KTH Engineering Sciences

A Modified Sharpe Ratio Based Portfolio Optimization

Pär Lorentz

Abstract

The performance of an optimal-weighted portfolio strategy is evaluated when transaction costs are penalized compared to an equal-weighted portfolio strategy. The optimal allocation weights are found by maximizing a modified Sharpe ratio measure each trading day, where modified refers to the expected return of an asset in this context. The leverage of the investment is determined by a conditional expectation estimate of the number of portfolio assets of the next-coming day. A moving window is used to historically measure the transition probabilities of moving from one state to another within this stochastic count process and this is used as an input to the estimator. It is found that the most accurate estimate is the actual trading day's number of portfolio assets and this is obtained when the size of the moving window is one. Increasing the penalty parameter on transaction costs of selling and buying assets between trading days lowers the aggregated transaction cost and increases the performance of the optimal-weighted portfolio considerably. The best portfolio performance is obtained when at least 50% of the capital is invested equally among the assets when maximizing the modified Sharpe ratio. The optimal-weighted and equal-weighted portfolios are constructed on a daily basis, where the allowed $\mathrm{VaR}_{0.05}$ is €300 000 for each portfolio. This sets the limit on the amount of capital allowed to be invested each trading day, and is determined by empirical $\mathrm{VaR}_{0.05}$ simulations of these two portfolios.

Keywords: Modified Sharpe Ratio, Portfolio Optimization, Transaction Cost, Conditional Forecasting, Performance Analysis, Transition Probability, Stochastic Count Process, Value-at-Risk.

Acknowledgements

I would like to start by thanking my parents for supporting me throughout my academic studies. To Emelie Barman, Fredrik Hansson and Mathias Wennberg, I am truly grateful for the views and opinions on my thesis work that you have shared with me throughout this period.

I would like to thank Ronnie Söderman and Niklas Hedman at Fortum Trading and Industrial Intelligence for offering me the opportunity to do my thesis work at Fortum. A special thanks to my thesis supervisor Niklas Hedman at Fortum, for your guidance and support which have significantly helped me keeping track of my work. Finally, I would like to thank my thesis supervisor Boualem Djehiche at the Department of Mathematics at KTH, for your valuable discussions and feedback.

Stockholm, September 2012
Pär Lorentz

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Fortum is an energy company operating in the Nordic countries, Russia, Poland and the Baltic Rim area. The company's activities cover generation, distribution and sales of electricity and heat as well as operation and maintenance of power plants. The Trading and Industrial Intelligence (TII) is Fortum's competence centre for commodities traded on the financial markets. It provides analysis and views of the current behavior of the commodities on the financial markets and functions as decision support to Fortum's business divisions, corporate strategy, M&A and external communications. Additionally, TII supports Fortum by carrying out asset backed trading, sales trading and fuel management for Fortum's whole portfolio, and proprietary trading. The latter consist of value-generating trading in

- Electricity forwards, futures and options primarily on the Nord Pool and EEX exchange markets.
- CO_2 emission allowances on the ICE/ECX exchange market.
- Financial coal and oil derivatives on the ICE and OTC exchange markets.

The proprietary trading portfolio consist of tradable assets, or strategies as they are commonly referred to at TII. They are constructed as a single contract, a spread contract or a basket of contracts of the specified financial instruments presented above. The set of instruments used to construct new portfolio assets, and at which trading days new/existing portfolio assets are activated/deactivated, is decided by TII's Case Group unit. The Case Group unit base their decisions on daily fundamental views and quantitative analysis of the financial markets mentioned above. This decision process generates the dynamics of the portfolio, i.e the number of assets that are included in the portfolio each trading day. This leads to a portfolio which in general consists of few to several assets, where capital is needed to be allocated each trading day. As a direct result of this, TII wants to construct a portfolio optimized allocation model which takes transaction costs into consideration and construct a model that gives a probable prediction estimate of the number of portfolio assets of the next-coming day. The latter is of importance for allocation planning, and when implemented in the optimization routine it will determine the leverage of the investment and lower the transaction costs of the portfolio between trading days. As of today, TII distributes its capital discretionary between portfolio assets each trading day and they do not have any prediction estimate, more than mere experience, on the future number of assets in the portfolio. It is well known that trading is exposed to risk which may lead to losses due to unexpected market behavior. In order to protect the daily investment from possible drawbacks, the company has regulated that the probability of losing more than €300 000 is at most 5%. It follows that a Value-at-Risk model is needed to simulate the riskiness of holding the portfolio between trading days, which also can be used to determine how much capital is allowed to be allocated to the portfolio each trading day.

The optimal-weighted portfolio is computed each trading day by maximizing the modified Sharpe ratio when transaction costs are penalized between trading days. Modified refers to the fact that the performance measure is changed in order to agree with TII's view on how each asset in the portfolio is expected to perform, i.e. the expected return of an asset. This optimal-weighted trading portfolio, defined as the *optimal trading strategy*, is in terms of performance compared to an equal-weighted trading portfolio, defined as the *benchmark trading strategy*, to determine the best candidate. The performance of each trading strategy is mainly based on annualized Sharpe Ratio and aggregated transaction cost of these two trading portfolios. The prediction estimate of the number of portfolio assets of the next-coming day is computed by using transition probabilities as input to a conditional expectation estimator. This is implemented in the optimization as a leverage on the investment of the two trading portfolios. The model that will be used to simulate the riskiness of holding these two trading portfolios over one day is the empirical Value-at-Risk at confidence level $\alpha = 0.05$.

