Option Pricing with Events at Deterministic Times

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Abstract

In this thesis we have investigated the affects of events at deterministic times on stock prices and option volatilities for medical companies. To do this we derive an extension of the Black & Scholes option pricing formula that incorporates apriori known events. From two implications of the model we have then analyzed if a sample of events for medical companies exhibit these model implied characteristics. From the model we have also derived a jump estimator that we analyze to see how the estimated jump correspond to actual event day volatilities for the companies. Our findings suggest that the model we look at seem to capture the effects of the events on the prices of options. In the last part of the thesis we look at two different delta hedging schemes for companies with events. From the analysis of the two different schemes we conclude that using a volatility where we don't take into account the jump volatility gives an on average lower hedging cost but at a much higher variance in the outcome.

Keywords: Implied volatility, jump estimator, scheduled event, delta hedging.

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Contents

Chapter 1

Introduction

Investing in medical or biotech companies is difficult. Not only are the company's products often very intricate and difficult to understand but there is also a regulatory uncertainty with investing in them. Each product that a medical company releases to the market must go through immense testing and evaluation. In the end it is up to the regulators to decide if the product is approved or not. If the drug or device that the company has been developing is approved it will affect the company earnings in the future since they now have rights to sell a new product. These approvals can thus potentially have large effects on the company and its stock price. Due to the binarity of approving or rejecting a drug or medical device and the importance for future earnings, the information may lead to a situation where the stock price either will jump up or down in a discontinuous way when it is given to the market. From an investors perspective it would thus be interesting to know the amount of uncertainty that is embedded in the release of regulatory decisions, before the information is released.

In the US these approvals and rejections are all handled by the Food and Drug Administration (FDA). Since the US market is the worlds largest market for pharmaceuticals and medical devices there is a large number of companies that want to launch their products here. As an example we can look at the release of the complete response letter for the company Biodel Inc. and their product VIAject. A complete response letter is in essence a rejection at that time but with the possibility that the drug can be approved in the future. The actual date for the release of this information had been given to the company and the market many months ahead by the FDA. The information of the complete response letter was given to the market before closing on the 1 of November but the last trading day before that had been on the 29 of October. The difference between the stock closing price on these two days was a drop of 41.32%.

For option pricing this event thus creates some problems. Since the stock is not behaving in a continuous way the Black & Scholes framework is not applicable without some adjustments. One of the assumed properties for the Black & Scholes model to actually work is that the stock has a continuous sample path. We can thus not expect this framework to hold when modeling the underlying stock. The second problem is that if we today where to price an option with maturity after the event this future jump will have to be accounted for in the estimation of the implied volatility for the option.

In this thesis we will investigate how a priori known events affect stock prices and option prices for medical companies trying to launch drugs in the US. To do this we will look at a model of the stock price that is an extension of the Black & Scholes model incorporating jumps at deterministically known times. With deterministically known jumps we will mean jumps that we know the timing of when they will happen. The distribution of the jumps will be modeled as normally distributed variables. We will from the model then derive a closed form option pricing formula and discuss the implications this formula have on implied volatilities. We will then analyze market data for companies that have had FDA (NDA) events to see how these events affect the actual prices of the stock and options and see if these model implications are apparent in the market pricing of call options.

The last part of the thesis will be analyzing how to delta hedge options on stocks that has deterministically timed jumps. We will here discuss different choices of volatility to be used to calculate the delta and test these strategies using monte carlo simulation of the stock sample path.

Chapter 2

Background

2.1 U.S. Food and Drug Administration

In the healthcare industry there is legislation for what a company needs to do before launching a new drug or device to the market. The agency that oversees the legislation in the US is called the U.S. Food and Drug Administration (FDA) and they are responsible for protecting the public health. In the US the legislation that companies need to follow to receive the right to sell and market a drug or device is called the Prescription Drug User Fee Act, PDUFA, and was enacted in 1992. This legislation authorizes the FDA to collect fees from drug manufacturers. The FDA then use these fees to pay for the process of approving drugs that are under investigation. The enactment has increased the speed of the review process making it possible to get new drugs to the market faster. Today FDA is normally given 10 months to review a new drug. If the drug is selected for a priority review a 6 month review period is allotted. All these time periods begin from the date a company sends in its New Drug Application (NDA). The NDA is the vehicle that propose to the FDA if a new pharmaceutical should be approved for sale and marketing. The NDA therefore provide information to the FDA concerning: whether the drug is safe and effective and if the benefits outweigh the risks; if the packing insert and labeling of the drug is appropriate; and if methods and controls used in manufacturing is adequate.[1]

In addition to their own investigation through the NDA the FDA use external Advisory Committees (Adcom) consisting of well-known academics and practitioners. The Adcom is used to review the various material together with the FDA's own staff. In the end the Adcom's also advice the FDA on the drug they are investigating. The advice from the Adcom is given to the market before the FDA decides to approve or not.[2]

The material that the FDA use to review a new drug are tests done by the company responsible for the drug. These tests are divided in three different phases: Phase I, II and III, with criteria's to proceed to the next phase. If the drug is not sufficient in for example the phase I trial the company is not allowed to proceed to the phase

[1]US Department of Health & Human Services: `http://www.fda.gov/Drugs/ResourcesForYou/Consumers/ucm143534.htm`

[2]US Department of Health & Human Services: `http://www.fda.gov/Drugs/ResourcesForYou/Consumers/ucm143534.htm`

II trial. The NDA is submitted by the company firstly after the phase III trial.[3]

2.2 Discussion of event impact on stock prices

Every step in the process of receiving an approval increases the probability that the company might get increased earnings. An increase, or reduction, of the probability that the company might get an approval to sell and market a drug affects the company's stock prize since stock prices usually are regarded as present values of future cash flows.[4] An approval to continue with a phase II trial, after a phase I trial, is a sign that the drug can potentially be approved. The market therefore reevaluates the future earnings and depending if it is positive or negative news the stock price usually increases or decreases. The fact that the drug passes through the trials does though not mean that the drug can be sold to the public. The last step in the process is always to send in the NDA and get an approval from the FDA. Once the NDA is submitted there can also be an Adcom meetings that can affect the price of the stocks. If the Adcom is negative towards a drug this increases the possibility that the FDA will not approve the drug, and vice versa if they are positive. The last step is then for the FDA to approve or to disapprove the drug. The FDA can, in addition to approving or disapproving, also give a complete response (CR) to the company. This means that the company needs to further analyze the drug or device in more clinical studies and tests.

The approval notification can have different impact on companies pending on how large the potential income from the new drug is given the company's current earnings. For a large medical company the approval of a small niche drug is most likely not going to have any large impact on the earnings of the company, and it will hence not have a large impact on the stock price. For smaller companies, and companies with perhaps no drugs in the market, these approval events and data publications can have a major impact and the stock price can have large jumps on this date.

[3]US Department of Health: & Human Services `http://www.fda.gov/Drugs/ResourcesForYou/Consumers/ucm143534.htm`

[4]*Corporate Finance by Johnathan Berk and Peter DeMarzo, 2007*, Chapter 9.3

Chapter 3

Previous research

3.1 Jump models

There are many scientific papers on the subject of incorporating jumps in the price process of stocks. Many of these articles model the jump occurrence as random. In this category of models we have the famous Merton article [11] from 1976. In this article Merton extends the Black & Scholes model with a Poisson process to capture abnormal price variations that the normal Black & Scholes model does not. In 2002 Kou [9] extended Mertons model such that the jumps have a double exponential distribution instead of a lognormal distribution. Both of these models choose a frequency for the jumps, i.e. the jumps are not known in advance.

We have found three papers on the subject of modeling known jumps, events that are known in advance of them happening, for equities. Abraham and Taylor [1] discuss the differences between scheduled and unscheduled events and their different impact on prices. They put forward a model, which they call the Event model, for option pricing that take into account both of these types of events. The model is a jump diffusion model with an added term for the scheduled jump.

In Dubinsky and Johannes [7] jumps in presence of earnings announcements are analyzed. To model the behavior of these events, i.e. earnings announcements, they develop two different jump models, one with constant diffusive volatility and deterministically timed jumps and one with stochastic volatility and deterministic jumps. The authors also describe two jump estimators that they derive from the model. We will in this thesis take a closer look at one of these estimators, namly the one they call the term structure estimator.

The third paper by Radchenko [12] considers the problem of finding hedging strategies of European call options for a one-dimensional model of assets prices driven by a Wiener process and jumps at earlier known time moments. The author begins by a asset pricing model and then moves on to decompose the model using a Föllmer-Schweizer decomposition that can be found in [8]. The Föllmer-Schweizer decomposition is then used to find the solution to a minimization problem where the author is trying to find the hedging strategy that minimizes the variance of a contingent claim on a stock. The theory and method in the last part of the Radchenkos paper is outside of the scope of this thesis and will not be used here. What we will use is the setup of the model that Radchenko uses in his paper.

Chapter 4

Models and theory

The goal of this chapter is to show how we can model the stock price dynamics of stocks with deterministically timed events. We begin with the Black & Scholes option pricing formula and the geometric browninan motion and then extend this model with jumps (events) at known times.

4.1 Black & Scholes option pricing formula

The Black & Scholes option pricing formula, derived by Fischer Black and Robert Merton in 1973 in their article *The Pricing of Options and Corporate Liabilities*, is derived in [3]. It is the system of stochastic differential equations

$$\begin{aligned} dB_t &= rB_t dt \\ dS_t &= \alpha S_t dt + \sigma S_t dW_t \end{aligned} \tag{4.1}$$

that is the starting point and fundamental building blocks in the derivation of the Black & Scholes option pricing formula. Equation (4.1) is called a geometric brownian motion and consists of a drift term α, a diffusion term σ and a brownian motion W_t.

Theorem 4.1 *The price of a European call option with strik price K and time of maturity T is given by the formula $\Pi(t) = F(t, S_t)$, where*

$$F(t, S_t) = S_t \Phi[d_1(t, S_t)] - e^{-r(T-t)} K \Phi[d_2(t, S_t)]. \tag{4.2}$$

Here Φ is the cumulative distribution function for the N(0,1) distribution and

$$\begin{aligned} d_1(t, S_t) &= \frac{1}{\sigma\sqrt{T-t}} \left[\log \frac{S_t}{K} + (r + \frac{1}{2}\sigma^2)(T-t) \right] \\ d_2(t, S_t) &= d_1(t, S_t) - \sigma\sqrt{T-t} \end{aligned}$$

From Equation (4.2) we can extract the Black & Scholes implied volatility (IV). The IV for an option is the market expectation of the volatility for the reminder of the life of the option. To get the IV for an option one solves the equation

$$p = c(s, t, T, r, \sigma, K) \tag{4.3}$$

where p is the market price of an option, s is the spot price of the underlying stock, t is the time today, T is the maturity of the option, r is the risk free interest rate, σ is the implied volatility and K is the strike price.

