Modeling Non-maturing Liabilities

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Abstract

Non-maturing liabilities, such as savings accounts, lack both predetermined maturity and reset dates due to the fact that the depositor is free to withdraw funds at any time and that the depository institution is free to change the rate. These attributes complicate the risk management of such products and no standardized solution exists. The problem is important however since non-maturing liabilities typically make up a considerable part of the funding of a bank. In this report different modeling approaches to the risk management are described and a method for managing the interest rate risk is implemented. It is a replicating portfolio approach used to approximate the non-maturing liabilities with a portfolio of fixed income instruments. The search for a replicating portfolio is formulated as an optimization problem based on regression between the deposit rate and market rates separated by a fixed margin. In the report two different optimization criteria are compared for the replicating portfolio, minimizing the standard deviation of the margin versus maximizing the risk-adjusted margin represented by the Sharpe ratio, of which the latter is found to yield superior results. The choice of historical sample interval over which the portfolio is optimized seems to have a rather big impact on the outcome but recalculating the portfolio weights at regular intervals is found to stabilize the results somewhat. All in all, despite the fact that this type of method cannot fully capture the most advanced dynamics of the non-maturing liabilities, a replicating portfolio still appears to be a feasible approach for the interest risk management.

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Chapter 1

Introduction

1.1 Background

Non-maturing liabilities are as the name implies characterized by the absence of a predetermined maturity. A typical example of this type of products are most deposit accounts, where the depositor is free to at any time deposit or withdraw money from the account, changing the account balance. Another dimension of uncertainty is added by the fact that the deposit rate may be changed at any time by the depository institution, i.e. the bank. Because of these attributes the risk management of non-maturing liabilities proves difficult. Also, non-maturing liabilities typically make up a large part of the funding of a bank which makes this problem important.

The risk management problem can be divided into two different parts; liquidity risk and interest rate risk. The liquidity risk arises from the fact that the future cash flows of the non-maturing accounts are unknown which means the bank does not know how much funds to have ready at any given time. The interest rate risk on the other hand occurs due to the unknown future rates and the prospect of changed rates having an impact on the profit and liquidity.

There are many unknown variables in this problem and they depend on each other in intricate ways. Costumers deposit money on the accounts and receive a deposit rate. The bank may however invest money in for example bonds at market rates, which are typically higher than the deposit rate. This spread between rates is a source of profit to the bank. The deposit rate is naturally dependant on the market rates since the bank continually adjusts the deposit rate to make a profit. But this dependence is not necessarily consistent or coherent, but varies depending on the situation. Many models addressing this issue assume that the relation between market and deposit rates is the same regardless if the rates increase or decrease, however this is usually not the case. Banks tend to be quicker to adjust the deposit rates in times of rising market rates than when rates are falling (see e.g. O'Brien (2000)). Also not only banks reacts to changes in rates, but costumers may react as well, at a change in rates wanting to for example withdraw their money and place them somewhere they believe is more lucrative. But not all costumers respond the same, some pay big attention to these changes and some do not react at all. Also the reactions tend to be somewhat delayed. Then there are of course a multitude of other reasons why a certain costumer behaves a certain way, personal circumstances, economic situation, perhaps time of the year etc.

So what is unknown today are the future market rates, which reflect the movements on the market and the economic situation, the future deposit rate which has a connection to the market rates, although how is unknown and may vary, and the future volumes on the accounts, which origin from the customer behavior and could be impacted by the rates. All these aspects have an influence on the future cash flows of the bank and therefore impact the risk management.

Typically, banks handle the risk of the non-maturing liabilities by making assumptions regarding the unknown maturity. This could either be very simple assumptions or assumptions based on deeper theory and derived from examining historic data. According to Kalkbrener and Willing (2004) many banks use the approach of dividing the total volume on the accounts into two parts; a stable part (core balance) and a floating part. In some literature this is referred to as *non-maturation theory*. This is reasonable since, due to the large number of costumers and the comparably small average volume of each costumer, most of the volume will remain with the bank as not all costumers will behave alike, withdrawing large amounts at the same time. The floating part is then seen as volatile and is assumed to have a very short maturity, while the stable part is assigned a longer maturity, or subdivided into portions allocated to different maturities.

Another approach that has grown more common for handling non-maturing products is the *replicating portfolio approach*. This approach is described further below but in short it is a way of assigning maturities and re-pricing dates to the non-maturing accounts by creating a portfolio of fixed income instruments that imitates the cash flows of the accounts. According to Maes and Timmermans (2005) most large Belgian banks rely on a *static replicating portfolio approach* to handle the interest rate risk of their non-maturing accounts. They also mention that some Belgian banks use or have been experimenting with more complicated modeling approaches such as *dynamic replicating portfolios* and models based on Monte Carlo simulations. These approaches are also described further below.

But so far there is not any general solution or framework in place for handling non-maturing products, instead different banks use their own way. The regulation provided by FSA (and FI in Sweden) is not very redundant and the evolution has been slow due to the complexity of this problem. Various papers and articles have been written on the subject and models have developed over the years, becoming theoretically more sophisticated and growing more realistic. However these models turn very advanced to capture the difficulties involved in this problem and the banks tend to be reluctant to implement such complicated models and instead stick to less complex ones.

1.2 Aim and Scope

A bank needs to handle the liquidity and interest rate risk for its non-maturing liabilities as well as for other products. The risk measurements of these non-maturing products need to be handled in a way so they can be incorporated in the general risk management framework used by the bank. Also the risk measurements for non-maturing products need to be in line of what is demanded from the authorities. However since these demands are very general there is room to use methods and measurements that would be best suited for a specific bank.

The aim of this paper is twofold. This thesis is carried out at Svenska Handelsbanken AB (publ), hereafter known as 'the Bank', and firstly it aims to provide an overview of models handling non-maturity liabilities to give a view of what could be done in this area, both regarding liquidity and interest rate risk management, and to be a foundation for further analysis performed at the Bank.

Secondly it aims to formulate a way to handle the interest rate risk management for some specific non-maturing liabilities by making an analysis of real world data. The goal is to calculate measures for the interest rate risk of these liabilities that fulfill the following conditions

- The modeling error should be as small as possible. Since these liabilities by definition lack maturity and re-pricing dates assumptions or approximations need to be made to be able to calculate interest rate risk measures and the goal is to keep these assumptions as well in line with reality as possible.

- The interest rate risk should be as low as possible. Different assumptions or modeling approaches would of course yield different measurements and the chosen way should be the least risky one.

- The profit, i.e. the spread between deposit rates and a market investment, should be as big as possible.

- The method used should be possible to implement in a realistic way.

The analysis is performed on data provided by the Bank and will focus on non-maturing liabilities in the form of savings accounts and on the replicating portfolio modeling approach.

The structure of this paper will be as follows. Section two will be an introduction to related literature in this area and a review of existing models and methods which deal with handling non-maturing products. Section three will be a more specific description of the risk management framework in place at the Bank, this will give a better view for further analysis and conclusions. Section four will be a display and an analysis of the data provided by the

Bank, i.e. volumes on different accounts, deposit rates, market rates and the relations between them. Section five will focus on the interest rate risk management with a display, implementation and evaluation of the chosen method. Finally section six will conclude the results.

Chapter 2

Theoretical Background

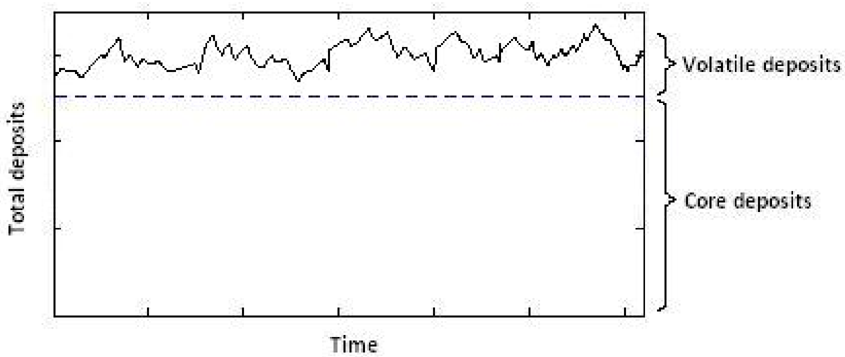
2.1 Overview of Methods and Models

This section aims to provide a brief overview of the different ways to model non-maturing products. In the following sections a few types of models are described in greater detail and after that follow a review of previous literature on the subject.

The most basic way to handle non-maturing products is to make assumptions regarding their maturity. This would result in approximated cash flows that are easier to handle than the original unknown ones. Of course the simplest way is to arbitrarily assume one maturity for all non-maturity liabilities. However this is not the safest or most effective way to go around. Assuming very short maturity may be accurate in the way that the future is unknown and technically, since customers have the option to immediately remove their funds, all may be gone tomorrow. As mentioned earlier this is however usually not the case in reality. So investing all deposited funds at short maturities would keep you from being unable to meet withdrawals, but is a costly approach since it also prevents more profitable investments on longer term. Assuming a longer maturity on the other hand could be more gainful but also increases the risk of having insufficient liquid funds.

A slightly more advanced approach is the use of the previously mentioned *non-maturation theory* (see e.g. OeNB (2008)) to split the volume into a stable part (core part) and a volatile part, where the stable component is assumed to have a long maturity and the volatile a short. Preferably this is done by examining historical data (in Figure 1 is an example of how it could look). This method could be cultivated further by examining the natural rate of decay and use this to assign a maturity profile, i.e. the *outflow rate method* (OeNB (2008)). This is done empirically by observing declines in volume, calculating an annual percentage outflow rate and then allocating the maturities accordingly.

Figure 1: Example of core vs. volatile deposits



To achieve more accurate modeling more advanced relations between the unknown variables can be employed. For example the use of regression to relate the deposit rate to the market rate as in the earlier mentioned *replicating portfolio approach*. This approach is described further below. Also these methods could be, and often are, combined. For example one could assign the core part of the volume to a suitable long maturity and construct a replicating portfolio for the volatile part. Or use a replicating portfolio with medium and long maturities for the core part, while keeping the volatile part at a very short maturity.

However all of these methods are *deterministic* in the sense that there is no randomness involved. In a deterministic model, even if the relations and formulas are advanced, only one possible scenario is considered, i.e. the one in the historical data the calculations are based on. In contrast, a *stochastic model* would allow you to consider multiple possible future scenarios. In a stochastic model the evolution of interest rates is described as a stochastic process, a term structure model, and from that the expected future cash flows are estimated via relations between the volumes, market rates and deposit rates.

This report will use the distinction framed by Bardenhewer (2007) who state that the models for non-maturing products fit into two main classes; replicating portfolio models and option-adjusted spread (OAS) models, where the OAS models are stochastic models based on stochastic interest rate term structures. Both types are described closer in following sections.

2.2 Replicating Portfolio Models

The replicating portfolio approach is a way to transform the non-maturity liabilities into a portfolio of fixed rate products with known maturities. This would provide cash flows that are easy to handle and use in calculation of for example interest rate risk. Basically the risk profile of the non-maturing liabilities would be approximated with the risk profile of the portfolio which could be easily calculated since it would consist of common instruments.