The plan of the thesis is as follows. In Chapter 2 the historical data is evaluated and its matrix structure is highlighted. Before presenting the results, a theoretical framework upon which the analysis rest is established in Chapter 3 . Some modifications are made here compared to standard theoretical procedure in the field in order to accommodate to the nature of the problem. In Chapter 4 the numerical results from the optimal trading strategy and the benchmark trading strategy are presented, and a comparison between the two models is made. This is followed by a discussion and conclusions in Chapter 5, and proposals on further studies for enhancement of established models and theory.

Chapter 2

Data Study

In order to understand how the theoretical framework is applied, it is necessary to introduce the reader to how the trading data is structured and give some comments on its properties. The data, consisting of two matrices, ranges from 2004-01-02 to 2007-11-01 and is created to reflect the true original trading data which is confidential. Dates that account for weekends and bank holidays are neglected and not incorporated in the data sets.

2.1 Matrix Representation

The two data sets are represented in matrix form consisting of 1000 rows and 931 columns, where a row indicates a specific trading day $t = 1, 2, 3, \ldots, 1000$ and where a column indicates a specific asset $a = 1, 2, 3, \ldots, 931$. The first matrix contains historical 1-day returns for each asset, defined as the *return matrix*, and the second matrix contains historical information showing which assets have been included in the portfolio each trading day, defined as the *information matrix*. The structure is illustrated in two matrices below.

$$\begin{bmatrix} -0.01665 & 0.01092 & 0.002106 & 0.00805 & \cdots & r_{931}^{1} \\ -0.04257 & 0.1264 & 0.05999 & 0.003021 & \cdots & r_{931}^{2} \\ -0.05284 & 0.04274 & -0.008734 & 0.005665 & \cdots & r_{931}^{3} \\ 0.01703 & -0.04218 & -0.01079 & -0.01104 & \cdots & r_{931}^{4} \\ 0.01309 & 0.02606 & 0.0007212 & 0.009992 & \cdots & r_{931}^{5} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{1}^{1000} & r_{2}^{1000} & r_{3}^{1000} & r_{4}^{1000} & \cdots & r_{931}^{1000} \end{bmatrix} \tag{2.1}$$

This is the structure of the *return matrix*, where the first four assets' historical 1-day returns of the first five recorded trading days are depicted and where r_a^t is the return of asset a at trading day t.

$$
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 \\
0 & I_2^6 & I_3^6 & I_4^6 & I_5^6 & \cdots & I_{n-k}^6 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \cdots & I_{n-k}^m & \cdots & I_{n-1}^m & I_n^m & \cdots & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & I_{n-1}^{m+1} & I_n^{m+1} & \cdots & I_{n+l}^{m+1} & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & I_{n+l}^{999} & \cdots & I_{929}^{999} & I_{930}^{999} & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & I_{929}^{1000} & I_{930}^{1000} & I_{931}^{1000}
\end{bmatrix} \qquad (2.2)
$$

This is the structure of the *information matrix*, illustrating the assets that have been included in the portfolio on a specific trading day and where $I_a^t \in \{1,0\}$ is the indicator function of asset a at trading day t. The indicator function takes the value 1 if asset a is active at trading day t and 0 otherwise.

By combining rows and columns in (2.2) it can be seen that on trading day $t = 1, 2$, there are no active assets in the portfolio. On trading day $t = 3$, the portfolio consists of asset $a = 1, 2$. On trading day $t = 4$, the portfolio consists of asset $a = 1, 2, 3, 4$. The same reasoning for the remaining rows in the matrix. As soon as an asset a is found to be active in matrix (2.2), it is possible to extract its corresponding historical 1-day returns from matrix (2.1).

2.2 Asset Returns

To get an intuition on how the assets perform over time and how they are distributed, a few figures will be examined below for illustrative purpose. Due to the fact that the *return matrix* (2.1) is very large, the full range of the data is not presented here. As a consequence, only a small data sample from arbitrarily chosen time points is presented for this empirical study.

The chosen time points under observation are trading day $t = 424$ and trading day $t = 995$ where each data sample consists of the returns from trading day t to trading day $t - 195$ of each asset.

The active assets in the portfolio on trading day t are extracted from the *information matrix* (2.2) and their corresponding historical 1-day returns are extracted from the *return matrix* (2.1).

2.2.1 Historical 1-Day Returns

By definition the historical 1-day return of asset a at trading day t is computed by equation (2.3), where p is the closing price of an asset.