4.2 Doléans-Dade exponential

The following proposition is from [6]

Proposition 4.2 *Let X_t be a Lévy process with Lévy triplet σ^2, ν, γ. There exists a unique cadlag process Z_t such that*

$$\begin{aligned} dZ_t &= Z_{t-} dX_t \\ Z_0 &= z \end{aligned}$$

Z is given by:

$$Z_t = z \exp\left\{X_t - \frac{1}{2}\sigma^2 t\right\} \prod_{0 \le s \le t} (1 + \Delta X_s) \exp\left\{-\Delta X_s\right\} \tag{4.4}$$

4.3 Geometric brownian motion with deterministically timed events

We now introduce the dynamics of a deterministic jump to the GBM model described in Equation (4.1). We do this by looking at the Lévy process X_t with differential

$$dX_t = \mu dt + \sigma dW_t + I\left\{t = s_j\right\} U_j \tag{4.5}$$

Here $I\left\{t = s_j\right\}$ is the indicator function being 1 if t is equal to the jump time s_j and $U_j \in (-1, \inf)$ is the jump distribution at the deterministic jump (news announcement) instant s_j. The other parts of this equation is the same as in the GBM model in Equation (4.1).

If we solve Equation (4.5) we get that

$$\begin{aligned} dX_t &= \mu dt + \sigma dW_t + I\left\{t = s_j\right\} U_j \\ X_t &= \int_0^t \mu ds + \int_0^t \sigma dW_s + \int_0^t I\left\{t = s_j\right\} U_j ds \\ X_t &= \mu t + \sigma W_t + \sum_{j : s \le t} U_j \end{aligned} \tag{4.6}$$

Using Equation (4.4) we see that the Doléans-Dade exponential for the process in Equation (4.6) is

$$Z_t = z\exp\left\{X_t - \frac{1}{2}\sigma^2 t\right\} \prod_{0\le s\le t} (1+U_j)\exp\{-U_j\}$$
$$Z_t = z\exp\left\{\mu t + \sigma W_t - \frac{1}{2}\sigma^2 t + \sum_{j:s\le t} U_j - \sum_{j:s\le t} U_j\right\} \prod_{0\le s\le t} (1+U_j)$$
$$Z_t = z\exp\left\{\mu t + \sigma W_t - \frac{1}{2}\sigma^2 t\right\} \prod_{0\le s\le t} (1+U_j) \tag{4.7}$$

If we set $U_j = \exp\{Y_j\} - 1$ we can see that the last Equation (4.7) can be written as

$$Z_t = z\exp\left\{\mu t + \sigma W_t - \frac{1}{2}\sigma^2 t\right\} \prod_{0\le s\le t} \exp\{Y_j\}$$
$$Z_t = z\exp\left\{\mu t + \sigma W_t - \frac{1}{2}\sigma^2 t\right\} \exp\left\{\sum_{0\le s\le t} Y_j\right\}$$
$$Z_t = z\exp\left\{\mu t + \sigma W_t - \frac{1}{2}\sigma^2 t + \sum_{0\le s\le t} Y_j\right\} \tag{4.8}$$

Letting Y_j be normally distributed we have a way to model stocks with events at deterministica times.

We rewrite Equation (4.8) with Z_t exchanged with S_T (to indicate that this is the process of the stock price S), setting z to S_t, letting T be the maturity time of a European call option and N_T^d the number of jumps between t and T

$$S_T = S_t \exp\left((\mu - \frac{\sigma^2}{2})(T-t) + \sigma(W_T - W_t) + \sum_{j=1}^{N_T^d} Y_j\right) \tag{4.9}$$

In [7], pages 11-12, there is a derivation of the equivalent martinguale measure for the process in Equation (4.9). Under the equivalent martingale measure $\mathbb{Q}$ discounted prices should be a martingale which means that they need to be both a martingale between jump times and that the pre-jump expected stock price of the post-jump stock price is equal to the pre-jump stock price. This indicates that the between the jump times the drift of S_t under $\mathbb{Q}$ is rS_t. If prices are to be $\mathbb{Q}$ martingale at jump times we need to have $E^{\mathbb{Q}}[S_{\tau_j}|\mathcal{F}_{\tau_j-}] = S_{\tau_j-}$, which means that there can not be any expected capital gain at a deterministic jump instant, $E^{\mathbb{Q}}[\Delta S_{\tau_j}|\mathcal{F}_{\tau_j-}] = 0$. This inturn leads to that $E^{\mathbb{Q}}[e^{Y_j}|\mathcal{F}_{\tau_j-}] = 1$. In this thesis we do not construct the martingale measure using the Girsanov Theorem, but draw from the conclusions in [7], and state that if $Y_j = -\frac{1}{2}(\sigma^{\mathbb{Q}})^2 + \sigma^{\mathbb{Q}}\epsilon$ where $\epsilon \sim N(0,1)$, discounted prices are martingales under $\mathbb{Q}$. This leads to the process in Equation (4.10)

$$S_T = S_t \exp\left((r - \frac{\sigma^2}{2})(T-t) + \sigma(W_T^{\mathbb{Q}} - W_t^{\mathbb{Q}}) + \sum_{j=1}^{N_T^d} Y_j \right) \tag{4.10}$$

4.3.1 Distribution of stochastic parts in the price model

Let us first look at the distribution of the two random parts in Equation (4.10).

The first part we analyze is the Wiener process, $W_T^{\mathbb{Q}} - W_t^{\mathbb{Q}}$. We will drop the superscript $\mathbb{Q}$ on the process from now on but think about W_t as $\mathbb{Q}$-Wiener process. We know that if we let any $0 \leq t < T$, the increments of $W_T - W_t \sim N(0, \sqrt{T-t})$.

The other random part in Equation (4.10) is due to the jump. This part is a non-random series of independent normal random varaibles. Since $Z_t = -\frac{1}{2}(\sigma_j)^2 + \sigma_j \epsilon$ we have that

$$\sum_{j=1}^{N_T^d} Z_j \sim N\left(-\frac{1}{2}\sum_{j=1}^{N_T^d}(\sigma_j)^2, \sqrt{\sum_{j=1}^{N_T^d}(\sigma_j)^2} \right)$$

We now look at the the distribution of these two parts, and the non-stochastic parts of the exponent in Equation (4.10), together. We will call this variable Y_T:

$$Y_T = (r - \frac{\sigma^2}{2})(T-t) + \sigma(W_T - W_t) + \sum_{j=1}^{N_T^d} Z_j$$
$$Y_T \sim N\left((r - \frac{\sigma^2}{2})(T-t) - \sum_{j=1}^{N_T^d}\frac{1}{2}(\sigma_j)^2, \sqrt{\sigma^2(T-t) + \sum_{j=1}^{N_T^d}(\sigma_j)^2} \right)$$

To simplify this for the continuing derivation we breake out $(T-t)$ from the standard deviation and set

$$\gamma = \sqrt{\sigma^2 + (T-t)^{-1}\sum_{j=1}^{N_T^d}(\sigma_j)^2}$$
$$\mu = r$$

From

$$Y_T \sim N\left[(\mu - \frac{1}{2}\gamma^2)(T-t), \gamma\sqrt{T-t} \right]$$
$$Y_T = (\mu - \frac{1}{2}\gamma^2)(T-t) + \gamma\sqrt{T-t}\epsilon \text{ where } \epsilon \sim N(0,1)$$

Having derived the distribution of Y_T we can easily derive a risk neutral pricing formula for a contingent claim on a S_T. The derivation of this pricing formula can be done in the same manners as for the pricing formula in Theorem 4.1 (we have done these calculations in the Appendix. From the derivation of the option pricing formula in the Appendix we state the following theorem:

Theorem 4.3 *The price of a European call option with strik price K and time of maturity T that have a underlying security with events(having log normal distribution with mean $-\frac{1}{2}\sigma_j^2$ and standard deviation σ_j) at deterministically known time is given by the formula $\Pi(t) = F(t, S_t)$, where*

$$F(t, S_t) = S_t \Phi[d_1(t, S_t)] - e^{-r(T-t)} K \Phi[d_2(t, S_t)] \tag{4.11}$$

Here Φ is the cumulative distribution function for the N(0,1) distribution and

$$d_1(t, S_t) = \frac{1}{\gamma\sqrt{T-t}} \left[\log(\frac{S_t}{K}) + (r + \frac{1}{2}\gamma^2)(T-t) \right] \tag{4.12}$$

$$d_2(t, S_t) = d_1(t, S_t) - \gamma\sqrt{T-t}$$

$$\gamma^2 = \sigma^2 + (T-t)^{-1} \sum_{j}^{N_T^d} (\sigma_j)^2 \tag{4.13}$$

As we can see this closed form solution in Equation (4.11) is very similar to the solution in Theorem 4.1. It is so similar because the random parts in Equation (4.10) are all normally distributed.

The difference between the two models, the model in Theorem 4.1 and the model in Theorem 4.3, is the form of the implied volatility. The deterministic jumps in the model creates predictability in the implied volatility. If we introduce a single jump (in our case this will be a NDA decision announcement) at t_j, $t < t_j < T$ we see that since $\gamma^2 = \sigma^2 + (T-t)^{-1}(\sigma_j)^2$ this implies two testable characteristics for the implied volatility for options on stocks with events at deterministic times.

1. Before an event annualized IV is $\gamma_{t_j-}^2 = \sigma^2 + (T-t)^{-1}(\sigma_j)^2$ and after $\gamma_{t_j}^2 = \gamma^2 = \sigma^2$. This therefore implies a discontinous decrease in the IV after the event.

2. The IV should increase into an event with a rate of $(T-t)^{-1}$.

4.4 Jump estimator

From the implied volatility structure, $\gamma^2 = \sigma^2 + (T-t)^{-1} \sum_{j}^{N_T} (\sigma_j)^2$ we will now look at jump estimators derived using this structure. It will be an ex-ante estimation of the jump, based on implied volatilities.

The estimator was developed in [7] and we will it derive it here again.