Depending on the aim of the modeling the portfolio could either be a fictive portfolio used to get estimated risk measures or it could be a real world portfolio to be used as a hedge or investing strategy.

The replicating portfolio should yield cash flows that as closely as possible matches the cash flows of the non-maturing product. This is done by the use of multiple regression between the market rates and the deposit rates. A few market instruments are chosen and the deposit rate is assumed to be a linear combination of these market rates plus a constant margin.

For example

$$R = w_1 \cdot r_{1m} + w_2 \cdot r_{6m} + w_3 \cdot r_{12m} + w_4 \cdot r_{2y} + w_5 \cdot r_{5y} + m$$

where R is the deposit rate, m is the constant margin, $(w_1, \dots, w_5)$ are the portfolio weights and $r_{1m}$is the 1-month market rate, r_{6m} the 6-month market rate etc.

Then the weights of the replicating portfolio are derived from looking at historical data. It becomes an optimization problem with the constraint that the sum of the weights must be one. Additional constraints or criteria are also needed to determine an efficient portfolio. One common constraint is that no short-selling is allowed, i.e. no negative weights. Other criteria that could be used are to maximize the margin or minimize the variation in the margin. When the weights are determined the volume of the non-maturing accounts is divided accordingly, yielding estimated future cash flows.

This method is called a *Static Replicating Portfolio Approach*. The weights are computed once and then used continually and maturing investments are re-invested at the same maturity, keeping the weights constant. A more realistic, and more complicated, strategy is a *Dynamic Replicating Portfolio Approach*. It is a stochastic approach to the standard replicating portfolio where future interest rate scenarios are simulated and used as a basis for determining an optimal portfolio, instead of simply using historical data. This approach also uses multistage optimization procedure to calculate the weights, allowing the weights to be changed continuously (see Frauendorfer and Shürle (2007)).

Two examples of static replicating portfolios in the literature are Maes and Timmermans (2005) and Bardenhewer (2007). Both are described below.

2.2.1 Replicating Portfolio of Maes and Timmermans (2005)

According to Maes and Timmermans (2005) most large Belgian banks rely on a particular variant of the static replicating portfolio model to estimate the duration or interest rate sensitivity of their non-maturing accounts.

The calculations are based on the idea of investing the volume from the accounts in a portfolio of fixed-income assets such that an objective criterion is optimized subject to the constraint that the portfolio exactly replicates the dynamics of the deposit balance over an historic sample period. Then the duration of the saving deposits is estimated as the duration of the replicating portfolio.

The criterion to be optimized is to select the portfolio that yields the most stable margin, represented by the portfolio that minimizes the standard deviation of the margin.

Problem formulation:

$$Min\ std\left(r_p - R\right)$$

Subject to the constraints:

- *(i)* $\sum_{\mathrm{i}=1}^{\mathrm{n}} w_i r_i = r_p$, *where* $\sum_{\mathrm{i}=1}^{\mathrm{n}} w_i = 1$
- *(ii)* *No short sales are allowed, i.e.* $w_i \geq 0, \forall i$
- *(iii)* *The volume of deposits is perfectly replicated by the portfolio investment at all sample dates*

where r_p is the return of the replicated portfolio, R is the deposit rate, and $\{w_1, \ldots, w_n\}$ is the vector of weights corresponding to the set of n available standard assets with different maturities, each with return r_i.

Another mentioned alternative for objective criterion is to maximize the risk-adjusted margin, measured by the margin's *Sharpe ratio*. The Sharpe ratio can be generally described as the expected return of an asset divided by the square root of the variance of the return. For this case the expected return would correspond to the margin, so the Sharpe ratio would be calculated as the ratio of the expected margin to the standard deviation of the margin. When calibrating the portfolio over a historical sample period the expected margin would be estimated as the average margin so the Sharpe ratio of the margin would be formulated as

$$S_m = \frac{E[m]}{\sqrt{var[m]}} = \frac{\bar{m}}{\sigma_m}$$

where $\bar{m} = \overline{r_p - R}$ is the average margin and $\sigma_m = std(m) = std\left(r_p - R\right)$ is the standard deviation of the margin.

The problem formulation of Maes and Timmermans (2005) also include that the total volume are divided into three parts; interest rate insensitive core deposits, volatile deposits and remaining balances. The core deposits are invested at a chosen long horizon (in their example seven years), the volatile deposits are invested at the interest rate risk free short horizon (one month) and the remaining balances are replicated by the portfolio. This would avoid the possibility of changes of volume to impact the interest rate risk.

2.2.2 Replicating Portfolio of Bardenhewer (2007)

Bardenhewer (2007) takes a slightly different approach. He takes volume changes more specifically into account and divide the total volume into a trend component and an unexpected component. The total volume is then replicated by a portfolio under the condition that the unexpected component of the volume by default is assigned to the shortest maturity or more specifically the rate of return of the market instrument with the shortest maturity (which is the one month market rate in this case). The portfolio is estimated by using ordinary least squares.

The trend could either be estimated from historical data or be determined by expert knowledge, i.e. someone with information or understanding of the product could formulate an expectation of future volume changes. If the trend is estimated a linear, quadratic or exponential trend could be applied depending on the properties of the data. A linear trend for example would look as follows.

Linear trend:

$$V_t = \beta_0 + \beta_1 \cdot \Delta_t + \sum_i \kappa_i \cdot (r_{i,t} - \overline{r_i}) + \delta \cdot (cr_t - \overline{cr}) + \varepsilon_t$$

where V_t is the total volume at time t, Δ_t is the time between 0 and t, β_0 and β_1 are the linear parameters to be estimated, $\sum_i \kappa_i \cdot (r_{i,t} - \overline{r_i})$ is a contribution to the trend incorporating influence on the volume caused by market interest rates, $\delta \cdot (cr_t - \overline{cr})$incorporates influences from the deposit rate and ε_t is the time *t* residual.

The terms $\sum_i \kappa_i \cdot (r_{i,t} - \overline{r_i})$ and $\delta \cdot (cr_t - \overline{cr})$ consists of parameters to be estimated, the κ_i:s and δ, $i \in \{1, \dots, I\}$ are the maturities of market rates, $r_{i,t}$ is the time *t* market rate with maturity i, $\overline{r_i}$ is the average rate with maturity i over the estimation period, cr_t is the deposit rate (or client rate) at time t, $\overline{cr}$ is the average deposit rate over the estimation period.

Another element included by Bardenhewer (2007) is that of using a moving average instead of rate of return for the market instruments used in the replicating portfolio. The moving average is then, for a market instrument with a maturity of *j* months, the average of the *j*-month market rate over the last *j* months.

$$ma_{j,t} = \frac{1}{j} \cdot \sum_{i=0}^{j-1} r_{j,t-i}$$

where $r_{j,t}$ is the *j*-month market rate at time *t*.

The moving average is implemented in reality by dividing each portfolio weight into monthly maturities. This means that for example the estimated weight corresponding to a market instrument with a maturity of one year, this could be for example 20%, would be divided into twelve parts each invested in the one-year instrument with monthly intervals. So each month one contract from each maturity or weight would mature and being re-invested at the same maturity. This would also yield a connection between the market rates and the deposit rates that indirectly would take into account that the lag of the deposit rate, that it usually is a bit slow in adapting to the market rates.

When a trend function is determined and using moving averages the estimated deposit rate is formulated as

$$cr_t = \frac{F_t(.)}{V_t} \cdot \sum_j w_j \cdot ma_{j,t} + \frac{A_t(.)}{V_t} \cdot r_{1,t} + \theta_0 + \eta_t$$

where $F_t(.) = F(\Delta_t, r_{i,t}, \overline{r_i}, cr_t, \overline{cr}, \hat{\beta}_0, \dots, \hat{\beta}_4, \hat{\kappa}_i, \hat{\delta})$ is the estimated trend volume at time t. The percentage of the total volume explained by the trend, $\frac{F_t(.)}{V_t}$, is allocated according to the portfolio weights that are to be estimated. w_j is the portfolio weight corresponding to maturity j and $ma_{j,t}$ is the moving average interest rate with maturity j at time t.

$A_t(.)$ is the balancing volume which is the volume not explained by the trend, $A_t(.) = V_t - F_t(.)$. That part of the total volume is by default contributing to the one-month weight and therefore multiplied by the one-month market rate at time t, $r_{1,t}$.

$\theta_0 + \eta_t$ is the spread between the observed deposit rate at time t and the deposit rate modeled as the yield of the portfolio plus the yield of the balancing volume invested at one month. θ_0 represents a constant factor that is to be estimated, this would correspond to the margin from the previous example of a replicating portfolio albeit with a minus sign, and η_t is the time t residual.

So with this formulation the optimization problem to obtain optimal weights consists of minimizing the volatility of this spread, i.e. keeping the fluctuations of the margin as small as possible, subject to the constraints

$$w_j \geq 0, \forall j \quad \text{and} \quad \sum_j w_j = 1$$

With this problem formulation the optimal weights can be estimated and when implemented are divided into the earlier mentioned monthly parts re-invested each month. And the future observed volume that falls outside the trend estimation is continually being handled by adding and withdrawing from the one-month weight of the portfolio, i.e. buying and selling one-month market instrument contracts, so the one-month part of the portfolio works as a buffer for unexpected volume changes.

The theory also includes one aspect to this problem called *Market Mix*. This is incorporated since this approach focuses on the bank's option to adjust the rate, via the dependencies between market and deposit rates. But there is also the customer behavior to account for and the risk of them withdrawing money, in particular the risk of them withdrawing larger amounts than may be covered by the one-month buffer. Therefore a liquidity constraint is added, known as a *Market Mix*. It is done by computing both optimal portfolio weights and liquidity constraints for each maturity used in the portfolio. These liquidity constraints are portfolio weights calibrated to the volume instead of the deposit rate, a simple way is to use the maximum historical volume change for each maturity. Then these two weight measures are compared in a matrix and if the liquidity constraint yields a larger weight than the original portfolio weight for a certain maturity, this is used instead.