$$r_a^t = \frac{p_a^t - p_a^{t-1}}{p_a^{t-1}}. \tag{2.3}$$

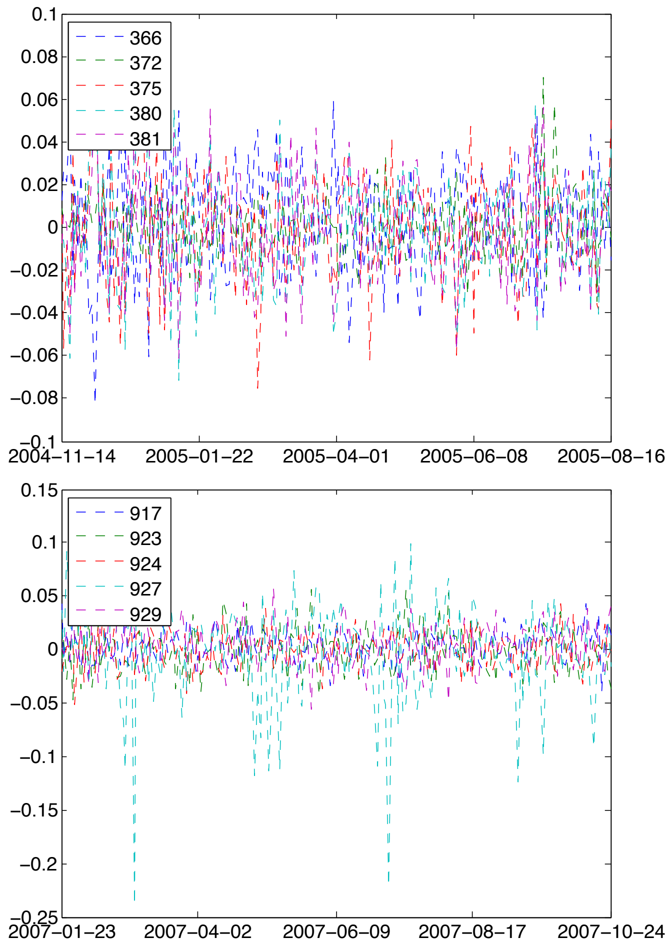


Figure 2.1: Historical 1-day returns.

Figure 2.1 illustrates the historical 1-day returns of those assets found to be active (top left corner in each graph) in the data samples conducted from trading day $t = 424$ (top graph) and trading day $t = 995$ (bottom graph). Notice that asset $a = 927$ occasionally exhibits high volatile outcomes compared to the other assets.

2.2.2 Cumulative Product Returns

The cumulative product return of asset a at trading day t for sample size k, where $t \geq k$, is defined by equation (2.4).

$$cpr_a^{t,k} = (1 + r_a^{t-k+1}) \cdot (1 + r_a^{t-k+2}) \cdots (1 + r_a^t). \tag{2.4}$$

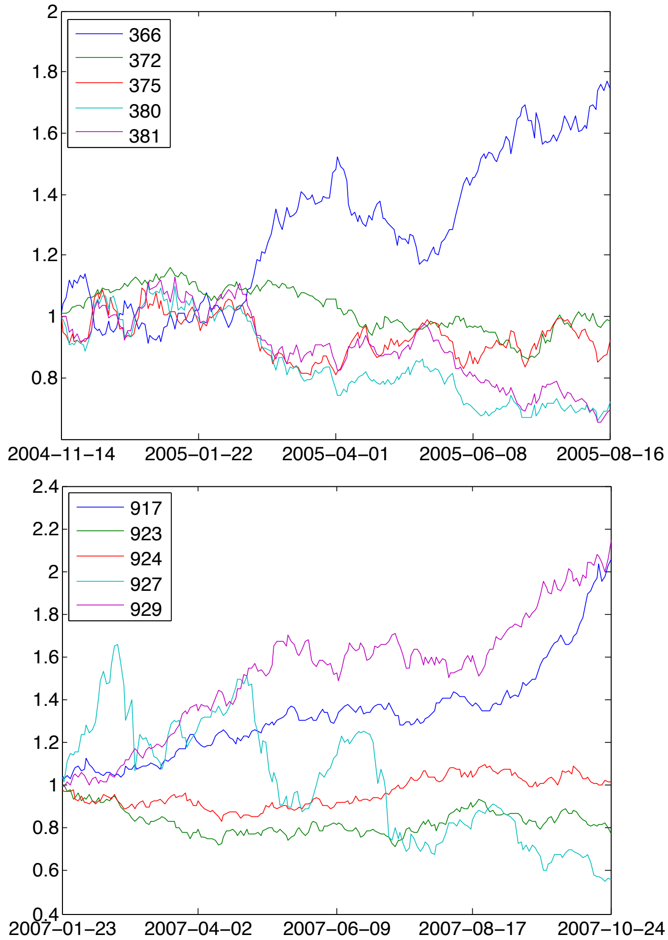


Figure 2.2: Cumulative product returns.

Figure 2.2 illustrates the cumulative product returns of those assets that were found to be active (top left corner in each graph) in the data samples conducted from trading day $t = 424$ (top picture) and trading day $t = 995$ (bottom graph). Bare in mind that assets are activated when they are expected to outperform their fundamental state, e.g. oversold. An example of this can be studied in the bottom graph for asset $a = 927$ which has experienced times of increase in value followed by decrease in value and vice versa.

2.3 Portfolio Dynamics

From the *information matrix* (2.2), it is possible to extract additional information from its structure property by analyzing the rows and columns in detail. Each row contains information on the number of assets that are included in the portfolio on a specific trading day t. From this it is possible to illustrate how the number of assets in the portfolio historically has varied over time and how it is distributed. Each column contains information on the number of trading days an specific asset a has been included in the portfolio. This measures an asset's historical tendency of surviving and its distribution can give an indication on how future, not yet activated assets, will tend to survive in the portfolio.

Combining these facts enables the possibility to investigate the time series of how many portfolio assets that: are active at trading day t, activates/deactivates at trading day t and survives from trading day $t-1$ to trading day t. From these time series the corresponding marginal distributions are created and evaluated.

2.3.1 Active Assets

The number of active assets is the amount of investable assets the portfolio consist of each trading day t and these numbers are obtained by adding together the elements of each row in (2.2). This generates a time series of 1000 data values, one for each trading day, which is illustrated in Figure 2.3 below.

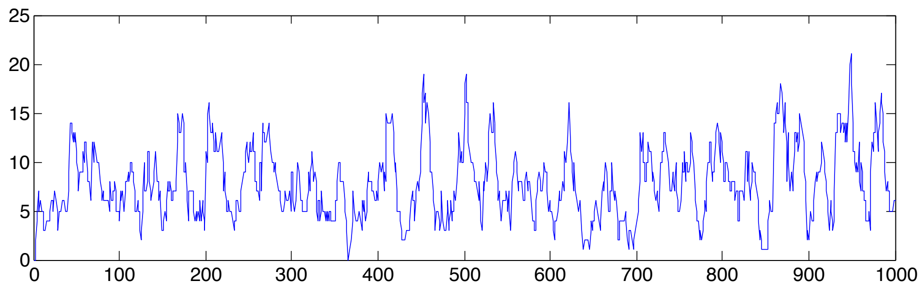


Figure 2.3: Time series of active assets.

