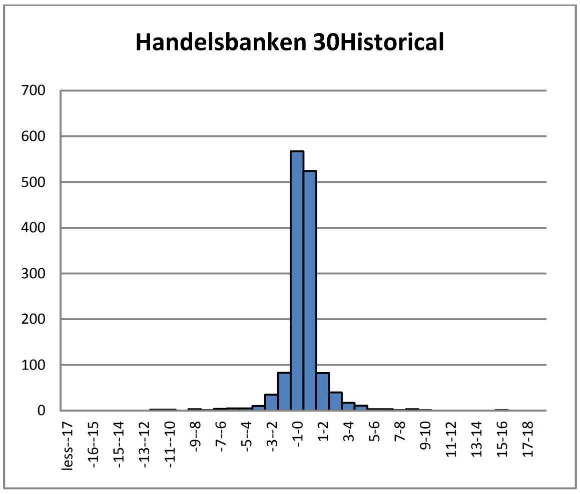

Figure 20 *Histogram over the daily changes in Handelsbanken 30 days historical*

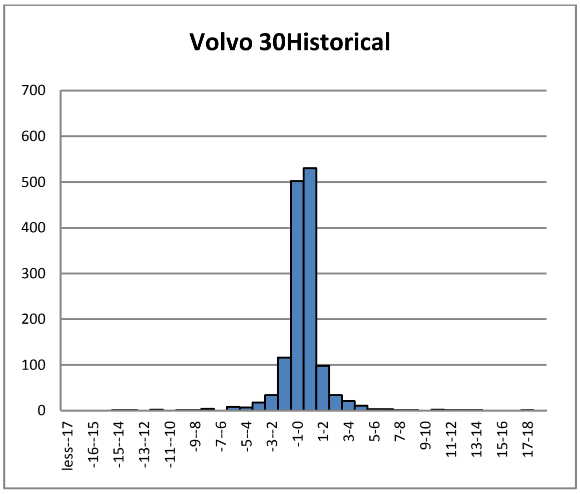

Figure 21 *Histogram over the daily changes in Volvo 30 days historical*

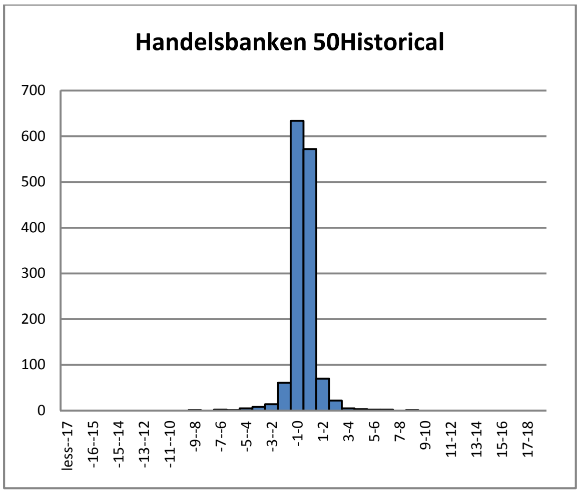

Figure 22 *Histogram over the daily changes in Handelsbanken 50 days historical*

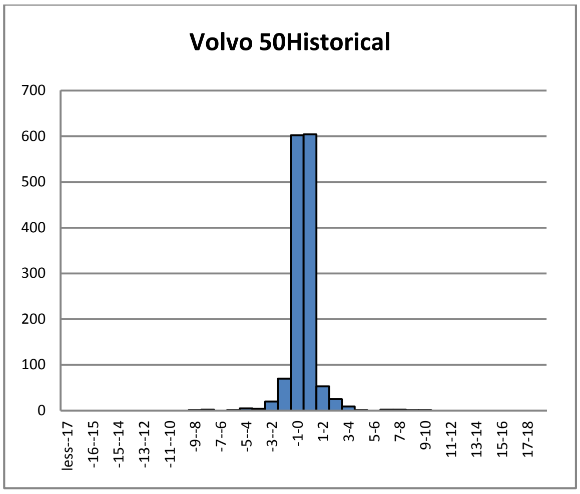

