Master Thesis

Sensitivity Analysis and Stress Testing in the Interest Rate Market

Bastien Grandet

Supervisors
Filip Lindskog
Boualem Djehiche

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Abstract

Recent improvements in trading techniques increased banks' exposure to market risk. This exposure needs to be managed correctly and properly. Quantifying risk and allocating sufficient capital to absorb potential losses is the main challenge: on the one hand not enough capital can put the bank in danger, and on the other hand too much capital has a negative effect on its competitiveness. It is clear that the upper bound is set by the management according to its own objective on benefits, therefore this report investigates what lower bound should be seen as appropriate

The starting point though is to estimate the bank's exposure to moves of the markets parameters, called *sensitivity to risk factors*. Easy in the past, this first step has become more complicated to complete since the volumes traded have increased dramatically, to a current average of 1 Billion EUR a day on the French equity market for example. The second step of risk management is to estimate potential losses and whether the bank can survive them. Calculations of VaR and Stressed VaR provide an approximation to the amount of capital the bank must allocate whereas stress tests check whether this capital is sufficient under feared economic downturns. These works do not ask for any breakthrough in probability but for a different and in my opinion wiser use of existing knowledges.

However risk management is still far from perfect but the current version is young. Research and critical evaluation of current standards will bring about needed improvements. This will be for the best.

Contents

Introduction

Throughout the recent history of finance, the harshest event have been unpredictable. For two years starting from summer 2007, events that had never happened before and were estimated to happen once every 200 years used to happen twice a week. Events such as the ones that have occurred during the last crisis could never have been forecast.

However, any risk manager have to deal with these issues. He has to set up some limits on the trading positions, so that even the worst losses he can plausibly imagine do not put the bank out of business. Not to be wrongly alarmist those unexpected losses have to be taken with a high and also credible confidence interval, so that they are sure at 99% for example that the bank will not suffer more losses that it can absorb. Theoritically it may seem rather easy, given that there is supposed to be a model fitting the market outcomes. There is of course no such model and the risk manager have to deal with either data and/or their knowledge of the market:

- Using purely historical data leads to historical measures of risk (VaR, CVaR and Stressed VaR).
- Using knowledge of the market to estimate possible outcomes leads to stress testing.
- Using insights into the market outcomes to come with a empirical model (Monte Carlo simulation).

Uses of these measure increased constantly from the mid 1990's to 2007 when it became a serious issue to manage risk properly, and has boomed ever since. This report will therefore deal with the way main banks manage their risk and drive their exposure to increasing threats, and the evolution brought about by the last crisis. Using stress testing is being the last trend.

I will consider the whole process of calculating these risk measures with a special focus on stress testing, starting from collecting market prices to analyzing risky positions. The process will not be exhaustive but will give a good insight into how the bank proceed all the way through.

In the first part I will deal with fundamentals issues that I had to tackle during this internship. First, one has to be aware that though mature and well known the interest rate market remains rather complex, and even linear products are challenging. Then I will tackle the issue of the sensitivities, helping to hedge a book, to calculate its profits and losses, the risk measures and among them estimate the stress test.

The second part will focus on risk measures themselves and deal with there limits leading to the introduction of stress testing. Advantages and drawbacks of any of them will be analyzed with a particular stress on the latter. Indeed the last part will be its application, focusing on two situations that cannot be taken into account using the historical or empirical measures: Stress testings have raised from this need to analyze special scenarii that may happen but that have not happened the way we fear yet. The perfect example is currently the threat of default from some European countries that cannot be estimated historically but may be quantified given current analyses. A special attention will also be paid on liquidity tightening.

Chapter 1

Background and Sensitivity Issues

During the internship I had to deal with a lot of different products which I needed to price, and estimate the inherent risk.
Estimating the PnL of an interest rates' derivatives book is pricing its asset through time. The pricing of plain or basis swaps raised lots of questions. One way of dealing with it is full pricing: i.e discounting future cash flows. For this we need to know all the curves at a given date, derive the forward rates and the discount factors and use them for the full pricing. The method is very accurate but demands time, and computing. One alternative is the use of sensitivities. They are calculated overnight on a daily basis too but the following calculations are far easier: an excel sheet is sufficient to calculate PnL, VaR, Stressed VaR and so on with the sensitivities, whereas the full pricing calculation is heavy. That is why the prior has become popular even though, as we will see, it is still approximative.

1.1 Interest Rate Vanilla Derivatives

The interest rate market is more crowded than the equity one. The most common traded assets remain swaps of different kinds, money markets, repos and bonds. Dealing with them brings about dealing with the very large amount of indexes, their maturities and their complicated correlations. Throughout the report I will focus only on vanilla derivatives, and therefore not take volatility into account in the followings.

1.1.1 Common Indexes

I will not make an exhaustive list of the indexes I have met for the past six months, but I will deal with the main ones, that are most traded.

- EONIA (Euro OverNight Iterest Average) : This is an average of the rates at which banks of the euro-zone loan or borrow for a 1-day period.

- TJC** : EONIA-type of rate with occurs in many countries (TJCUS in the US, TJCCN in China, TJCSG in Singapore and so on).

- EURIBOR (Euro InterBank Offered Rate) : With EONIA it is the other reference rate of the Euro zone. It is the rate at which a bank lend to other banks. This goes alongside with EUR6M (6-Month Euribor) or STIBOR (in Sweden), LIBOR (in many countries) or CIBOR (in Denmark) for example.

- BS*** : Basis spread used in pricing a cross-currency basis swap (see later). As spread of a reference index, the index BSEUR is a spread over EURIBOR 6M.

- TR*** : Zero Coupon Bonds rates for a given country's treasury bills.

- RP*** : Repo rates.

- SP*** : Spread considered as from a reference curve. For example, some treasury index are quoted as a spread above EURIBOR.

The traded rates where up to 300 in the vanilla books, in more than 60 currencies.

1.1.2 Notations

The products traded on the interest rate market are mostly exchanges of cash flows at different maturities, we can therefore use the same notations from now on. We denote :

- $(T_i)_{i=1,\dots,n}$: Time at which floating cash flows are paid.$(\bar{T}_i)_{i=1,\dots,p}$ for fixed ones. Naturally $T_n = T_p$

- $\delta_i = T_i - T_{i-1}$ and $(\bar{\delta} = \bar{T}_i - \bar{T}_{i-1})$

- $(C_i)_{i=1,\dots,n}$: Value of the floating cash flows.$(\bar{C}_i)_{i=1,\dots,p}$ for fixed ones.

- $PV(X,t)$: Present value of a future cash flow X seen at time t. If we consider $t=0$ it will not be mentioned.

- $\Pi(t)$: Price of a financial product. For a swap Π_f is the price of the floating leg and $\bar{\Pi}$ the one of the fixed one.

- $p(T,t)$: Discount factor at time t for a cash flow received at time T. $p(T)$ at $t=0$.

- $r(t,T)$: Zero rate with a maturity T as seen at time t.
- $f(t,T_1,T_2)$: Forward rate between T_1 and T_2 as seen at time t.
- $f(t,T)$: Forward spot rate at time T as seen at time t.

1.1.3 Common Products

Most of these indexes are traded under the form of swaps and money markets. The latter will not be taken care of because it hardly brings about any risk and is usually an overnight operation.

Plain Vanilla Swaps

A plain vanilla swap is an agreement between two parts to exchange a fixed rate against floating one (commonly EURIBOR). The inputs of a swap are:

- A fixed rate $\bar{r}$.
- A floating rate r.
- A notional N.
- Two sequences of time $(\bar{T}_i)_{i=1,...,m}$ and $(T_j)_{j=1,..,n}$ for the payments of respectively the fixed and floating cash flows.

A swap is an exchange of cash flows between two counterparts. One receive fixed flows whose are previously known, seen as interest at a fixed rate on the notional N, and pays interest at a floating rate, usually a reference rate plus a spread if necessary[1]. Pricing a swap is more complicated than it looks like. The trickiest part is no discounting the cash flows, but calculating the discount factors.
The first step of the method is to collect swaps (not only if we want to be complete) on the market and estimate the values of the underlying indexes from these prices, which are fair prices. The present value at time 0 of such a product is 0, therefore equalizing the two legs leads to the knowledge of the discount factors, then the forward rates and so on. f_i denotes $f(0,T_{i-1})$, i.e the forward spot rate fixed at T_{i-1} at which the floating cash flow received (or paid) at time T_i is paid. So that $C_i = f_i\delta_i N$ and $\bar{C}_i = \bar{r}\delta_i N$.

The continuous relationship between the forward spot rate and the discount factors is:

$$f(0,T) = -\frac{\partial \ln p(0,T)}{\partial T} \tag{1.1}$$

[1]Interbank swaps usually cost the brokerage fees, a spread is applied when the product is sold to a company. It reflects it creditworthiness, and is seen as a premium. From the bank's point of view this is the price of the swap.

Formula (1.1) drives the calulations of the forward rates. An example on EURIBOR 6M is shown on figure 1.1.

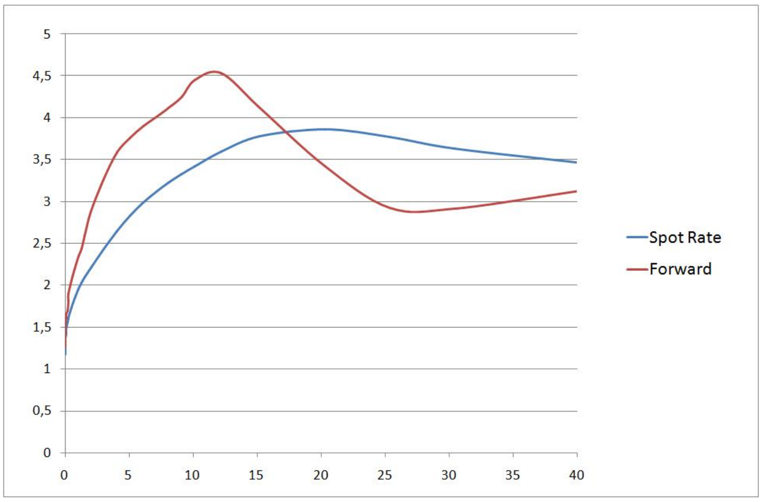


Figure 1.1: Spot and forward rates of EURIBOR 6M

The discrete approximation version using our notations is:

$$
\begin{aligned}
f_i &= -\frac{\ln p(0,T_i) - \ln p(0,T_{i-1})}{T_i - T_{i-1}} \\
&= \frac{\ln \overbrace{\frac{p(0,T_{i-1})}{p(0,T_i)}}^{\approx 1}}{\delta_i} \\
&= \frac{\frac{p(0,T_{i-1})}{p(0,T_i)} - 1}{\delta_i} \\
\Rightarrow f_i.\delta_i.p(T_i) &= p(T_{i-1}) - p(T_i)
\end{aligned}
\tag{1.2}
$$

We notice that $C_i = N.f_i.\delta_i.p(T_i)$, which will simplify the future calculations, therefore:

$$
\begin{aligned}
\Pi_f &= \sum_{i=0}^{n} C_i = \sum_{i=0}^{n} N(p(T_{i-1}) - p(T_i)) \\
\Pi_f &= N(P(T_0) - P(T_n)) = N(1 - p(T_n)) \\
(\Pi_f &= N(1 - p(T_p)))
\end{aligned}
\tag{1.3}
$$

The value of the fixed leg is obvioulsy :

$$\bar{\Pi} = \sum_{i=1}^{p} N.\bar{r}_n.\bar{\delta}_i.p(\bar{T}_i)$$

Equalizing Π_f and $\bar{\Pi}$ for a Swap of maturity T_n with pays coupons $N.\bar{r}_n$:

$$\begin{aligned} 1 - p(T_p) &= \sum_{i=1}^{p} \bar{r}_n.\bar{\delta}_i.p(\bar{T}_i) \\ p(T_p) &= 1 - \bar{r}_n \sum_{i=1}^{p-1} \bar{\delta}_i.p(\bar{T}_i) - \bar{\delta}_p.\bar{r}_n.p(T_p) \\ \Rightarrow & \boxed{p(T_p) = \frac{1 - \bar{r}_n \sum_{i=1}^{p-1} p(\bar{T}_i)\bar{\delta}_i}{1 + \bar{\delta}_p.\bar{r}_n}} \end{aligned} \tag{1.4}$$

In a perfect world where swaps with any given maturity are quoted, this simple formula is sufficient to get any points of any curve. However in practice such quotations do not exist, so we have to use interpolation methods that will be detailed later.