This graph shows explicit indications of a stochastic behavior in the time series, where the number of active assets oscillates each trading day between a minimum of 0 to a maximum of 21 assets. The marginal distribution, i.e. the number of times the portfolio consist of an unique number of active assets $(0, 1, 2, \ldots, 21)$, is generated from this time series and illustrated in Figure 2.4 below.

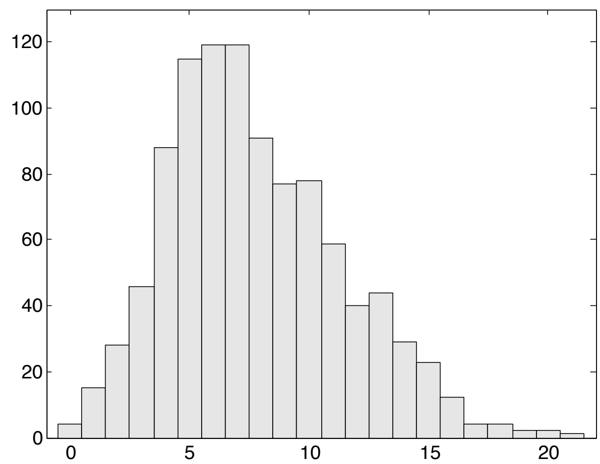


Figure 2.4: Marginal distribution of active assets.

This graph illustrates the outcome frequency for each time the portfolio consists of an unique number of active assets, indicating a rather large negatively skewed distribution which peaks at 6 possibly 7 active assets.

2.3.2 Surviving Assets and Their Survival Time

The surviving assets are the active assets that survives from trading day $t-1$ to trading day t in the portfolio and these numbers are extracted from (2.2). This generates a time series of 1000 data values, one for each trading day, illustrated in Figure 2.5 below.

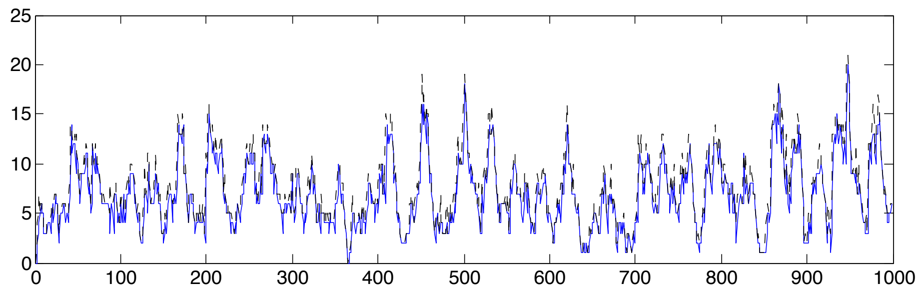


Figure 2.5: Time series of surviving assets compared to active assets.

This graph compares the amount of assets surviving from trading day $t-1$ to trading day t (blue line) with the actual number of active assets in the portfolio (black dashed line). The number of surviving assets oscillates each trading day between a minimum of 0 to a maximum of 20 assets, and not surprisingly it behaves similarly to that of the time series of the active assets. The corresponding marginal distribution of this time series can be seen in Figure 2.6 below.

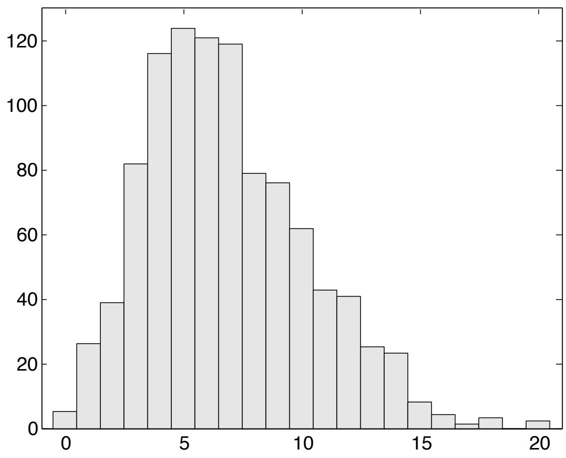


Figure 2.6: Marginal distribution of surviving assets.

This distribution illustrates the outcome frequency of the surviving assets, which peaks at 5 surviving assets and exhibits a similar negatively skewed distribution as that of Figure 2.4.

Additional properties can be extracted from matrix (2.2) by adding all elements in each column, resulting in the number of days each asset has been active in the portfolio, i.e. each asset's tendency of surviving. A time series of 931 data values is generated, one for each asset, representing the survival time of an asset. This is illustrated in Figure 2.7 below.

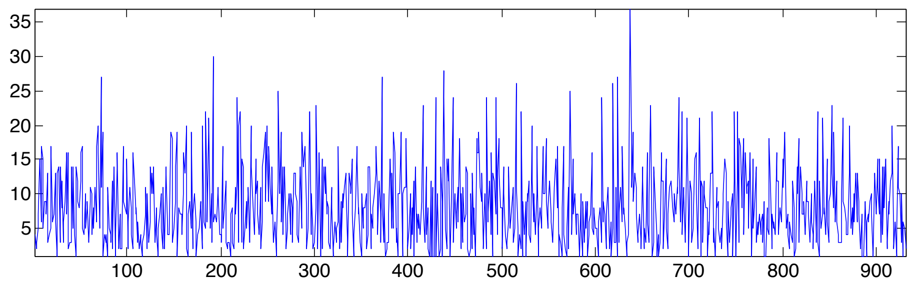


Figure 2.7: Time series of assets' survival time in the portfolio.