Figure 23 *Histogram over the daily changes in Volvo 50 days historical*

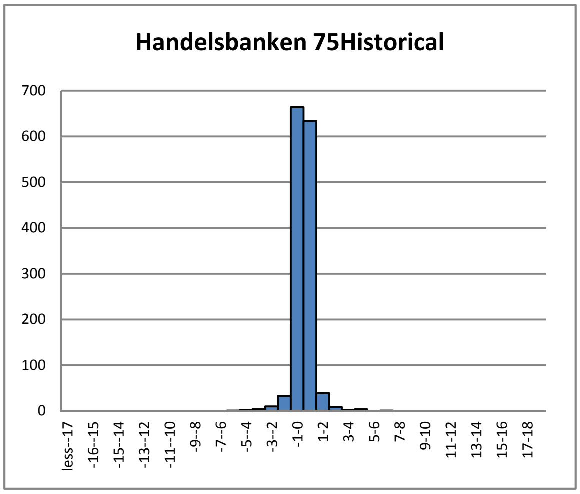

Figure 24 *Histogram over the daily changes in Handelsbanken 75 days historical*

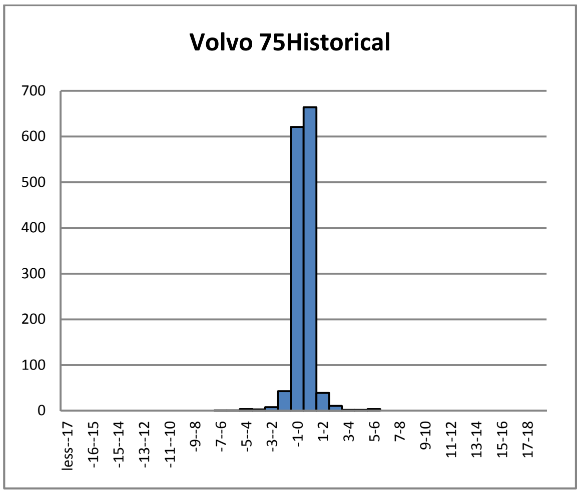

Figure 25 *Histogram over the daily changes in Volvo 75 days historical*

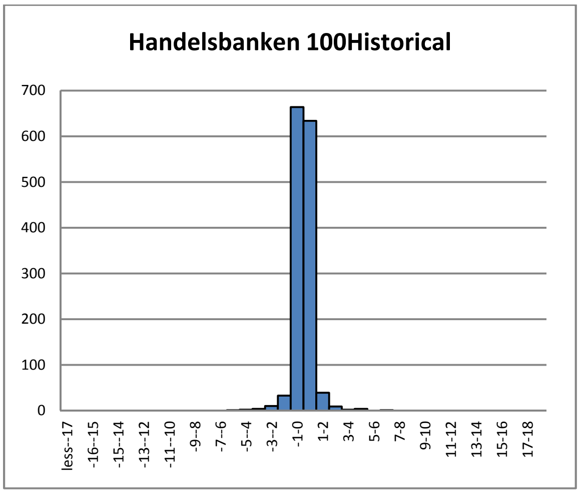

Figure 26 *Histogram over the daily changes in Handelsbanken 100 days historical*