Repos

A Repo (Repurchasement Agreement) is a way of raising funds. It is a loan where the counterpart *gives* a security as collateral. The rate at which the loan is done is called the repo rate. The most famous repos in the departement are Treasury repos where the security is a bond. The rates are quoted on the market and depend highly on the quality of the security (a loan collateralized with a portuguese bond will be higher than one with a german bond) and the quality of the counterpart, which can be charged a spread if it is judged likely to default.

Cross-Currency Basis Swap

Cross currency swap are basically swaps with each leg in a different currency. The advantage is the same as a plain swap. One company need to borrow in a foreign currency, and therefore uses the competitive advantage that can provide a domestic company, that would like to borrow itself in the other currency at a prefered rate.
At a first glance the pricing does not look that much different than a plain one's, discounting future cash flows. However the market thinks differently: there is a spread between the market price and the plain full pricing *'swap-wise'* of the product. Its outlook is that exchanging interest rates from one currency to another adds some risk.
Actually this risk is surprisingly not linked to the exchange rate as one may suppose. This comes from the fact that the aim of a cross-curency swap

is to get money in the foreign currency to invest it in assets quoted in the same currency. Therefore, the exchange rate, spot or forward as no impact on the price of the product. Then whether or not the company takes the exchange rate to calculate the PnL of these products is an internal issue[2] . So what this extra risk may be? The spread in the price reflects the difference of liquidity available in each currency which may bring about a rise in the interest rate, and therefore impact the PnL. The spread is actually quoted on the market and known as a Basis Spread with respect to a reference index. It is basically a liquidity premium[3] seen over a given period of time. We denote this spread as s_n, which may be positive or negative given the currencies involved in the exchange.
We do not lose generality by quoting the basis swap against a reference currency, therefore they are officially quoted against USD Libor. The price of JPY-vs-EUR basis swap can be obtain from the one a JPY-vs-USD and a USD-vs-EUR. Amounts in USD will never be exchange actually. s_n is therefore the liquidity premium for the *foreign* currency against the US Dollar[4]. These products are therefore quoted as a spread over USD-Libor[5].

To proceed the princing of the products, using the discount curve as in the former section is inconsistent. Therefore the practitioners use a different curve $p^*(t,T)$. This discount factor should hold for any product quoted in a given currency[6]. By the definition of the discount factor (as in the former section):

$$\sum_{i=1}^{n} \delta_i (f_i + s_n) p^*(T_i) = 1 - p^*(T_n)$$

Which gives the following bootstrapping relation:

$$\boxed{p^*(T_n) = \frac{1 - \sum_{i=1}^{n-1} \delta_i (f_i + s_n) p^*(T_i)}{1 + \delta_n (f_n + s_n)}} \tag{1.5}$$

The f_i's are calculated in the domestic market using (1.4) and (1.2). There is however something disturbing here. The disount factor $p^*(T_i)$ bears

[2]To make things clear, let us think about a bank that quotes its PnL in Euro that is involved in a cross-currency-swap in Swiss Franc and US Dollar. It is interested only in EUR/X exchange rate which has clearly no influence on the price of the swap.

[3]For example in late 2010, a tightening in the Swedish krona liquidity brought about a important rise in the SEK Basis ($\pm 50bp$ for several days in a row) which impacted heavaly the Interest Rate Linear book of the bank to about 6M€

[4]It also represents the difference between the supply and the demand for a given currency compared to the US dollar. Thus it reflects the cost of funding is this currency.

[5]The market quotes it over USD, in this bank they switch it as a spread above the domestic currency reference index against USD. The index BSEUR is therefore a spread over EURIBOR. Of course BSUSD is 0

[6]wich means that we should find $p^*(t,T) = p(t,T)$ when $s_n = 0$

information on s_i which is not supposed to play a role in pricing a basis swap with maturity T_n. Using relation (1.3) this cancels out so it becomes less of a problem. Pricing Cross Currency Basis Swap is a real issue in the banking institution, and the way to handle them varies from one bank to another: some take $p^*(T_n)$ as the discount factor that would be obtained using the modified forward rate $f_i^* = f_i + s_i$.
Since most of the traded Cross Currency Basis Swaps pay a cash flow only at the maturity, the problem disappears[7]. If the bank wants several coupons it can enter in several contracts, the price will then be higher.

1.1.4 Interpolating the Curves

These theoritical results are really handy, although in practice it becomes a bit messy. Values of an interest rate given a maturity is usually not quoted on a market place, but may be needed by the bank to price the assets it owns. For that it takes the quoted prices of different product to generate zero curve of this given rate. This implies interpolating (and extrapolating when necessary) the curve from the points than can be easily calculated, the section above was the first step. There exists several methods giving different accuracies: the ones that are manageable with a hand are of course far from perfect, interpolating precisely the curves is done during the night in the system. The detailed calculations are given in the appendix A. In practice the steps are different:

- First, a patchwork of quite a numerous amount of product is used to get a precise rate for as many maturities as possible, therefore calibrating those rates on the quoted prices. Given short or long maturity, one uses the most liquid products, as follows:
  - Short term: Mainly money markets
  - Middle term: Futures
  - Long term: Swaps, Cap-and-Floor's, Swaptions...
- For swaps the interpolation is more complicated that the ZT or linear I have myself used. With only around 30 swap prices available, involving time in a more accurate methodology would not have paid.

The easiest data to interpolate are discount factors from which we can obtain the zero and forward rates.
The goal of interpolationg is that given two maturities T_i and T_{i+1} when we have accurately calculated the rate, we want to calculate the same rate at a third date $\tilde{T} \in [T_i, T_{i+1}]$, so that:

$$p(t, \tilde{T}) = f(p(t, T_i), T_i, p(t, T_{i+1}), T_{i+1})$$

[7]My opinion is that those are the most traded because the other one are harder to price and would imply arbitrage opportunities or potential losses for the bank

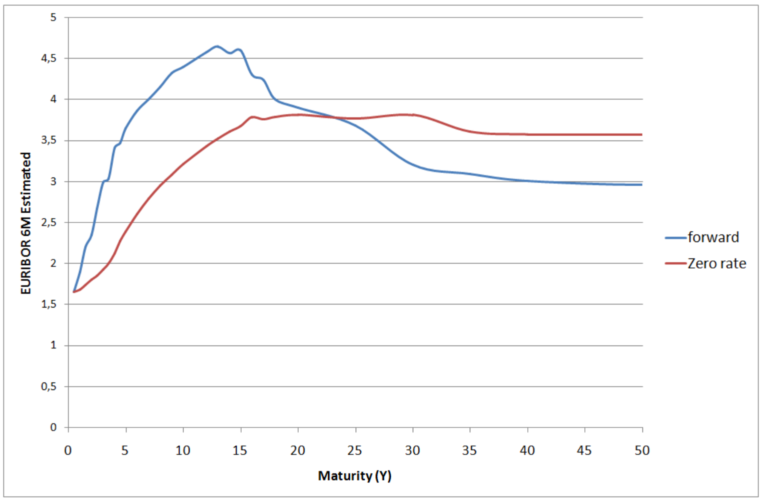


Figure 1.2: Interpolation of EURIBOR 6M from swap prices

There are several ways of choosing f:

- Linear : $p(t,\tilde{T}) = p(T_j)\frac{T_{j+1}-\tilde{T}}{T_{j+1}-T_j} + p(T_{j+1})\frac{\tilde{T}-T_j}{T_{j+1}-T_j}$
  Results obtained with this method are displayed in figure 1.2.
- ZT : $p(\tilde{T}) = \frac{T_{j+1}.p(T_{j+1})-T_j.p(T_j)}{T_{j+1}-T_j} + \frac{1}{\tilde{T}}.\frac{T_{j+1}T_j}{T_{j+1}-T_j}(p(T_j) - p(T_{j+1}))$
- More sophisticated models with recurcive formulas.

The only curve we have approximated is the discounted curve[8]. Then the zero rate is easy to obtain as:

$$r(T) = -\frac{\ln(p(T))}{T}$$

The forward rates can be calculated using (1.1). The discretisation of the formula leads to a lack of precision if there are not enough maturities that are calculated with a high accuracy (i.e calibrated with the prices). That is why these curves are not satisfying in my calculations. Things get easier with the basis indexes since the spread (over the reference forward spot rate) is explicitely quoted in the market.

1.2 Dealing with Sensitivities

Though easy in theory, calculating these sensitivities turns out to be harder when it has to be handled in practice. Unlike the equity Vanilla derivatives, interest rates ones' sensitivities on an index have to be calculated for the

[8]Even though in practice we observe that $\forall i, f_i \approx \bar{r}_i$

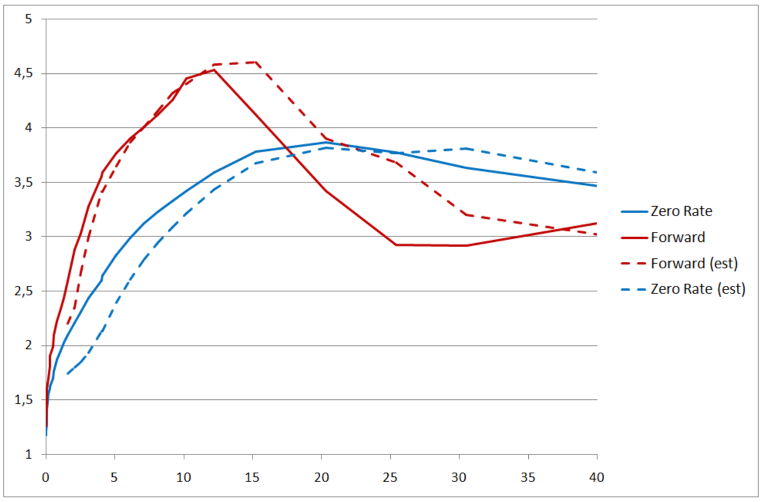


Figure 1.3: Comparison: Market Curves vs Calculated (Linear interpolation)

discount factor and the index itself for each maturity (sensitivity Δ contains also the information of ρ, the sensitivity on the discount rate).
There are two main methods used in practice to calculate them:

- Sensitivities to a move of the prices of the products used to get the yield curve : These are used for the calculation of the PnL. For example estimating the price of a swap may require to know the price of the 2Y-future on EURIB6M. Then the price of the swap moves if the future does. The 2-year sensitivity of this swap will then be the impact of a 1bp-move on the future price.

- Sensitivities to a move on the yield curve of the given index : These are used for calculating the VaR. This method is easier because the data necessary were more accessible and it makes more sense on a theoritical point of view. Therefore I will use this method from now on.

