A Quantitative Analysis of Liquidity and Funding Value Adjustments

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Abstract

This thesis considers the expected loss and gain of capital due to allocated capital within the context of collateralized and un-collateralized trading relationships. The potential cost or gain of capital is arising from the mitigation of counterparty credit risk on account of the Credit Support Annex issued by ISDA. A consequence of the mitigation is that two new market risks arises, continuous cash flow streams between the involved parties, liquidity value adjustment and funding value adjustment. These are derived mathematically and quantitative analyzed with the aim to quantify and transfer the two risks to a third party in order to achieve a more neutral market risk. The impact of the two new risk are significant. Investigating a 1 year IRS the total empirical expected extra cost becomes 0.22 basis points and another 0.54 basis points at risk at the 1% quantile.

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1 Introduction

Today's market is highly competitive among the dealer houses and are getting more competitive by the day. The market has become more transparent giving the clients the possibilities to more easily narrowing the bid-offer spreads. A fraction of a basis point price difference can in today's market be the distinction between getting a transaction and losing it to another dealer house. This applies mainly to vanilla interest derivatives but also more complex derivatives. Even though the initial margin on a traded derivative can be small or even absent the transaction itself are often valuable to the dealer house. Dealers have prioritized to get the deal done with less or none margin instead of a greater margin risking losing the deal. In the interest rate market this too has lead to decreasing bid-offer spreads with less margin as a consequence.

Some small risks has prior to the financial crisis often been neglected or overseen in hope of getting a trade done. Post the financial crisis and in the repercussions of the Lehman crash an enhanced focus in the market have been risk mitigation. Risk measures have gained yet more focus when financial regulators enforce stricter regulations such as Basel II and Basel III. Fluctuations in the market could give significant impact of the present value of derivative transactions and the effects could be enormous, especially in an already stressed market environment as in the case of the financial crisis. It is essential to identify and quantify the risks in order to avoid potential future losses. To be competitive in the market and at the same time controlling the risk the ability of accurate valuation of derivatives has become more important than ever before.

1.1 Problem Statement

When a market maker or dealer house, here mentioned the Dealer (D), enters into an OTC derivative transaction with any Counterparty (C), the Dealer hedges the equivalent transaction in order to eliminate the market risk exposure. This hedge is often made in the interbank market with a third party, the Hedging Counterparty (HC). There are however several risks the Dealer needs to take into account, some of these arising from the possibility of any of the parties C or HC goes default.

In this thesis we will investigate the consequences arising to the Dealer of mitigating the counterparty credit risk of HC while ignoring the counterparty credit risk of C. The consequences we will investigate are two market related risks that arise from the mismatch of predetermined contracts ensuring the payments spite any default event, usually CSA issued by ISDA, among the parties involved. This in turn gives rise to a mismatch of cash flows between the parties involved due to capital posted on a collateral account. The aim of this thesis is to estimate the two risks called liquidity value adjustment (LVA) and funding value adjustment (FVA) and the expected costs of these for a standard interest rate derivative. More precisely we will look at, seen from the Dealers perspective, how LVA and FVA changes the value of the derivative and what risks the transaction incorporates.

2 Theory

2.1 Interest Rates

An interest rate is the cost of borrowing money, alterantively the gain of lending money, for a certain period of time. Interest rates are usually expressed as a percentage of the notional for a period of one year.

2.1.1 Zero Coupon Bond, Discount Factor and Interest Rate

A *zero coupon bond* (ZCB) is a debt instrument paying no or "zero" coupons, hence the name. It is a contract where the seller of the contract undertakes, at the expiration date of the contract or equivalently the time of maturity (T), to pay back the buyer the initial price of the bond plus some interest. The initial price is determined at time t. The ZCB is essentially a loan issued by a debtor and bought by a lender for a price lower than its nominal value of 1 currency unit. At time T the debtor pays back the lender the nominal value of the ZCB.

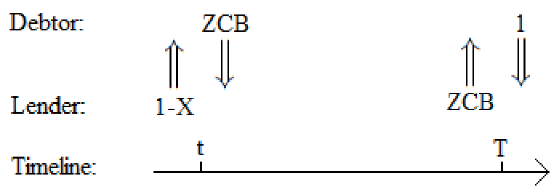

Figure 1: Payment schedule of a Zero Coupon Bond. The graph displays the cashflow between the Lender and Debtor.

The price of the ZCB is also called *discount factor* and is denoted by $P(t,T)$. The set-up and notations are the same as in [1]. The time of maturity, T, and the initial spot price, determined at time t, together determines the current interest rate R(t,T).

$$R(t,T) = \frac{1}{T-t}\left[\frac{1}{P(t,T)} - 1\right] \tag{1}$$

The price of the ZCB and hence the interest rate is determined by the supply and demand on the market. Debtors could e.g. be states, financial institutions, agencies or corporates. The Zero Coupon Bond is the simplest type of bond but there are however many different types of bonds. The most common has periodic coupon payments with fixed coupon amounts alternatively floating coupons indexed to an index. By stripping the bond of the coupons the bond can be broken down into several separate cash flows. Each cash flow can separately be viewed as a ZCB and the interest rate can then be calculated accordingly. Instead of ZCB the notation discount factor and $P(t,T)$ will henceforth be used throughout the text.

2.1.2 Yield Curve

The yield curve is a function expressing the relationship between interest rate and time to maturity and is often called the Interest Rate Term Structure or merely Term Structure. The term structure displays the cost to borrow money and is type specific for any given borrower and currency. It indicates the interest rate of money borrowed today, held until maturity.

The discount factors are important building blocks when constructing the curve. The interest rates for specific maturities are used in the curve and the intermediary steps are interpolated. The Term Structures could have many different shapes and appearances but the most commonly form is an increasing and convex function of time. Other building blocks of the term structure could be any type of bond and also, especially for longer maturities, expectations of future macro economic data.

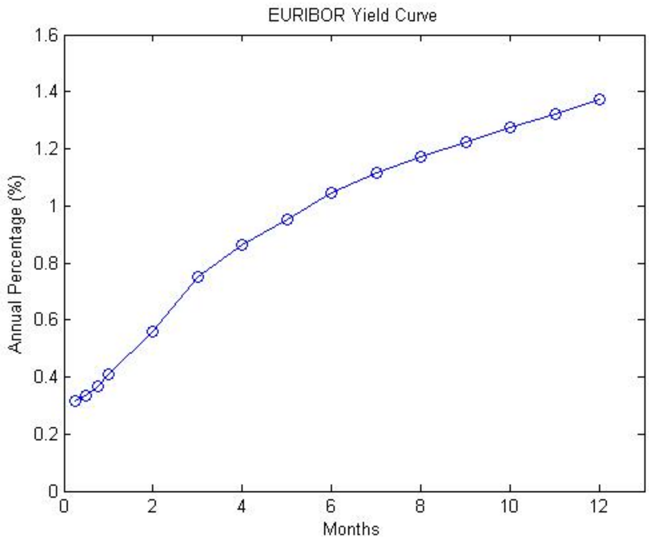

Figure 2: The graph displays the yield (o) of EURIBOR spanning from 1 week to 12 months. The line is the interpolated curve building up the yielcurve. The market data is extracted 2012-04-12 from [3].

The yield curve is a momentarily snapshot in time of the interest rates. Like the price of the discount factor $P(t, T)$, the yield curve is constantly changing due to the supply and demand on the market.

2.1.3 EURIBOR and EONIA

EURIBOR is short for *Euro Interbank Offered Rate*. The EURIBOR rates are based on the average interest rates at which a panel of more than 50 European banks borrows fund from one another. There are different maturities, ranging from one week to one year.

The EURIBOR rates are considered to be the most important reference rates in the European money market. The interest rates do provide the basis for the price and interest rates of all kinds of financial products like interest rate swaps, interest rate futures, saving accounts and mortgages.

EONIA is short for Euro Over Night Index Average. The EONIA rate is the 1-day interbank interest rate for the Euro zone. In other words, it is the rate at which banks provide loans to each other with a duration of 1 day (see [2]). In this report EONIA can be considered as the 1 day EURIBOR rate.