We start by looking at the implied volatility of two at the money (ATM) options with different maturities, expiring after the event (jump). If there is a single event before the options mature the IV of the ATM option at time t is (in annualized units) $\gamma_{t,T-t}^2 = \sigma^2 + (T-t)^{-1}(\sigma^Q)^2$. If we have two options with different maturity, $T_1 = T^1 - t$ and $T_2 = T^2 - t$ where $T_1 \leq T_2$, we thus must have that $\gamma_{t,T_1}^2 > \gamma_{t,T_2}^2$, since both σ and σ_j are constant in the model. If we have the two market IV, here called γ_{t,T_i} for these two options we can thus solve this equation system

$$\begin{cases} \gamma_{t,T_1}^2 = \sigma^2 + T_1^{-1}(\sigma_j)^2 \\ \gamma_{t,T_2}^2 = \sigma^2 + T_2^{-1}(\sigma_j)^2 \end{cases}$$

Solve the second equation for σ^2 and insert in the first to get

$$\gamma_{t,T_1}^2 = \gamma_{t,T_2}^2 - T_2^{-1}(\sigma_j)^2 + T_1^{-1}(\sigma_j)^2$$

Now solve for $(\sigma_j)^2$ which we now will call $(\sigma_{term})^2$

$$\begin{aligned} (\sigma_{term})^2(T_1^{-1} - T_2^{-1}) &= \gamma_{t,T_1}^2 - \gamma_{t,T_2}^2 \\ (\sigma_{term})^2 &= \frac{\gamma_{t,T_1}^2 - \gamma_{t,T_2}^2}{T_1^{-1} - T_2^{-1}} \end{aligned} \tag{4.14}$$

As noted above γ_{t,T_i} is the market Black Scholed implied volatility with expiration in $T_i = T^i - t$ days, where t is today. These IV can be calculated from options prices in the market.

From this Black & Scholes model with deterministically timed jumps we have created an estimator of the implicit jump size that the options market is pricing.

Chapter 5

Emperical study of NDA events

In this chapter we will look at NDA events for drug companies that have applied to launch products in the US. We will hence only focus on events for companies that are in the last phase of their filing for a new drug or medical device. We begin this section with a case study to go through how a NDA event can affect a company's stock price and IV. In this section we also describe the method of how we will find the time series estimator.

5.1 Event case study and data discussion

To better understand the dynamics of an NDA event and the jump estimator we have developed we provide a case study of the company Biodel Inc.

On the 1 of November 2010 Biodel Inc. received a complete response (CR) on its NDA for its drug Linjeta and the company provided the news to the market during trading hours in the US on the same day. Since the company did not have any steady cash flow during the period the CR was almost as severe as a rejection for the market since it would mean more expenses for Biodel Inc., and the market started wondering if the company could afford these extra costs.

The stock price of Biodel Inc. for the period 4 months before and up to the event (and a week after) is shown in the Figure 5.1.

We can see that the on the day of the event the stock depreciated 41.32%, going from \$3.63 on the 29 of October to \$2.13 on the 1 of November. We can also see from Figure 5.1 that the distribution looks reasonably normally distributed if it would not have been for the large jump on the day of the event.

Let's now look at what the information that the company gave to the market leading up to the 29 of October. On the 30 of December 2009 Biodel Inc. announces that is has submitted an NDA to the FDA. On the 12 of February Biodel Inc. releases the results from company's two phase III studies, from which the conclusion is drawn that Linjeta (then called VIAject) was more effective than human insulin and the fast acting analogue insulin lispro. It is not until the 1 of March that the FDA announces that they have accepted to review the drug. In the press release on the 1 of March the FDA states that they expect time of action to be on the 30 of October 2010, 8 months later. At this time there is no open volume in any call option on the Biodel

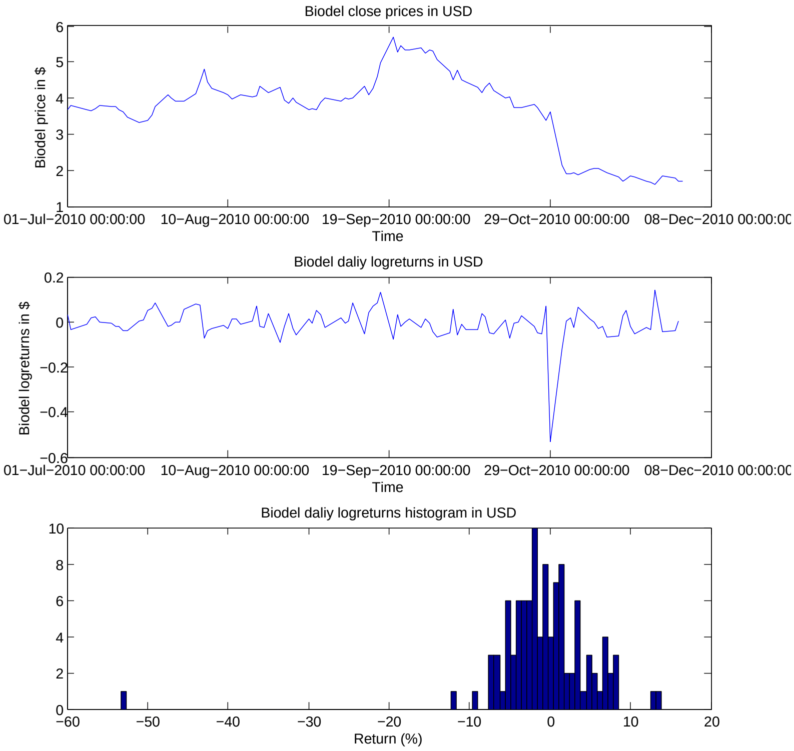

Figure 5.1: Return for the Biodel Inc. stock for a period of 4 months prior to the event until 1 week after.

Inc. stock with maturity in either November or December, i.e. maturity after the proposed release of the information. During the time between the first of March and the 30 of November no substantial evidence that Linjeta would be either approved or unapproved is given to the market. The FDA did not have an advisory committee discussing the drug. On the 1 of November, before the markets open for trading in the US, Biodel Inc. announces that they have received a complete response letter from the FDA. In the letter the FDA asks for new phase III studies and more data related to stability and manufacturing.[1]

In the Figure 5.2 the ATM IV is given for Biodel Inc. 4 months before the event leading up to the event and 1 month after. The IV we have plotted in Figure 5.2 is taken from Bloomberg. Bloomerg calculates these IV's from a weighted average of the volatilities of the two options with strike price closest to the spot price of the underlying stock each day. The contracts used are the closest pricing contract month that is expiring at least 20 business days out from today. The reason for choosing this ATM IV data is beacuse the model that we have derived above does not take into account any smile or skew effects that has been shown to exist for IV's.[2] If we therefore look at only the ATM volatilities these effect should not distort any affects that the events might have on the IV. If we would have chosen to look at a fixed strike, the potential drift of the spot price away from the strike price of the option could cause these smile effects to increase the IV, an effect that we do not want.

From the Figure 5.2 we can see that the IV is increasing leading up to the event, it peaks a few days before the event, and then there is a drop on the day of the

[1]Biodel press releases: `http://investor.biodel.com/releases.cfm`
[2]`http://en.wikipedia.org/wiki/Volatility_smile`

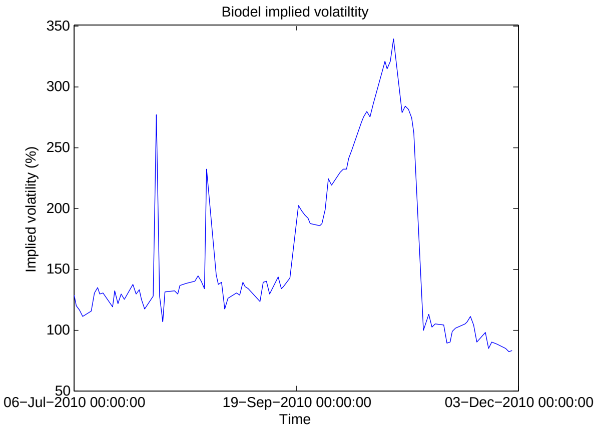

Figure 5.2: Implied volatility of at the money options in Biodel 4 months ahead of the event and 1 month after.

event. Since the contracts used are the closest pricing contract month at least 20 business days out from today we should not expect any increase in the IV until one month ahead of the event if there are open contracts with maturity in September. The reason for this is that if the closest pricing month date is before the event, the event should not affect the price of the option given our model discussed in the previous chapter. From 5.2 we can see that the IV doesnt increase much before the 19 of September (apart from the two spikes that we think are bad data points). The reason for this is probably that before this date the maturity of the options used are before the event date. We still think that Figure 5.2 is a good way visualize the time series affect of the IV due to that for the last month the maturity of the options have to be after the event, but it should have been even better if we could show the time series ATM IV for contracts having maturity after the event. In the month prior to the jump we see that the IV is increasing and we also notice the large volatility in absolute terms, reaching almost 350%, a few days before the event.

We will now look the jump estimator for ATM options in Biodel Inc. The implied IV's used in these calculations will be backed out from the actual market prices of options using Equation (4.3).

Since the release of the information was on the 1 of November, we set t to be that day. We will look at prices on the day before, *t-1*, which is on the 30 if October. Since the stock at that time stood at \$3.63 this will be ATM. The maturity for the closest options was on the 19 of November, approximately 15 trading days away. This leads T_1 to be 0.0595 years.[3] The maturity for the next closest option was on the 17 of December, approximately 40 trading days away. This lead T_2 to be 0.1587 years. Since there are no options with strike price \$3.63 we have chosen to look at the two closest options, strike price at \$3 and \$4. We have then averaged over the implied volatilities of these two options to account for possible skew effects. This is though just the case for the options maturing in November because prices for the \$3 strike was not available for options maturing in December. For this maturity we have chosen just to use the option with strike at \$4. Biodel has never had any dividend so this parameter is set to zero. The interest rate is chosen as the 1 year t-bill rate on the 29 of October, 0.0022% (which is approximetly the same in both continuous

[3] We use that 1 year is 252 days

and yearly compounding). The corresponding Black-Scholes implied volatilities for these options were 331.21% (this is the average of the \$3 and \$4 strike options) and 244.89%. The jump estimator then becomes 68.82%.

If we compare the jump estimator to the realized return on the event day we see esitmator 68.82% is larger than the actual time series jump of 41.32%.

One problem with this metholodgy is that if we do not use the exact same strike for the options with different maurity we could get smile or skew effects that distort the maturity effect we are trying to look at. We therefore did the same calcualtion for only the \$4 strikes for the two maturities and the new jump estimator then became 66.02%. Using only the IV for the \$3 strike for the first maturity and the \$4 strike for the second maturity resulted in a time series estimator of 71.56%. The difference between the using the different strikes is not that large in comparison to the difference between the times series estimator and the actual time series jump of the stock price.

5.1.1 Results from the event study and further questions

From the case study of Biodel we firstly notice that the FDA announced on the 1 of March that they would announce the result of the NDA on the 30 of October. The market therefore knew about this event before hand and should therefore take this into account when pricing the options. As we showed in Figure 5.2 we could also see that the rolling ATM volatility increased leading into event, and drops sharply after it has occured, which is what we are expecting. In this example the markets therefore seem to be pricing in the possibility of a jump. From the options we also estimated the jump estimator to be 68.82%.

From the above results it would be interesting to look more closely at a larger number of companies with NDA events. We are most interested in seeing if he market is anticipating the event, if the jump size can be estimated using our term structure estimator and how this estimated jump volatility compares to volatlity of the underlying stock on the event day. As a pseudo problem we are also interested in what factors that may affect the jump size. We have discussed that the size of the company can have an effect so we analyze this some more.