1.2.1 Fixed Income Products

The easiest case fixed coupon-paying bonds. The price is given actualising the future cash flows with the zero rate $r(T)$, representing the creditworthiness of the bond issuer. Indeed a cash flow at a given maturity is worth less if it is supposed to be received from Greece than from Germany.

With the notations as above :

$$\begin{aligned}\Pi(t) &= \sum_{i=0}^{n} PV_i(t) \\ &= \sum_{i=0}^{n} C_i.p(t,T_i) \\ &= \sum_{i=0}^{n} C_i.e^{-r_i.(T_i-t)} \\ &= \Pi(r_i,t)\end{aligned}$$

We notice that the value of a fixed income product is sensible to a variation of the underlying interest rate. Using Itô formula, we get

$$\mathrm{d}\Pi = \underbrace{\frac{\partial \Pi}{\partial t}}_{\theta}\,\mathrm{d}t + \sum_{i=1}^{n} \underbrace{\frac{\partial \Pi}{\partial r_i}}_{\Delta_i}\,\mathrm{d}r_i + \sum_{i=1}^{n} \underbrace{\frac{1}{2}.\frac{\partial^2 \Pi}{\partial^2 r_i}}_{\gamma_i} < \mathrm{d}r_i > \tag{1.6}$$

The correlation between the different buckets i are neglected in theory. In practice there is a method of calculation that allows to take into account this slight dependence[9], especially when it comes to swaps where the relation (1.3) bears the information. However further calculations are not relevent for these types of products.
Plus we start noticing that even if it seems easy to calculate there are some problems raising : What if the value of the rate is not known for a maturity T_i?

- Do we calculate the sensitivities using the interpolated rate?

- Do we calculate the sensitivity on the closest known buckets and get the one we want as a weighted average of both? This means that if we know $p(t, T_i)$ and $p(t, T_{i+1})$ and the maturity of a cash flow lies in between, the latter will be shared out among the buckets i and $i + 1$. It took me too much time to find a good documentation, so I chose to use the prior for the followings, which gives quite good matches with the data avalaible in the market and in the system (luckily this turned out to be the same one used in practice). This implies however that equation (1.6) does not hold any more given that the dates at which we calculate the sensitivities are not necessarily the same as the ones when the cash flows are exchanged. Actually the bucket i bears a part of the sensitivity of all the flows exchange between the dates T_{i-1} and T_{i+1} since they are discounted with a factor calculated from the closest

[9] It becomes important for exotic products with high γ and σ.

two buckets (see linear interpolation of discount factors).

Actually if linear interpolation is used, sharing out cash flows or interpolate discount factors gives the same results. So setting C_i as the sum of shares of cash flows which have sentivities on the bucket i, we find again (1.6). And further calculations give for a coupon paying bond[10]:

$$\begin{aligned} \theta &= \sum_{i=1}^{n} C_i r_i e^{-r_i T_i} \\ \Delta_i &= -C_i T_i e^{-r_i T_i} \\ \gamma_i &= \frac{1}{2} C_i T_i^2 e^{-r_i T_i} \end{aligned} \tag{1.7}$$

1.2.2 Plain Vanilla Swaps

This part will be trickier since the sensitivity in the zero rate r_i hangs in both the discount factor and in the forward rate:

$$f_i = -\frac{\partial \ln(p(T_i))}{\partial T_i} \quad = \frac{\partial (r_i T_i)}{\partial T_i} = r_i + T_i \frac{\partial r_i}{\partial T_i}$$

The discrete form is therefore:

$$\begin{aligned} f_i &= r_i + T_i \frac{r_{i+1} - r_i}{T_{i+1} - T_i} \\ f_i &= r_i \left(1 - \frac{T_i}{T_{i+1} - T_i}\right) + r_{i+1} \frac{T_i}{T_{i+1} - T_i} \end{aligned} \tag{1.8}$$

Since in most of the cases the rates r_i and r_{i+1} will need to be interpolated, a floating cash flow received or paid at time $\tilde{T} \in [T_{i-1}, T_i]$ will be sensitive to variations on the buckets T_{i-1}, T_i and T_{i+1}. There is no special need to get an explicit formula for to theoritically get the sensitivities. The methodologies used to calculate the latter are detailed in Appendix B, using two different methods.

1.2.3 Cross Currency Basis Swaps

Cross Currency Basis Swaps are priced using an alternative discount factor $p^*(T_i)$ where lie sensitivities on f_i, s_i (albeit slight) for every intermediate maturities and also f_1, f_n and s_n after the simplification (1.3). There is no explicit formula as (1.8) available since their exists no such thing for the

[10] The repartition of the cash flows does not add any sensitivity, since the dependence to t cancels out

$p^*(T_i)$'s. Therefore the easiest method is to shift the input data and see the impacts on the prices.
However as I said earlier, most of the Cross Currency Basis Swaps traded are over a single period, which means that the discount factor is given by the quoted spread. The volume traded is high enough to estimate the discount factors for a sufficient amount of maturities. Therefore those can be estimated as follows:

$$\begin{gathered} T_n(f_n+s_n)p^*(T_n) = 1 - p^*(T_n) \\ \boxed{p^*(T_n) = \frac{1}{1+T_n(f_n+s_n)}} \end{gathered} \tag{1.9}$$

From this we can derive explicitely the sensitivity on the floating leg (the one bearing the spread). Though a formula of the sensitivity to the spread (1.10) is bearable the on to the zero rate is too heavy to be interesting.

$$\begin{aligned} \Delta_{s_n} &= \frac{\mathrm{d}C_n}{\mathrm{d}s_n} \\ &= \frac{\mathrm{d}\left((f_n+s_n)p^*(T_n)\right)}{\mathrm{d}s_n} \\ &= p^*(T_n) + (f_n+s_n)\frac{\mathrm{d}p^*(T_n)}{\mathrm{d}s_n} \\ &= \frac{1}{1+(f_n+s_n)T_n} + \frac{(f_n+s_n)T_n}{(1+(f_n+s_n)T_n)^2} \\ \Delta_{s_n} &= \frac{1}{(1+(f_n+s_n)T_n)^2} \end{aligned} \tag{1.10}$$

Theses calculations are derived as information they are not used in practice. Nevertheless, we observe that for very short maturities $\Delta_{s_n} \approx 1$. The positions are harder to hedge and can bring about consequent losses (cf swedish exemple above).

1.2.4 Analyses

Using (1.3) we notice that if the first floating cash flow is received at time T_1, then:

$$\Pi_f = N\left(p(T_1) - p(T_n)\right) \tag{1.11}$$

The sensitivities are borne on the first and last payments[11], independently on the intermediary cash flows, whereas the fixed cash flows are sensible to a move of the zero rate for any maturity.

$$
\begin{aligned}
\mathrm{d}\Pi_f = &\overbrace{N\left(r_1 p(T_1) - r_n p(T_n)\right)}^{\theta} \mathrm{d}t - \overbrace{NT_1 p(T_1)}^{\Delta_1} \mathrm{d}r_1 + \overbrace{NT_n p(T_n)}^{\Delta_n} \mathrm{d}r_n \\
&+ \underbrace{\frac{1}{2} NT_1^2 p(T_1)}_{\gamma_1} \mathrm{d} < r_1 > + \underbrace{\frac{1}{2} NT_n^2 p(T_n)}_{\gamma_n} \mathrm{d} < r_n >
\end{aligned}
$$

These sensitivities are very handy to further calculations. Calculating the VaR with full pricing takes a night with powerful computers, whereas sensitivity analyses[12] take a couple hours on my own computer. The price to pay is of course accuracy. From now on however we will deal with the risk issues only on the sensitivities point of view. Typically they are calculated on 33 buckets going from 1 day to 50 years.

1.2.5 Methods

There exist several methods to approximate the sensitivities, which can not be calculated theoritically. This estimation is useful for three purposes:

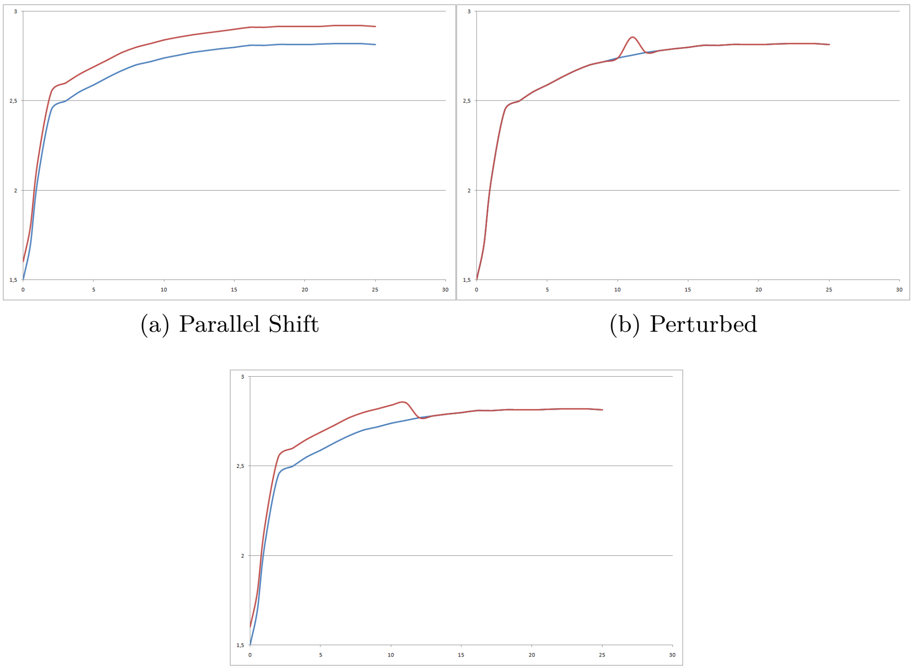
- PnL: Calculate the PnL of the book.
- Risks: Estimating the risks inherent to a book and the losses it can imply.
- Hedge: Gives a quick and handy outlook and how any product should be hedged.

The actual estimation is provided is appendix B, one of the following three methods is usually used (figure 1.4):

- Parallel shift: Gives the sensitivity of the price of the product to a parallel shift of the curve. About 80% of the market movements can be considered as *parallel*. So this method gives handy results but are not very precise and useful quantitatively: what if there is a huge move on the short term only (i.e liquidity tightening for example)
- Perturbed: The method lies in shocking all the buckets one after the other and observing the impact on the prices. It is accurate enough for vanilla products but gives poor results for exotic one (where convexity is more important)

[11] Not be mistaken, the rate at which the first cash flow of the floating leg is paid is known at time 0, the sensitity referred as *1* is the sensitivity to forward spot rate at time T_1 at which the second cash flow will be exhanged

[12] Applying past shocks to current sensitivities



(a) Parallel Shift

(b) Perturbed

(c) Cumulative

Figure 1.4: Different Curve-shifting Methods

- Cumulative: Shocks the buckets one after another but keeps in memory the impacts of the former shocks.

The sensitivity on the bucket in the move of the price following the shift. The cumulative sentivity is a move of price compared to the prior shift (i.e cumulative shift until the prior bucket). This method is the only one that provides good results for exotic products. Linear products are manageable with the perturbed methodology. Actually the linearity of the products implies that whichever method one uses (perturbed or cumulative) one obtains the exact same results.

Chapter 2

Risk Measures : Power, Limits and Alternatives

Now we have seen how banks manage their data, I will develop how they use them to managing their risk. Since the early 90's three factors increased the risk which banks are exposed to:

- The rise of derivatives allowing to have more exposure with the same amount of money involved.
- Progress in IT that boosts the volume of trades.
- Spread of credit risk through securitisation.