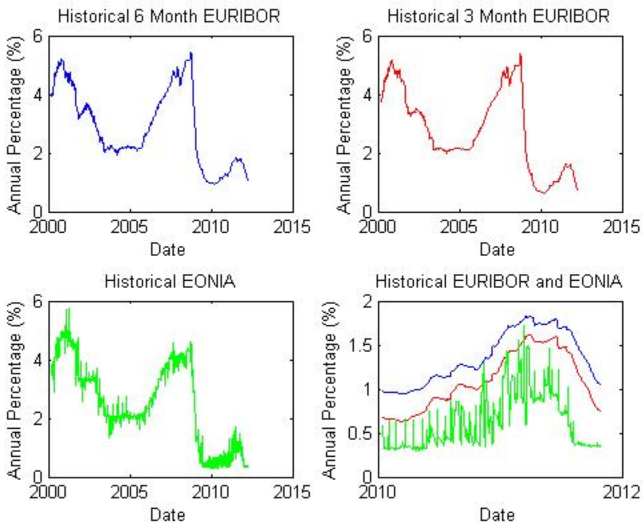

Figure 3: The first three subfigures plots the historical fixings of 6 months EURIBOR, 3 months EURIBOR and EONIA between the dates 2000-03-13 and 2012-04-12. The fourth subfigure displays the three interest rate fixings from the date 2010-02-01 to 2012-04-12. The data is extracted from [4].

These rates are calculated and published on a daily basis except for weekends and European Bank Holidays.

2.1.4 Forward Interest Rate

A forward interest rate or just the forward rate is the rate to be exchanged at time T_i for the period T_{i-1} to T_i, determined at time t where $t \leq T_{i-1} < T_i$.

$$L(t; T_{i-1}, T_i) = \frac{1}{T_i - T_{i-1}} \left[\frac{P(t, T_{i-1})}{P(t, T_i)} - 1 \right] \tag{2}$$

The time $T_i - T_{i-1}$ is the fraction of time in years and will be denoted as

$\tau_i = T_i - T_{i-1}$ henceforth. At time t this rate is the current "fair" forward rate. For derivation and a more detailed discussion please see [5] or alternatively [6].

2.2 Derivatives

A derivative is a contract, written between two or more parties, with a value that is determined by the performance of the underlying asset. It can be seen as a financial instrument or a mathematical function whose characteristics are derived from any underlying variable or variables. The most commonly used underlying variables are interest rates, stock prices, money exchange rates or commodities but there is an infinite variety of variables that can be used. The most common types of derivatives traded are futures, forwards, options and interest rate swaps. The main reasons to use derivates are to hedge market risk, take leveraged positions in order to speculate in the market and not the least to seek arbitrage opportunities.

Over-the-counter (OTC) derivatives are contracts that are traded, and privately negotiated, directly between two parties, without going through an exchange or other intermediary. Products such as swaps, forward rate agreements, exotic options - and other exotic derivatives - are almost always traded in this way. The OTC derivative market is the largest market for derivatives, and is largely unregulated with respect to disclosure of information between the parties, since the OTC market is made up of banks and other highly sophisticated parties, such as hedge funds. Reporting of OTC amounts are difficult because trades can occur in private, without activity being visible on any exchange. According to the Bank for International Settlements, the total outstanding notional amount is US $707 trillion, as of June 2011, see [7]. Of this total notional amount, 67% are interest rate contracts. Because OTC derivatives are not traded on an exchange, there is no central counter party. Therefore, they are subject to counter-party risk, like an ordinary contract, since each counter party relies on the other to perform.

2.2.1 Interest Rate Swap

An *interest rate swap* (IRS) is a derivative in which the counterparties involved periodically exchange interest rate payments on a notional amount. There are many types of Interest Rate Swaps IRS's such as Fixed-to-Floating IRS, fixed-to-fixed IRS, floating-to-floating IRS, forward starting IRS, cross currency IRS and many more complex IRS combinations of these. In this context fixed refers to a predetermined fixed interest rate and floating to a stochastic interest rate. However in this paper we will only consider one type, namely the fixed-to-floating IRS also called the plain vanilla IRS. This swap is the least complex and the most liquid and commonly used by the market.

The plain vanilla interest rate swap is an agreement between two parties where the two parties periodically exchange interest rate payments at known future dates generated by predetermined period of time and notional amount. One party, the *Receiver*, pays a floating interest rate to the other party, the *Payer*, whom will pay back a fixed interest rate to the Receiver. The two names Reciever and Payer originate from who will receive or pay the fixed rate. The floating interest rate can follow any interest rate index, for instance 6 months EURIBOR.

Each party's consecutive stream of interest payments is called payment leg and there are in total two payment legs involved in a plain vanilla IRS, the Receiver's floating payment leg and the Payer's fixed payment leg. The *present value* (PV) of the Receiver's floating interest rate payments is the sum of every future cash flow discounted back to today. Let t be the date today and $T_a, \ldots, T_b$ be the dates for the floating leg payments, then

$$PV(Receiver(t; T_a, \ldots, T_b)) = Not \times \left[\sum_{k=a}^{b} P(t, T_k) \tau_k L_k(t) \right] \tag{3}$$

where Not is the notional amount, $L_k(t) = L(t; T_{k-1}, T_k)$ and $\tau_k = T_k - T_{k-1}$ accordingly.

The PV of the Payer's fixed interest rate payments is the sum of every future cash flow discounted back to today. Let t be the date today and $T_c, \ldots, T_d$ be the dates for the fixed leg payments, then:

$$PV(Payer(t, S(t); T_c, \ldots, T_d)) = Not \times \left[\sum_{j=c}^{d} P(t, T_j) \tau_j S(t) \right] \tag{4}$$

where $S(t)$ is the fixed rate.

Seen from the Payer's point of view the total value of the IRS then becomes:

$$IRS(t, S(t); T_a, ..., T_b; T_c, ..., T_d) = \tag{5}$$

$$Not \times \left[\sum_{k=a}^{b} P(t, T_k) \tau_k L_k(t) - \sum_{j=c}^{d} P(t, T_j) \tau_j S(t) \right] \tag{6}$$

The total value of the IRS seen from the Receiver's perspective equals the negative equation above. In order for the two parties to agree on the swap the initial net present value has to be at least 0. Since they both cannot have positive values at the same time the only remaining option is a value of 0, $IRS(t, S(t); T_a, ..., T_b; T_c, ..., T_d) = 0$. An initial value different from 0 would give rise to arbitrage opportunities. The fixed rate $S(t)$ is the unknown in this equation but by putting equation (6) equal to 0 and rearrange one gets:

$$S(t) = \frac{\sum_{k=a}^{b} P(t, T_k) \tau_k L_k(t)}{\sum_{j=c}^{d} P(t, T_j) \tau_j} = K \tag{7}$$

Here K is a constant and $IRS(t) = 0 \Rightarrow S(t) = K$. The constant K is the "fair" value of the fixed interest rate and the rate to be used in the IRS. Hence the value of the swap seen from the Payer's perspective then becomes:

$$IRS(t, K; T_a, ..., T_b; T_c, ..., T_d) = \tag{8}$$

$$Not \times \left[\sum_{k=a}^{b} P(t, T_k)\tau_k L_k(t) - \sum_{j=c}^{d} P(t, T_j)\tau_j K \right] \tag{9}$$

The IRS can be evaluated continuously until the maturity of the swap and the value will fluctuate depending on the performance and behavior of the underlying interest index and its forward rates. Hereinafter the following notation will be used:

$$IRS(t, K; T_a, ..., T_b; T_c, ..., T_d) = IRS(t) \tag{10}$$

When the IRS derivate has a positive value it is said to be *in-the-money* (ITM) and when negtive, *out-of-the-money* (OTM). When one party is ITM then the other party is OTM and vice versa.

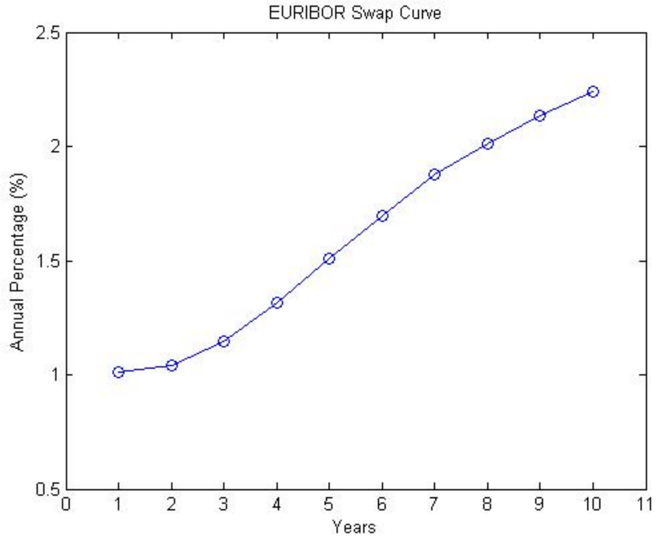

Figure 4: The graph displays the 6 months EURIBOR interest rate swap levels (o) spanning from 1 year to 10 years. The line is the interpolated curve building up the swap curve. The market data is extracted 2012-04-12 from [9].