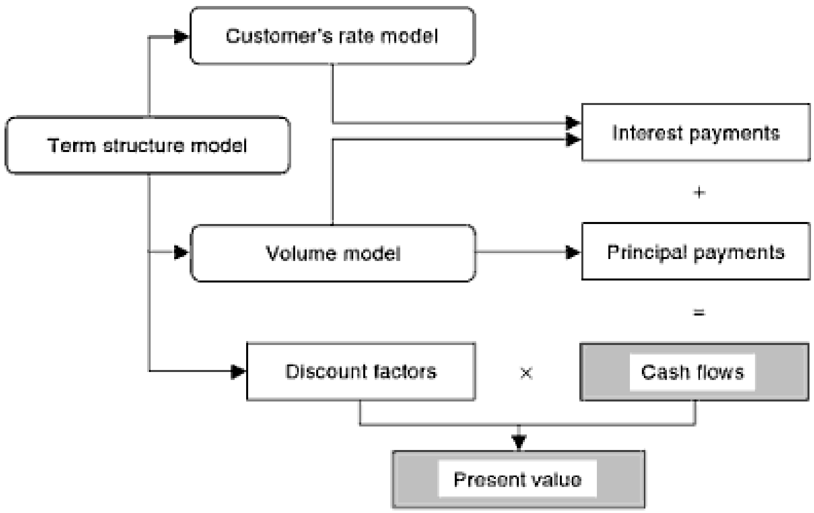
2.3 OAS Models

The option-adjusted spread can be explained as a spread that represents the added value of having an option. Consider for example the difference between a callable and a non-callable bond. The callable bond is more expensive due to the fact that there is an option involved, and the OAS describes the spread the holder of the bond receives for providing this option. Or in other words the OAS is the spread that must be added to the market rate so that the market value of the option equals the option value. For non-maturing liabilities an OAS model aims to capture and model the options that are embedded in them, i.e. the customers' option to add or withdraw money anytime and the bank's option to change the deposit rate, and attempts to capture the value of these options. So it is a way of viewing non-maturing products as highly complicated options and applying option pricing theory to deal with them.

OAS models are stochastic models and their foundation is the term structure model which will generate possible interest rate scenarios. One important term structure model is the *Vasiček interest rate model* which is a one-factor short term, mean reverting model based on Brownian motion. Other term structure models used for modeling non-maturing products have been various extensions of the Vasiček model such as the *Cox-Ingersoll-Ross model (CIR)* and the *Hull-White model*, along with rate models based on the *Heath-Jarrow-Morton framework (HJM)*.

To capture the options in an OAS model the further requirements are a deposit rate model and a volume model. The deposit rate model typically depends on the market rate, usually with some time lag. The volume model aims to depict the volume changes on the accounts and also shows the customer behavior. It could depend both on market rates and other factors, like seasonality or time lag. From this interest rate scenarios can be generated stochastically by use of Monte Carlo simulation and future cash flows can be estimated. (Some models in the literature have derived closed form solutions for the present value instead of using simulation.) Then, when the cash flows are estimated, the present value of the non-maturity liabilities could be calculated. This is done frequently in the literature, where most models are valuation models, although by somewhat different approaches. As mentioned by Bardenhewer (2007) the value of the embedded options could either be part of the present value or added implicitly as a spread. Both approaches fall under his definition of OAS models. However, according to some literature, the OAS approach is just the latter approach, i.e. discounting the expected future cash flows at a discount rate which includes a spread to account for the riskiness of the cash flows. See for example Maes and Timmermans (2005) who more or less refer to what is here called OAS models as *Net present value Monte Carlo simulation models* and then make the distinction between the *OAS approach* and the *contingent claim* or *no-arbitrage approach*. The latter is done by manipulating the expected cash flows by subtracting a risk premium that reflects the risk, and then these certainty equivalent cash flows may be discounted at the risk-free rate.

Figure 2: The functionality of an OAS model (source Bardenhewer (2007))



2.4 Related Literature

Here follow a review of previous literature on this subject. The aim of different models and methods in the literature differ somewhat. Some articles describe valuation models that aim to calculate a present value of the future cash flows associated with the non-maturing products, some want to find a hedging strategy, some focus on the management of liquidity risk and some on interest rate risk. Also not everyone look at the exact same type of instruments but the bottom line problem remains the same; modeling non-maturing products.

Two rather well-known articles regarding the valuation of non-maturing liabilities are Hutchison and Pennacchi (1996) and Jarrow and Van Deventer (1998). Hutchison and Pennachi (1996) analyze valuation in an equilibrium framework and derive an analytic formula for duration (interest rate sensitivity) where interest rates are assumed to follow a square root mean-reverting process consistent with the Vasiček term structure model. Jarrow and Van Deventer (1998) provide an approach to valuation and hedging of demand deposits and credit card loans based on arbitrage-free pricing methodology. They obtain a closed form solution for the value where the market rate is assumed to follow an extended Vasiček term structure model and the deposit rate and volume are expressed as deterministic functions of the market rate. They hedge by using equivalent interest rate swaps. This can be seen as an OAS model according to Bardenhewer (2007). However according to Frauendorfer and Shürle (2007) these earlier arbitrage-free and equilibrium methods are often based on simplifying assumptions to provide the closed form solutions, especially regarding the relations between deposit rate, volume and market rates.

O'Brien (2000) also develops an arbitrage-free model for valuation but models deposit rate and volumes as autoregressive processes. The term structure model used is a one factor CIR model. He also takes into account the typical 'stickiness' of deposit rates, that banks tend to be quicker to adjust the rate in times of rising market rates than in falling, by studying alternative specifications for the deposit rate with asymmetric adjustments to the market rate changes.

The Jarrow and Van Deventer model is extended to a general case including simulation in Kalkbrener and Willing (2004) who propose a three-factor stochastic model for risk management of non-maturing liabilities. It is a very broad modeling approach with a two factor HJM model for market rates. The deposit rate is modeled as a function of the short rate and the volume is described by a log-normal diffusion model dependant on another stochastic factor. The value and interest rate sensitivity is computed by the use of Monte Carlo simulation of the processes and based on that a replicating portfolio is constructed for the interest rate risk management.

Maes and Timmermans (2005) investigate the dynamics of Belgian saving deposit volumes and rates and discuss models to manage the interest rate risk, such as static replicating portfolio models, Monte Carlo valuation models and dynamic replicating portfolio models.

Bardenhewer (2007) discusses and compares replicating portfolio models and OAS models for non-maturing products, mainly from a liquidity risk management perspective. He also discusses how a stochastic optimization approach to a replicating portfolio would increase the accurateness and flexibility of the model by using multiple interest rate scenarios and by allowing the weights to change continuously. This could be described as a combination of an OAS and a replicating portfolio model. He also mentions that the complexity and computational demands of such a model are demanding, making implementation very difficult. An example can be seen by Frauendorfer and Shürle (2007) who specify such a dynamic replicating portfolio approach based on stochastic optimization and compare the approach to a basic static replication.

Paraschiv and Schürle (2010) seek to improve ways of modeling deposit rates and volumes of non-maturing accounts. The deposit rate model is used to test if there are asymmetric relations between the deposit rate and market rate and it is suggested that the volume can be explained by the spread between the deposit and market rates. They also mention that these results can be integrated in a dynamic replicating portfolio approach or a valuation model.

Blöchlinger (2010) introduce a model that is similar to Kalkbrener and Willing (2004) as it is an OAS valuation model that uses a replicating portfolio to hedge the interest rate risk. He uses a one-factor Hull-White market rate model, a deposit rate process which includes a factor to reflect the decision making of the bank and a volume process that includes an additional stochastic factor. However this model also comes closer to a dynamic replication model as criterions are defined for occurrences under which the portfolio weights would be readjusted.

Chapter 3

Risk Management at the Bank

This section provides a brief overview of the risk management framework in place at the Bank. The Bank is organized as such that the non-maturing liabilities in the form of savings accounts are administrated by the regional offices and individual units. However the aggregate funding of the Bank is supervised at the Treasury department in the head office. Therefore the connection between the market rate and the deposit rate is divided into two steps, via internal rates set by the Treasury department. Clients deposit money on their accounts with the offices at a deposit rate. Then the Treasury borrows the deposited money from the regional offices at the internal rate that is normally higher than the deposit rate; this way the money is pooled at the Treasury and the office makes a profit from the margin. The Treasury on the other hand is not trying to make a profit; their aim is to ensure liquidity and fund all of the loan business of the Group. The Bank is rather loan-heavy in the sense that the total of all loans amounts to much more than the total of all deposits made by costumers. According to the 2010 Annual Report, 546’173 million SEK were deposits and borrowing from the public, whereas 1’481’678 million SEK were loans to the public, i.e. 2.71 times as much. So the money deposited on savings accounts goes into the funding of the loan business but do not cover all of it, in 2010 deposits by the public only made up 26% of the total liabilities of the Bank. The gap is funded by the Treasury doing other deals, such as commercial paper programs, interbank borrowing and covered bond issues.

The non-maturing liabilities in the form of savings accounts, despite the difficulty in modeling them, are a rather stable source of funding. Moreover the Treasury performs daily checks of the liquidity and plans the funding in advance, keeping a liquidity reserve that would enable the Bank to manage for at least twelve months without borrowing any new funds in the financial markets. Actually the liquidity reserve, comprising of e.g. assets with central banks that can be converted to liquidity at short notice and of unutilized issue amount for covered bonds, exceeded SEK 500 billion at the end of 2010 which in itself was almost as much as the total of deposits and borrowing from the public. Also, if clients started to withdraw larger amounts of money this would likely be spotted by the Treasury and adapted to. Therefore the risk of the Bank running out of cash due to costumers withdrawing money is small; however there could be profit to obtain by improved modeling and of course bad modeling or too optimistic assumptions could still cause problems.

When measuring the liquidity, maturity gap analysis is employed. That is, the future timeline is divided into a number of predefined time bands or ‘buckets’. Then the future cash flows for all assets and liabilities, i.e. maturing loans, interest payments, deposits from savings accounts, bonds, other deals etc. are placed in these buckets, sorted by categories. This is done daily; reports are collected from each unit and are aggregated into a measurement on Group level. For liabilities like savings accounts where there exists no contractual maturity or re-pricing dates, assumptions has to be made. In these reports the non-maturing liabilities are grouped as ‘deposits from the public’ and are placed in buckets according to the current liquidity assumption.

The interest rate risk is also measured by the use of a bucketing system, where all interest rate risk sensitive assets and liabilities are placed in buckets based on price sensitivity, i.e. value change for a specified change in underlying interest rate. This is calculated based on the future cash flows for all interest sensitive products and is too reported by all units and aggregated. Then it is up to the Treasury to handle business in a way that makes sure that the resulting interest rate risk in each bucket is kept low enough.

Chapter 4

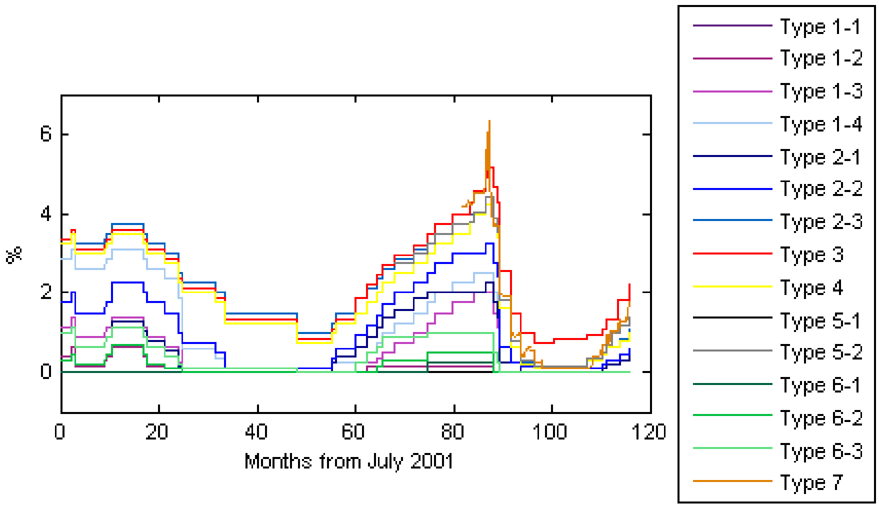
Analysis of the Data

4.1 Description of the Data

All data used is provided by the Bank and consists of daily historical market rates, deposit rates and deposit volumes.