The last crisis and its consequences made institutions realize that their management was not appropriate. Therefore they increased their need of capital linked to the following risk measure. They can be separated in two categories: the objective and subjectiv ones.

2.1 Common Measures and Limits

There are common measures that every banks have been using for a while. Although they have shown some limits their apparent objectivity have made them popular among risk managers, plus VaR and stressed VaR are used as benchmark measures within the banking system: under Basel III the minimal amount capital C allocated to market risk is

$$\begin{aligned} C &= 3(\mathrm{VaR}_{1\%,10\mathrm{days}} + \mathrm{StressedVaR}_{1\%,10\mathrm{days}}) \\ &= 3\sqrt{10}(\mathrm{VaR}_{1\%,1\mathrm{day}} + \mathrm{StressedVaR}_{1\%,1\mathrm{day}}) \end{aligned} \tag{2.1}$$

2.1.1 Value-at-Risk and CVaR

Even if they are well-known I recall their definition:

- **Value-at-Risk** : It represents an amount of losses the bank is not supposed to exceed given a level of confidence α, usually 99%. It is denoted as VaR_α.
- **Conditional Value-at-Risk** or **Expected Shortfall** : It is the mean of the worst $1-\alpha$ losses. It is denoted as ES_α

If L is the distribustion of losses:

$$\mathrm{VaR}_\alpha = F_L^{-1}(1-\alpha)$$
$$\mathrm{ES}_\alpha = E[L|L < \mathrm{VaR}_\alpha]$$

The Value-at-Risk is the most popular risk measure in the banking system. It is so for several reasons: it is easy to calculate, reflects the positions, is beyond contestation[1] and is the oldest one[2] and therefore calibrated and easily trackable.
The Condiational VaR helps to consider information on tail risk that are not in the VaR. It is not easily readable and since it has not spread enough in the banking system, it makes comparison from one insitution to another complicated. Most of them do not use it.

I computed a program that calculates the VaR using just sensitivities, the steps are as follows:

- First calculate the sensitivities of all the products of a given trading book to all the different indexes on all the 33 buckets (this is computed at night time in the system)
- Then the aim is to estimate how the interest rates have moved overnight over the past year, and calculate the PnL that such moves would imply on the current book.
  - In the system, they take the cruves as they were at days d and $d+1$, calculate the Marked to Market at both dates, and the difference is the PnL as it would be if we meet the same markets conditions as day d[3].
  - The sensitivity methodology is different. We export the the values of interest rates on the considered bucket at days d and $d-1$, then estimate the shocks that occured on this interest rate this time (the methodology to calculate the shocks is given later). Applying these shocks on the sensitivities gives a really good estimate of the hypothetical PnL.
- The last step is just sorting the scenarii from worst to best. From there calculating α-VaR and α-CVaR for every α becomes easy.

[1] It is apparently not impacted by the subjective views of the risk managers.
[2] Computed the for the first time in 1993 at JP Morgan Chase
[3] The same methodology applied to calculate the daily PnL

Importing 200 indexes' values over 261 scenarii for the 33 buckets plus calculating the shocks and the impact on the PnL with the sensitivities made the file heavy and not easy to use[4]. However the results were good and fitted the system.

Albeit easy and handy the VaR shows quickly its limit. First the time horizon is limited, in the sense that we restrict future possible outcomes to the ones that occured over the past year. For example the VaR of the Interest Rates books decreased a lot in late may although the positions were still the same and there were no sign of lower risk. The outlook of PIGS' default would even lead us to the opposite conclusion. The reason is that, time passing by, the scenarii of May 2010 became excluded from VaR calculation whereas they were the worst ones. This observation raises some questions about the legitimacy of this risk measures (VaR and CVaR). While relatively wise risk manager would consider a growing risk on the bond market, the supposed objective and undiscussable risk measure tells the opposite.

Shocks Calculation

This also deals with the relevance of the VaR. We denote by *shock* the way to quantify the overnight moves of an interest rate. There are two methods that seem legitim and come first in the mind:

- **Absolute**: This is the raw overnight increase of the index in basis point. It seems at a first glance perfectly logic. However, there have been quite a huge volatility over the past two years. EONIA for example have been from around 2% to 0,25% and to 1,25% again. An increase of 10bp in the first case makes no sense in the second one: it reperensents respectively and increase of 5% and 40%.
- **Relative**: That is why we consider also the relative increase, as a percentage of the value of the rate, that we multiply with today's values. However, we get the opposite problem as with the absolute method. When the ECB increase the short term rate of 25bp, it represents 100% in the second case and and 12,5% in the first one.

Then the risk manager have to use their knowledge of the markets to decide which method to choose. This partly reduces the powerful objectivity of the VaR and CVaR.

Back-Testing

One way of validating the model used to calculate the VaR is Back-Testing, i.e checking whether the daily PnL is actually over the VaR of the day. The model is consider validated if *back-testing exception* happen less than $\alpha\%$

[4] It took 2 to 3 hours to run completely

of the time (i.e twice a year for a 1%-VaR). For example UBS got around 30 back-testing exceptions in the yeary 2008 alone and BNP-Paribas which had the lowest level of exceptions got 6. The model of the VaR is clearly not appropriate when it comes to tail risk, the theory behind it gave some clues and the events confirm them.

2.1.2 Stressed VaR

To tackle the issue of the *one-year* data, stressed VaR was created. The methodology is the same as the common VaR except that the period of time over which we analyse the underlyings' moves is fixed and taken in a period of high stress that we fear they may happen again. The banks chooses this period to be the one that would imply the heaviest losses given current positions. For example I suppose that now in the main banks the stressed period lies somewhere between 2007 and 2008.

Then the methodology to calculate shocks and PnLs is exactely the same as in last section, with the same pros and cons. Even if choosing the period of time gives more legitimacy to the accuracy of the model, this implies that all the possible outcomes have already happened sometime in the past. Plus the stress VaR period(s) are the same for every activities in the bank: 2007-2008 scenarii would strongly impact the equity activities whereas bonds trading activities would need a *2010-2011* kind of period.

Anyway these measures and their alternatives (CVaR, Monte-Carlo VaR and so on) give a fairly good outlook on the risk inherent into traders' positions but they can not be relied on when it comes to check on the cushion of capital supposed to absord potentially huge losses. However in theory, taking C as in (2.1) should ensure that a pure trading book would survive for one more year with a confidence interval of 99%. Indeed since we observe that in practice we have often $StressedVaR_{\alpha,1day} \geq VaR_{\alpha,1day}$ we get:

$$\begin{aligned} C &\geq 3\sqrt{10}.2VaR_{1\%,1day} \\ &\approx \sqrt{360}VaR_{1\%,1day} \\ C &\geq VaR_{1\%,1year} \end{aligned} \tag{2.2}$$

since it is known that $VaR_{\alpha,Ndays} = \sqrt{N}.VaR_{\alpha,1day}$. The Basel agreements require that the bank ensures survival over one year with a confidence interval of 99,9%. However that has only to do with the banking book[5], whose implied losses are far more important[6]

[5]loans, investment bonds, ABS and so on

[6]For example, losses on ABS cost the bank 15bn€during the year 2008

2.2 Coming to Stress Testing

2.2.1 Use and Relevance

The introduction of stress testing makes perfect sense following the idea: we fear an event to happen, let us say, the greek state goes to default with a haircut of 60% on its debt.
This gross scenario is easy to solve: we multiply the exposure the bank has on the greek state by 0.6 and we get our loss. That would mean neglecting the impacts such a huge event may have on the economy and thus the markets. Following default, as it did for Lehman Brothers, it will not be clear who had which positions on Greek bonds[7], and lead to a confidence crisis and thus a liquidity crisis, increasing interbank interest rates. Confidence on other fragile states will also decrease bringing up the spreads on Spanish, Portuguese, Italian and Irish bonds among other. This may impact the equity market too as the investors flee risky positions to safety ones, bringing down equity prices and up assets like gold or swiss franc. This to show that the consequences of a macroeconomic scenario on the markets can be very diverse.
The next question is how to quantify some book's exposure to such a scenario? Clearly nothing likely has happened during the past year (so no use for the VaR) and even before[8] (so no use for stressed VaR either). That is the time when economist, risk quants and managers come to work together on elaborating a model quantifying this exposure, turning the macroeconomical events into moves on data used to price the assets of the book. This can be:

- Market Data : A stress scenario whichever it is will have an impact on the data quoted on the markets. Equity usually goes down and interest rates rise in such a case.
- Non-observable data: They can move highly under stress and impact asset prices. For example stress increases volatility and correlation which need therefore to be recalibrated to estimate the losses they imply.
- Models: On the edges the models are less precise, and the model validation departments establish reserves that need to be applied when thoses edges are met. Plus under stress the calibration is no longer available and a new one modifies the parameters of the model (typically α and ρ for the SABR model)

To summarize, proceeding to stress testing means that under a given scenario, one needs to estimate the impacts on the parameters which the

[7]though one of the goal of the last stress tests round was to force the banks to show their unveal on the Greece

[8]Some states actually defaulted but none that had that much impact on the European market

book has sentivities on. For the books I was dealing with those were: δ_{rate}, δ_{basis}, γ_{rate}, γ_{basis}[9], fx-rates (plus α, ρ_{SABR}, and σ when dealing with inflation products)

However this is not enough. One important part of stress testing is to estimate a confidence iterval of such an chain of events happening. Such an estimation is not easy but as we will see in the next chapter some probabilistic tools may help us.

2.2.2 Objective vs Subjective views

As I mentioned earlier the power of VaR and Stressed VaR is that it is fairly objective (except the shock calculation) and makes comparison of the risks taken by different banks easier to drive and easier to audit. However, their range is clearly limited and it cost a crisis to realize that they were not enough to estimate unexpected losses on the trading books. This cleared the passage to subjective (comprehensive) risk management, which aim is to estimate possible market outcomes from unlikely but still plausible macroeconomic stresses. This demands more work, calculation and care, and depends highly on the risk manager's subjectivity but the results are much more significant to his eyes.

[9] Since the dependency of the price to the spread s_n is linear (i.e the impact on the discount factor is really low), γ_{basis} is close to 0

Chapter 3

Applications

3.1 Probabilistic Issues

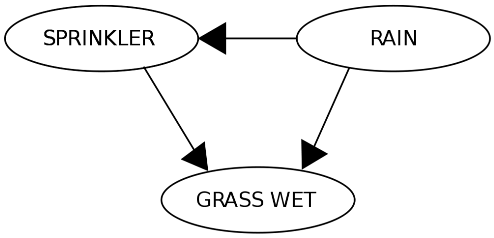
It is one thing to imagine macroeconomic outcomes, but it is nothing if one cannot estimate the likelihood of this happening. This part I could unfortunately not have access. I will anyhow try to draft the main steps one has to follow to complete this estimation.

The utlimate goal of this quest is to get the joint probabilities of different events to happen. The task is far from easy since the information needed are more numerous than the one in our possession. One way of dealing with this duality is to use bayesian nets (an example is drawn in figure 3.1).
A bayesian net is a acyclical directional graph that draws causal links between events happening. As every graphs there are nodes, which represent the events, and edges which represent the conditional linnks between nodes. Each node can be seen as a function of boolean: the input being the realisaton or not of the events whose edge points on the node, the output the probability of the event being true or false.
To illustrate the methodology, let us take the simple example in figure 3.1.