5.2 Data and description of sample

We have chosen to look at 41 events, all of which are NDA events for companies trying to launch a product in the US. Only NDA events are choose due to the fact that accuratly timing the date for Adcoms is hard since they are usually not announced in advance to a great extent. The event dates have been chosen from a database of NDA events, collected by Bernstein Research, and are NDA events that happened between 2009 and January 2011. For each company and event we have checked press releases from the companies to determine the exact date and timing of the release of the information to the market.

From the original list of events we have choosen to look at companies that had a total call option volume on the day, or week, before the event day that was greater than zero. This means that there actually where call options that were traded in these companies in the days prior to the event. We do this to try to sort out companies that may have stale call option prices that hence may be different from the actual market price of the options. We have from these companies and events chosen to look at companies with a market capitalization of less than 10 billion USD. We have chosen to not include larger companies due to the cumbersome and time consuming process of checking all the event dates. Choosing to look at smaller companies also comes from our belief that these events should have a greater impact on the earnings, and hence the stock price, than if we would have looked at a sample of companies with larger market capitalization.

For the time series changes of IV's we have chosen to use Bloomberg calculated ATM IV, the same type of data as we used in the case study and for the same reason as in the case study. When we calculate the actual IV used to construct the term structure estimator, we will use the call option prices and back out the implied volatility from Equation (4.3), again in the same manner as we did in the case study.

5.3 Stock movement on event day

For all the stocks in the sample we have collected closing prices of the stock around the event date. If the new information is given to the market after closing hours on day t the return is calculated for $t+1$, hence by $(S_{t+1} - S_t)/S_t$. This is given in Figure 5.3. The largest positive single day return in this sample is 625.9% and the largest negative single day return is -74.9%. We have cut the y-axis at 65% since the next largest return was 65%. Of the total 41 companies 17 did not move more than 5% on the day of the event and 24 companies moved more than 10%, in absolute terms.

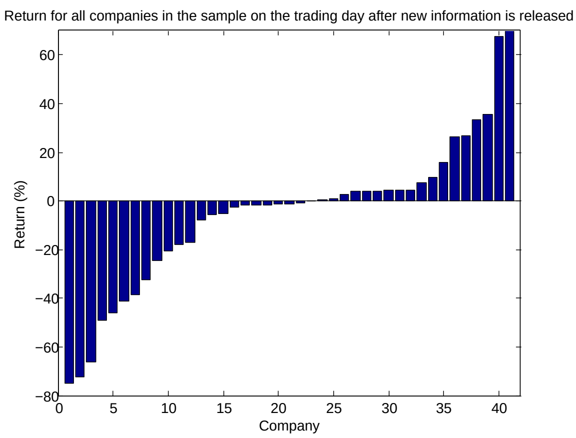

Figure 5.3: Plot of percentual return on the day of the event for all companies in the sample.

Of the total number of companies in Figure 5.3 23 had a negative return on the day of the event and 18 had a positive return. For the sample there is hence a bias towards negative jumps. The average jump size in the sample is 8.4% but the median jump -1.3%. The large difference in mean and median jump size is due to the 625.9% jump described above. If we remove this price jump the mean would become -7.0%.

Table 5.1 present the distribution between favorable and unfavorable NDA decisions for the sample.

Decision	Number of outcomes
Favorable	12
Unfavorable	4
CR	25

Table 5.1: Statistics of different outcomes from the NDA for the companies in the sample.

There is a large number of CR (complete response) notifications in the sample. As we discussed above this can be seen as a milder rejection of the drug since there is still a possibility that the drug can be approved. The small number of unfavorable decisions indicates could that the process of filing for a new drug is rather well constructed, companies unsure of recieving a favorable decision may be inclined not to file in the first place.

The fact that there are more negative decisions (unfavorable and CR) should also explain why there are more companies having negative return than positive on the event day. Even though a CR is not a rejection it is still not a good outcome from the NDA since it usually leads to more costs for the company.

5.4 Implied volatility analysis

5.4.1 Implied volatility difference and actual event day return

We have in Figure 5.4 calculated the percentage difference between the ATM IV on the day of the event, day t and the day before the event *t-1*, for the companies in the sample. The calculation is $(IV_{t-1} - IV_t)/IV_t$. A large positive difference means that the IV the day before the jump is larger than after the jump.[4]

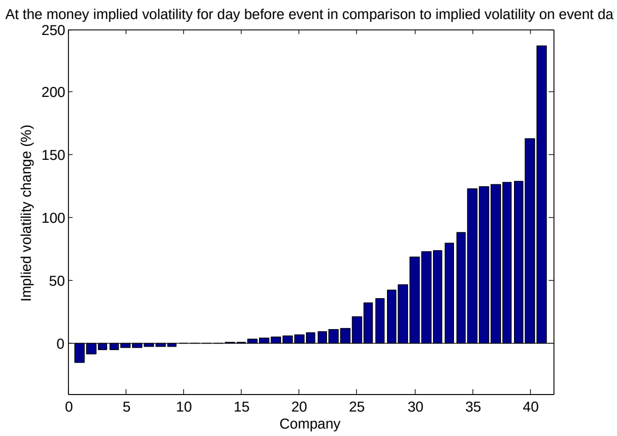

Figure 5.4: Percentage difference between the companies implied volatility on the day before the event day and on the day of the event.

From Figure 5.4 we see that most of the companies have a large positive difference

[4]The IV data is taken from Bloomberg and is calculated in the same manner as in the case study and used for the same reason

in the IV between the days. In the sample of 41 companies there are 17 that have a volatility change that is larger than 20%. The mean change in the sample is 39.6%.

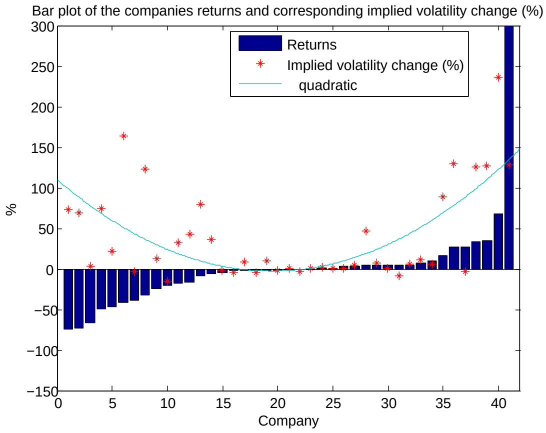

Figure 5.5: Plot of both the percentage change in the stock and the percentage difference between the companies implied volatility on the day of the event and on the day before the event day.

In Figure 5.5 we can see that for most of the companies that have had a jump in the stock price, positive or negative, the implied volatility decreases. If there is no jump in the stock price on the day of the event we could interpret this as the information given did not lessen the uncertainty about the outcome of the NDA. This means that the market could still be concerned over the future volatility of the underlying stock, and will hence not decrease the IV. If this would have been a case were the stock jumps unanticipated the volatility should increase, not decrease, since this should indicate that the IV could be too low.

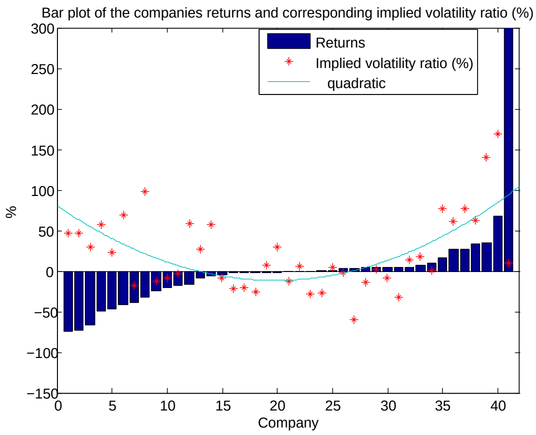

Figure 5.6: Plot of both the percentage change in the stock and the percentage difference between the companies implied volatility on the day of the event and an average of prior implied volatilities.

In the Figure 5.6 we look at the relation between the mean IV and the IV on the event day. The mean is here calculated as the mean of the volatility for at least 2 months prior to the event day. For some companies it was not possible to find ATM IV:s that long back, so their means are calculated for a shorter time period. We can see that for the larger jumps there is a indication that the IV on the day before the event is larger than the mean IV prior to the jump, pointing to the fact that the IV have increased leading up to the event.

5.4.2 Implied volatlity difference and market capitalization

From Figure 5.7 the relationship between the market capitalization and the IV difference in percent is plotted. The volatility difference in percent is as in calculated as in Figure 5.4. As expected the IV change is larger for smaller companies since the events are usually more important for their earnings, in respect to their current earnings.

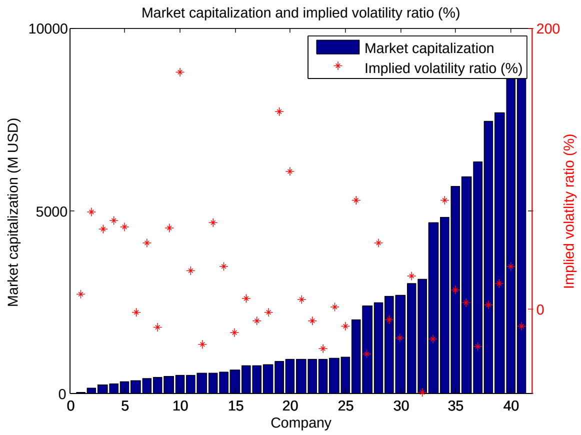

Figure 5.7: Plot of market the capitalization and the percentage difference between the companies implied volatility on the day of the event and on the day before the event day.

5.4.3 Jump estimator

We now look at the jump estimate derived from the options in these companies. The calculations will be the same as in the Biodel Inc. case. We have just chosen one strike and taken the same strike for both maturities. The strikes for the options are chosen as close to the spot price of the stock on the day before the event as possible, and the two maturities chosen are the two shortest of the available options. Some companies did not have active trading in the shortest options on the day before the event. If this was the case we moved back one day to see if there were prices given quoted, which we did until we found good prices.[5]

We have in Table 5.2 listed the IV for the two options used and the term structure estimator of the jump volatility. In Table 5.2 we see that 12 companies have NaN for the term estimator. This is due too that the longer maturing option volatility (IV_2) is higher than the shorter (IV_1) for these companies. Since Equation (4.14) is the square root of the difference between the first and the second IV's, this number becomes a complex number if the second IV is larger than the first. The reason for the first IV to be smaller than the second can be that there is some other event that

[5]All option prices have been collected from Bloomberg.

the market is pricing in that is after the first option expires. Since the term structure only can handle one jump for the period it is evaluating, it will not be applicable in these situations.

As we can could expect from the earlier analysis there are some companies were the jump estimator is rather large. Look at company 5 for example. For this company the IV was 372.21% for the shortest maturing option and 238.75% for the longest option. Together with the information about how many days the options had until maturity the term structure estimate was 96.53%, which is the highest in the sample. This indicates that the market is assigning a implied jump of almost 100% for one trading day between the two maturities of the options.