2.3 Trading Governance and Relationships

In a typical OTC derivative transaction there are three parties involved, a *counterparty* (C), a *dealer* (D) and a *hedging counterparty* (HC). Party C initiates the transaction by entering a trade with D which in turn tries to hedge the identical trade against HC. The reason D hedge the position is to be neutral to market risk. D acts here as a market maker. In reality, it is common that D splits the transaction into several parts and hedge separate cash flows to different HCs. However in this paper we will, without any loss of generality, simplify to only one HC. The OTC derivative transaction schedule looks like the following:

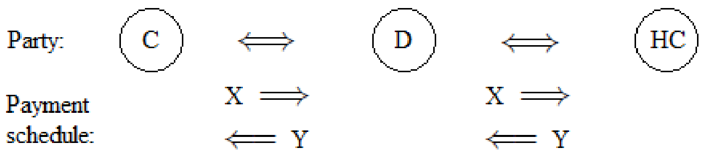

Figure 5: Payment schedule between the parties Counterparty, Dealer and Hedging Counterparty. The figure displays the ideal cashflow between the parties when the Counterparty initate an OTC transaction.

The payments made by C goes to HC via D and in the same manner the payments made by HC goes to C via D. According to the schedule above D is market risk neutral.

2.3.1 Trading Realtionship under ISDA Master Agreement

The *International Swap and Derivative Association* (ISDA) have established an agreement called the ISDA *Master Agreement* (MA). It is designed to enable uncomplicated OTC derivative transactions and mitigate the counterparty credit risk for the parties involved. The ISDA MA is internationally one of the most established master contracts among trading parties. It is a contract reached between two parties, in which the parties agree to most of the terms that will govern future transactions or future agreement. A master agreement permits the parties to quickly negotiate future transactions or agreements, because they can rely on the terms of the master agreement, so that the same terms need not be repetitively negotiated, and to negotiate only the deal-specific terms (see [9]).

2.3.2 Trading Realtionship under Credit Support Annex

One consequence of an OTC derivative trade between two parties is that it incorporates unwanted counter party credit risk for both of the parties. The parties are inclined to mitigate this exposure on each other's credit risk and this can easily be achieved by including an annex to their existing ISDA MA called *Credit Support Annex* (CSA). The CSA is a legal document and it defines under which terms collateral should be posted or transferred between the two parties. The collateral amount posted is based on the current aggregated net PV of all their outstanding trades towards each other. The party that has an aggregated negative PV of the outstanding trades, also called the Pledgor, has the obligation to post eligible collateral according to the CSA predetermined regulations. The counterparty, with equal but positive PV, also called the Secured Party, is the receiver of the collateral.

The most important terms in the CSA states and defines thresholds, minimum transfer amounts, collateralized percentage γ, transfer periodicities, currencies and eligible securities. Collateral must be posted if the net PV exceeds the threshold limit. Minimum transfer amount is the least amount of collateral that will be transferred. The collateral will only be transferred if the PV has changed more than the minimum transfer amount since the prior collateral transfer. The collateralized percentage γ denotes what percentage of the net PV that will be collateralized. The periodicity states when and how often the trades should be evaluated and collateral transferred. Currencies states what currencies are eligible and are to be used as collateral. Instead of collaterizing the PV in money eligible securities can be used e.g. bonds.

If only one of the parties has, according to the contract, obligations to the other party the contract is called unilateral CSA. That means that one of the parties never has to post collateral whilst the other party never will receive any collateral. When both parties have obligations towards each other the contract is called bilateral CSA. In a bilateral CSA the two parties doesn't necessarily have identical obligations towards each other. I.e. thresholds, minimum transfer amounts, periodicity, currencies and eligible securities may all differ.

The pledgor is typically paid some interest on the allocations that the Secured Party holds. What type of interest rate applying is predetermined in the CSA. The interest rate used often depends on the periodicity in the CSA, if the CSA has daily payments a typical interest rate for the collateral would be daily too e.g. EONIA. For weekly CSA periodicity the interest rate index used could be 1 week EURIBOR, for monthly CSA periodicity 1 month EURIBOR and so forth. It is the secured party that pays the interest rate in exchange for keeping the collateral.

2.3.3 Current Trading Relationsship

A common situation, and the situation considered in this paper, is a negotiated ISDA MA between the parties C and D and a negotiated ISDA MA with CSA between the parties D and HC.

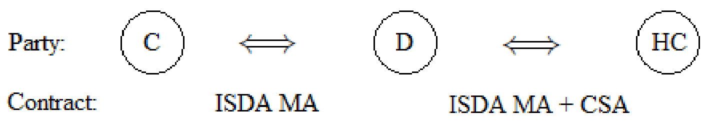

Figure 6: Displays the current contractual trading relationship between the parties Counterparty, Dealer and Hedging Counterparty.

The consequence of that D has dissimilar contracts with C and HC will give rise to a potential mismatch of cashflow streams.

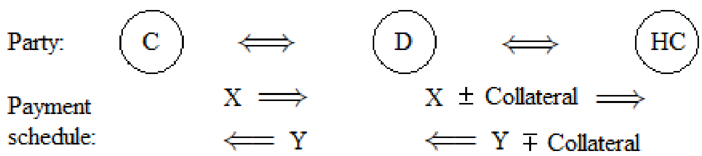

Figure 7: Payment schedule between the parties Counterparty, Dealer and Hedging Counterparty contingent the current contractual trading relationsship shown in Fig. 6. The consequence becomes a mismatch of cashflow streams for the Dealer.

D will either post or receive collateral depending on the current market condition due to the underlying asset of the derivative. Hence D is no more market risk neutral.

2.4 Liquidity Value Adjustment

Between parties that have ISDA MA CSA, in this example between D and HC, a potential cost (or gain) arises called *liquidity value adjustment* (LVA). The LVA originates from different deposit and lending rates when capital is being allocated. The following section have similar methodology and set-up as [10] with minor differences in notation.

The Secured Party holds collateral on which it pays the Pledgor a *collateral rate* ($O(t)$). This collateral rate to be paid will most likely be a significant cost for the Secured Party. To avoid the cost the Secured party will in turn lend or make a deposit to the rate of $R(t,T)$ of the collateral to its Treasury or alternatively the market. Here T is the next rebalancing time of the collateral. At time T the LVA of a derivative then becomes the time fraction τ of the difference between the depository rate R and the collateral rate O times a percentage, γ, of the derivative's PV.

$$LVA(T)_{Derivative} = \left[\tau^{C}[R(t) - O(t)]\gamma PV(Derivative(t))\right] \tag{11}$$

The previous equation states the value for a single rebalancing period of the LVA when the interest of the collateral is paid at time T. The indexation C refers to the collateral. For multiple consecutive future rebalancing periods the PV of the LVA for the Secured Party becomes the sum over all the discounted future expected values of the previous equation (11).

$$LVA(t)_{Derivative} = \sum_{j=1}^{N} P^{D}(t, t_j) \tag{12}$$

$$\times E_{D}^{t_j}\left[\tau_j^{C}[R_j(t) - O_j(t)]\gamma Derivative(t_j; T_a, T_b)\right] \tag{13}$$

Here N is the number of rebalancing periods and the indexation D refers to the discounting curve of the interest rate R(t). Equation (12-13) states the discounted expected outcome of the LVA at time t whilst equation (11) states the realized LVA at time T.

Definition: *The LVA is the discounted value of the difference between the risk-free rate and the collateral rate paid (or received) on the collateral, and it is the gain (or loss) produced by the liquidation of the net PV of the derivative contract due to the collateralization agreement.* [10]

Suppose the Pledgor could borrow money, to post as collateral, from its treasury at the same rate as above $R(t,T)$. Then the LVA from the Pledgors' perspective becomes as equation (12-13) but with the opposite sign due to the reversed order $[O_j(t) - R_j(t)]$. Hence the LVA may alternate between positive and negative for both parties given that the underlying derivative may alternate between positive and negative PV.