The aim is to estimate the probability of the grass being wet or not. Although the question seems easy the answer is not trivial without historical *data*.

- The first step is to draw a list of all the events that may influence the result. Here if we observe that the grass is wet this means that either it rained or the sprinkler was activated. Both we therefore be the *parent* events of the *child* "wet grass".
- The second step is to estimate the probability of each events happening in the following order:
  1. Get the marginal probabilities of the "first-order" parents. Here the only event of this kind is "Rain" (i.e nothing can cause rain)

RAIN	SPRINKLER T	SPRINKLER F
F	0.4	0.6
T	0.01	0.99



RAIN T	RAIN F
0.2	0.8

SPRINKLER	RAIN	GRASS WET T	GRASS WET F
F	F	0.0	1.0
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.99	0.01

Figure 3.1: Bayesian net drawn on a simple example

2. Get the conditional proabilities of the secont-order parents, and so on until the last order.
3. Putting all the results in one, we obtain the probability we are seeking.

The trickiest part is to estimate the probability quantitatively. As we will see, it is already messy with only three variable easily linked:
$G = 1$ and $G = 0$ denotes the event of the grass being respectively wet or not. Same R and S denotes the outcomes of the other events. The joint probability is given by:

$$\begin{aligned} P(G, S, R) &= P(G|S, R)P(S, R) \\ &= P(G|S, R)P(S|R)P(R) \end{aligned} \tag{3.1}$$

The goal of (3.1) is to express the joint probabililties as a function of the conditional and marginal probability we know[1]. The results are drawn in table 3.1. For example the last line is derived from (3.2).

$$\begin{aligned} P(G = 1, S = 1, R = 1) &= P(G = 1|S = 1, R = 1)P(S = 1|R = 1)P(R = 1) \\ &= 0.99 * 0.01 * 0.2 \\ &\approx 0.002 \end{aligned} \tag{3.2}$$

Id	R	S	G	$P(G\|R,S)$	$P(S\|R)$	$P(R)$	P
p_1	0	0	0	1	0.6	0.8	0.48
p_2	0	0	1	0	0.6	0.8	0
p_3	0	1	0	0.1	0.4	0.8	0.032
p_4	0	1	1	0.9	0.4	0.8	0.288
p_5	1	0	0	0.2	0.99	0.2	0.0396
p_6	1	0	1	0.8	0.99	0.2	0.1584
p_7	1	1	0	0.01	0.01	0.2	0.00002
p_8	1	1	1	0.99	0.01	0.2	0.00198

Table 3.1: Joint Probabilities

Therefore the probability of the grass being wet is given by:

$$p_2 + p_4 + p_6 + p_8 = 0.45$$

This example was simple but shows roughly how things work. I could have taken a financial example but I have not the necessary background to estimate myself the probability and the links of such events. The bayesian nets however are really powerful for several reasons:

- They are really easy to draw and clear to read.
- They are easy to modify, if we want to take into account more or less events, it the example we could have added:
  - Threat of draught, in which case it would be forbidden to use the sprinkler.
  - The season, it is more likely to rain in fall than in summer.
  - and so on...

Using the cleverest tricks to establish the marginal, conditional and even joint probabilities among events will do us very little good unless we have picked our stress scenarios in an intelligent way:

- Top Down approach: Economists draw plausible macroeconomic scenarii, and then derive the impact of those on the trading and banking books
- Bottom Up approach: Risk managers determine the weaknesses of the portfolios (i.e *where it hurts*) and derive scenarii that stress these vulnerabilities.

For example, if we consider greek default, this event will be at the very top of the graph. It would cause increase in CDS' spreads, a downturn in other bonds prices, that would themselves imply moves on the markets. The

[1]If there are two parents of the first order, then they are conditionally independent, a the final expression is roughly the same.

following steps are quantitative and macroeconomic analyses which I deal with it the next sections.

3.2 Modelizing Stresses

As I said before the main use of stress testing is to estimate the behaviour of a portfolio under some situation. Once the hardest part of determining quatitatively the stresses, the rest is calculation.

3.2.1 Former Crises

The banks like to see how their books would react if similar moves as former crises happen. For some periods like 1987, 1994 or 1998 stressed VaR is useless. There are not enough data available and quantitatively speaking the moves that occured at this time have nothing to do with the one that may happen now. The aim of stress testing is not the reproduce exactly the same market condition but to consider the same macroeconmic context and estimate the consequences that thesse conditions would have on the markets. In one word one has to recalibrate these moves to fit the current market conditions

The crises reproduced by many banks for the use of stress testing are :

- **1987**: Most of the developed world's indexes fell sharply (S&P downturn was up to 35% is 3 days), as shown in Figure 3.2
- **1994**: The december mistake. This follow a sharp devaluation of the mexican peso, which spread to assets quoted in USD. Since this one was not considered as systemic, the effect on the global economy were limited.
- **1998**: Russian Crisis (see later)

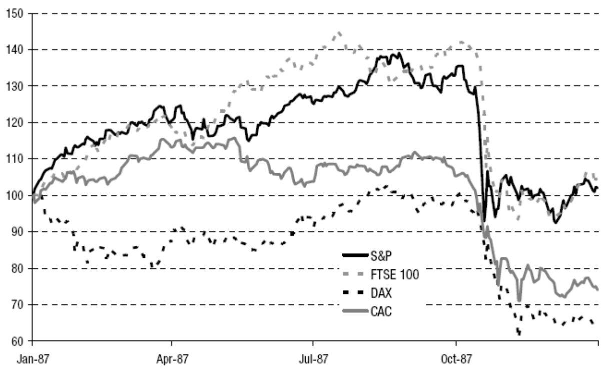


Figure 3.2: The Black Monday

A special care will be given to two crises: The russian crisis of 1998 and the last one in 2008 (Lehman)

Russia 1998

The first step is to analyse the macroeconomic data provided by the economists, the effect that these conditions had on the markets at the time and estimate which they would have now.

Economists report:
In mid-August 1998, Russia faced a severe cash-flow problem as investors withdrawn from the government debt market. As foreign reserves dropped quickly, Russian government announced a restructuration of rouble-denominated government's debt and an increase of the exchange rate. In 10 days, from August the 17th, Russian spreads doubled. The consequences on the global markets were the following:

- *Rapidly, a panic regarding other emerging markets appeared: 50% relative shocks on their credit are observed on average.*
- *After few days of a "flight to quality" process (gold, raw material), European and American spreads brutally rose because of an extended fear of a systemic crisis (as it has happened with Greece over Spain and Italy and now France). Meanwhile, equity markets suffered a severe adjustment and volatilities rose significantly.*

This first analyse sets qualitatively the starting point of the work. The next step is to determine which market paramaters would, on one side affect the prices of the bank's assets and on the other side how they would be affected by such a crisis-like event. Such parameters are called *risk factors*:

- Equity: Spot, volatility, correlation, dividends
- Interest rates: Spot (other rates can be derived), volatility (hardly any products trade correlation)
- Foreign Exchange: Spot, volatility
- Credit: Spread, volatility
- Commodities: Spot, volatility (energy, base metals, precious metals)

Their moves implied by the crises lie in table **??**.

This gives a scale on which we should condiser the moves, the quantitative approach is harder. It should be a compromise between the *plausible* and the *alarmist*. The higher the moves are taken, the higher would be the losses on the portfolio put less plausible they will be. If we take these moves too likely, they would be nothing like *stresses* but more like common moves, bringing no more information than VaR. I can not bring many details on this quantitative approach of risk because I am not allowed to. However I

Underlying	Risk Factor	Impact
Equity	Spot	−
	Volatility	++
	Correlation	+
	Dividend	0
Interest Rates	Spot (G11)	−
	Spot (Em'ing)	+++
	Volatility	+
FX	Spot (USDvsRUB)	++
	Spot (USDvsOthers)	-
	Volatility	+
Credit	Spot	+++
	Volatility	++
Commodities	Spot (En, BM, PM)	0
	Vol (En, BM)	0
	Vol (PM)	++

Table 3.2: Impact of the 1998 Russian crisis on market factors

will display some results in the next part, removing the scale and numerical outputs[2].

Nevertheless, there is a last step before purely quantitative analysis, we need to define the time window on which the stresses apply. It is clear that the effect of a crisis do not appear in one day, therefore there are two approaches:

- We consider only the period of trouble, considering only the positions of the portfolio, mutliplying sensitivity by market moves. The russian scenario was determined using these stresses, over a 10 business days period, when the market moves were highest.
- Or we consider a wider window, even the overall period of crisis, assuming that the bank can refund and close out the positions. This brings about new analyses on the cost of funding under the crises for example, that is the approach under which ECB stress test were lead.

Regarding histrical crises however, one favour the first approach, since their use is more to give a idea than they challenge the capital of the bank.

[2]Especially because the following 2008 stress project has just been completed.

Following Lehman: October-November 2008 and International Tensions

Unfortunately I had no access to the process under which the stresses were quantitatively determined, but only to the results, which I used on a weekly basis on interest rates.

After the crisis, some sovereign debt were seen as relatively *risky*, so the price of their bonds fell (like French and Italian, Figures 3.3 and 3.4) rising the zero rate and the spread above the reference index, which a better indicator of the creditworthiness of a country. Others like Germany were seen as a safe investment, so their rate decreased same for the spread (figures 3.5 and 3.6).
Plus EURIBOR was falling, following moentary policies of the central bank to restaure confidence

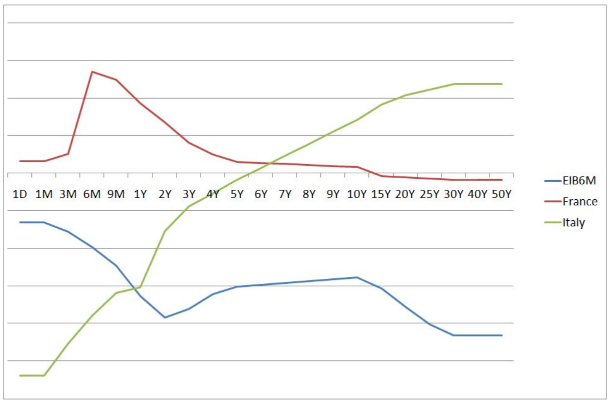


Figure 3.3: Impact on Italian and French treasury rates

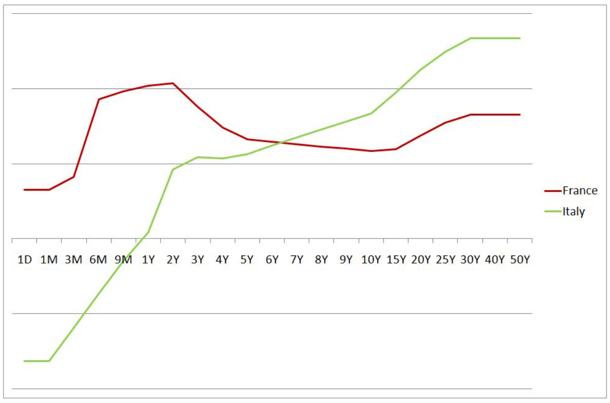


Figure 3.4: Impact on Italian and French spreads over EURIBOR 6M

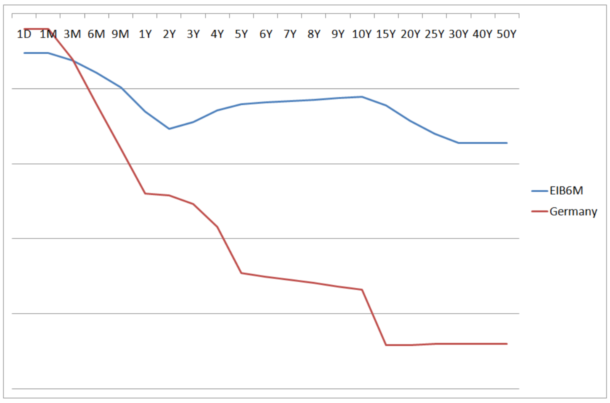


Figure 3.5: Impact on German treasury rates

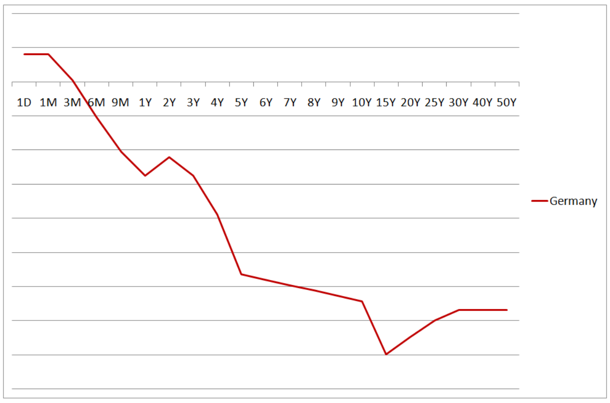


Figure 3.6: Impact on German spreads over EURIBOR 6M

I chose first the examples of the bonds the examples of bonds since they are the most challenging interest rates derivatives right now. For example the shocks on the PIGS treasury rates are negative. This result may question the use of historical stress testing when the economy is already troubled. This argument is countered however by the fact that these scenarii are set to be used for years, and even though the seem totally unrealistic now their use in the future could turn out to be precious.