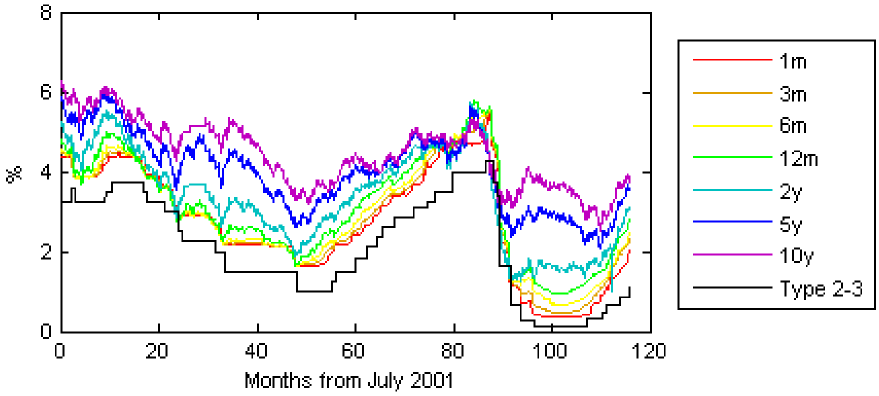
All interest rate data are obtained over a sample period ranging between July 1, 2001 and February 28, 2011, i.e. a period of 116 months. The data for deposit rates consists of historical rates for seven different account types. For some of the account types costumers also receive different deposit rates depending on the volume deposited by the particular costumer, generally slightly higher rates if the funds exceed certain levels. This gives a total of fifteen different deposit rates although some are very similar. In Figure 3 the deposit rates are shown, and are as can be seen re-priced on irregular intervals, generally more often as the rates go up than when they go down.

Figure 3: Deposit rates for different account types



The market rate data consists of seven rates with different maturities, i.e. 1-month, 3-month, 6-month and 12-month SEK Stibor rates as well as 2-year, 5-year and 10-year SEK Swap rates. The market rates can be seen in Figure 4 along with one of the deposit rates for illustration. As can be seen the longer term market rates are higher than the shorter term rates and the deposit rate is generally lower than the market rates and seems to roughly follow the evolution of the shorter market rates.

Figure 4: Market rates and a deposit rate



The data available for deposit volumes is also daily, however for a slightly shorter sample period of 31 months ranging between Aug 1, 2001 and Feb 28, 2011.

4.2 Analysis of the Data

4.2.1 Relations Between Interest Rates

Relations between interest rates, in particular between deposit rate and market rates, should be investigated to give an idea of how the bank's re-pricing behavior is connected to movements in the market.

But first the relations between deposit rates for different account types are briefly investigated. The multitude of deposit rates raises the question of which one to use in the analyses since the data is not redundant enough to know how big parts of the volume that correspond to which rate. However as can be seen in Figure 3 most of the deposit rates are very similar in shape, although differs in altitude. The big differences occur when rates for some account types flatten out as they get close to or hit a natural floor, since the rates cannot go below zero.

The correlations between the deposit rates were examined. This was computed using the measure of *'Pearson's correlation'* that defines the correlation between two random variables X and Y as

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$$

Where a result of *0* indicates that the variables are uncorrelated, *+1* indicates a perfect positive linear relationship between the variables and *-1* indicates a perfect negative linear relationship.

The result can be seen in Table 1 where $\rho(X,Y)$ is calculated, the observations of X and Y being represented by the previously described historical daily data series for the deposit rate types seen in the table.

Table 1: Correlations between deposit rates for different account types

Y / X	Type 1-1	Type 1-2	Type 1-3	Type 1-4	Type 2-1	Type 2-2	Type 2-3	Type 3	Type 4	Type 6-1	Type 6-2	Type 6-3
Type 1-1	-	-	-	-	-	-	-	-	-	-	-	-
Type 1-2	-	1.00	0.66	0.80	0.55	0.62	0.66	0.63	0.67	0.10	0.85	0.80
Type 1-3	-	0.66	1.00	0.89	0.88	0.96	0.92	0.95	0.94	0.68	0.88	0.90
Type 1-4	-	0.80	0.89	1.00	0.72	0.85	0.90	0.88	0.92	0.37	0.84	0.88
Type 2-1	-	0.55	0.88	0.72	1.00	0.96	0.83	0.89	0.83	0.69	0.84	0.91
Type 2-2	-	0.62	0.96	0.85	0.96	1.00	0.92	0.97	0.93	0.66	0.87	0.93
Type 2-3	-	0.66	0.92	0.90	0.83	0.92	1.00	0.95	1.00	0.49	0.81	0.89
Type 3	-	0.63	0.95	0.88	0.89	0.97	0.95	1.00	0.97	0.58	0.83	0.89
Type 4	-	0.67	0.94	0.92	0.83	0.93	1.00	0.97	1.00	0.50	0.82	0.89
Type 6-1	-	0.10	0.68	0.37	0.69	0.66	0.49	0.58	0.50	1.00	0.56	0.50
Type 6-2	-	0.85	0.88	0.84	0.84	0.87	0.81	0.83	0.82	0.56	1.00	0.94
Type 6-3	-	0.80	0.90	0.88	0.91	0.93	0.89	0.89	0.89	0.50	0.94	1.00

The reason there are no correlations for account type 1-1 is because the deposit rate for this type was zero over the entire sample period. Also the types 5-1, 5-2 and 7 are omitted in the table as they only have existed for a much shorter period of time than the others. A conclusion from the table is that when types 1-2 and 6-1 are not involved, the correlations between the remaining are *0.80* or more with a just a few exceptions. And those two types are the most flattened out types apart from type 1-1 and are very close to zero the whole sample period. All types that are not as close to zero seem to be rather heavily correlated which is also what could be seen in Figure 3.

For comparison six of the deposit rates with various degrees of 'flatness' is displayed in Figure 5 and their correlations against the market rates are calculated in Table 2.

Figure 5: Six deposit rates with increasing 'flatness'

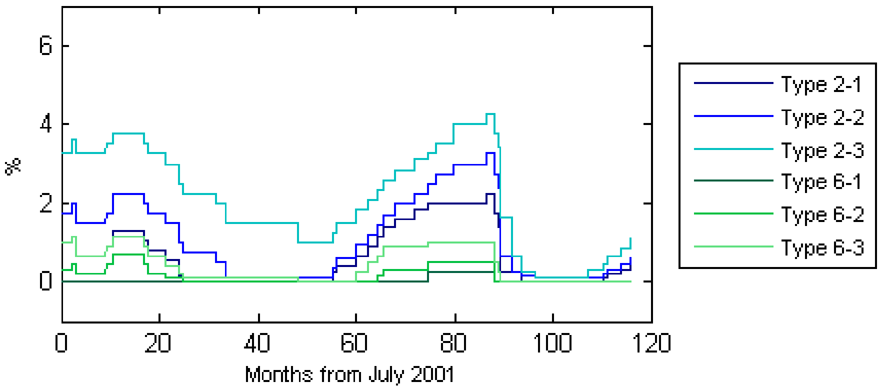


Table 2: Correlations between six deposit rates (sorted by level of 'flatness') and the market rates

X \ Y	1m	3m	6m	12m	2y	5y	10y
Type 2-3	0.99	0.99	0.98	0.96	0.92	0.86	0.77
Type 2-2	0.91	0.92	0.93	0.93	0.84	0.72	0.58
Type 2-1	0.82	0.85	0.87	0.87	0.77	0.62	0.45
Type 6-3	0.88	0.89	0.90	0.90	0.87	0.79	0.67
Type 6-2	0.79	0.81	0.82	0.81	0.78	0.70	0.60
Type 6-1	0.51	0.54	0.56	0.55	0.42	0.29	0.17

The conclusion is not surprisingly that the more flattened a deposit rate is, the less correlated to the market rates it becomes. However the differences are not very extreme unless a deposit rate is very flat, as the last case in Table 2 shows, and that is only the case regarding four of the fifteen account types, i.e. types 1-1, 1-2, 5-1 and 6-1.

Based on this most of the further analyses in this report will be performed using the rate for account type 2-3 as a non-flat arbitrarily chosen deposit rate, and seen as a reasonable approximation for the reality in following examples.

Table 2 also shows that the deposit rates display higher correlations against the short term market rates than against the long term rates, which was also the conclusion drawn from Figure 4. The chosen deposit rate (Type 2-3) is extremely highly correlated to the first four market rates, especially the 1m rate, but is strongly connected to the remaining market rates as well which is reasonable because the market rates display strong relations between themselves. This can be seen in Table 3 where the market rate correlations are shown and which are generally high, especially between all short term rates and also between all pairs of subsequent market rates.

Table 3: Correlation between market rates and a deposit rate

Y / X	Type 2-3	1m	3m	6m	12m	2y	5y	10y
Type 2-3	1.0000	0.9906	0.9851	0.9775	0.9635	0.9242	0.8645	0.7721
1m	0.9906	1.0000	0.9965	0.9895	0.9777	0.9394	0.8736	0.7748
3m	0.9851	0.9965	1.0000	0.9974	0.9893	0.9442	0.8670	0.7555
6m	0.9775	0.9895	0.9974	1.0000	0.9961	0.9490	0.8670	0.7472
12m	0.9635	0.9777	0.9893	0.9961	1.0000	0.9612	0.8787	0.7510
2y	0.9242	0.9394	0.9442	0.9490	0.9612	1.0000	0.9572	0.8566
5y	0.8645	0.8736	0.8670	0.8670	0.8787	0.9572	1.0000	0.9636
10y	0.7721	0.7748	0.7555	0.7472	0.7510	0.8566	0.9636	1.0000

4.2.2 Deposit Volume vs. Interest Rates

Relations between deposit volumes and interest rates should be investigated as well, to see whether costumer behavior is impacted by rate changes. The deposit volume was therefore plotted together with the deposit rate, a short term (3m) and a long term (5y) market rate. No clear connections could be seen, however the volume seemed to be impacted by a growing trend that probably is a reflection of increasing businesses rather than the costumer behavior. To get a better view of the connections the volume was therefore assumed to have a linear trend computed according to

$$V_t = F_t + \varepsilon_t$$

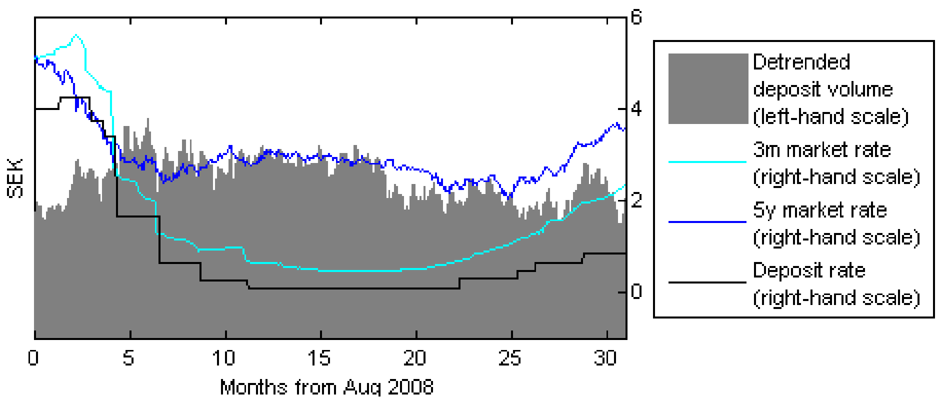
and

$$F_t = \alpha + \beta \cdot \Delta t$$

where V_t is the actual historical volume at time t, F_t is the estimated linear trend at time t, ε_t is the time t residual, and α and β are constants being estimated using ordinary least squares.