Let $Q(t) = R(t) - O(t)$ and by assuming independency between $Q(t)$ and $\gamma Derivative$ the following yields:

$$LVA(t)_{Derivative} = \sum_{j=1}^{N} P^D(t,t_j) E_D^{t_j}[\tau_j^C Q_j] E_D^{t_j}[\gamma Derivative(t_j)] \tag{14}$$

If the derivative is a payer IRS then the second expectation becomes

$$E_D^{t_j}[\gamma IRS(t_j;T_a,Tb)] = Not \times C_D^{a,b}(t) \frac{E_D^{a,b}[\gamma(S_{a,b}(t) - K)]}{P^D(t,t_j)} \tag{15}$$

where $E_D^{a,b}$ is the expectation taken under the swap measure, with numeraire equal to $C_D^{a,b}(t) = \sum_{j=a+1}^{b} P^D(t,T_j)\tau_j^S$, where the index S refers to the payment leg. So the LVA can be written:

$$LVA_{IRS(t;T_a,T_b)} = Not\times \tag{16}$$

$$\sum_{j=1}^{N} P^D(t,t_j) \left[E_D^{t_j}[\tau_j^C Q_j(t)] \frac{C_D^{a,b}(t) E_D^{a,b}[\gamma(S_{a,b}(t) - K)]}{P^D(t,t_j)} \right] \tag{17}$$

Hence the complete LVA for an IRS becomes:

$$LVA_{IRS} = Not\times \tag{18}$$

$$\sum_{j=1}^{N} P^D(t,t_j) \left[E_D^{t_j}[\tau_j^C Q_j(t)] \frac{\sum_{j=a+1}^{b} P^D(t,T_j)\tau_j^S E_D^{a,b}[\gamma(S_{a,b}(t) - K)]}{P^D(t,t_j)} \right] \tag{19}$$

2.5 Funding Value Adjustment

When in need of cash a party will borrow money from its own treasury to a *funding rate* ($F(t)$). When in excess of money the same party will deposit the money to its treasury to the rate of $R(t)$. In the similar manner as with the LVA and the existence of an ISDA MA CSA between the parties, the *Funding Value Adjustment* (FVA) arises from a mismatch in collateral rate and funding rate, more precisely when the funding rate is greater than the collateral rate, $F(t) \geq R(t)$. As before, the following section have similar methodology and set-up as [10] with minor differences in notation.

With the same reasoning and approach as previous section the FVA, with the same IRS as derivative, can be written:

$$FVA_{IRS(t;T_a,T_b)} = \sum_{j=1}^{N} P^D(t,t_j) E_D^{t_j}\left[\tau_j^C [F_j(t) - R_j(t)]\gamma IRS(t_j;T_a,T_b)\right] \tag{20}$$

Let $U_j(t) = F_j(t) - R_j(t)$ and if $U_j(t)$ and $IRS(t;T_a,T_b)$ are independent the FVA can be rewritten to:

$$FVA_{IRS(t;T_a,T_b)} = \sum_{j=1}^{N} P^D(t,t_j) E_D^{t_j}[\tau_j^C U_j(t)] E_D^{t_j}[\gamma IRS(t_j;T_a,T_b)] \tag{21}$$

$$FVA_{IRS(t;T_a,T_b)} = Not\times \tag{22}$$

$$\sum_{j=1}^{N} P^D(t,t_j)\left[E_D^{t_j}[\tau_j^C U_j(t)] \frac{C_D^{a,b}(t) E_D^{a,b}[\gamma(S_{a,b}(t) - K)^-]}{P^D(t,t_j)}\right] \tag{23}$$

where $C_D^{a,b}(t) = \sum_{j=a+1}^{b} P^D(t,T_j)\tau_j^S$ as before and $E[X^-] = E[min(X,0)]$. The reason for the negative expectation $E[X^-]$ is that the lending rate equals the collateral rate whenever the PV of the derivative is positive. Hence, merely a negative expected value of the derivative will make an impact on the FVA. Thus the complete FVA of an IRS can be written as:

$$FVA_{IRS} = Not\times \tag{24}$$

$$\sum_{j=1}^{N} P^D(t,t_j)\left[E_D^{t_j}[\tau_j^C U_j(t)] \frac{\sum_{j=a+1}^{b} P^D(t,T_j)\tau_j^S E_D^{a,b}[\gamma(S_{a,b}(t) - K)^-]}{P^D(t,t_j)}\right] \tag{25}$$

2.6 Total Value of an IRS

From D's perspective equations (8-9), (18-19) and (24-25) yields the total value of an IRS when D is the fixed rate payer:

$$IRS(t) = Not\times \tag{26}$$

$$\left[\sum_{k=a}^{b} P^D(t,T_k)\tau_k L_k(t) - \sum_{j=c}^{d} P^D(t,T_j)\tau_j^S K\right] + LVA_{IRS} + FVA_{IRS} \tag{27}$$

3 Simulation and Calculation Methodology

The Dealer and Counterparty will initiate a derivative transaction consisting of a 1 year plain vanilla interest rate swap at current market price. C will pay a fixed percentage of the notional to D, which will in return pay back a floating interest rate. These payments are scheduled semi annually, i.e. in 6 and 12 months from today. The two floating rate fixings occur 6 months prior to each payment. That is the first fixing is already known at the agreement of the swap. The underlying floating interest rate will be a simulated 6 months EURIBOR.

The Dealer will initiate a second transaction, an opposite but else identical transaction with HC, that is pay the fixed rate and receive the floating rate, all else being equal. I.e. notional, payment schedule and interest rate index are as before. Since D and HC have a negotiated CSA the daily difference of the present value of the 1 year IRS will be posted daily as collateral. The cash flows that originate from this collateral will be generated $N = 5000$ times and the outcomes analyzed further on.

To get a realistic scenario the current market prices and historical interest rate fixings will be used as a base for the simulations.

3.1 Assumptions Considered

For the above mentioned scenario the following assumptions will be made;

1. No interest rate can become negative.

2. The short rate interest model Cox-Ingersoll-Ross (CIR) will be used to simulate future interest rate paths for 6 months EURIBOR and EONIA. The parameters belonging to each interest rate are calculated from the historical fixings.

3. The Funding rate is set to $EONIA + 1.00\%$.

4. The C and HC negotiated CSA possess the following properties;

Type:	Bilateral
Periodicity:	Daily
Eligible Currency:	EUR
Eligible Securities:	EUR currency
Threshold Amount:	0 EUR
Collateralized PV, γ:	100%
Minimum Transfer Amount:	0 EUR
Collateral rate:	$0.5*$ EONIA

5. There exist no CSA between the parties C and D.

6. The counterparty credit risk for party C will not be encountered for in the price and or value of the swap.

7. Furthermore no transaction costs will exist.

3.2 Simulating Interest Rates

The interest rate EONIA is simulated with daily time steps N times from the date 2012-04-13 with 12 months horizon until 2013-04-12. The parameters σ, α and θ used to model the future interest are given by the historical EONIA data from the date 2004-02-12 until 2012-04-12 and the model CIR. The simulation start value of EONIA is set to 0.3500%.

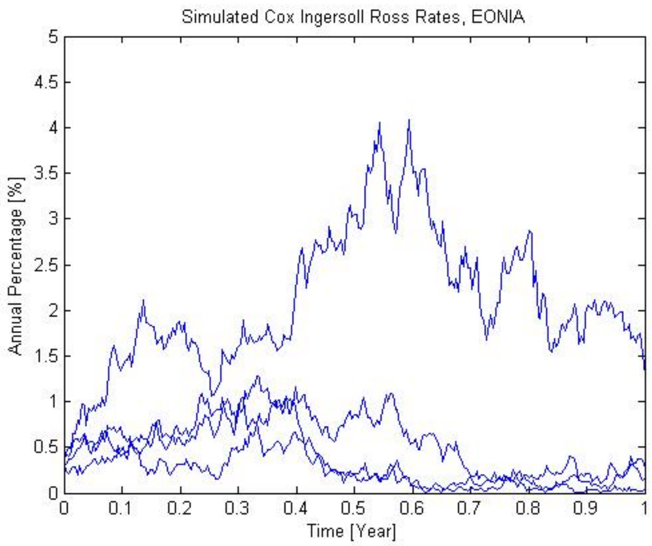

Figure 8: Simulated EONIA paths.

For each of the N simulated EONIA paths the corresponding CIR term structure is calculated with the same approach as [11]. The graph below shows the discounting curve for one of the simulated EONIA paths.