The last column in Table 5.2 is the actual change in the share price on the event day. From the table we can see that of the 41 events 19 had term estimators that where higher than the actual change in share price and 10 had lower term estimator than the change in the share price.

5.5 Conclusions from examining events

From the analysis above we can conclude that the market is aware of the events and they are pricing call options in accordance to the characteristics of the model that we have discussed. Since the IV of call options seem to follow the characteristics we have found the term strucuture estimate can become a effective tool to use when assesing and investing in companies with deterministically timed events. Since these events have such a big impact they should also be accounted for when calculating risk measures for these companies. Looking at historical data could underestimate the future risk of investing in the company. Using the jump estimator one could therefore extract a jump estimator to be used when modeling the stock with deterministically timed jumps.

Company	IV_1(t-1)	IV_2(t-1)	Term	Stock price change
1	263.34	192.86	55.11	-72.5
2	211.5	168.88	52.62	67.4
3	203.91	107.38	59.25	-32.27
4	36.58	34.71	3.23	4.06
5	372.21	238.75	96.53	-41.32
6	252.50	185.64	57.86	26.43
7	70.19	66.40	10.08	-46.17
8	140.68	79.21	17.94	3.96
9	372.78	161.47	39.31	35.38
10	50.58	74.79	NaN	-38.50
11	60.23	62.34	NaN	4.46
12	55.62	37.45	6.48	-1.32
13	221.38	164.36	43.27	-74.96
14	161.57	126.55	35.48	26.65
15	29.88	25.29	4.69	-2.52
16	227.95	121.97	30.33	15.99
17	105.05	122.41	NaN	-24.71
18	25.16	26.01	NaN	0.50
19	21.77	24.05	NaN	-1.29
20	197.89	109.45	15.4	-65.97
21	171.8	118.17	24.15	-7.98
22	59.32	59.30	0.45	-1.6
23	76.46	73.33	8.95	9.76
24	32.64	31.86	1.97	-5.17
25	56.07	56.52	NaN	4.33
26	62.45	64.29	NaN	3.87
27	244.41	175.09	55.04	-49.22
28	47.00	38.38	12.42	-1.83
29	171.15	106.97	22.96	-18.04
30	124.41	111.93	18.28	-1.057
31	110.85	79.58	10.57	-0.98
32	46.03	46.87	NaN	4.4
33	223.23	146.83	42.16	-16.92
34	89.62	57.60	20.21	-0.16
35	405.45	259.50	74.33	625.93
36	61.63	40.07	12.43	0.86
37	68.19	71.22	NaN	-20.44
38	197.41	212.56	NaN	-5.82
39	82.02	92.70	NaN	2.65
40	222.59	172.90	70.34	33.29
41	49.10	53.12	NaN	7.30

Table 5.2: Implicit volatilities for the companies the day before the event and term structure estimate, all values in percent. The first two columns are implicit volatilities for the two shortest maturing at the money options. The third column is the term structure estimate. The last column is the actual time series percentage change in the stock price for the event day.

Chapter 6

Hedging events

6.1 Delta hedging

The delta of an option is the ratio of the change in the price of the option to the change in the change in the price of the underlying security, assuming all other variables remain unchanged. Matimatically delta is represented by

$$\Delta = \frac{\partial C}{\partial S}$$

Delta is therefore the slope of the curve that relates the option price to the underlying security. A delta of 0.6 therefore means that if the stock price change by a small amout the option price change about 60% of that amount.[1] We should here stress the importance that the stock only can move in a small amount for the delta to accuratly describe any move in the option price.

The delta of an European call option is given by

$$\Delta = \frac{\partial C}{\partial S} = \Phi(d_1(t, S_t))$$

where $d_1(t, S_t)$ is given in Theorem 4.1.

From a practical perspective the idea of delta hedging is to keep a portfolio delta neutral to hedge away any directional trading risk, i.e. risk associated with the movement of the underlying security. If a trader has sold a call option he can create this delta neutral portfolio by firstly calculating the delta of the option using the equation $\Phi(d_1(t, S_t))$. The only parameter in this equation that the trader needs to estimate is the volatility to be used. Usually this estimation is done by using the implied volatility estimated from the market, or estimating from historical volatility of the underlying. The trader then use this delta, calculated from the estimated volatility, to buy or sell the underlying stock to create a delta neutral portfolio. Therefore, if the trader has sold a call option and the delta of this option is 0.5, the trader needs to buy 0.5 positions of the underlying to create a delta neutral portfolio. This portfolio of a call option and position in the underlying is then rebalanced as many times as the trader wants. The trader can, due to constraints, not hedge continously

[1] *Fundamentals of Futures and Options Markets (6th edition) by John Hull, 2008*, Chapter 15.4

and need to do it discretly during the life of the option. Since we have derived the closed form solution for the call price given events in Theorem 4.3 we can look at the corresponding delta this model implies.

Using our closed form solution for the price of a call option we see that we have a slightly different delta formula when we model the stock price with jump that is deterministic in time. For this model the delta is again given by:

$$\Delta = \frac{\partial C}{\partial S} = \Phi(d_1(t, S_t))$$

but here

$$d_1(t, S_t) = \frac{1}{\gamma\sqrt{T-t}}\left[log(\frac{S_t}{K}) + (r + \frac{1}{2}\gamma^2)(T-t)\right]$$

where

$$\gamma^2 = \sigma^2 + (T-t)^{-1}\sum_{j=1}^{N_T^d}(\sigma_j)^2$$

Since we have shown that we can use options in the companies to estimate the jump we will now look at two different schemes to delta hedge the options using the fact that we can separate these two different volatilities, the jump volatility and the stocks underlying volatility.

6.2 General delta hedging scheme with daily rebalancing

Lets assume we have a call option $C_0 = C(S_0, \sigma, 0)$ at time 0, where σ is the estimated volatility used to price the option. Let a trader sell this option at time 0. He will then collect the cash C_0 at time zero. The delta of the portfolio is now the delta of the option. To make the portfolio delta neutral the trader needs to buy delta number of stocks in the underlying security. The trader thus have to borrow this amount and pay the interest rate r. When this is done the portfolio is delta neutral, if we assume that the stock only make small moves in each time step. If the trader wants to rebalance the portfolio on a daily basis he need to calculate the delta of the option the next day, using the new stock price. The trader then needs to rebalance the portfolio depending on the difference between the first and the second day delta. He either needs to buy or sell $\Delta(S_1, t_1, \sigma) - \Delta(S_0, t_0, \sigma)$ number of stocks at the spot price at that day. The cashflows for the whole process is thus:

$$
\begin{aligned}
&B_0 = -\Delta(S_0, t_0, \sigma)S_0 \\
&B_1 = e^{r\Delta t}B_0 - (\Delta(S_1, t_1, \sigma) - \Delta(S_0, t_0, \sigma))S_1 \\
&B_2 = e^{r\Delta t}B_1 - (\Delta(S_2, t_2, \sigma) - \Delta(S_1, t_1, \sigma))S_2 \\
&\ \dots \\
&\ \dots \\
&\quad \text{if } S_T > K \\
&B_T = e^{r\Delta t}B_{T-1} - K + (1 - \Delta(S_{T-1}, t_{T-1}, \sigma))S_T \\
&\quad \text{if } S_T \leq K \\
&B_T = e^{r\Delta t}B_{T-1} + \Delta(S_{T-1}, t_{T-1}, \sigma))S_T
\end{aligned}
$$

At the last day the option will either be in our out of the money. If the option is in the money the trader needs to pay the difference between the strike price and the stock price at time T to the owner of the option. If the stock is out of the money at time T he will just recieve the stocks value times his position (delta) on day $T-1$.

The total cashflow B_T is thus the cost of the delta hegde. If we could hedge continously, and discounted with the interest rate r, this value would be the same as the value of the option at time t_0.[2]

6.3 Discretization of Black & Scholes with deterministically timed jumps using Euler Scheme

To be able to simulate the model in Equation (4.10) a discretization of the model is needed.

Starting with Equation (4.10) we see that the log price $X_t = log(S_t)$ is

$$
X_t = X_0 + (r - \frac{1}{2}\sigma^2)t + \sigma W_t + \sum_{j=1}^{N_T^d} Y_j \tag{6.1}
$$

The discretization of Equation (6.1) can then be done via a Eueler-sheme. This Eueler-scheme can be written as

$$
X_{t+\Delta t} = X_t + (r - \frac{\sigma^2}{2})\Delta t + \sigma\sqrt{\Delta t}W - \frac{1}{2}\sigma_j^2 I\{t+\Delta t = s_j\} + \sigma_j I\{t+\Delta t = s_j\}\epsilon \tag{6.2}
$$

Here both W and ϵ are $N(0,1)$ distributed and independent. $I\{t + \Delta t = s_j\}$ is as before the indicator function, being one if $t + \Delta t$ is the jump time s_j.

[2] *Fundamentals of Futures and Options Markets (6th edition) by John Hull, 2008*, Chapter 15.4

6.4 Hedging stocks with one deterministically timed event

We now look at the problems of delta hedging call options with one deterministically timed event for the underlying stock. If a large jump realizes a portfolio that is setup with delta based on the previous trading day will not be immune to this directional move, due to the fact that the delta discussed above only is effective for small moves in the stock price. Even though we realize that we can't be delta neutral over the over the jump, we are interested in analyzng what happends if we use the delta hedging scheme in Section 6.2

If we assume that the stock follows the the model in Equation (6.2), and that we are looking at discrete times, we can see that the extra voltility and jump only affects one day, the day of the jump. The stock therfore moves like a stock without jumps and σ as volatilty term during all the other days during the time span we are looking at. But when pricing the option we must take this jump into consideration before the actual jump. So which volatility should we use to hedge the option?

Our first choice is to use the markets implied volatility, in our case it would be γ from Equation (4.13). This choice of volatility thus have a pre jump volatility and a post jump volatility. We are going to use the same scheme as we have discussed and shown in the Section 6.2 above to hedge the option. This strategy should leave the total portfolio delta neutral up until the day before the event and after the event.

The second method will be using only the σ, i.e. the volatilty without any jumps, and use this volatility during the whole hedging scheme. We can think of this as the actual volatility of the stock if the jump would not happened. This can also be seen as chosing to hedge with the historical return volatility of the stock. This choice of volatility used to calculate delta will not leave the portfolio delta neutral. These kind of simulations and questions have been dicussed and analyzed in [2] for stocks that does not jump.

6.5 Simulation setup

To hedge the options we will asume the following values for the variables necessary to simulate the underlying process and estimate the cost of the delta hegde. We will make no distinction between the drift in the stock price and the interest rate.

S_0	K	σ_j	σ	T	T_{jump}	r	Δt
100	100	0.7	0.4	2/12	20	0.05	T/40

Table 6.1: Values for the parameters used in the delta hedging simulations.