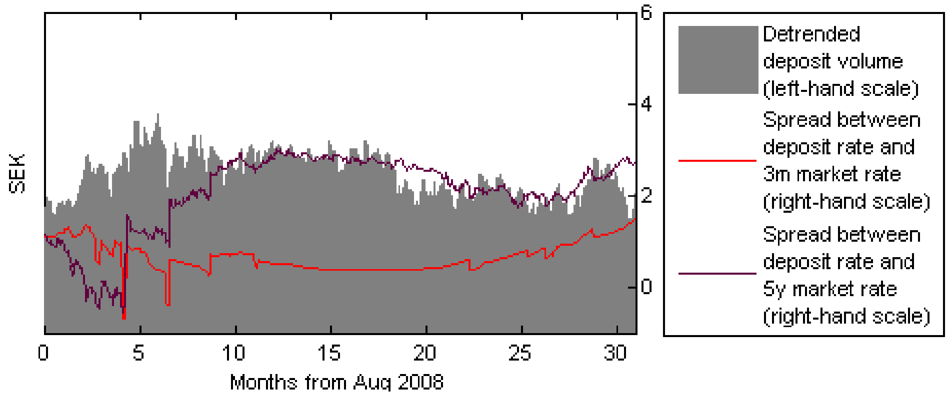
In Figure 6 the de-trended deposit volume, $D_t = V_t - \beta \cdot \Delta t$, can be seen together with the interest rates. But again there are not really any visible connections to be seen.

Figure 6: De-trended volume vs. deposit and market rates



In Figure 7 the de-trended volume is instead compared to the spread between deposit rate and market rate, again one short and one long. Here an asymmetric relationship between the volume and the spread between deposit rate and 3m market rate seems to be slightly indicated. As the 3m-spread declines the volume increases and vice versa. This could perhaps be due to costumers preferring to place their money on savings account over other investments if the rates are rather similar, but if they starts to differ more, reflected by an increasing spread, then other investments might start to seem more interesting causing costumers to withdraw their money from the accounts.

Figure 7: De-trended volume vs. spread between deposit rate and market rates



In Table 4 the correlations of de-trended volume against interest rates and spreads are shown. The connections seem rather weak overall even though there is a slight negative correlation against the 3m-spread of *-0.36*.

Table 4: Correlation between volume and interest rates

X \ Y	3m market rate	5y market rate	Deposit rate	Spread; 3m-deposit rate	Spread; 5y-deposit rate
De-trended volume	-0.21	-0.14	-0.14	-0.36	0.11

However the results are very vague and are not enough to prove a clear relation between volume and rates. Also preferably this analysis should be made based on a longer sample interval since this particular sample period started in the middle of the financial crisis in 2008, when the whole market was rather volatile.

Chapter 5

Interest Rate Risk Management

5.1 Problem Formulation

For interest rate risk management the risk origins from the fact that interest rate changes may cause unforeseen deviations in future cash flows resulting in losses. However, for non-maturing liabilities in the form of savings accounts, the interest rate outflows to customers have the potential to be matched by interest rate inflows from investing the deposited funds at a market rate. This means that the risk is related to the uncertainty of the margin or spread in between. Also some interest rate risk may arise from the option of the costumers, that they may withdraw funds at any time, causing future interest payments to disappear.

What is sought in regards of interest rate risk management is an estimation of the time to next reset date which in reality is unknown. That would enable for calculation of estimated future interest cash flows and also consequently present value and interest rate sensitivity. This would then be put together with all other interest sensitive products of the Bank in an interest sensitivity bucketing system enabling matching between these liabilities and other assets and liabilities, making it possible to monitor the interest rate risk and manage the overall net position. The interest rate risk is lowest when assets and liabilities match as closely as possible, since that would result in netted future interest cash flows close to zero. Therefore the aim is to provide as exactly estimated future reset dates as possible, keeping the modeling error down, so the matching will be accurate. These estimations should also be made while striving to maximize the net interest income. And then there is also the earlier mentioned risk that funds may be withdrawn to take into account. This could be formulated as minimizing the deviation between the model and reality while maximizing the profit, under a liquidity constraint.

As mentioned the management of interest rate risk needs to handle unexpected variations of the margin. One way of doing this is to try and keep the margin as stable as possible while investigating a way of approximating the deposit rate plus the margin with a portfolio of market rates. That way the properties of the non-maturing liabilities could be approximated with the properties of the portfolio. This is the ***static replicating portfolio approach***. In this case it would be a fictive portfolio which would be used to provide estimated measurements and interest cash flows.

5.2 Model Proposed

5.2.1 Optimization

The optimization of the replicating portfolio will be performed by minimizing the standard deviation of the margin between the portfolio return and the deposit rate using the formulas in section 2.2.1., i.e.

$$Min\ std(r_p - R)$$

Subject to

$$\begin{cases} \sum_{i=1}^{n} w_i r_i = r_p \\ \sum_{i=1}^{n} w_i = 1 \\ w_i \geq 0, \quad \forall i \end{cases}$$

where r_p is the return of the replicated portfolio, R is the deposit rate, n is the number of market rates with different maturities, w_i is the portfolio weight corresponding to the i:th maturity and r_i are the market rates.

The resulting margin is $m = r_p - R$.

The estimated deposit rate is the estimated portfolio return minus the mean margin which is an average of the margin between the estimated portfolio return and the real deposit rate for the estimation time interval.

$$\hat{R} = \hat{r_p} - \bar{m}$$

Minimizing the standard deviation of the margin is a way to get the estimations as close to reality as possible. This optimization will be compared by the alternative formulation of maximizing the Sharpe ratio:

$$Max\ \frac{\bar{m}}{\sigma_m}$$

where $\bar{m} = \overline{r_p - R}$ is the average margin and $\sigma_m = std(m) = std(r_p - R)$ is the standard deviation of the margin.

5.2.2 Lag Consideration

As previously mentioned one typical behavior of the deposit rate is that it tends to lag, i.e. be a bit slow in adapting to the market rates. This could be taken into account by as done by Bardenhewer (2007) (see section 2.2.2) the use of a moving average of each market rate instead of the market rates themselves. The moving average at time t is the average of the j-month market rate over the last j months:

$$ma_{j,t} = \frac{1}{j} \cdot \sum_{i=0}^{j-1} r_{j,t-i}$$

This alternative formulation will be examined as well, to get a view of what impact it would have on the optimization results.

5.2.3 Liquidity Constraints

All results obtained from the above approaches will only depend on the connection between market rates and the deposit rate which mirrors the bank's re-pricing behavior. The interest rate risk could also be influenced by the costumer behavior, i.e. the costumers' option to withdraw money. To avoid this some kind of liquidity constraint is needed. One way is for example to make assumptions as to how large part of the volume could be seen as volatile and use the replicating portfolio approach on the remaining part. Another way is to use the *Market Mix* described in section 2.2.2., investigating relative volume outflows for different time intervals and compare these outflows to the weights from the portfolio. The relative volume outflows will here be estimated by estimating a trend function and then investigating how big the relative negative volume changes could be for different time intervals for the de-trended volume.

A linear trend for the volume will be estimated according to

$$V_t = F_t + \varepsilon_t$$
and
$$F_t = \alpha + \beta \cdot \Delta t$$

where V_t is the actual historical volume at time t, F_t is the estimated trend at time t, ε_t is the time t residual, and α and β are constants.

α and β will be estimated by using ordinary least squares.

Then the de-trended volume changes ε_t will be investigated for intervals of the same length as the different maturities of the components in the replicating portfolio. If the historical decline for a certain interval is greater than what is covered by the optimal portfolio weights the weights will be changed to be able to handle such decreases in volume.

5.2.4 Duration

A duration measure will be employed in the evaluation of the method as a way of viewing and comparing the results, being a measurement that is rather easily interpreted and compared for different cases.

In this case the duration will be an approximate measure of the average time to re-pricing for all the non-maturing liabilities which is approximated by the duration of the estimated replicating portfolio, i.e. interest rate reset maturity. This will simply be calculated as the weighted average maturity for the portfolio:

$$D = \sum_{\mathrm{i}=1}^{\mathrm{n}} w_i m_i$$

where w_i is the portfolio weight corresponding to the i:th maturity m_i.

5.3 Implementation

5.3.1 Original Model

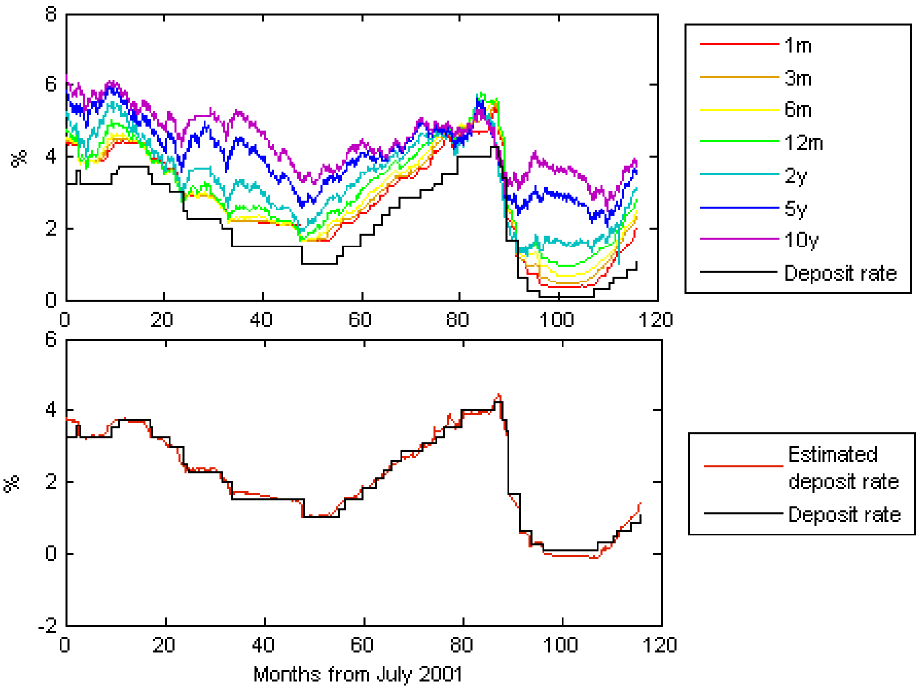
Since the correlations between different deposit rates are rather high, see section 4.2.1, the deposit rate chosen to be used in the implementation was the same one as in previous examples (Type 2-3).