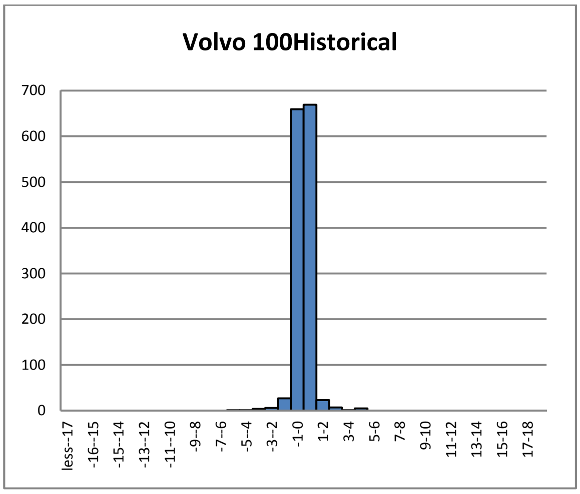

Figure 27 *Histogram over the daily changes in Volvo 100 days historical*

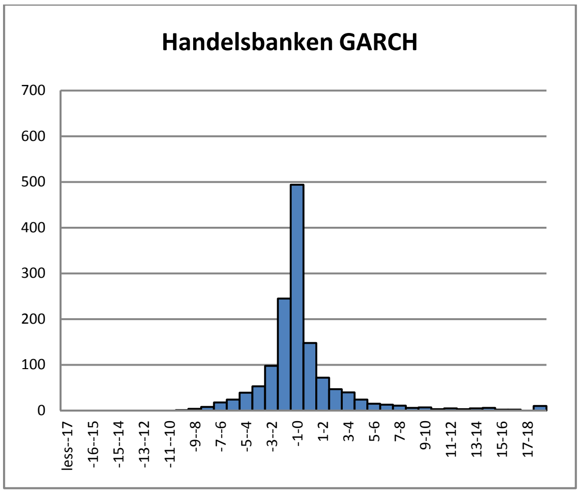

Figure 28 *Histogram over the daily changes in Handelsbanken GARCH*

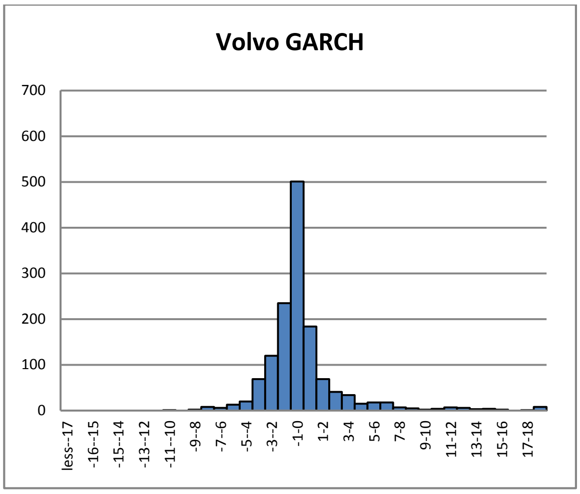

Figure 29 *Histogram over the daily changes in Volvo GARCH*

13 Conclusions

This thesis is written in a valuation and risk perspective, where different volatility models are evaluated based on how well they are imitating the implied volatility. The aim of the thesis is to investigate how to manage volatility when options are illiquid and the implied volatility is unknown. In order to make it possible to evaluate the models all assets in this investigation have liquid options. In the investigation both the models average deviations from the implied volatility and the daily changes in the volatility are studied.

When studying the implied volatility graphs it can be concluded that the data contain errors. The implied volatility 30 days to maturity had heavy periodic changes before 2008. Those were probably caused by the absence of liquid options to interpolate between. Therefore, the data set was cut in the beginning of 2008, meaning that only data from 2008 and after have been used in the investigation of the implied volatility 30 days to maturity.

The goodness of fit may vary over time. At one moment one model may seem to perform better than the other models while the situations can be reversed at a later point in time. The performance of a model may also vary from asset to asset.

A weakness of the models investigated in this thesis is that the underlying asset must be actively traded. If that is not the case the models will not work since they all are depending on daily returns of the underlying asset, although active trading in the underlying asset often is more common than trading in options of the asset.

For the GARCH and the historical models there are only one time series that is compared to three implied volatility time series with different times to maturity. This is of course a weakness of these models, since the implied volatility is varying with time to maturity. During the estimation period there are daily deviations between the time series of the implied volatility with different times to maturity, but the average value of the implied volatility time series with different times to maturity is similar. The deviations can for example depend on an upcoming days of report for the issuer of the underlying asset. When evaluating different strike levels, the problem with only one value of the volatility for each asset will be even more extended. By using the scaling factor model a volatility value can be derived for each implied volatility value available for the index, making it possible to create a whole volatility surface for an asset out of the volatility surface of an index. Investigating the whole volatility surface will be an issue for further research, since this investigation is only on at the money options.

The GARCH model is difficult to evaluate. During this investigation it has been concluded that the GARCH model is overreacting on high returns. After a chock, the volatility is decreasing but in a more moderate speed. This tendency can also be seen in the histograms over the daily changes in the volatility, where the two highest staples in figure 28 and 29 are representing small decreases in the daily volatility.

The scaling factor models, particularly the one with 100 observations, had for Handelsbanken less fit to the implied volatility than what the unmodified OMX index had. The scaling factor was too high, causing a too high value of the volatility. The scaling factor model was, however, still performing better than the historical models. It is however, interesting that an unmodified index implied volatility is performing better than the models. The conclusion of this is that the scaling factor is only contributing noise to the data when the values of the implied volatility of the index and the asset are close to each other.

When studying Volvo, it was shown that the scaling factor models were performing even better for this underlying asset compared to what the historical models did. Also, when studying the daily changes, the scaling factor models performed well compared to the historical models. The scaling factor model using 1000 observations to calculate the scaling factor has in this investigation an overall better results than the other models.

If a historical volatility model is to be used, it can be concluded that 50-75 observations seems to be most appropriate to use to imitate the implied volatility. Only when studying daily changes in the volatility it may be better to use a shorter estimation period, since the standard deviation of shorter estimation periods is higher in general. The difference is, however, small and the models are still considerably inferior compared to the scaling factor models.

14 Further research

In this thesis, the implied volatility is presumed to be the correct volatility measurement since the models are evaluated based on how well they are imitating the implied volatility. The thesis is not investigating whether the implied volatility is giving the best valuation and/or risk calculation.

Further, since the investigation is made on large companies with liquid options, no conclusions can be made on how well the models will perform on small companies with less liquid options. It is for illiquid options that usage of another model than implied volatility is needed.

Since the scaling factor is in many respects performing better than the historical model, research in this model on other underlying assets types would be interesting and also to see how the model would imitate the whole volatility surface.

15 Biography

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