The parameter T_{jump} is the timing of the jump and is here assumed to be in the middle of the maturity of the option. The choice of looking at a jump that is about one month into an option comes from the fact that we did not find many options with events and maturities after these events that had longer time until the event would happen. The reason for this could be that traders are unwilling to quote prices in these options further away from the events. The choice of the jump volatility σ_j and normal volatility σ are chosen arbitrarily.

We will in the below analysis firstly look at three different scenarios. One were the stock has negative jump, one where the stock has a positive jump and one were the stock does not jump.

6.6 Scenario 1: negative jump

6.6.1 Hedging negative jump with γ

We start by looking at the scheme of hedging the call options with the actual implied volatility that the options market is pricing the options with. This is hence $\gamma^2 = \sigma^2 + (T-t)^{-1}\sum_{j=1}^{N_T^d}(\sigma_j)^2$ in our model, where σ is the normal volatility of the option and σ_j is the jump volatility. We here assume that we can know this volatility γ. What this indicates is that we know σ and σ_j. We also see that this γ volatility is not constant, even though σ and σ_j are constant. After the jump the volatility γ is equal to σ.

The scenario we will go through is going to be a discrete hedging scheme of an option expering in two months that has a jump at day 20, as specified in Table 6.1. We are hence looking at hedging the option daily for 40 days. From the discretization above we simulate the stocks path and will in the below table show the result of the delta hedging scheme.

Day	Share price	Delta	Shares purchased	Cost of shares	Cumulative cost including interest	Interest
0	100	0.6447	0.6447	-64.4678	-64.4678	0.0134
1	104.2153	0.6657	0.021	-2.1865	-66.6677	0.0139
2	104.4672	0.6667	0.001	-0.1097	-66.7912	0.0139
3	104.5658	0.667	0.0003	-0.0319	-66.837	0.0139
4	102.5895	0.6571	-0.0099	1.0159	-65.8351	0.0137
5	102.4951	0.6565	-0.0007	0.067	-65.7817	0.0137
•	•	•	•	•	•	•
16	105.3783	0.6687	-0.008	0.8388	-67.5425	0.0141
17	104.6113	0.6648	-0.0039	0.4088	-67.1478	0.014
18	105.7461	0.6701	0.0053	-0.5649	-67.7266	0.0141
19	101.2702	0.6476	-0.0225	2.2804	-65.4603	0.0136
20	75.6159	0.01	-0.6376	48.2151	-17.2588	0.0036
21	73.2755	0.0038	-0.0062	0.4552	-16.8072	0.0035
22	73.3916	0.0031	-0.0007	0.0483	-16.7624	0.0035
23	74.629	0.0039	0.0008	-0.0568	-16.8227	0.0035
•	•	•	•	•	•	•
36	79.4382	0	0	0.0013	-16.5778	0.0035
37	82.0341	0	0	0	-16.5813	0.0035
38	84.0169	0	0	0.0004	-16.5844	0.0035
39	84.528	0	0	0.0001	-16.5878	0.0035
40	83.0243	0	0	0	-16.5912	0.0035

Table 6.2: Delta hedging scheme for sample path with negative jump using γ as the volatility parameter for delta.

As we can see in the table the interestng part is on day 19 and 20. On day 20 the stock jumps from 101.27 to 75.62. The delta of the option is reasonably stable around 0.65 before the jump, and there is hence not much rebalancing in the portfolio. After the jump the delta goes down to close to zero and almost the whole position in the stock is sold.

6.6.2 Hedging negative jump with σ

The next strategy we will look at is using the volatility of the underlying stock if we would discard the jump. In the equation $\gamma^2 = \sigma^2 + (T-t)^{-1} \sum_{j=1}^{N_T^d} (\sigma_j)^2$ this would correspond to the volatility σ. This is therefore lower than the γ before the jump but equal to it after the jump. On the day before the event day, day 19 in Table 6.3, the delta should therefore be lower than in Table 6.2 using γ as volatility parameter. After the jump on day 20 almost the whole position in the stock is sold.

Day	Share price	Delta	Shares purchased	Cost of shares	Cumulative cost including interest	Interest
0	100	0.5528	0.5528	-55.2777	-55.2777	0.0115
1	104.2153	0.6506	0.0979	-10.1998	-65.489	0.0136
2	104.4672	0.6569	0.0062	-0.6485	-66.1511	0.0138
3	104.5658	0.6598	0.0029	-0.3063	-66.4712	0.0138
4	102.5895	0.6144	-0.0454	4.6526	-61.8324	0.0129
5	102.4951	0.6123	-0.0021	0.2136	-61.6316	0.0128
•	•	•	•	•	•	•
16	105.3783	0.6974	-0.0381	4.0195	-72.3218	0.0151
17	104.6113	0.6789	-0.0185	1.9329	-70.404	0.0147
18	105.7461	0.7122	0.0333	-3.5163	-73.935	0.0154
19	101.2702	0.5804	-0.1318	13.3488	-60.6016	0.0126
20	75.6159	0.01	-0.5704	43.1289	-17.4853	0.0036
21	73.2755	0.0038	-0.0062	0.4552	-17.0338	0.0035
22	73.3916	0.0031	-0.0007	0.0483	-16.989	0.0035
23	74.629	0.0039	0.0008	-0.0568	-17.0493	0.0036
•	•	•	•	•	•	•
36	79.4382	0	0	0.0013	-16.8051	0.0035
37	82.0341	0	0	0	-16.8086	0.0035
38	84.0169	0	0	0.0004	-16.8118	0.0035
39	84.528	0	0	0.0001	-16.8152	0.0035
40	83.0243	0	0	0	-16.8187	0.0035

Table 6.3: Delta hedging scheme for sample path with negative jump using σ as the volatility parameter for delta.

6.6.3 Comparing the two strategies for negative jump

We start by noticing that the hedging cost after day 20 are the same for both strategies. This is what we expected to find since after the jump $\gamma = \sigma$. It is therefore what happends before day 20 that is interesting to look at. We see that for the γ strategy the delta is higher for day 0. This is because the volatilty used in calculate the delta is higher than for the σ strategy. The γ strategy therefore makes us buy more shares at day 0 than the σ strategy. As we see on day 19 the cumulative costs is 60.6 for the σ strategy but for the γ strategy approximetly 65.5, but we are holding more shares in the γ strategy. On the next day, day 20, the stock jumps down to 75.6. After this jump, on day 20, the delta of the two strategies are the same so from then on the heding costs will be the same. From the the γ strategy we can see that the cost for the 0.6476 shares on day 19 is $65.4603/0.6476 = 101.0814$, but for the σ strategy the average cost on this day was 104.41. It has therefore up until day 19 costed more per share for the σ hedging scheme. Since we have the same delta on day 20 and afterwards, this means that when the almost all the positions are sold on day 20 we will have a larger cost for the hedging strategy using σ.

In this example of a negative jump we therefore see that the sample path that the stock take leading up to the jump is the factor that decides wich strategy that is the least costly.

6.7 Scenario 2: no jump

The second scenario we will look at is when the stock price does not jump.

6.7.1 Hedging no jump with γ

We once again start by looking at the γ strategy. We have exactly the same setup as before expect that we are looking at a sample path without the jump. Looking at day 20 in Table 6.4 we can see that the stock does not jump on this day. By looking at day 40 we can see that the stock ends in the money ($115.8425 > 100$) and that the whole position in the stock is sold on that day. Since we have sold an option we have to give the option holder the the 15.84 that is over the strike price 100. We hence only recieve 100 on the last trading day.

Day	Share price	Delta	Shares purchased	Cost of shares	Cumulative cost including interest	Interest
0	100	0.6447	0.6447	-64.4678	-64.4678	0.0134
1	98.9104	0.6388	-0.0059	0.5825	-63.8988	0.0133
2	100.4673	0.6467	0.0079	-0.7959	-64.708	0.0135
3	102.3209	0.6559	0.0092	-0.9451	-65.6665	0.0137
4	104.1099	0.6646	0.0087	-0.903	-66.5832	0.0139
5	103.576	0.6618	-0.0028	0.2895	-66.3076	0.0138
•	•	•	•	•	•	•
16	92.1413	0.5978	0.0148	-1.3607	-60.3375	0.0126
17	92.7467	0.6011	0.0033	-0.3087	-60.6588	0.0126
18	97.337	0.6269	0.0258	-2.5093	-63.1808	0.0132
19	92.5481	0.5995	-0.0274	2.5363	-60.6577	0.0126
20	92.2335	0.2721	-0.3273	30.1926	-30.4777	0.0064
21	91.8534	0.2535	-0.0187	1.715	-28.7691	0.006
22	95.8248	0.382	0.1285	-12.3125	-41.0876	0.0086
23	97.294	0.432	0.0501	-4.8717	-45.9678	0.0096
•	•	•	•	•	•	•
36	105.1129	0.8432	0.2064	-21.6927	-89.4724	0.0186
37	108.5678	0.9696	0.1264	-13.7225	-103.2136	0.0215
38	113.312	0.9997	0.0302	-3.4167	-106.6518	0.0222
39	115.1728	1	0.0003	-0.032	-106.706	0.0222
40	115.8425	1	-1	100	-6.7283	0.0014

Table 6.4: Delta hedging scheme for sample path with negligible jump using γ as the volatility parameter for delta.

6.7.2 Hedging no jump with σ

Using only σ we get the result in Table 6.5. We can here again see that the option is in the money on the last trading day and that the stock position is closed on that day. But, once again, since we have sold a call option we need to pay the difference between the strike and the stock price to the holder of the option, only leaving us with 100 on the last day.

Day	Share price	Delta	Shares purchased	Cost of shares	Cumulative cost including interest	Interest
0	100	0.5528	0.5528	-55.2777	-55.2777	0.0115
1	98.9104	0.5251	-0.0276	2.7334	-52.5559	0.011
2	100.4673	0.563	0.0379	-3.8047	-56.3715	0.0117
3	102.3209	0.6078	0.0448	-4.5868	-60.97	0.0127
4	104.1099	0.6502	0.0424	-4.4099	-65.3927	0.0136
5	103.576	0.6384	-0.0118	1.2244	-64.1819	0.0134
•	•	•	•	•	•	•
16	92.1413	0.2931	0.064	-5.9014	-31.3977	0.0065
17	92.7467	0.3059	0.0128	-1.1858	-32.5901	0.0068
18	97.337	0.4505	0.1446	-14.071	-46.6678	0.0097
19	92.5481	0.2883	-0.1622	15.0088	-31.6687	0.0066
20	92.2335	0.2721	-0.0161	1.4895	-30.1858	0.0063
21	91.8534	0.2535	-0.0187	1.715	-28.4771	0.0059
22	95.8248	0.382	0.1285	-12.3125	-40.7955	0.0085
23	97.294	0.432	0.0501	-4.8717	-45.6757	0.0095
•	•	•	•	•	•	•
36	105.1129	0.8432	0.2064	-21.6927	-89.1796	0.0186
37	108.5678	0.9696	0.1264	-13.7225	-102.9206	0.0214
38	113.312	0.9997	0.0302	-3.4167	-106.3588	0.0222
39	115.1728	1	0.0003	-0.032	-106.413	0.0222
40	115.8425	1	-1	100	-6.4351	0.0013

Table 6.5: Delta hedging scheme for sample path with negligible jump using σ as the volatility parameter for delta.