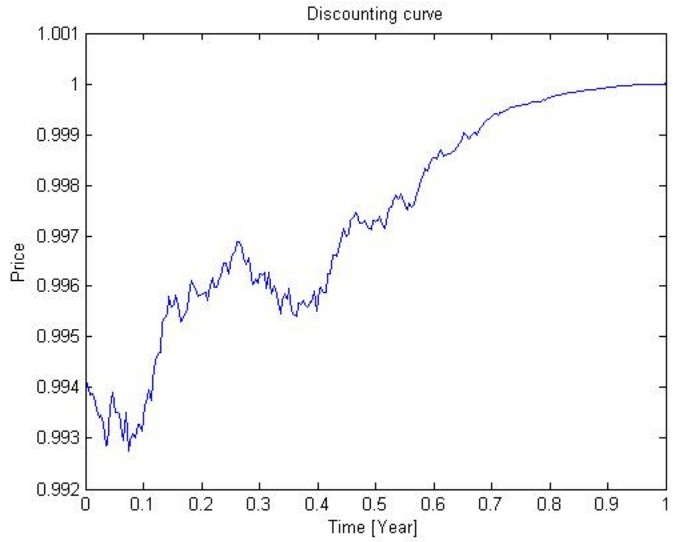

Figure 9: Discounting curve created from a simulated EONIA path.

The forward interest rate for 6 month EURIBOR is approximated by the simple forward rate described in [6]. The N simulated EONIA paths will in turn build up N number of forward paths. These paths construct the basis for the valuation of the swaps. The produced paths are all assigned the starting value equal to the market price of the 6 month forward rate of the 6 month EURIBOR, namely 0.9343% as of the closing price of 2012-04-12. The graph displays the daily market level for the interest rate 6 month EURIBOR between 2012-10-12 and 2013-04-12. At 2012-04-12 or time 0 on the axis the graph presents the 6 months forward rate for 6 month EURIBOR, at time 0.25 the 3 months forward rate and at time 0.5 the 0 day forward rate or equivalent the 6 month EURIBOR fixing.

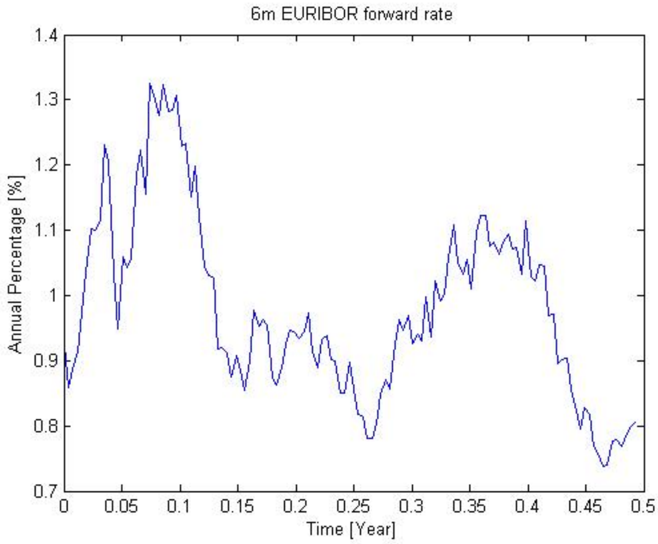

Figure 10: The forward rate of the underlying 6 month EURIBOR. The graph displays the evolution in daily timesteps of a 6 month forward of 6 month EURIBOR.

3.3 Swap

The 1 year swap rate is the closing price of 2012-04-12 taken from [8]. There are two fixings of the floating interest rate in the 1 year swap and the first fixing is decided initiating the transaction. The second fixing will take part 6 months in the future letting this fixing be the only random in this swap. By taking the difference of the swap price and the simulated 6 months EURIBOR the future value of the IRS can be calculated, hence the amount that needs to be posted as collateral by any of the two counterparties.

The difference between the 1 year swap rate and the initial 6 months EURIBOR fixing multiplied by the period 6 months and notional compose the first amount that needs to be posted as collateral. This amount count solely for the first swap payment and this amount is posted as collateral until the first payment.

The difference between the 1 year swap rate and the future 6 months EURIBOR fixing multiplied by the period 6 months and notional compose the second amount that needs to be posted as collateral. This amount belongs to the second payment and this amount is posted as collateral until the second payment. The total collateral amount to be posted can be expressed as the sum of the two previous mentioned differences.

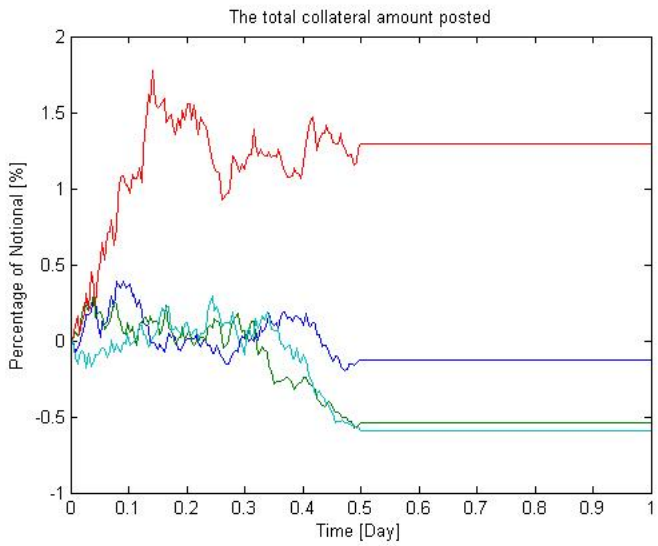

Figure 11: The total collateral amount posted or received for each interest path during 1 year due to the CSA. Positive value means the Dealer has to post collateral and negative value the Dealer will recieve collateral. After 6 months the floating interest rate fixing is determined for the rest of the time period, hence the constant lines. The graph displays the daily evolution of the total collateral.

3.4 LVA and FVA

When the PV of the transaction is negative to the Dealer, and therefore has to post collateral, the daily LVA becomes the difference between borrowing at EONIA and lending at the collateral rate multiplied the notional and by the time fraction of one day. If the PV of the transaction is positive to the Dealer the difference will be the opposite, namely borrowing at collateral rate and lending at EONIA. Fig.12 displays the daily realized LVA calculated from equation (11) for one simulated interest rate path.

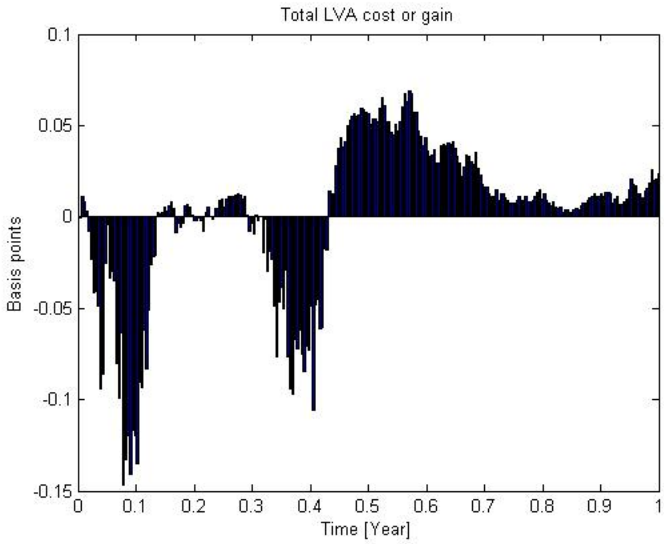

Figure 12: The daily LVA cost or gain spanned over one year for one simulated interest rate path. Positive value means a cost to the Dealer and a negative value means a gain to the Dealer. The graph is expressed in basis points per year of the notional.

For the Dealer the FVA arises when the PV of the transaction is negative. The daily FVA then becomes the difference between the funding rate and EONIA multiplied the notional and by the time fraction of one day.