The graph illustrates the survival time of each asset, ranging from a lowest of 1 to a highest of 37 trading days, indicating high randomness. The corresponding marginal distribution of this time series can be seen in Figure 2.8 below.

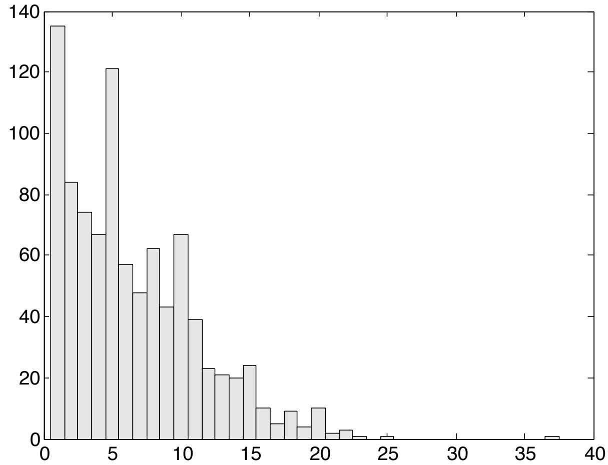


Figure 2.8: Marginal distribution of assets' survival time.

This graph indicates somewhat a decay in frequency as the number of trading days increases, except for some distinct outliers at 1, 5, 8 and 10 trading days.

2.3.3 Birth and Death of Assets

The birth and death of assets are the number of portfolio assets activated and deactivated at each trading day and these values are also obtained from (2.2). This generates two time series consisting of 1000 data values each, one for each trading day, which is illustrated in Figure 2.9 below.

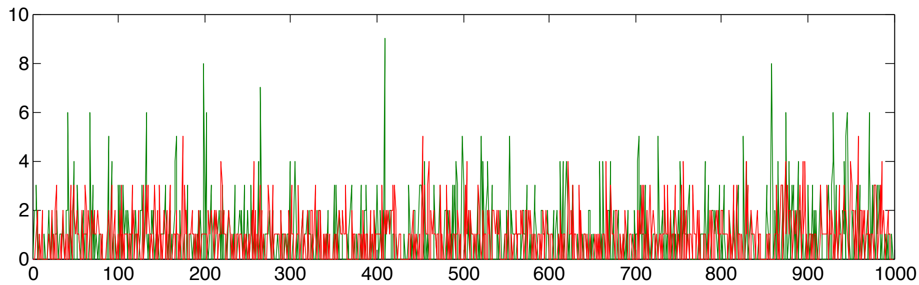


Figure 2.9: Time series of death and birth process.

The graph compares the number of births (green line) and deaths (red line) at each trading day t. In general, the death process seems to exhibit less extreme movements compared to the birth process. The two processes however oscillate heavily, between a minimum of 0 to a maximum of 5 assets for the death process and a minimum of 0 to a maximum of 9 assets for the birth process. The corresponding marginal distributions of these two time series are shown in Figures 2.10 and 2.11 below.

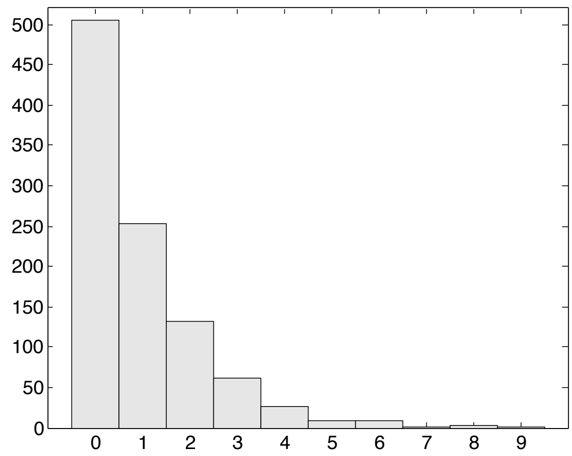


Figure 2.10: Marginal distribution of birth process.

The marginal distribution of the birth process clearly indicates that it decays in frequency as the births increase in number. Notice that approximately 50% of the time there are no births occurring in the time series, i.e. the probability of a new asset not being activated on a trading day is historically approximately 50%.

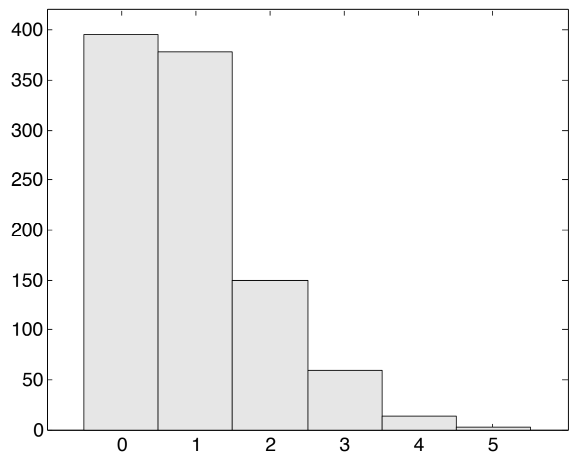


Figure 2.11: Marginal distribution of death process.

The marginal distribution of the death process indicates that the two most frequently occurring outcomes correspond to those of 0 and 1. These two events occur historically at approximately 40% and 37% of the time respectively. The combined probability of these two events covers 77% of the outcomes and the remaining mass is distributed between the remaining outcomes as the graph illustrates.