6.7.3 Comparing the two strategies for no jump

From the Tables 6.4 and 6.5 we see that when we are using γ the delta is higher than using σ and we are hence at day 0 buying more shares using the γ strategy. It is now also interesting to note that since the stock is moving down coming up to day 20 the delta for the σ strategy is also decreasing, but for the γ strategy it is still reasonably high, and this bacause the volatility used to calculate the delta is higher. On day 20 the delta for the γ strategy drops sharply. The average cost for the stocks held for the σ strategy at day 20 is 110.9364 and for the γ strategy 112.0066. We therefore here see that the σ strategy is less costly in this scenario since the remaining costs for hedging are the same for both strategies. The difference in the end is though not that large, 6.4351 for the σ strategy and 6.7283 for the γ strategy.

6.8 Scenario 3: postitive jump

We will now look at the last scenario where the stock takes a large upward jump at the event time.

6.8.1 Hedging positive jump with γ

In Table 6.6 we can see that the jump at day 20 is large, the stock price moves from 99.5858 to 204.2846. At day 20 the stock is therefore in the money and the delta is 1 from this day and no more rebalancing is done. The only change going down to day 40 is that the interest is increasing the hedging cost. At day 40 the stock is still in the money so the position is closed and we recieve 100 since we have to give the difference between the strike price to the holder of the option.

Day	Share price	Delta	Shares purchased	Cost of shares	Cumulative cost including interest	Interest
0	100	0.6447	0.6447	-64.4678	-64.4678	0.0134
1	101.3342	0.6513	0.0067	-0.6746	-65.1558	0.0136
2	101.7924	0.6535	0.0021	-0.2174	-65.3868	0.0136
3	104.1349	0.6649	0.0114	-1.1921	-66.5925	0.0139
4	109.6737	0.6907	0.0257	-2.8234	-69.4298	0.0145
5	109.0758	0.6878	-0.0028	0.3106	-69.1336	0.0144
•	•	•	•	•	•	•
16	101.3694	0.6487	0.0051	-0.5173	-65.302	0.0136
17	101.5549	0.6495	0.0008	-0.0776	-65.3932	0.0136
18	101.7142	0.6501	0.0006	-0.064	-65.4708	0.0136
19	99.5858	0.6388	-0.0113	1.1238	-64.3607	0.0134
20	204.2846	1	0.3612	-73.7855	-138.1596	0.0288
21	198.9328	1	0	0	-138.1884	0.0288
22	196.7012	1	0	0	-138.2172	0.0288
23	198.3447	1	0	0	-138.246	0.0288
•	•	•	•	•	•	•
35	177.5459	1	0	0	-138.592	0.0289
36	173.4704	1	0	0	-138.6209	0.0289
37	169.925	1	0	0	-138.6498	0.0289
38	170.8202	1	0	0	-138.6786	0.0289
39	166.8036	1	0	0	-138.7075	0.0289
40	163.679	1	-1	100	-38.7364	0.0081

Table 6.6: Delta hedging scheme for positive jump using γ as the volatility parameter for delta.

6.8.2 Hedging positive jump with σ

On day 20 in Table 6.7 delta is one since the option is deep in the money and the strtegy thus tells us to buy a full position in the share. For the σ strategy we again see that the option is in the money at maturity when the position in the stock is closed, but again we only recieve 100 since the rest is paid to the option holder.

Day	Share price	Delta	Shares purchased	Cost of shares	Cumulative cost including interest	Interest
0	100	0.5528	0.5528	-55.2777	-55.2777	0.0115
1	101.3342	0.5844	0.0316	-3.2064	-58.4956	0.0122
2	101.7924	0.5952	0.0108	-1.0972	-59.6049	0.0124
3	104.1349	0.6501	0.0549	-5.7171	-65.3345	0.0136
4	109.6737	0.7648	0.1147	-12.583	-77.931	0.0162
5	109.0758	0.7558	-0.009	0.9858	-76.9615	0.016
•	•	•	•	•	•	•
16	101.3694	0.5833	0.0309	-3.1288	-59.6649	0.0124
17	101.5549	0.5891	0.0058	-0.5902	-60.2675	0.0126
18	101.7142	0.5943	0.0053	-0.5345	-60.8145	0.0127
19	99.5858	0.5243	-0.07	6.9715	-53.8556	0.0112
20	204.2846	1	0.4757	-97.1695	-151.0364	0.0315
21	198.9328	1	0	0	-151.0679	0.0315
22	196.7012	1	0	0	-151.0993	0.0315
23	198.3447	1	0	0	-151.1308	0.0315
•	•	•	•	•	•	•
36	173.4704	1	0	0	-151.5407	0.0316
37	169.925	1	0	0	-151.5723	0.0316
38	170.8202	1	0	0	-151.6038	0.0316
39	166.8036	1	0	0	-151.6354	0.0316
40	163.679	1	-1	100	-51.667	0.0108

Table 6.7: Delta hedging scheme for sample path with positive jump using σ as the volatility parameter for delta.

6.8.3 Comparing the two strategies for positive jump

In this scenario we can see that there is a great difference between the two strategies. The γ strategies total cost is 38.7364 but for the σ strategy the total cost is 51.667. In this case the higher delta of the γ strategy lead to that this strategy held a larger position in the stock prior to the large positive jump. This lead to that this strategy had bought more of the stock at the prices before day 20 than in the σ strategy case. This leads to that when the jump has occured the γ strategy does not need to buy as many shares as in the σ case, and the cost hence becomes lower using this strategy.

6.9 Repeated simulation of the hedging strategies

Now that we have shown how we setup the different strategies, and how they behave in different scenarios, we want to test them against each other using monte carlo simulation. Since we are interested in options that have roughly a two month maturity we will stick to the model setup that we have above and do 20'000 simulations of the stock price to test the different strategies against each other.

In Figure 6.1 we have the distribution of returns for the different sample paths. We can from this see that there are some returns that are extrem, returns were the stock increases 10 times its initial value. The mean return for the sample is 0.18%.

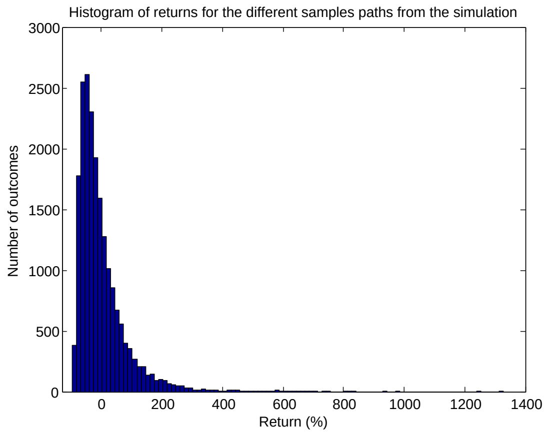

Figure 6.1: Histogram of the returns for the sample paths for a stock simulated from the Black & Scholes model with deterministic events.

We will now test our two different hedging schemes on the simulated sample paths. When performing the delta hedging schemes we will in this part discount the total cost of the hedging strategy for all the sample paths. We will then, for each hedging strategy, take the average of all the discounted hedging costs to get the mean cost of the strategy, in today's money value.

If we use the pricing model in Theorem 4.3, and the parameters in Table 6.1, we get the price for the option to be 28.3705. This price is of course independent of the sample path, but the delta hedging schemes are not, which we have shown in the previous sections.

Having evaluated the hedging schemes on the simulation we got the result that using only σ strategy gave the lowest hedging cost of 28.0664. The strategy using the γ strategy gave the mean hedging cost 28.3440.

If we look at other measures for the strategies we can see that even though hedging using only σ is the least costly strategy on average, it also has the largest standard deviation. This strategy has 29.94 in standard deviation in comparison to the 21.67 using γ. Hedging using only σ is also the most risky strategy if we look at the 99 percentile of the distribution of hedging costs for the strategy. The 99-percentile of the distribution for the hedging costs using only σ was 150.84 but only 108.87 using γ.

It is now interesting to see what happens if we move the event time. If we move the event to day 30 we get that the cheapest strategy is the σ strategy again with an average hedging cost of 28.2548. The γ strategy had a hedging cost of 28.3212.

To see the differences between the different strategies we will here present a table with different parameters of T_{jump} and the jump volatility σ_j.

T_{jump}	Strategy	Jump size		
		0.5	0.7	1.0
Day 10	Sigma	**21.0699**	28.4411	**38.6901**
	Gamma	21.1072	**28.4307**	38.7659
Day 20	Sigma	21.1813	**28.0664**	**38.7543**
	Gamma	**21.1261**	28.3440	38.8935
Day 30	Sigma	**20.8370**	**28.2548**	**38.8047**
	Gamma	21.0405	28.3213	38.8065

Table 6.8: Matrix of discounted hedging costs using the different hedging schemes. Columns vary in jump size and rows vary in timing of jump. Bold letters mean that this is the least costly strategy given the parameter setting for that simulation.

We first notice that for all jump sizes in Table 6.8 that the jump is close to the maturity of the option, i.e. on day 30 in our example, the least costly strategy is to use the σ scheme. It is also interesting to note that the lowest cost was given when using only σ when the jump volatility was the largest, i.e. 1.0.

Again we have that the risk, given as volatility and percentile (or VaR), is larger for the strategy using only σ. Simulating with the jump at day 20 and the jump volatility σ^Q we got that the standard deviation for the σ strategy was 54.5437 and the 99 percentile was 243.2197. For the γ strategy the standard deviation was only 32.3972 and the 99 percentile 155.8839. For the small gain that one could earn by hedging using the σ strategy in comparison to the large difference in risk, the conclusion can only be that using γ to hedge should be the method to choose.

6.10 Conclusion from delta hedging strategies

In this analysis we have looked at using a regular delta hedging scheme with two different choices of volatilty to use when calculating the delta to hedge the option with. One choice have been to use the full volatility γ and the second to only use σ. The choice of σ is done to see what would happend if one did not take the jump into consideration when delta hedging.

From our analysis of the two different strategies of hedging stocks with jumps we find that using the markets volatilty γ does not give the on average cheapest hedging costs, but it is the least volatile and risky alternative. If one would only use σ to hedge the option the portfolio would not be delta neutral before the jump but this strategy was the one that had the lowest hedging cost in the most number of situations. The negative side of chosing this strategy is though that the variance is much higher.