Optimizing the portfolio using the formulas in section 5.2.1 gave the weights, margin and standard deviation of margin seen in Table 5. As can be seen optimizing the portfolio based on the criterion of maximizing the Sharpe ratio resulted in both a higher mean margin and a higher standard deviation of margin. The estimated deposit rate based on the optimization criterion of minimizing the standard deviation can be seen in Figure 8.

Table 5: Margin and optimal portfolio weights for standard case

Optimization criterion	Margin	Std	1m	3m	6m	12m	2y	5y	10y
Min standard deviation	0.8196	0.1852	87%	0%	1%	0%	0%	0%	12%
Max Sharpe ratio	1.0098	0.2061	51%	22%	7%	0%	0%	0%	21%

Figure 8: Market rates seen with deposit rate & Estimated deposit rate



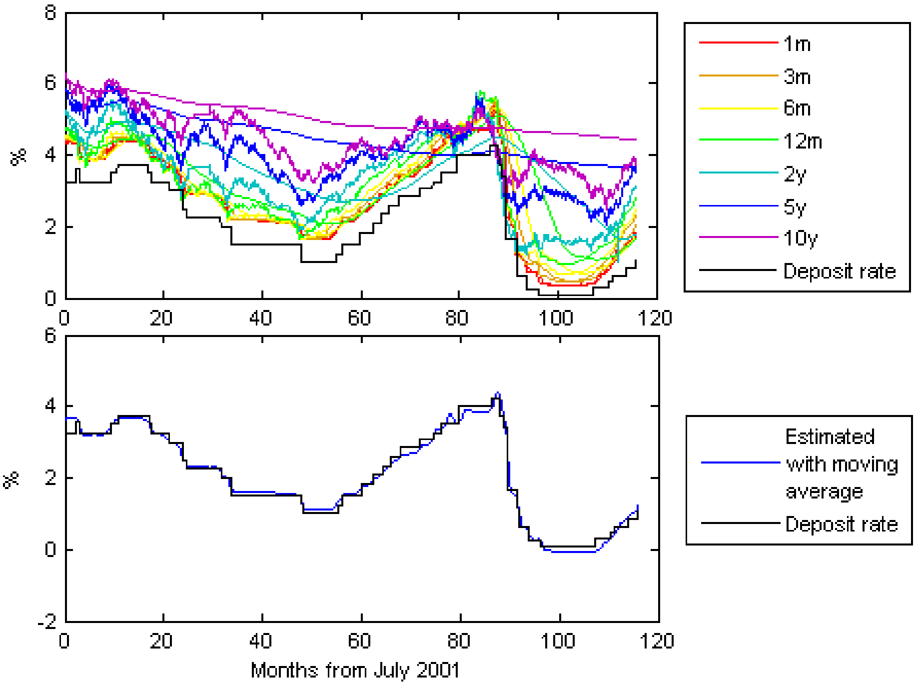
5.3.2 Including Moving Averages

An optimal portfolio was also computed when moving averages were used instead of the market rates, according to the formula in section 5.2.2. This gave the weights, margin and standard deviation of margin seen in Table 6. For the case of minimizing standard deviation using moving averages instead of market rates resulted in both higher margin and lower standard deviation. So both from a profit maximizing and a least modeling error point of view using moving averages seems to be the better choice in this case. For the case of maximizing Sharpe ratio using moving averages lowered the margin slightly, but on the other hand it also lowered the standard deviation quite a lot. The moving averages and estimated deposit rate based on the optimization criterion of minimizing the standard deviation can be seen in Figure 9.

Table 6: Margin and optimal portfolio weights using moving averages

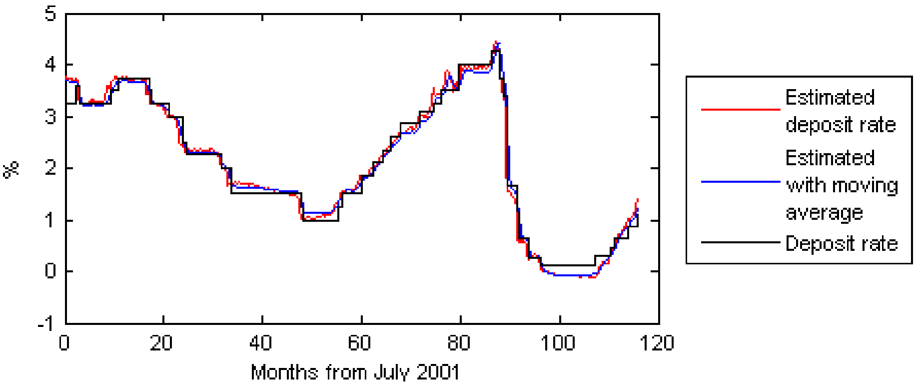
Optimization criterion	Margin	Std	1m	3m	6m	12m	2y	5y	10y
MIN STANDARD DEVIATION:									
Using market rates	0.8196	0.1852	87%	0%	1%	0%	0%	0%	12%
Using moving averages	0.8415	0.1600	90%	0%	0%	0%	1%	0%	9%
MAX SHARPE RATIO:									
Using market rates	1.0098	0.2061	51%	22%	7%	0%	0%	0%	21%
Using moving averages	0.9633	0.1705	85%	0%	0%	0%	0%	0%	15%

Figure 9: Market rates and moving averages & Estimated deposit rate using moving averages



For comparison both estimated deposit rates can be seen in Figure 10. Using moving averages seem to give a slightly less volatile result with less lag, especially in falling interest rate environments.

Figure 10: Estimated deposit rate both using market rates and moving averages



5.3.3 Including Liquidity Constraints

In Table 7 the calculated optimal weights can be seen and are also cumulated. These cumulated weights are then compared to cumulated liquidity constraints. The liquidity constraints are as mentioned in section 5.2.3 calculated as the maximum relative outflow, measured by the maximum relative negative change in the de-trended volume for each interval based on the historical volume data. This could for example be found to be 30% for the 3m period meaning that 30% of the volume could be gone in three months, not taking the trend into account. Then the corresponding portfolio weight is not allowed to be below this liquidity constraint since those funds could actually have been removed by then. If it is below, then the liquidity constraint is used instead to cover for the potential loss of volume. However it is the cumulated constraints that should be compared for each time period since funds weighted to a shorter period would be re-priced again before that and can cover a longer maturity as well.

Table 7: Cumulated optimal portfolio weights

	1m	3m	6m	12m	2y	5y	10y
OPTIMAL WEIGHTS:							
(1) Min standard deviation	87%	0%	1%	0%	0%	0%	12%
(2) Max Sharpe ratio	51%	22%	7%	0%	0%	0%	21%
CUMULATED WEIGHTS:							
Row (1) cumulated	87%	87%	88%	88%	88%	88%	100%
Row (2) cumulated	51%	72%	79%	79%	79%	79%	100%

In this case the cumulated liquidity constraints were calculated and did not exceed the cumulated optimal weights for either of the cases. Therefore the original weights cover possible withdrawals from costumers as well as are adapted to future reset dates by a replicating portfolio.

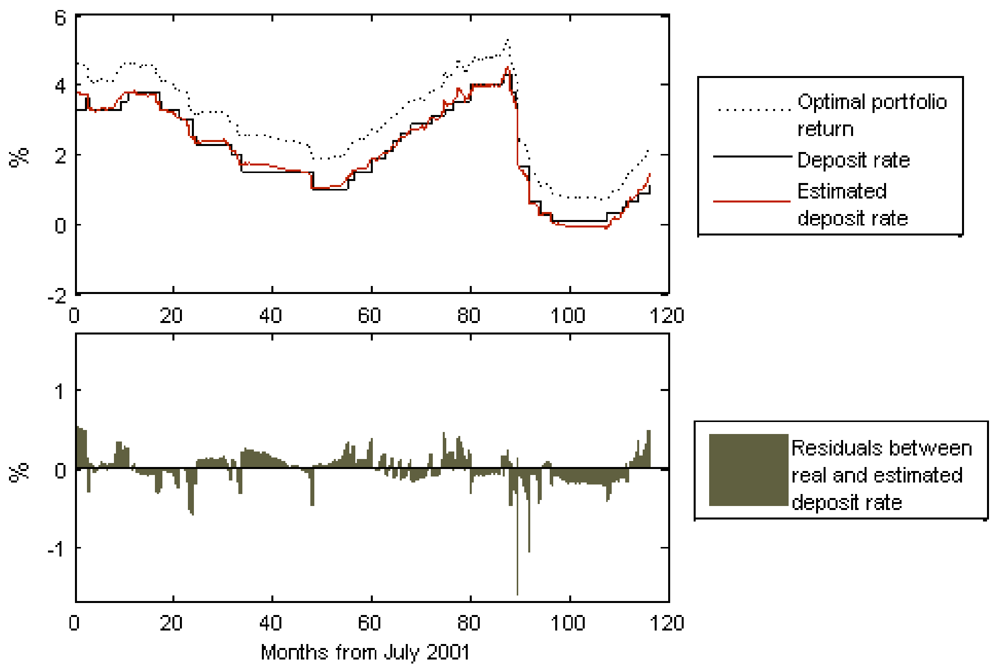
5.4 Evaluation

5.4.1 Investigating Residuals

To get a better picture of the model error the deviations from the mean margin were investigated. That is, the negative deviations since a positive deviation from the mean would only mean a higher margin than anticipated which would be no loss.

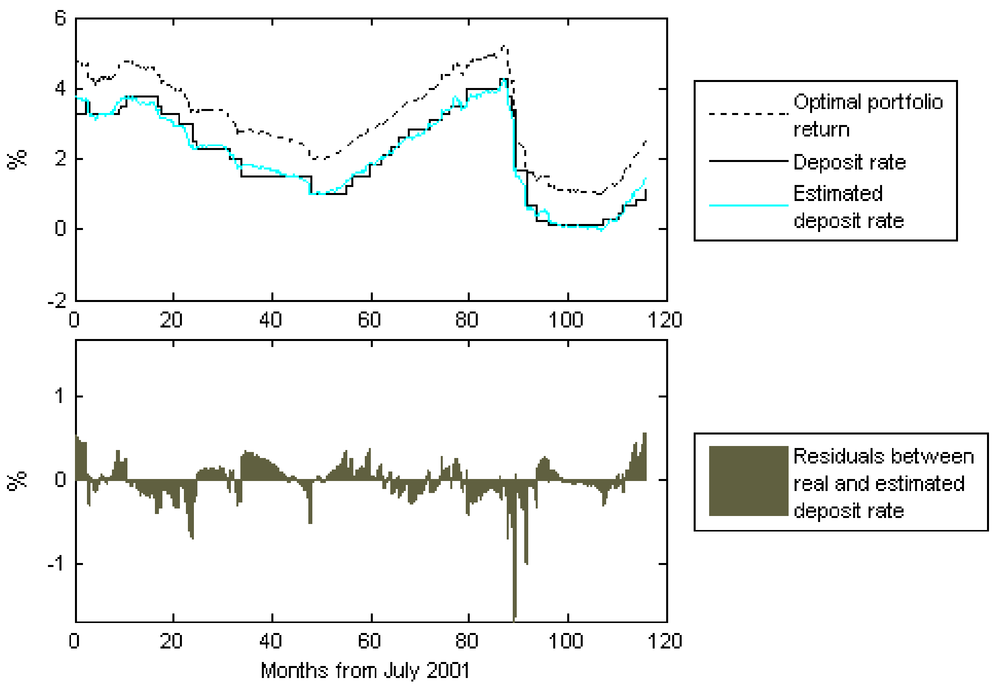
In Figure 11 the estimated portfolio return is displayed for the case of minimizing standard deviation, along with the deviations from the mean margin. Here the lag of the deposit rate is more visible resulting in peaking negative deviations from the margin when there is a larger drop in rates. That is because the deposit rate seemingly adapts slower in a falling interest environment so when there is a drop in market rates, and subsequently in the portfolio return, the deposit rate tends to stay on the same level a little while before it drops as well. In a rising interest environment the deposit rate is changed more often, keeping up with the market rates better, so here the deviations are smaller.