Chapter 3

Theoretical Background

The purpose of this chapter is to give the reader a thourough explanation of the methods and theories that have been used in order to obtain the numerical results later on in this thesis. These theories and methods are originally designed to apply on problems that in some cases differ from the problem that is faced in this thesis. As a consequence, it is necessary to make some few adjustments to the existing theories and methods in the field in order to fit to the framework of this thesis.

The reader is encouraged to make notice of one important interpretation that holds throughout this chapter. Up to trading day t everything is assumed to be known except the returns of the active assets in the trading portfolio for that day. They are first known at the end of each trading day since returns are computed from closing prices.

3.1 Time Series Analysis

In order for transaction costs to possibly be lowered and allocation planning to be applicable it is of importance to predict the most probable outcome on the number of active assets in the portfolio at trading day $t+1$. By extracting the dependency structure from the count process of active assets, birth of assets and death of assets it is possible to predict the number of portfolio assets of trading day $t+1$, when the number of portfolio assets at trading day t is given. Since the data is generated from stochastic count processes, it is reasonable to use a model that is proved to work with this type of behavior. Thus, the predictor that will produce this 1-day estimate will be based on transition probabilities together with conditional expectation theory.

3.1.1 Stochastic Processes and Transition Probability Matrices

It is stated in [1] that a stochastic process is a family of random variables X_t for time points t such that

$$\mathbf{X} = \{X_t : t \in T\}, \tag{3.1}$$

where T is some index set of the process. Typical cases are when T belongs to a subset or the whole set of nonnegative integers $\mathbb{N}_0$ (which case the process is said to have discrete time) and when T belongs to the set of real numbers $\mathbb{R}$, usually $[0, 1]$ or $[0, \infty)$ or $(-\infty, \infty)$, (which case the process is said to have continuous time). The stochastic process is in itself called discrete or continuous depending on the state space S, which is the set of attainable values of the process. A stochastic process with state space S is defined on the same probability space $(\Omega, \mathcal{F}, \mathbf{P})$. For each fixed $t \in T$, X_t is a random variable which is mapped from $\Omega \to S$. Furthermore, for each fixed $\omega \in \Omega$ it holds that the mapping $t \to X_t(\omega)$, defined on the index set T, is called a realization, trajectory or sample path of the stochastic process in 3.1.

The data sets which contain values of the active, birth and death of assets are outcomes from three different stochastic processes which all are discrete both in time and state space. Let them be represented by the stochastic processes $\{A_t : t = 1, 2, 3, \ldots, 1000\}$ (active assets), $\{B_t : t = 1, 2, 3, \ldots, 1000\}$ (birth of assets) and $\{D_t : t = 1, 2, 3, \ldots, 1000\}$ (death of assets). Evidently the state space S of the stochastic processes A_t, B_t and D_t belongs to the set of nonnegative integers $\mathbb{N}_0$ where t as usual indicates a trading day in this framework. These stochastic processes have each a discrete state space that evolves in discrete time and logically, as figures 2.3 and 2.9 indicates above, the outcome space of each stochastic process A_t, B_t and D_t is limited to their own unique set of attainable nonnegative integers. They are all subsets of $\mathbb{N}_0$ and given by $S_A = \{0, 1, 2, \ldots, 21\}$, $S_B = \{0, 1, 2, \ldots, 9\}$ and $S_D = \{0, 1, 2, \ldots, 5\}$ if the full range of the data set from each stochastic process is used.

The dependence structure within each sample path can be successfully revealed by estimating the probabilities of transition from one state to another, i.e. the probabilities $\mathbf{P}(A_{t+1} = a_{t+1} | A_t = a_t, \ldots, A_1 = a_1) = \mathbf{P}(A_{t+1} = a_{t+1} | A_t = a_t)$, $\mathbf{P}(B_{t+1} = b_{t+1} | B_t = b_t, \ldots, B_1 = b_1) = \mathbf{P}(B_{t+1} = b_{t+1} | B_t = b_t)$ and $\mathbf{P}(D_{t+1} = d_{t+1} | D_t = d_t, \ldots, D_1 = d_1) = \mathbf{P}(D_{t+1} = d_{t+1} | D_t = d_t)$. The idea of transition probabilities is originally a property of discrete time Markov chains and the full range of those theories are presented in [2] and [3]. However, they do not fully apply in this framework since the full range of the data is not used when estimating the transition probability matrices. They are estimated each trading day t by using a moving window of size w, where all transitions from one state to another in the sample paths are monitored from trading day $t+1-w$ to trading day t. Thereby the behavior of each stochastic process is captured and its transition probabilities can be summarized in three different transition probability matrices ($\hat{P}_{A_t}, \hat{P}_{B_t}, \hat{P}_{D_t}$), each with the matrix structure illustrated in table 3.1 below.

		x_{t+1}						
	state	0	1	2	3	4	$\cdots$	k
	0	$p_{1,1}$	$p_{1,2}$	$p_{1,3}$	$p_{1,4}$	$p_{1,5}$	$\cdots$	$p_{1,k+1}$
	1	$p_{2,1}$	$p_{2,2}$	$p_{2,3}$	$p_{2,4}$	$p_{2,5}$	$\cdots$	$p_{2,k+1}$
	2	$p_{3,1}$	$p_{3,2}$	$p_{3,3}$	$p_{3,4}$	$p_{3,5}$	$\cdots$	$p_{3,k+1}$
x_t	3	$p_{4,1}$	$p_{4,2}$	$p_{4,3}$	$p_{4,4}$	$p_{4,5}$	$\cdots$	$p_{4,k+1}$
	4	$p_{5,1}$	$p_{5,2}$	$p_{5,3}$	$p_{5,4}$	$p_{5,5}$	$\cdots$	$p_{5,k+1}$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	l	$p_{l+1,1}$	$p_{l+1,2}$	$p_{l+1,3}$	$p_{l+1,4}$	$p_{l+1,5}$	$\cdots$	$p_{l+1,k+1}$