Now we have roughly seen the way to proceed to create a stress scenario out of historical macroeconomic outcomes, we will see how to do for an *out of the box* scenario. I will stress on liquidity issues which are out of the range of VaR calculation and also the way to allocate reserves to the PnL.

3.2.2 Liquidity issues

I will care first about the liquidity issue following a confidence crisis. In this case the short term interbanking rates rise (typically EONIA and TAM). We then consider an increase of the cost of funding not only on the position the bank has now, and to the one it needs to refund (see later for ECB's stress tests).

What about market liquidity though? This one is observed among other things in the spread between the bid and ask prices (refered from now on as the spread bid/ask) setting a bid/ask reserve. This reserve represents the cost of hedging or closing out the positions. The idea comes from the fact that the PnL was calculated and the position hedged considering price at par (i.e the mean between the bid and ask prices), this needs to be corrected with a bid ask gap, widenning along when liquidity tightens. Therefore the reserve increases along with illiquidity, and has to be taken as a risk. Two choices are possible:

- Consider it as a potential risk and set a limit on it considering normal or stressed spreads.
- Calculate reserves on a regular basis, on the the positions of the portfolio. This reservres would impact the PnL of this portfolio, creating a compromise between the benefits of the positions and their exposure to illiquidity.

A liquidity tightening brings about moves of every risk factors: prices go up, volatility increases due to nervosity, same with correlation, implying a reserve for every greek. Therefore an estimation of the reserve demands sophisticated models, which I would not dare detail here. However,linear interest rate products would be concerned only by a rise in the price itself of the product (i.e the fixed rate for a swap, for example) and not its volatility and even its γ since we saw that this one is really low. This makes the model simpler. What follows will be a digression from the stress testingbut it is clear that once we get the methodology to estimate the liquidity risk, by applying diverse spreads bid/ask, we will get a range of possible outcomes. The problem will be which spread to choose, which question quants or consulting teams are supposed to answer to.

Linear products are not traded on a single market, therefore there need to be reserves for each one of them. A common step is to share out sensitivities within n buckets. The earliest maturity is usually $1/2$ Month - 1 Month, otherwise it is more of a funding issue. The spread bid/ask with reference to bucket i and currency j is δ_{ij}. T_i is the maturity of the product, whichever it is (FRA, swaps...)

Spreads are determined by index from a pool of products, they are normalised so that the sensitivity of a product with a spread δ_{ij} is 1.

$$\delta_{\text{quoted}} = \frac{\delta_{\text{actual}}}{|\Delta|}$$

In the upcoming sections, I will denote by *price* the surplus implied by the bid offer spread, i.e $\Delta_i.\delta_{ij}$.

Rate positions

First there is something we have to care about considering IRD linear products, it is easy to show but it can lead to some mistakes. Indeed, unlike the equity market, when $\Delta < 0$ the position is long. For example, when you own a bond, if the zero rate increases the price decreases which means that you are Δ-negative.
From now on we will consider a portfolio with the sensitivities as is figure 3.7.

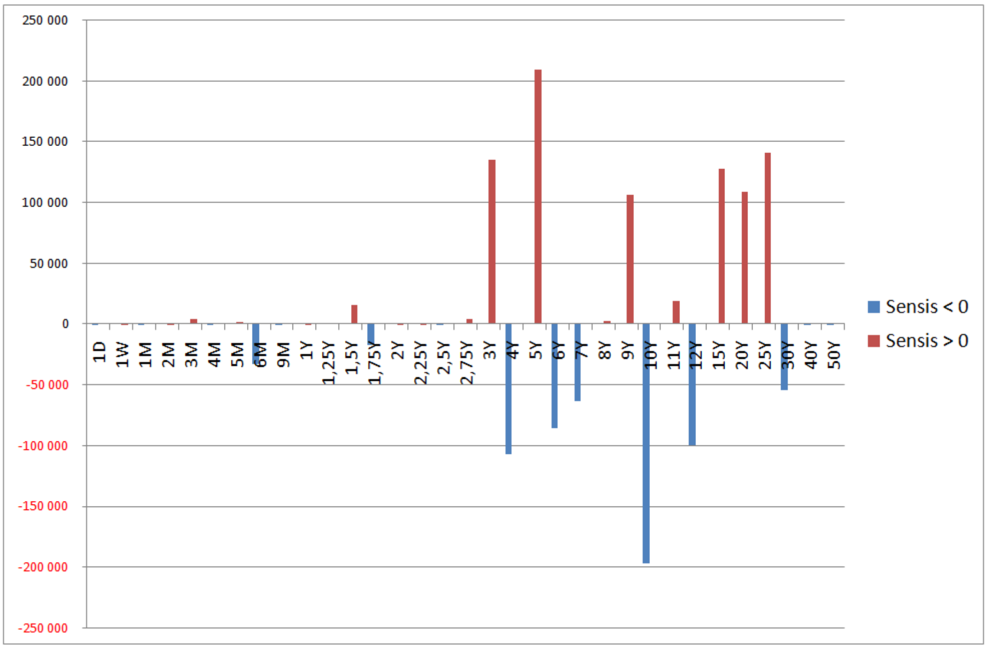


Figure 3.7: Sensitivities of the portfolio

Let us consider first a portfolio with positions on only 2 buckets. A simple strategy would be to cancel the sensitivities Δ_1 and Δ_2 buying now 1-maturity products with sensitivities $-\Delta_1$ and $-\Delta_2$. However we can also see that we could consider entering in some contracts, with price 0 (actually, considering the spread bid/ask, this amount will be some constant $\Pi > 0$) now, that would make us take a long position on one maturity where the sentitivity is positive and another one the other way round. The amount of the position would cancel the lowest sensitivity and reduce the other one in absolute value.

For example we take a portfolio with sensitivities on only two maturities T_1 and T_2: Δ_1 and Δ_2 such that $\Delta_1 \Delta_2 < 0$ and $|\Delta_1| < |\Delta_2|$. We consider the product with price Π as above, which sensitivity on the bucket i is given by $\Delta_{\Pi,i}$:

The reserve R imlplied by simply closing out with one maturity products is:

$$R = \frac{\delta_1}{2}|\Delta_1| + \frac{\delta_2}{2}|\Delta_2|$$

Let's consider the other strategy. The first step *costs* $\frac{\delta_2}{1}|\Delta_1|$ bringing the sensitivity on the first bucket to 0 and the second one to $_1 + \Delta_2$ (cf (3.3)).

$$\begin{aligned} \Pi &= \Pi_1 + \Pi_2 \\ \Rightarrow \mathrm{d}\Pi &= \mathrm{d}\Pi_1 + \mathrm{d}\Pi_2 \\ \Rightarrow \Delta_{\Pi,1} + \Delta_{\Pi,2} &= 0 \\ \text{since we want } \Delta_2 + \Delta_{\Pi,2} &= 0 \\ \text{we have: } \Delta_{\Pi,1} &= \Delta_2 \end{aligned} \tag{3.3}$$

The overall reserve is therefore:

$$R' = \frac{\delta_2}{2}|\Delta_1 + \Delta_2| + \frac{\delta_2}{2}|\Delta_1|$$

- If $\Delta_1 < 0 \Rightarrow |\Delta_1| = -\Delta_1$, $\Delta_2 > 0$ and $|\Delta_1 + \Delta_2| = \Delta_1 + \Delta_2$. Hence $R' = \frac{\delta_2}{2}\Delta_2$

- If $\Delta_1 > 0 \Rightarrow |\Delta_1| = \Delta_1$, $\Delta_2 < 0$ and $|\Delta_1 + \Delta_2| = -\Delta_1 - \Delta_2$. Hence $R' = -\frac{\delta_2}{2}\Delta_2$

Hence:

$$R' = \frac{\delta_2}{2}|\Delta_2| < R$$

Then, when there are more buckets we do as follows:

1. We identify whether we have higher positions on the long or short side[3]. Following the figure 1.8, the portfolio is more sensitive to increase in the rate.

[3] this amount needs to be weighted by the spread, since we want the cheapest strategy

2. We enter successively in contracts that close out the lowest positions.
3. Once the lowest side is closed out. We cancel the remaining positions by the taking one-maturity products.

If we set:

$$I_+ = \{i, \Delta_i > 0\} \quad \text{and} \quad I_- = \{i, \Delta_i < 0\}$$

We solve this problem as we solved the one in the previous part total reserve is therefore:

$$R = \underbrace{\sum \frac{\delta_j}{2} \Delta_i}_{(1)} + \underbrace{\sum \frac{\delta_j}{2} \Delta_j + \Delta_i}_{(2)} \tag{3.4}$$

Where (1) represents the price entering two-maturity products, and (2) is the price of closing out the remaining positions.

(3.4) becomes, for our example:

$$\boxed{R \approx \sum_{i \in I_+} \frac{\delta_i}{2} \Delta_i} \tag{3.5}$$

The problem is totally symmetric, therefore we derive the complete expression of the reserve R_j on the index j:

$$\boxed{R_j = \max \left(\left| \sum_{i \in I_+} \frac{\delta_{ij}}{2} \Delta_i \right| ; \left| \sum_{i \in I_+} \frac{\delta_{ij}}{2} \Delta_i \right| \right)} \tag{3.6}$$

A numerical application of the example set in figure 3.7 gives:

$$\left| \sum_{i \in I_+} \frac{\delta_{ij}}{2} \Delta_i \right| = 787968$$

$$\left| \sum_{i \in I_-} \frac{\delta_{ij}}{2} \Delta_i \right| = 594410$$

$$R_{\text{EIB6M}} = 594410$$

The method is rough, but gives a good estimate of the dependence to the liquidity of the market.

Other products

The idea is the same for other products. There are slight differences from one product to another. For example, if you consider a bond, you may want to hedge swedish bonds with finnish ones, for example. The idea is just that one calculates the reserve by type of issuer, which reduces the amount of the reserve.