Figure 11: Optimal estimated portfolio return & Deviation from mean margin (min std)



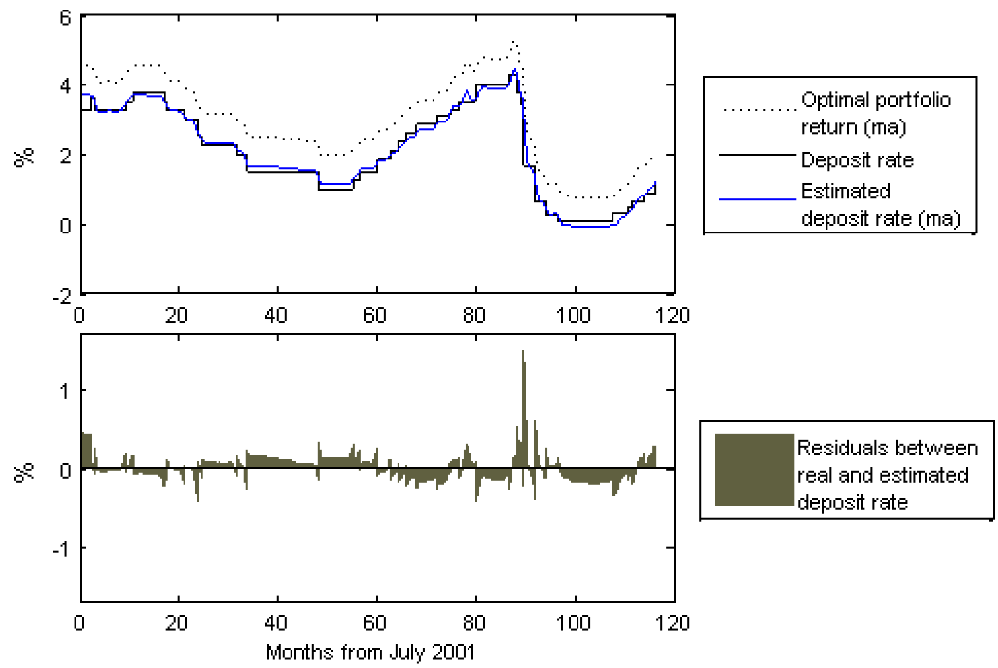
In Figure 12 the same thing can be seen, but for the case of maximizing Sharpe ratio. The residuals look almost the same as in the first case, but are slightly bigger which is expected due to the higher standard deviation in this case.

Figure 12: Optimal estimated portfolio return & Deviation from mean margin (max Sharpe)



In Figure 13 the same can be seen but with calculations based on moving averages (ma) instead of market rates. The optimization criterion is of minimizing standard deviation. Here the large negative deviations of both first cases do not show.

Figure 13: Optimal estimated portfolio return (ma) & Deviation from mean margin



To get a better view the residuals for both cases of minimizing standard deviation, with market rates and with moving averages, can be seen compared in Figure 14 and in Table 8 the maximum negative deviation, as well as the approximate *99.5%* empirical quantile, can be seen compared to the previously estimated standard deviations for each case. As anticipated the negative deviations are a lot smaller in the case of using moving averages.

Figure 14: Comparison of residuals

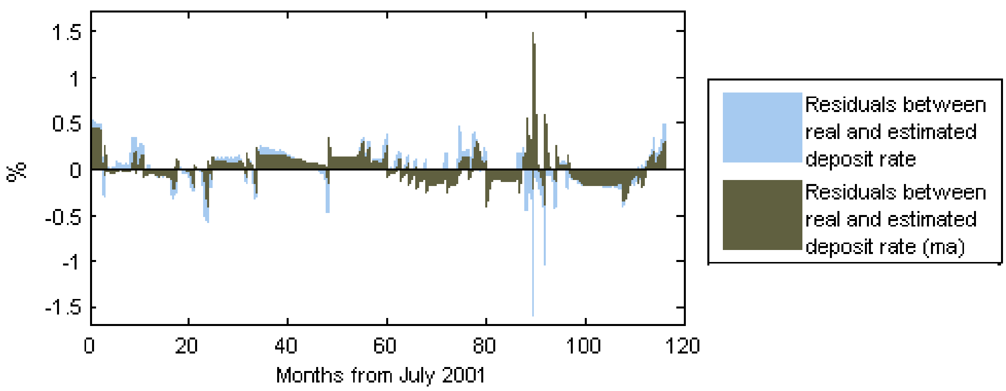


Table 8: Comparison of maximum negative deviations and standard deviations

Optimization criterion	Standard dev	Max neg dev	99.5% quantile
MIN STANDARD DEVIATION:			
Using market rates	0.1852	-1.5949	-0.5597
Using moving averages	0.1600	-0.4217	-0.3616
MAX SHARPE RATIO:			
Using market rates	0.2061	-1.7045	-0.6938

5.4.2 Margin vs. Standard Deviation

To obtain a higher desired margin the weights could be optimized with added margin constraints. This enables an understanding of the dependence between average margin and the standard deviation of margin. A higher margin would yield a bigger standard deviation. So if minimizing the model error, in this case by minimizing the standard deviation of the margin, is the most important criteria then the original portfolio should be used. But if one is willing to accept a higher standard deviation, bigger margins could be obtained. Figure 15 show the standard deviation grow with rising margin and how the portfolio weights change with rising margin. As can be seen obtaining higher margins would push the weights out to longer maturities, i.e. a higher duration for the total portfolio, which is logical since the market rates get higher with increasing maturity. The optimal portfolio with the lowest standard deviation is marked in blue in the picture. If a higher margin is desired one possible choice would be the portfolio obtained by maximizing Sharpe ratio, marked in cyan. Table 9 shows the varying margins, along with corresponding standard deviations and weights.

Figure 15: Standard deviation vs. mean margin & Corresponding portfolio weights

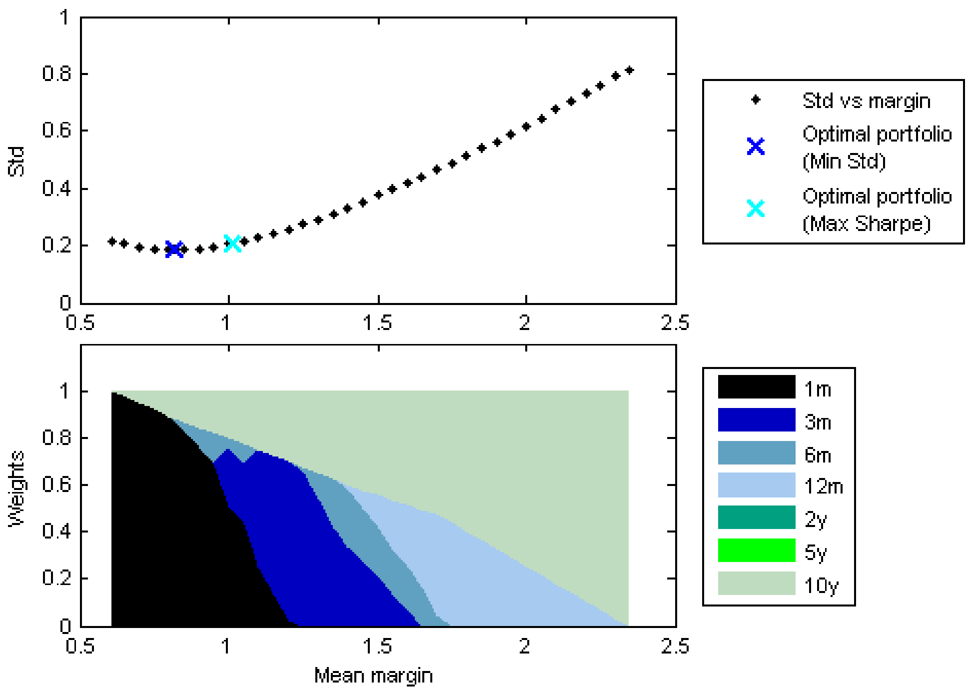


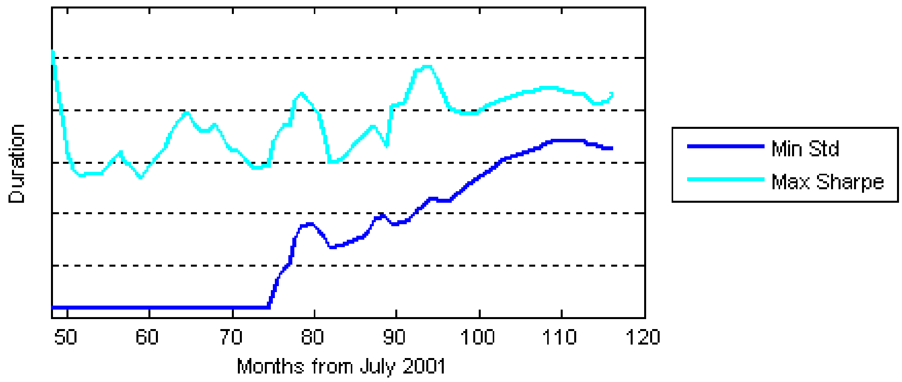
Table 9: Optimal portfolio weights for varying margin