Chapter 7

Conclusion

This thesis has analyzed the effects of jumps in the sample paths of stocks in health-care and biotechnology companies. The jumps we were interested in examining was jumps that were triggered by regulatory events, events that is known before they happen.

To conduct the study of the jumps in these companies we therefore derived a model to simulate the sample path of a stock with deterministically timed jumps. We did this by extending the Black & Scholes model with a sum of variables modeled as normal distributions. From the model we then derive a closed form option pricing formula that is very similar to the Black & Scholes option formula, but were we have separated the volatility into two terms. We called this volatility γ and it was made up of both the normal volatility of the underlying stock and a jump volatility.

The setup of the model indicates two testable characteristics for the IV used to price options on companies that have one event within the time span from today until maturity. The characteristics are that volatilities should increase leading up to the event and decrease discontinuously after the event.

To see if we could use the model to predict the size of the jump we looked at a jump estimator that we called the term structure estimator that use the information of two options with different maturities to calculate the jump volatility.

To see if the market actually behaves like the modeled derived we looked at 41 NDA events during the years 2009 until February of 2011. Using both stock prices, ATM implied volatilities and option prices we could see that the market seem to be pricing the options according to our characteristics of the volatility. We also found that using the term structure estimator we could find the implied jump for the companies and that this jump volatility was substantial for many companies. Using this estimator could therefore be very useful when evaluating if to invest in the companies.

The last part of the thesis was concerned with the problem of delta hedging options in companies with deterministically timed events. From the option pricing formula we derived we saw that the delta of the option could be calculated in the same manner as the usual Black & Scholes delta, but that the volatility used in the calculation could be divided into two separate volatilities, a normal volatility and a jump volatility. Due to the fact that the stock price might jump on the event day we conclude that it is not possible to maintain a delta neutral position, using a stock and an option, over the jump. Since the new jump model gave us the possibility to separate the two different IV's we were interested in analyzing the differences in results from using these two different volatilities in a delta hedging scheme.

From analyzing the two different choices of delta we found that the using the market's volatility, γ in our case, was the least risky alternative. Using only σ was on average the least costly alternative, but also much more risky. The small gain that the strategy had over the γ strategy did not justify the larger risk for this strategy.

Since we have seen that these jump effects can have great impact, the analysis and results in this thesis should be useful for traders, portfolio managers and risk analysts. From a risk perspective the discretized model now makes it possible to simulate the returns of companies with deterministically timed jumps. Using this model one should get more accurate risk values for VaR and expected shortfall. For portfolio managers the possibility to back out the implied jump could help them to better asses if their beliefs are in line with the market.

Chapter 8

Appendix

8.1 Proof of opition pricing formula

From Theorem 7.8 in [3] we know that the arbitrage free price of the claim $\Phi(S_T)$ is given by $\Pi(t;\Phi) = F(t, S_t)$ where F is given by

$$F(t, S_t) = e^{-r(T-t)} E^Q_{t,S_t}[\Phi(S_T)]$$

where the Q-dynamics of S are described by equation (4.11).

Now, letting $S_T = S_t e^{Y_T}$, where S_t is deterministic, we have

$$F(t, S_t) = e^{-r(T-t)} \int_{\infty}^{\infty} \Phi(S_t e^{(\mu - \frac{1}{2}\gamma^2)(T-t) + \gamma\sqrt{T-t}\epsilon}) f(\epsilon) d\epsilon$$

where $f(\epsilon)$ is the probability density function of a normal distribution.

Letting $\Phi(S_T) = \max[S_T - K, 0]$ we get that

$$\begin{aligned}
&E^Q_{t,S_t}[\max(S_t e^{Y_T} - K, 0)] = \\
&= \left[\max(S_t e^{Y_T} - K, 0) = S_t e^{Y_T} - K \text{ if } \epsilon \geq \frac{\log(\frac{K}{S_t}) - (\mu - \frac{1}{2}\gamma^2)(T-t)}{\gamma\sqrt{T-t}} = d \right] \\
&= 0 \cdot Q(S_t e^{(\mu - \frac{1}{2}\gamma)(T-t) + \gamma\sqrt{T-t}\epsilon} \leq K) \\
&+ e^{-r(T-t)} \int_d^{\infty} \Phi(S_t e^{(\mu - \frac{1}{2}\gamma^2)(T-t) + \gamma\sqrt{T-t}\epsilon)} - K) f(\epsilon) d\epsilon \\
&= e^{-r(T-t)} \int_d^{\infty} (S_t e^{(\mu - \frac{1}{2}\gamma^2)(T-t) + \gamma\sqrt{T-t}\epsilon)} - K) f(\epsilon) d\epsilon \\
&= e^{-r(T-t)} \frac{1}{\sqrt{2\pi}} \int_d^{\infty} (S_t e^{(\mu - \frac{1}{2}\gamma^2)(T-t) + \gamma\sqrt{T-t}\epsilon)} - K) e^{\frac{-\epsilon^2}{2}} d\epsilon \\
&= S_t e^{-r(T-t)} \frac{1}{\sqrt{2\pi}} \int_d^{\infty} e^{(\mu - \frac{1}{2}\gamma^2)(T-t) + \gamma\sqrt{T-t}\epsilon)} e^{\frac{-\epsilon^2}{2}} d\epsilon - e^{-r(T-t)} \frac{1}{\sqrt{2\pi}} \int_d^{\infty} K e^{\frac{-\epsilon^2}{2}} d\epsilon
\end{aligned} \tag{8.1}$$

We now look at the two integrals in equation (8.1) seperatly, and we begin with the second integral.

$$
\begin{aligned}
&e^{-r(T-t)}\frac{1}{\sqrt{2\pi}}\int_d^{\infty} Ke^{\frac{-\epsilon^2}{2}}d\epsilon = \\
&= e^{-r(T-t)}\frac{1}{\sqrt{2\pi}}\left(\int_{-\infty}^{\infty} Ke^{\frac{-\epsilon^2}{2}}d\epsilon - \int_{-\infty}^{d} Ke^{\frac{-\epsilon^2}{2}}d\epsilon\right) \\
&= e^{-r(T-t)}K(1-\Phi(d))
\end{aligned}
$$

where $\Phi(d)$ is the value of the cumulative normal distribution function at d.

We now solve the first integral in equation (4.12).

$$
\begin{aligned}
&S_t e^{-r(T-t)}\frac{1}{\sqrt{2\pi}}\int_d^{\infty}(e^{(\mu-\frac{1}{2}\gamma^2)(T-t)+\gamma\sqrt{T-t}\epsilon})e^{\frac{-\epsilon^2}{2}}d\epsilon = \\
&= S_t e^{-r(T-t)}e^{(\mu-\frac{1}{2}\gamma^2)(T-t)}\frac{1}{\sqrt{2\pi}}\int_d^{\infty} e^{\gamma\sqrt{T-t}\epsilon}e^{\frac{-\epsilon^2}{2}}d\epsilon \\
&= S_t e^{-r(T-t)}e^{(\mu-\frac{1}{2}\gamma^2)(T-t)}\frac{1}{\sqrt{2\pi}}\int_d^{\infty} e^{\gamma\sqrt{T-t}\epsilon+\frac{-\epsilon^2}{2}}d\epsilon \\
&= S_t e^{-r(T-t)}e^{(\mu-\frac{1}{2}\gamma^2)(T-t)}e^{\frac{\gamma^2(T-t)}{2}}\frac{1}{\sqrt{2\pi}}\int_d^{\infty} e^{-\frac{\gamma^2(T-t)}{2}+\gamma\sqrt{T-t}\epsilon+\frac{-\epsilon^2}{2}}d\epsilon \\
&= S_t e^{-r(T-t)}e^{(\mu-\frac{1}{2}\gamma^2)(T-t)}e^{\frac{\gamma^2(T-t)}{2}}\frac{1}{\sqrt{2\pi}}\int_d^{\infty} e^{-\frac{1}{2}(\epsilon-\gamma\sqrt{T-t})^2}d\epsilon \\
&\left[\text{ let } y=\epsilon-\gamma\sqrt{T-t} \text{ and } dy=d\epsilon\right] \\
&= S_t e^{-r(T-t)}e^{(\mu-\frac{1}{2}\gamma^2)(T-t)}e^{\frac{\gamma^2(T-t)}{2}}\frac{1}{\sqrt{2\pi}}\int_{d-\gamma\sqrt{T-t}}^{\infty} e^{\frac{-y^2}{2}}dy \\
&= S_t e^{-r(T-t)}e^{(\mu-\frac{1}{2}\gamma^2)(T-t)}e^{\frac{\gamma^2(T-t)}{2}}\frac{1}{\sqrt{2\pi}}\left(\int_{-\infty}^{\infty} e^{\frac{-y^2}{2}} - \int_{-\infty}^{d-\gamma\sqrt{T-t}} e^{\frac{-y^2}{2}}\right)dy \\
&= S_t e^{-r(T-t)+\mu T}e^{\frac{1}{2}(T-t)(\gamma^2-\gamma^2)}(1-\Phi(d-\gamma\sqrt{T-t}) \\
&= S_t e^{-r(T-t)+\mu T}(1-\Phi(d-\gamma\sqrt{T-t})
\end{aligned}
$$

Once again $\Phi(d-\gamma\sqrt{T-t})$ is the value of the cumulative normal distribution function at that point.

So far we thus have this solution:

$$
E^Q_{t,S_t}[\max(S_t e^{Y_T}-K,0)] = S_t e^{-r(T-t)+\mu}(1-\Phi(d-\gamma\sqrt{T-t}))-e^{-r(T-t)}K(1-\Phi(d))
$$

Since,

$$
1-\Phi(d-\gamma\sqrt{T-t}) = \Phi(-(d-\gamma\sqrt{T-t})) = \Phi\left(\frac{1}{\gamma\sqrt{T-t}}\left[\log(\frac{S_t}{K})+(\mu+\frac{1}{2}\gamma^2)(T-t)\right]\right),
$$

and

$$
1-\Phi(d) = \Phi(-(d)) = \Phi\left(\frac{1}{\gamma\sqrt{T-t}}\left[\log(\frac{S_t}{K})+(\mu-\frac{1}{2}\gamma^2)(T-t)\right]\right)
$$

we set

$$d_1(t, S_t) = \frac{1}{\gamma\sqrt{T-t}}\left[\log(\frac{S_t}{K}) + (\mu + \frac{1}{2}\gamma^2)(T-t)\right]$$
$$d_2(t, S_t) = d_1 - \gamma\sqrt{T-t}$$

We then have that

$$E^Q_{t,S_t}[\max(S_t e^{Y_T} - K, 0)] = S_t e^{-r(T-t)+\mu(T-t)}\Phi(d_1) - e^{-r(T-t)}K\Phi(d_2)$$

but since $\mu = r$ we state Theorem 4.3.

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