3.3 ECB stress testing

Stress testing have become particularly famous for the unspecialised audience since the ECB got involved in the process. I will therefore have a word about the process and its consequences.

As I developed partly above, there are two approaches to lead stress tests. The overall ECB process was to challenge the survival of banks. Therefore they considered the approach were funding was available, with the cost of funding being stressed too. Actually the central bank lead two separated analyses. The first one without any funding possible, where it turned out that a serious number of banks defaulted (Spanish and Irish mostly), was the one I had to deal with. The second one used the same results, with some adjustment considering the funding, its availability and its cost.

3.3.1 Method

During the process I had been along my manager to lead the sensitivity calculation of the impacts of the stresses on the linear books. The job took a long time but was interesting as we saw ECB's forecast. There were two secnarii:

- **Baseline**: This scenario was slightly stressed. It means that it could happen with a relatively high probability and fortunately, under this scenario, no bank defaulted.
- **Adverse**: This scenario is far more pessimistic and less plausible, but still not neglectable. The goal is the give a confidence interval of the losses the institution banking system suffer.

The ECB developed those macroeconomic scenarii. The goal was do determine which impact these would have and the underlying interest rates. Once they transmitted their insights to the banks themselves, the latter dedicated quant teams to provide the digits my team would use to calculate the impact. Tables 3.3 and 3.4 summarize in a simplified manner the stress applied on different currencies and markets.

Currency	Mat	Baseline	Adverse
USD	3M	10	135
	2Y	55	160
	10Y	100	180
EUR	3M	70	195
	2Y	55	160
	10Y	43	125
GBP	3M	120	245
	2Y	100	185
	10Y	70	125
Others	3M	85	210
	2Y	60	180
	10Y	35	150

Table 3.3: Non emerging Markets (Absolute basis, bp)

Zone	Mat	Baseline	Adverse
Asia	3M	15	35
	2Y	20	40
	10Y	25	45
Eastern	3M	20	45
	2Y	10	30
	10Y	-5	20
LatAm	3M	10	20
	2Y	10	20
	10Y	5	10

Table 3.4: Emerging Markets (Relative basis, %)

The curves are shifted for the maturities 1 day to 50 years, so the interpolation techiques were the followings:

- From 0 to 3M and 10Y to 50Y: the extropolation was constant.
- From 3M to 2Y and from 2Y to 10Y: the interpolation was linear.

Let's take EURIBOR 3M as an example, it is shifted following the EUR directives, which gives figures 3.8 and 3.9.

This method is nothing new and it has been applied to the bank for a couple of years now. Every week we drove analyses of the reaction of the linear portfolio to the same kind of stresses, even stronger. The role was not to provide an actual and useful measure of risk like the ECB wanted to but give the management an idea about which side the value of the portfolio would move if things went really wrong, or rather, extreme (wrong depends on the positions you have).

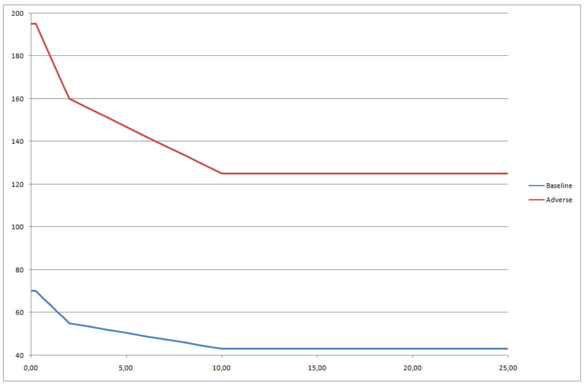


Figure 3.8: Shift applied to EURIBOR 3M under stress

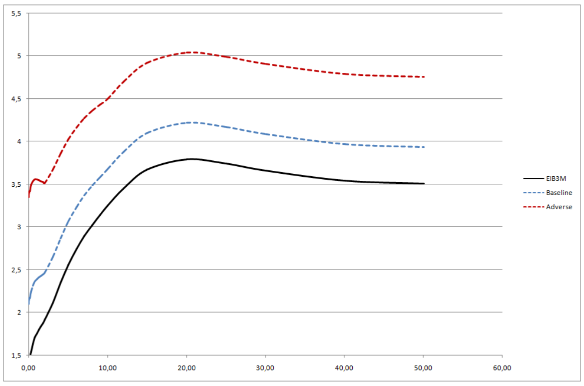


Figure 3.9: EURIBOR 3M shifted

3.3.2 Results and critics

Since the methodolgy was different for the bonds[4], I had to deal only with linear products on *simple* interest rate, which did not provide significant results compared to the ones the bank published. Credit derivatives and bonds would have been hardly affected along with equity exotics as the volatitly increased.

As it can be found on the EBA's website[5] the risk weighted assets of the bank account for more than 560bn€and the core tier one capital for more than 46bn€. So the approximate result of potential losses up to -45M€under the adverse scenario that I calculated would not affect too much the diagnosis. However I would like to make some comments about the way the

[4]the trading bonds were taken with the same methodology but more detailed whereas the sovereign bonds held to maturity were stressed with a higher probability of default and possible haircut (f.e 40% on Greece)

[5]European Banking Authority

calculation were driven[6].

First the methodology was very opaque. There were some index we were supposed to consider because it was stressed a special way so another team had to take care of them, another time it was our job, then not any more... It happened often, so we had to drive new computation on so on. That made me think that maybe there are some products that were left out or taken twice. One thing that has been left out for sure is the impact of stresses on the basis index. I read the methodology several times and they are not mentioned at all. Since they represent a spread they clearly should not be shifted as any interest rate but since they represent a liquidity risk between two currencies they needed to be taken into account.

For example, a tightening in European monetary policy combined to a mistrust between eurozone banks would lead to a lower liquidity of the Euro hence to high moves of the BSEUR index. I made some calculation on my side and applying similar shocks as the other euro index, it would increase the potential losses up to 200M€.

However, the calculations were driven under control of the EBA so the results are trustworthy. Even if to most of the economists' opinion the stresses were too light, they exposed european banks' positions on sovereign bonds and risky assets, a move that may bring back confidence to the euro area.

[6] My present knowledge on interactions between macroeconomy and interest rate markets would not allow me to make any comments ont stresses themselves

Conclusion

This report stresses partly on the difficulties to get accurate data from the market prices, although they are used for PnL, Risk and so on. One may think that since the products I dealt with are linear, the hedging and thus the risk are easily manageable. However since these products are very widespread the margins are low so the volume needs to be high.That is why the accuracy is an important issue.

It becomes then legitimate to question the results if we are not even sure if the underlying data are correct. For the equity market the spot prices of stocks are quoted as for many options. Here it is not the case, one can not trade explicitely EURIBOR 6M at a maturity 2 years. This rate lies in the value of some products, which are use to calibrate our interpolation model. Of course I exagerate a bit, the market is very mature and so are the estimation techniques, but the traded volumes are higher and higher, so needs to be the accuracy of these techniques.

This caution may also be taken about the risk the bank publishes. Most of them are subject to subjective views, even the value at risk. One can modify the methodology, the shifts from one date to another. I am critical on purpose and these modifications are perfectly justifiable and make sense[7], but look suspicious for somebody that in not used to the job. I was lucky to work there right after such a huge crisis and at the beginning of a new one: it was a turning point in risk management (at least considering bonds and interest rate markets) so I was able to see the limits of the measures, the iterative modifications, their alternatives and so on.

One thing I observed is that when everything goes fairly well, risk management works perfectly, but when things start to get dirty, so when we need most a strong management and reliable measures, they do not work so well anymore. That is why the opinion has shifted slightly to stress testing. They are not really more reliable, but they give a better insight of what can happen under high stresses: whether the bank has enough equity to

[7]The proof is that there were no back-testing exception for 6 months.

absord losses, to summarize if the bank can survive to events the analysts and economists estimate plausible. This is just a way to get out of the raw mathematical approach that proved not to be sufficient. As we saw they also need to be taken with caution, but they provide a new, useful outlook. However, as the ECB stresses show: you can still choose the stresses you want to find the result you want: bad enough to look credible but no too much, to bring back confidence.

I would like draw one personnal conclusion to this work. Following the research I made and given the huge amounts involved in the banking system, the methods to handle them seem rather messy, but so far work. Before I thought that the work similar to most of my daily tasks were set to be automated, it turned out that nothing is ready. I will now work in research in a quantitative credit/market risk team that links theory and implementation and get involved in all the chain which will give me a better insight of risk managament.

As a last comment I would like to tell that the contents of this document are not exactly the ones I was thinking about in the first place. The main reason is that I had a restricted access to some data among them the models used to derive the stress amounts[8], which I really would have liked to analyse. Moreover I was thinking of driving myself my own stress tests, but it demanded skills in macro and microeconomy that I do not have. Plus since the process is new, most of the research are driven inside the banks and not public yet. I just saw the emerged part of it leaving most of the process unknown, even though I manage to found some interesting books and papers beside my work. That is why the two last chapter are less *scientific* than expected, and why I stressed more on the data gathering, which I experienced.

[8]Even when I did (cf 1998 crisis) I could not disclose completely the methods.

Appendix A

Index Inderpolation - Excel File

The goal of this part is to create an Excel file that gives the yield curve of a given interest rate from the market prices of some derivatives collected on Bloomberg. The access to Bloomberg was restricted to me so it was impossible to collect a large amount of data to have a perfect match with the yield calculated in the system, however, the methodology remains the same and the algorithm in detailed below.

As we saw in chapter one, we get the discount factors from the swap fair price using the bootstrapping method.

Let us consider that we want to get the yield curve of the index EURIB6M with a 60-year horizon. Therefore we need to get the discount factors for every 6-month interval from now until 2071. Unfortunately, swaps with a maturity of 55 and a half years are not traded. We therefore need to interpolate from the formula (1.4). We notice that linear interpolation gives handy results albeit not very accurate. In the first place the calculation will be made using this method, we will analyse another one later.
Suppose that we want to get $p(t, \tilde{T})$ for some given $\tilde{T}$. The closest swap maturity at which data are available are supposed to be T_i and T_{i+1}. A general extrapolation is :

$$p(t, \tilde{T}) = f(p(t, T_i), T_i, p(t, T_{i+1}), T_{i+1})$$

A.1 Linear Interpolation

A.1.1 Methodology

With the notations set above we have as earlier :

$$p(t,\tilde{T}) = p(t,T_j)\frac{T_{j+1}-\tilde{T}}{T_{j+1}-T_j} + p(t,T_{j+1})\frac{\tilde{T}-T_j}{T_{j+1}-T_j}$$

or more generally,

$$p(t,\tilde{T}) = p(t,T_j)f(\tilde{T},T_j,T_{j+1}) + p(t,T_{j+1})f(\tilde{T},T_j,T_{j+1})$$

Proceding iteratively from the equation (1.4) we can explicitely get the discount factors at a time when there exists a swap whose price is quoted on the market. Let this time be T_n and T_k the latest time before T_n when we have found a quoted swap.

$$\begin{aligned} p(T_n) &= \frac{1 + r\sum_{j=1}^{n-1} p(T_j)\delta_j}{1 + r\delta_n} \\ &= \frac{1}{1+r\delta_n}\left(1 + r\underbrace{\sum_{j=1}^{k} p(T_j)\delta_j}_{\text{already known}} + r\underbrace{\sum_{j=k+1}^{n-1} p(T_j)\delta_j}_{\text{to extrapolate}}\right) \end{aligned}$$

Let us calculate the right hand term :

$$\begin{aligned} \sum_{j=k+1}^{n-1} p(T_j)\delta_j &= \sum_{j=k+1}^{n-1}\left(p(T_k)\frac{T_n-T_j}{T_n-T_k} + p(T_n)\frac{T_j-T_k}{T_n-T_k}\right)\delta_j \\ &= p(T_k)\sum_{j=k+1}^{n-1}\frac{T_n-T_j}{T_n-T_k}\delta_j + p(T_n)\sum_{j=k+1}^{n-1}\frac{T_j-T_k}{T_n-T_k}\delta_j \end{aligned}$$

Entering all in one we get :

$$p(T_n) = \frac{1}{1 + \frac{r\sum_{j=k+1}^{n-1}\frac{T_j-T_k}{T_n-T_k}\delta_j}{1+r\delta_n}}\left(1 + r\sum_{j=1}^{k} p(T_j)\delta_j + r.p(T_k)\sum_{j=k+1}^{n-1}\frac{T_n-T_j}{T_n-T_k}\delta_j\right) \tag{A.1}$$

This barbarian equation can be simplified in this case. Calculating the yield of EURIBOR 6M we set: $forall j, \quad \delta_j = 0.5 \quad \text{and} \quad T_j = j/2.$
Once we get T_n, we can easily calculate the intermediate maturities. This shows us a recursive way to obtain all the points of the curve. The algorithm is then rather simple, starting from $t = 0$ where the value of the rate is known, we move up in the maturities until the next one when there is a quoted swap and so on.