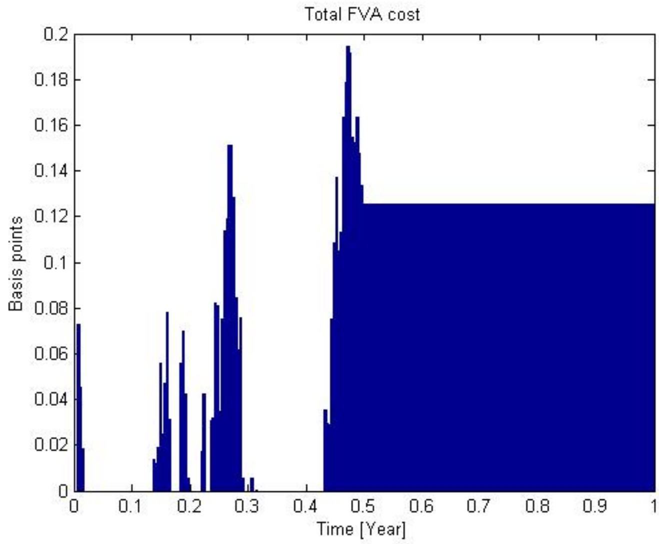

Figure 13: The daily FVA cost spanned over one year for the identical simulated interest rate path as in figure 18. A positive value means a cost to the Dealer. The graph is expressed in basis points per year of the notional.

The total daily cost or gain of the allocations is the sum of the LVA and FVA.

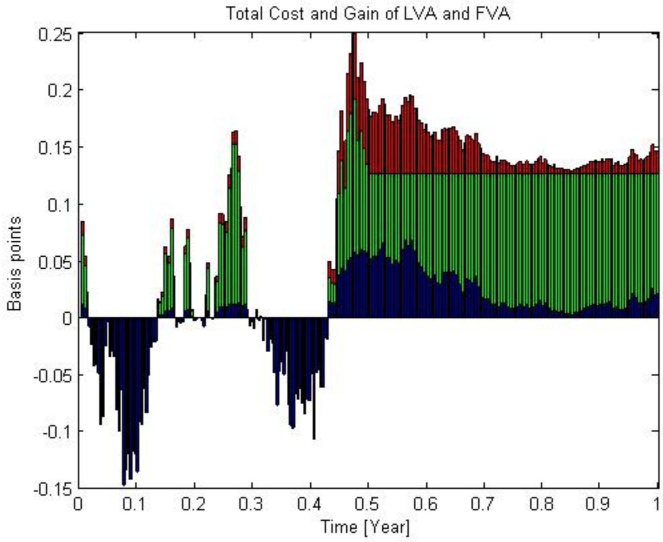

Figure 14: Displays the sum of the LVA, Fig.12, and FVA, Fig.13. Equivalently the total daily cost or gain to the Dealer due to the collateral. The blue bars are the LVA costs to the dealer, green bars FVA cost and the red bars are the sum of the two.The graph is expressed in basis points per year of the notional.

3.5 Present Value

The daily costs and gains are being discounted and summed up for every simulated interest rate path. These N sums are the present values for the LVA and FVA.

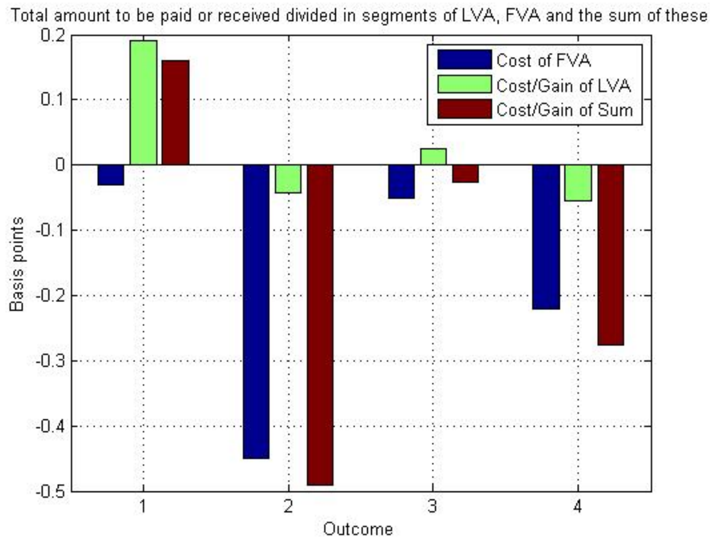

Figure 15: The total discounted cost or gain for 4 interest rate paths divided into parts of LVA(green), FVA(blue) and the sum of these (red). The present values are expressed in basis points of notional. A positive value means, to the Dealer, a positive present value and a negative value a negative present value.

4 Results

The CIR parameters extracted from EONIA are shown in Table 1 together with the start value equal to the EONIA fixing of 2012-04-12.

Table 1: Parameters for the CIR interest rate with corresponding start value.

Interest Rate:	Speed θ	Level μ	Volatility σ	Start value r_0
EONIA	0.72	1.75	1.71	0.3500%

4.1 LVA Outcomes

The total outcomes of the PV of the LVA are plotted as a histogram in figure 24.

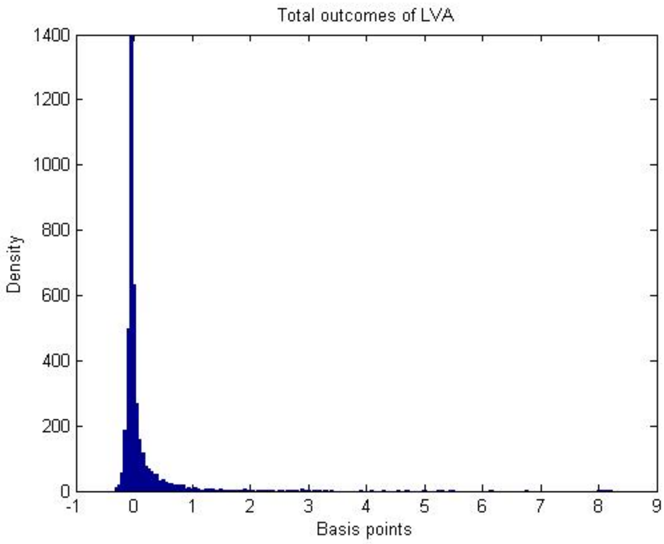

Figure 16: LVA outcomes of the N simulated interest rate paths. Positive value is a gain to D and negative value a cost. The histogram is expressed in basis points per year of the notional.

The empirical quantiles of 1% and 99% are visible at the tails of the histogram in Fig. 17. The former to the left and the latter to the right.

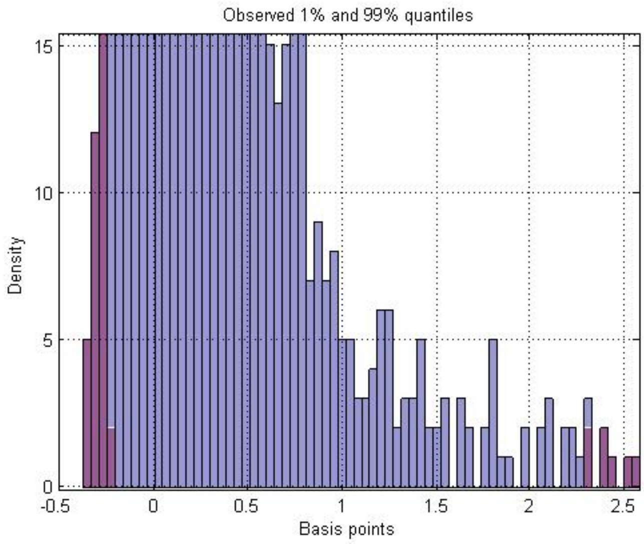

Figure 17: The empirical quantiles of 1% and 99% are here displayed at the tails in red. The outcomes are identical to those as in Fig. 16.

The observed empirical mean, median, 1% and 99% quantiles of the LVA can be seen in Table 2.

Table 2: Empirical values of the LVA.

Measure	Mean	Median	1% quantile	99% quantile
LVA	0.07 bp	-0.04 bp	-0.25 bp	2.29 bp

4.2 FVA Outcomes

The total outcomes of the PV of the FVA are plotted as a histogram in Fig.18. Since the FVA never results as gain to D, the histogram becomes skew. The empirical quantiles of 1% and 99% are visible at the tails of the histogram in Fig. 19. The former to the left and the latter to the right.