Table 3.1: Transition probability matrix

This table illustrates the structure of the transition probability matrix which is estimated each trading day t. The arbitrary variable x is replaceable with a, b and d which corresponds to the active, birth and death of assets respectively. Each probability is estimated simply by dividing the number of times state $i-1$ has moved to another state $j-1$ by the total amount of movements from state $i-1$, i.e.

$$p_{i,j} = n_{i-1,j-1} / \Sigma_{a=1}^{k+1}(n_{i-1,a-1}), \tag{3.2}$$

where $n_{i-1,j-1}$ is the number of transitions made from state $i-1$ to state $j-1$ for $i = 1, 2, 3, \ldots, l+1$ and $j = 1, 2, 3, \ldots, k+1$. It holds that $\Sigma_{j=1}^{k+1} p_{i,j} = 1$ for every i, if all of the states $\{0, 1, 2, \ldots, l\}$ transitions to one of the states $\{0, 1, 2, \ldots, k\}$ at least one time. However, if the state that correspond to row i has not been found within the sample path, then all the columns of that row will be filled with zeroes. As table 3.1 illustrates, the variables k and l are the highest transition states that are found within the sample path, which define the size of the transition probability matrix. The size of the moving window w affects the probability distribution of transitions and it is thereby needed to be specified from statistical analysis. In order to fully understand the properties of the transition probability matrix (3.1), it is convenient to illustrate it by a figure of arbitrary transition state chains followed by three case examples.

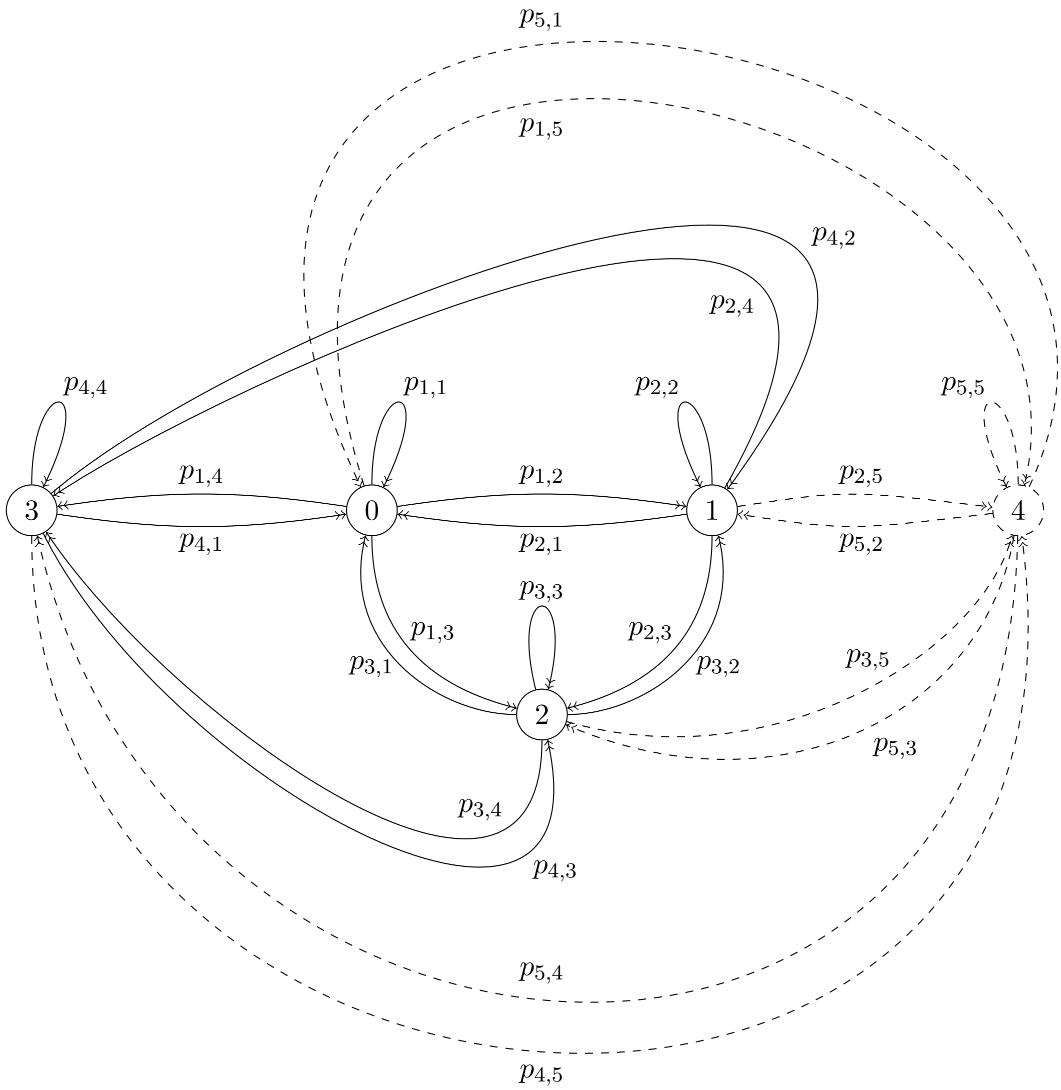


Figure 3.1: Transition state chain.