Margin	Std	1m	3m	6m	12m	2y	5y	10y
0.6086	0.2147	100%	0%	0%	0%	0%	0%	0%
0.7000	0.1952	95%	0%	0%	0%	0%	0%	5%
0.8000	0.1855	89%	0%	0%	0%	0%	0%	11%
0.9000	0.1892	76%	0%	8%	0%	0%	0%	16%
1.0000	0.2042	51%	25%	4%	0%	0%	0%	21%
1.1000	0.2275	26%	49%	0%	0%	0%	0%	25%
1.2000	0.2570	2%	68%	0%	0%	0%	0%	30%
1.3000	0.2920	0%	54%	11%	0%	0%	0%	35%
1.4000	0.3318	0%	34%	25%	1%	0%	0%	40%
1.5000	0.3746	0%	21%	21%	14%	0%	0%	44%
1.6000	0.4192	0%	7%	18%	27%	0%	0%	49%
1.7000	0.4651	0%	0%	5%	42%	0%	0%	53%
1.8000	0.5132	0%	0%	0%	40%	0%	0%	60%
1.9000	0.5645	0%	0%	0%	33%	0%	0%	67%
2.0000	0.6182	0%	0%	0%	26%	0%	0%	74%
2.1000	0.6739	0%	0%	0%	18%	0%	0%	82%
2.2000	0.7310	0%	0%	0%	11%	0%	0%	89%
2.3000	0.7893	0%	0%	0%	3%	0%	0%	97%
2.3458	0.8160	0%	0%	0%	0%	0%	0%	100%

5.4.3 Out-of-Sample Analysis

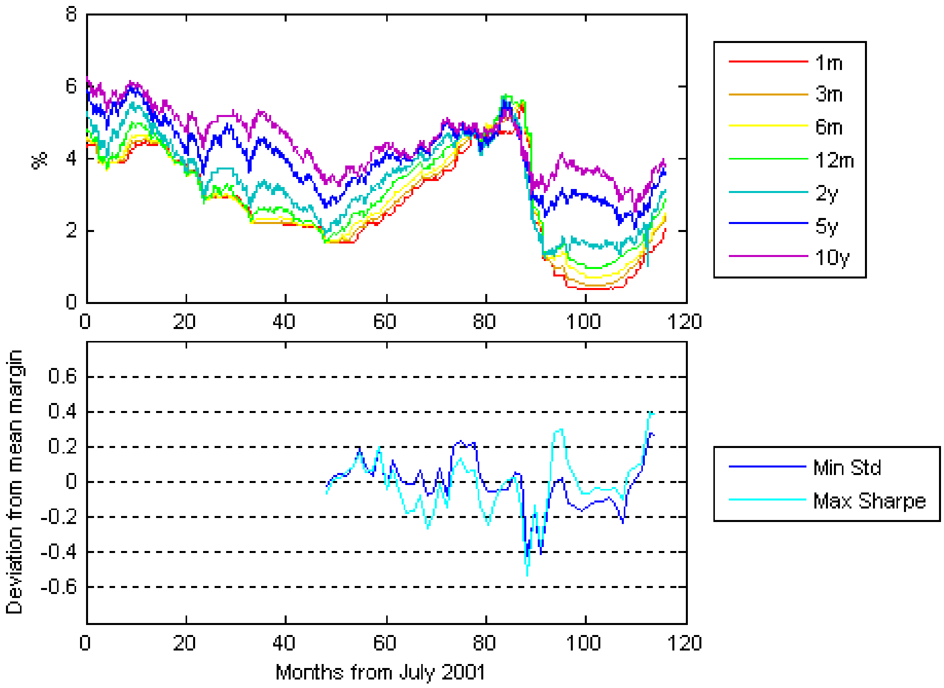
To see how the results of the optimization change over time the duration (see section 5.2.4) of the optimal portfolio was calculated based on an historical estimation interval of four years that was continually moved. This was done both for the criterion of minimizing standard deviation and for maximizing Sharpe ratio and the results can be seen in Figure 16. As can be seen the duration corresponding to the minimum standard deviation fluctuates less but ranges over a larger span and stays consistently on a lower level. So from this point of view maximizing Sharpe ratio seems to be a more stable choice of optimization criterion.

Figure 16: Duration over time



Also the deviation from mean margin was calculated with changing optimization interval. For this the same estimation interval of four years were used as a basis for calculating optimal portfolio weights. Then the optimal weights were used on an 'evaluation interval' of one month, directly succeeding the estimation interval, and an average deviation was calculated between the optimized margin and the observed margin. These calculations were performed while both intervals were continually moved and the deviations were plotted for each point. In Figure 17 the deviations from mean margin can be seen together with the market rates to get a view of the market situations corresponding to the evaluations.

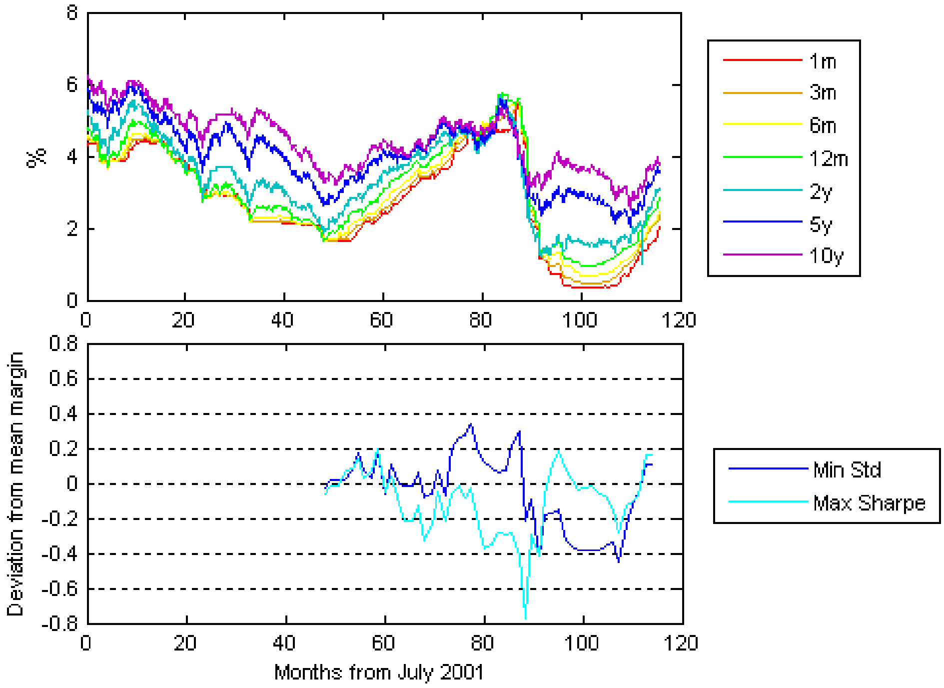
Figure 17: Deviation from mean margin over time, both estimation interval and evaluation interval moving



The deviations fluctuate for both cases, slightly more for the optimization criterion of maximizing Sharpe ratio, although the differences are not that significant. The maximum negative deviation for the Sharpe case were *-0.52* compared to *-0.42* for minimizing the standard deviation. Those rather extreme cases occur at around ninety months where the evaluation interval, which is the subsequent month from each plot point, includes extreme drops in the market rates. But mostly the deviations actually stay inside the standard deviations previously measured for the whole sample period which were around *0.2*. So considering that the estimated margin for the whole period was *0.2* higher in the case of Sharpe ratio maximization, that would probably be a better choice based on this analysis as well, despite the slightly larger fluctuations.

A similar analysis was made but this time the estimation interval was held constant as the first four years of the available historical data, while the one-month evaluation interval was moved forward. The results can be seen in Figure 18.

Figure 18: Deviation from mean margin over time, estimation interval fixed and evaluation interval moving



Here the results are more volatile for both cases, the deviations start off rather small close to the estimation interval but grow when the market situation changes. The deviations for the case of minimizing standard deviation stay roughly under *0.4* but for the other case the fluctuations are bigger, especially the negative ones, which is a bit concerning. The maximum negative deviation for the Sharpe case were *-0.77* compared to *-0.45* for minimizing the standard deviation. So based on this the weights probably need to be updated regularly, as in the previous example, to provide safe results and to effectively adapt to changing market situations.

Chapter 6

Conclusions

Using a replicating portfolio for the interest rate risk management of non-maturing liabilities might very well be an acceptable choice. The advantages are that the idea behind the theory is rather straightforward, that historical data both for deposit and market rates are usually quite easy to get and that the obtained weights directly yield estimated cash flows that could be used to obtain any measurements that are to be used. However the results are somewhat ambiguous and the choice of an optimal portfolio is not a straightforward one. Different sample intervals seem to have rather big impact on the output and the results fluctuate a lot. Also the deposit rate movements and traits beyond that of its correspondence to the market rates cannot be taken into account in this model. Neither can more advanced relations between the variables involved.

However there is some flexibility in how to define the model, for example the possible choice or trade-off between low risk and high payoff that was illustrated by the connection between margin and standard deviation seen in Figure 15. Simply minimizing standard deviation might be a bit too narrow-minded. In fact maximizing the Sharpe ratio would probably be a better choice since the difference in standard deviation is not that big while the gain in average margin is considerable. And it is still an optimal portfolio in a view, not the least risky one, but the one with optimal ratio of riskiness to gain. Furthermore the analyses in section 5.4.3 point to both a gain in stability and in profit for using the maximum Sharpe ratio criterion since the duration calculated over time both stays on a higher level and fluctuates less. The results may however turn unreliable unless portfolio weights are reevaluated at regular intervals, as the deviations from expected margin seem to increase with time.

As could be seen in section 5.4.1 there is a danger in the negative deviations at interest rate drops which in worst case leads to temporarily negative margins before the deposit rate catches up to the market rates. But these differences were not that big between the optimal portfolios of minimizing standard deviation and maximizing Sharpe ratio, as could be seen in Table 8. Using moving averages instead of market rates seem to yield safer results in the analyses since it to some extent avoids these large negative deviations in case of a sudden rate drop. But the use of moving averages would be implemented through rolling tranches which means that the rate of e.g. the 10y weight of the portfolio is not in fact the current ten year rate but the average over the last ten years. So it would be good as a real life investment or hedging strategy, but unsuitable for obtaining theoretical measures as in this case. Besides it takes time to build up such a portfolio; it would for example take ten years before all parts of the 10y weight are invested in ten year contracts at monthly intervals. So perhaps it is not an as costly effective approach as it seems.

As an alternative modeling approach an OAS model might be the next step, where the advantage would be to be able to examine more scenarios by the use of simulation. It would also make it easier to model for example the 'stickiness' of the deposit rate, that it tends to adapt faster to the market rates in a falling interest environment than in a rising, as was also what could be seen in Figure 3. This, and other traits such as for example the natural interest rate floor of zero, could be taken into account by proper formulating of a deposit rate model. As could any believed connection between rates and volumes in a volume model. However, careful implementation of all parts of the OAS model would be required which is not the easiest task and the question is how much such a formulation would gain in effectiveness of the model. Even though the advanced theories in the literature are very interesting, since they are rather complicated with many different parts, there are many ways where calculations or assumptions could go wrong. And in the end it all comes down to available data and how much it can be trusted because even the most complicated and realistic model needs to be calibrated in some way. So there will always be the choice of what estimation interval to choose. Still there might be a good idea to investigate different modeling options and variations and compare the results to reduce the risk of a bad modeling choice or wrongful implementation. Also one should probably be prepared to update or change the model in case of for example very volatile market situations and not trust any solution to be completely absolute, as reality, due to the nature of non-maturing liabilities, is dynamic.

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