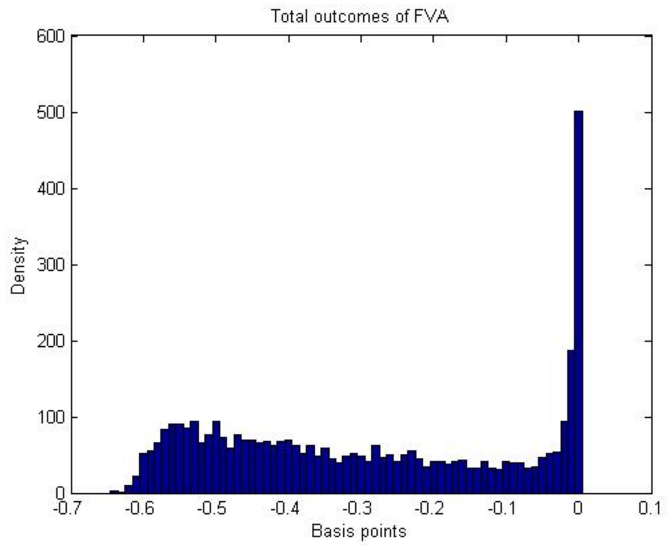

Figure 18: Displays the FVA outcomes of the N simulated interest rate paths. Negative value is a cost to D.

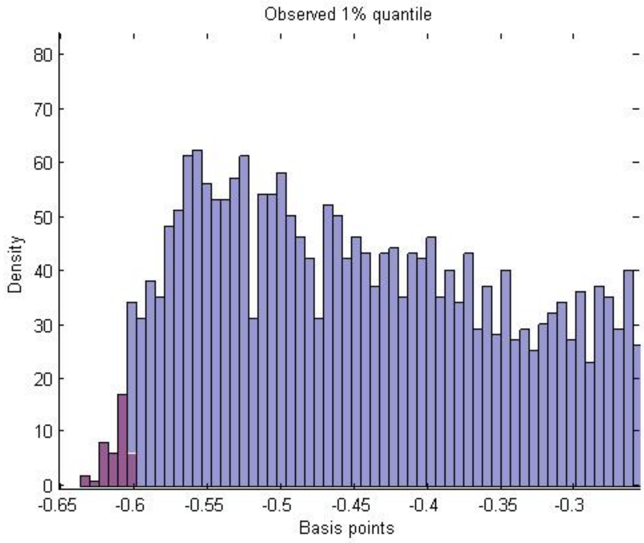

Figure 19: Displays the FVA outcomes of the N simulated interest rate paths. The outcomes are identical to those as in Fig. 18. Negative value is a cost to D. The empirical 1% quantile is displayed to the left in red.

The observed mean, median and the 1% quantile of the FVA can be seen in table 3.

Table 3: Observed empirical values of the FVA.

Measure	Mean	Median	1% quantile
FVA	-0.29 bp	-0.31 bp	-0.60 bp

4.3 Result of LVA and FVA

The two histograms of LVA and FVA are plotted in the same graph in Fig. 20.

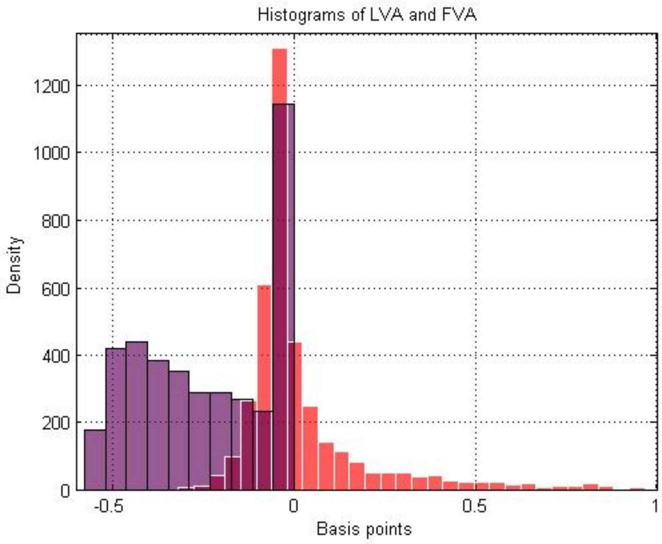

Figure 20: The histograms of the LVA and FVA outcomes are here both displayed in the same graph. The FVA is displayed in thicker bars and the LVA in thinner bars. The histograms are expressed in basis points per year of the notional.

For each set of interest rate paths the sum of LVA and FVA is plotted as a histogram in Fig. 21. The empirical quantiles of 1% and 99% are visible at the tails of the histogram in Fig. 22. The former to the left and the latter to the right.

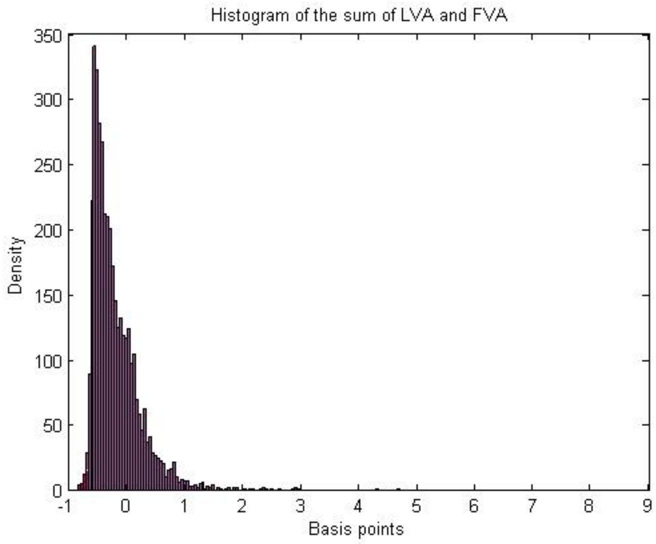

Figure 21: Displays the total present value for each simulated interest rate path, i.e. the LVA and FVA are summed up for each path. The histogram is expressed in basis points per year of the notional.

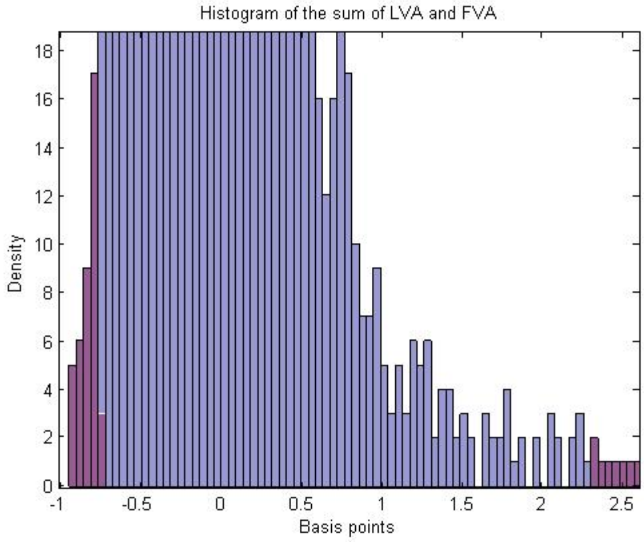

Figure 22: The empirical quantiles of 1% and 99% are visible at the tails of the histogram. The outcomes are identical to those in Fig. 21.

The observed empirical mean, median, 1% and 99% quantiles of the sum of LVA and FVA can be seen in Table 4.

Table 4: Observed empirical values of the sum of LVA and FVA.

Measure	Mean	Median	1% quantile	99% quantile
LVA+FVA	-0.22 bp	-0.38 bp	-0.76 bp	2.29 bp

The outcomes of the sums of LVA and FVA are plotted in a sorted order.

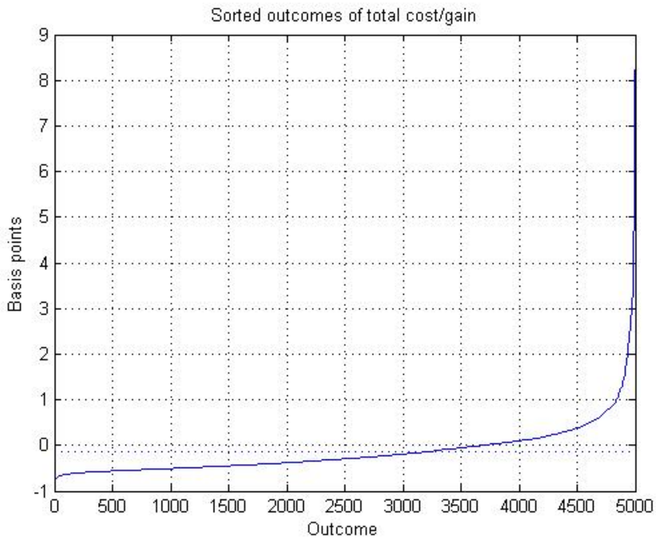

Figure 23: The total discounted sum of LVA and FVA of 5000 simulated interest rate paths, plotted in an increasing sorted order. The horizontal dashed line displays the mean of the of the samples. The positive and negative values are to the Dealer gains and costs respectively. The graph is expressed in basis points per year of the notional.