A.1.2 Calculations

I used this methodology to interpolate the curve of EURIBOR 6M based on swaps of the following maturities: 1Y, 2Y, 2.5Y, every year from 3Y to 21Y, 24Y, 25Y, 30Y, 35Y, 40Y, 50Y and 60Y stored in $\mathbf{T} = (T_i)_{i=1,\dots,29}$. r_i is the fixed rate of the swap with maturity T_i. The more data we have the more accurate is the result, but it was impossible for me to collect more. Using A.1 in this specific case we get for all k and n with $n > k$ and T_k and T_n in $\mathbf{T}$:

$$p(T_n) = \frac{1}{1 + \frac{r \sum_{j=k+1}^{n-1} \frac{j-k}{n-k}}{2+r}} \left(1 + \frac{r}{2} \sum_{j=1}^{k} p(T_j) + \frac{r}{2} p(T_k) \sum_{j=k+1}^{n-1} \frac{n-j}{n-k} \right)$$

Hence:

$$\begin{aligned} \sum_{j=k}^{n-1} \frac{n-j}{n-k} &= \frac{n(n-k) - \overbrace{\sum_{j=k}^{n-1} j}^{=\frac{(n-k)(n-k-1)}{2}}}{n-k} \\ &= n - \frac{n-k-1}{2} = \frac{n+k+1}{2} \end{aligned}$$

likewise

$$\begin{aligned} \sum_{j=k}^{n-1} \frac{j-k}{n-k} &= \frac{\sum_{j=0}^{n-k-1} j}{n-k} = \frac{(n-k-1)(n-k)}{2(n-k)} \\ &= \frac{n-k-1}{2} \end{aligned}$$

hence

$$p(T_n) = \frac{1}{1 + \frac{\frac{r}{2}(n-k-1)}{2+r}} \left(1 + \frac{r}{2} \sum_{j=1}^{k-1} p(T_j) + \frac{r}{4} p(T_k)(n+k+1) \right) \tag{A.2}$$

We enter (A.2) recursively to construct the cruve from the derivatives' prices. These calculations can become very irksome when it comes to exotic swap with non contant payment timetable or a floaing fixed-leg (i.e the fixed cash flows are non contant albeit fixed at time 0). In practic, a patchwork of different products is used to estimate the curve making explicit formulas indigestible, but completely feasible with a computer.

A.1.3 ZT Interpolation

The method is defined as (with all the same notations) :

$$p(\tilde{T}) = \frac{T_{j+1}.p(T_{j+1}) - T_j.p(T_j)}{T_{j+1} - T_j} + \frac{1}{\tilde{T}}.\frac{T_{j+1}T_j}{T_{j+1} - T_j}(p(T_j) - p(T_{j+1})) \quad \text{(A.3)}$$

For reasons of simplicity we will apply (A.3) directly to the case of interpolating an EURIBOR 6M curve in the same condition as in A.1. This gives with $\tilde{T} = T_j = j/2$ and encircled by $T_k = k/2$ and $T_n = n/2$:

$$p(T_j) = \frac{np(T_n) - kp(T_k)}{n-k} + \frac{1}{j}\frac{nk}{n-k}(p(T_k) - p(T_n))$$

We notice at this point that the term j at the denominator will make things impossible to have an explicit formula. Simplificating the equation above we get:

$$p(T_j) = \left(\frac{k(n-j)}{j(n-k)}\right) p(T_k) + \left(\frac{n(j-k)}{j(n-k)}\right) p(T_n) \quad \text{(A.4)}$$

Which leads to:

$$p(T_n) = \frac{1}{1 + \frac{\frac{nr}{n-k}\sum_{j=k}^{n}\frac{j-k}{j}}{2+r}} \left(1 + \frac{r}{2}\sum_{j=1}^{k-1} p(T_j) + \frac{kr}{(n-k)} p(T_k) \sum_{j=k}^{n} \frac{n-j}{j}\right) \quad \text{(A.5)}$$

We therefore have the recursive formula necessary to compute the yield curve.

Appendix B

Sensitivity Calculation - Excel File

Once we get the curves the next step is to calculate the sensitivities for a given product to any variation on the yield curve. As said above there exist two main method of calcultions : perturbed and cumulative. I therefore computed a program to get them both for both Plain Vanilla Swaps and Cross Currency Basis Swaps.

As we saw in section 1.4 there is an easy way of simplifying the calculation, however I calculataed the sensitivities with the intuitive formula method and the simplified one. Actually the latter is more efficient only to sort out a theoritical explicit formula, which is not the aim of this part. To price a swap owned by the bank the input of the macro may be:

- The notional N.
- The dates of the first and last cash flows, for both the floating and the fixed leg.
- The frequency of payment of each leg[1].
- The fixed rate
- The zero curve of the floating rate.
- The rate at which the first floating cash flow will be paid.
- The time quotations 30/360, actual/360 or actual/actual.
- A coefficient α equal to 1 if the floating is paid and -1 if it is received.
- The method used to drive the calculations (*cum* for cumulative and *per* for perturbed.

[1]If the simplified method is used, then the floating leg's payments frequency is useless

B.1 Perturbed Sensitivities

The method here is rather simple and basic : For every maturity available (i.e calculated from the market data) we stress the rate with one basis point and observe the impact on the price of the product.

Theoritical Sensitivities

Using the simplified version of the actualized cash flows, approximating the sensitivities becomes rather easy:

$$\Pi = \Pi_f + \bar{\Pi} = \alpha N \left(\bar{r}_n \sum_{i=1}^{n} \bar{\delta}_i p(\bar{T}_i) - (p(T_1) + p(T_n)) \right)$$

There is no easily readable explicite formula after spliting the actual cash flows and discount factors between the known maturities. It will be done by the computeur: for every i it will find the closest ones and share out the cash flows. Therefore the price Π will be expressed as follow:

$$\Pi = \sum_{j=1}^{k} C_j p(\hat{T}_j)$$

where k is the total number of known maturities $(\hat{T}_i)$ and C_j the corresonding cash flow which may be equal to 0. Then applying (1.7) we have our theoritical sensitivities.

In Practice

The first step is to calculate the price of the swap with the actual market data. This implies calculate the forward spot rate curve and the discount curve and then quantify and discount the future cash flows. The relatively tricky part is to share the cash flows between the two closest maturities. Once this is possible with the actual data, we apply the same methodology with the stressed data iteratively: we stress the first earliest calculated rate, estimate the impact on the price, then we stress only the second and so on. Wether we une the normal or simplify version does not change the methodology but only the repartition of the cash flows and therefore the results.

B.2 Cumulative Sensitivities

As I said earlier, this method provides no improvement for linear products. Indeed since the products are linear their sensitivities to shifts on the yield curve is linear too. To summarize we have:

$$\text{Price}_{\text{yield}}(\text{shift}_A - \text{shift}_B) \approx \text{Price}_{\text{yield}}(\text{shift}_A) - \text{Price}_{\text{yield}}(\text{shift}_B) \quad \text{(B.1)}$$

when the shifts are not taken on the same maturities[2].

Taking the example of the cumulative method, shift_A would be the one in figure **??** and the shift_B the same one with the former maturity. Then it is clear that there is nothing new in taking this method upon the perturbed one: The only convexity comes from the discount factor.
If the reader is not convinced by this only fact he or she can manipulate the excel program called *sensisswaps.xls*, to see that however different maturities are shifted the interaction with each other are almost null.

B.3 Gamma

The conclusions are also driven by (B.1). Since the convexity lies in the discount factor, it will not have a very high impact, plus it is a term of second order, therefore it will be even lower. It is important only in case of very high stresses (say 150bp) which are taken squared, however, the effect remains very low compared to Δ. As an information, with the perturbed method the gamma is given by:

$$\gamma = \frac{\Delta_{\text{up}} - \Delta_{\text{down}}}{2.\text{shift}} \quad \text{(B.2)}$$

Where Δ_{up} and Δ_{down} respectively refer to the price given an upward and downward shift. It is basically the differential of Δ.

[2]Otherwise it would mean that taking $\text{shift}_A = \text{shift}_B$ the price given a non-shifted curve is 0...

Appendix C

Codes and notations

Currencies and groups

Currency groups for the ECB stress tests, the distribution is based on geography but also on market similarities.

Main Currencies	
EUR	Euro
USD	United States Dollar
GBP	United Kingdom Pound
Non Emerging	
AUD	Australian Dollar
CAD	Canadian Dollar
CHF	Swiss Franc
DKK	Danish Krona
JPY	Japanese Yen
NOK	Norwegian Krona
NZD	New Zealand Dollar
SEK	Swedish Krona
Latin America	
ARS	Argentina Peso
BRL	Brazil Real
CLP	Chilian Peso
COP	Colombian Peso
MXN	Mexican Peso
PEN	New Peruvian Sole
VEB	Venezualan Bolivar

Asia	
EGP	Egyptian Pound
IDR	Indonesian Rupiah
ILS	Israeli Shekel
INR	Indian Rupee
KRW	South Korean Won
MAD	Moroccan Dirham
MYR	Malaysian Ringgit
PHP	Philippines Peso
SGD	Singapore Dollar
THB	Thai Baht
TND	Tunisian Dinar
TWD	Taiwan Dollar
VND	Viet Nam Dong
ZAR	South African Rand
Eastern	
BGN	Bulgarian Lev
CSD	Serbian Dinar
CZK	Czech Krona
EEK	Etonian Kroon
HRK	Croatian Kuna
HUF	Hungarian Forint
ISK	Iceland Krona
KZT	Kazakhstan Tenge
LTL	Lithuanian Litas
LVL	Latvian Lat
PLN	Polish Zloty
RON	New Romanian Leu
RUB	Russian Ruble
TRY	New Turkish Lira
UAH	Ukraine Hryvna
Misc (follow USD)	
AED	Unidet Arab Emirates Dirham
BHD	Bahraini
CNY	Chinese Yuan
HKD	Hong Kong Dollar
KWD	Kuwaiti Dinar
OMR	Oman Rial
QAR	Qatari Riyal
SAR	Saudi Riyal
UAH	Ukraine Hryvna

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