Examining the distributions of the LVA and FVA the outcomes of these are plotted against each other.

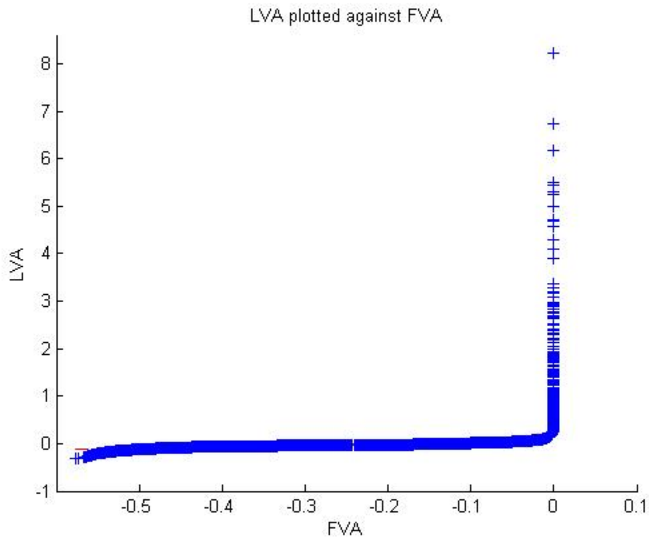

Figure 24: The graph displays the outcomes of LVA plotted against the associated outcomes of the FVA. The graph displays the total N outcomes.

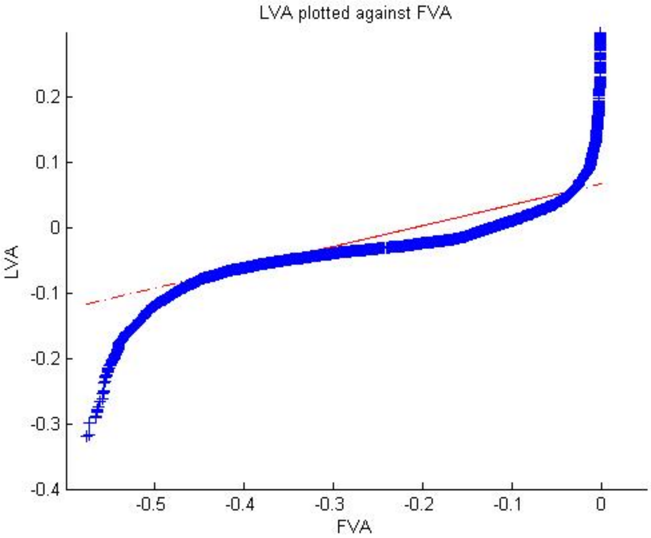

Figure 25: The graph displays a closer view of some of the outcomes in Fig. 24.

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5 Conclusion

An implication of the result is that the initial spot price of the swap is an inaccurate price for D to trade at. In order to become independent of the exposed expected risks, D would need to transfer the expected cost of LVA and FVA, derived from the CSA, to party C at the initiation of the swap. Otherwise the expected net present value of the swap would become negative instead of neutral or positive to party D. The extra cost C would need to receive is an expected value of the extra cost calculated prior of the deal but the true outcome D would finance is stochastic. This makes the transaction hard to hedge since there still exists an intrinsic market risk that not will be passed on to C. Although a possible solution for D to mitigate both the LVA and FVA could be to enter into a similar trade, but with the reversed payment relationship. To practically enter into a perfectly mirrored trade is however implausible. The risk is significantly higher for the FVA than the LVA and a potential gain is absent. For the LVA the potential gain exist albeit small.

An easy and standard way of letting C finance D's costs is to add these costs as a spread on the fixed rate. The fair value of the transaction for party D is thus the initial market fixed rate $+spread$. In this case a spread of 0.22 basis points that is the expected value of the sum of the costs LVA and FVA.

According to the results above the adjustments of LVA and FVA might seem negligible. However the scenario is based on a 1 year swap with only one random future fixing. A spread of 0.22 basis points could make a significant impact when the notional is large or multiple swaps being traded.

In Fig. 11 one can see there is an advantage for D to pay the fixed rate to HC. The graph shows a lower limit equal to the negative swap price at -1.0105%. This means that the worst case scenario for D is that the floating index of reference converges towards the lower limit of 0%. By this there exists a limit of the absolute maximum amount party D potentially might allocate as collateral, namely the sum of the discounted differences between the future fixed interest rate payments and the expected floating rate fixings. On the contrary there is no limit of the amount that D may get allocated by party HC, provided the difference between the interest index of reference and the discounting rate rises indefinitely. None of these rates are restricted by upper finite limits. The capital D then receives as collateral is the sum of the discounted differences between the expected floating rate fixings and the fixed interest rate payments. The implication, of being either payer or receiver, is more clearly seen in Fig. 16 and Fig. 17 and comparing the empirical 1% and 99% quantiles of the LVA outcomes of the collateral account. The fixed rate payer D's LVA 1% quantile equals the floating rate payer HC's 99% quantile and D's 99% quantile equals HC's 1% quantile, due to one party's negative collateral exposure equals the other party's positive exposure. The results show that at the 1% and 99% quantiles the amount to risk as collateral is greater for the floating rate payer than the fixed rate payer. Would D instead pay the floating rate and in return receive the fixed rate the opposite relationship would yield.

The analysis has been seen from D's perspective. The results have stated the IRS's additional expected cost and risk associated for the swap transaction for party D. The fair value for party D is thus fixed rate level $+spread$. The fair value of the IRS for party C is still the initial market price. Since the parties C and D don't have any negotiated CSA among themselves neither of the parties have to post any collateral towards the other. C thus becomes independent of any extra expected costs and risks associated with the transaction, (aside from the consideration that D goes default). Hence the swaps value differentiates for the parties C and D. According to C, D's suggestion to pay C the floating rate and in return receive a fixed rate of market price $+spread$ is too expensive. This is the swaps value to party D. C's own suggestion, to pay D a fixed rate equal to the market price and in return receive the floating rate is according to D too expensive. This is the swaps value to C. The parties have different views and will not agree upon the transactions fair value, however they will agree on a fair price the moment they enter the contract of the IRS.

The LVA and FVA results and conclusions above are all direct implications of the negotiated CSA between the parties D and HC. Would C and D have the identical CSA negotiated between them the risks of both LVA and FVA would cancel out for party D.

A short interest rate model would be appropriate to use with the choice of an IRS with maturity of one year with a future fixing 6 months into the future. The CIR model is explaining possible interest rate paths in a feasible manner in the short term and therefore a suitable model to use for the analysis. The above results all originate from the short interest rate model CIR and assumptions of the collateral and funding rate. With the same reasoning as above the reader can test scenarios with interest rate model, funding and collateral rate, index of reference, maturity, fixing frequency and currency of own choice.

References

[1] T. Björk. Arbitrage Theory in Continuous Time.
Oxford, 1998. (228-230)

[2] European Banking Federation. http://www.euribor-ebf.eu/euribor-eonia-org/about-eonia.html

[3] Bloomberg Terminal. Ticker: EURIBOR Index. More information available at http://www.bloomberg.com

[4] Bloomberg Terminal. Ticker: EUR006M Index, EUR003M Index, EONIA Index. More information available at http://www.bloomberg.com

[5] F. Mercurio. LIBOR Market Models with Stochastic Basis. Available at http://papers.ssrn.com/ 2010.

[6] T. Björk. Arbitrage Theory in Continuous Time.
Oxford, 1998. (230-233)

[7] Bank for International Settlements, BIS. Amounts outstanding of over-the-counter (OTC) derivatives by risk category and instrument. Available at http://www.bis.org/statistics/derstats.htm

[8] Bloomberg Terminal. Ticker: EUSA1 Curncy - EUSA10 Curncy. More information available at http://www.bloomberg.com

[9] International Swaps and Derivatives Association Inc, ISDA. More information available at http://www2.isda.org/functional-areas/legal-and-documentation/

[10] A. Castagna. Pricing of Derivatives Contracts under Collateral Agreements: Liquidity and Funding Value Adjustmenst.Iason research paper. Available at http://iasonltd.com/resources.php, 2012.

[11] T. Björk. Arbitrage Theory in Continuous Time.
Oxford, 1998. (261)