This figure illustrates the transition between states generated by some arbitrary sample path accompanied by the corresponding transition probabilities as that of the transition probability matrix (3.1). Notice that this transition chain is not restricted in any way and can be made arbitrarily large just by adding more states.

For the case study, it is assumed that the sample path of the arbitrary stochastic process $\{X_t : t = 1, 2, 3, \ldots, 14\}$ is $x_t = [0\ 1\ 3\ 2\ 3\ 4\ 3\ 5\ 7\ 4\ 3\ 5\ 6\ 2]$ in one realization. By using a moving window $w = 6$ trading days, measuring the transition between states from trading day $t+1-w$ to trading day t, it is possible to estimate the transition probability matrix (3.1) at each trading day t. The structure of this transition probability matrix is as follows.

Case 1) $l < k$

In the first case, it is assumed that $t = 6$ which gives $x_6 = 4$ from the sample path above. It can be seen that a transition has occurred from states $\{0, 1, 2, 3\}$ at least one time, but not from state 4 to another state. From this reasoning, it is clear that $k = 4$ and $l = 3$ which evidently yields that the transition probability matrix $\hat{P}_{X_6}$ in Table 3.1 has four rows and five columns. As a last remark, notice that $\Sigma_{j=1}^{k+1} p_{i,j} = 1$ for every i in this case.

Case 2) $l = k$

The case when the transition probability matrix is symmetric is just a matter of moving one time step to trading day $t = 7$, which now yields $x_7 = 3$ from the sample path above. It can be seen that a transition has occurred from states $\{1, 2, 3, 4\}$ at least one time but not from state 0 to any other state. It is clear that $k = l = 4$ which yields that the symmetric transition probability matrix $\hat{P}_{X_7}$ consists of five rows and five columns. However, notice that $\Sigma_{j=1}^{k+1} p_{i,j} = 0$ for $i = 1$, since state 0 has never transitioned to another state in this case.

Case 3) $l > k$

The last case is accomplished by moving to trading day $t = 14$, which gives $x_{14} = 2$ from the sample path above. Now a transition has occurred at least one time from the states $\{3, 4, 5, 6, 7\}$ to another state, but not from states $\{0, 1, 2\}$ to any other state. This yields that $k = 6$ and $l = 7$, which generates the transition probability matrix $\hat{P}_{X_{14}}$ that consist of eight rows and seven columns, where $\Sigma_{j=1}^{k+1} p_{i,j} = 0$ for $i = 1, 2, 3$.

3.1.2 Conditional Forecasting

As the probability distribution of the stochastic count processes undergoes conditioning, it is necessary to use a model that accounts for this. In [1], it is stated that if the random variables X and Y are jointly distributed, then it holds that the conditional expectation of Y given that $X = x$ is

$$\mathrm{E}[Y|X = x] = \sum_y y p_{Y|X=x}(y). \tag{3.3}$$

For each stochastic count process, i.e. active, birth and death of assets, the corresponding transition probability matrix contains the estimated probabilities of moving from one state to another. They are seen as the true probabilities defining the probability space of the stochastic count processes A_t, B_t and D_t when standing at trading day t. This is interpreted as follows, given that $A_t = a_t$, $B_t = b_t$ and $D_t = d_t$, the random variables A_{t+1}, B_{t+1} and D_{t+1} are distributed according to $\hat{P}_{A_{t+1}|A_t=a_t}(a_{t+1}), \hat{P}_{B_{t+1}|B_t=b_t}(b_{t+1})$ and $\hat{P}_{D_{t+1}|D_t=d_t}(d_{t+1})$ respectively. These are the conditional probabilities of moving from one state to another and by applying (3.3) we construct two predictors, 1) where the count process properties of the active assets are used and 2) where the count process properties of the birth and death of assets are used. The first predictor is defined as

$$\hat{H}_a^{t,t+1} = \mathrm{E}[A_{t+1}|A_t = a_t] \quad = \begin{cases} \sum\limits_{a_{t+1}} a_{t+1}\hat{P}_{A_{t+1}|A_t=a_t}(a_{t+1}), & \text{if } w > 1, \\ a_t, & \text{if } w = 1, \end{cases} \tag{3.4}$$

which corresponds to the active assets predictor, where $\hat{H}_a^{t,t+1}$ is the predicted number of active assets in the portfolio of trading day $t+1$ when the number of active assets at trading day t is given. Notice that this predictor does not generate integer values and it is therefore assumed that it is allowed to round the predicted value in order to fit to the count data. Thus, if $\lceil \hat{H}_a^{t,t+1} \rceil - \hat{H}_a^{t,t+1} \leq 0.5$ it follows that $\hat{H}_a^{t,t+1} = \lceil \hat{H}_a^{t,t+1} \rceil$ (rounded to the closest integer upwards), and otherwise $\hat{H}_a^{t,t+1} = \lfloor \hat{H}_a^{t,t+1} \rfloor$ (rounded to the closest integer downwards). The second predictor is defined as

$$\begin{aligned} \hat{H}_{b,d}^{t,t+1} &= \mathrm{E}[A_{t+1}|A_t = a_t, B_t = b_t, D_t = d_t] \\ &= \mathrm{E}[A_t + B_{t+1} - D_{t+1}|A_t = a_t, B_t = b_t, D_t = d_t] \\ &= \mathrm{E}[A_t|A_t = a_t, B_t = b_t, D_t = d_t] + \mathrm{E}[B_{t+1}|B_t = b_t] \\ &\quad - \mathrm{E}[D_{t+1}|D_t = d_t] \\ &= \begin{cases} a_t + \sum\limits_{b_{t+1}} b_{t+1}\hat{P}_{B_{t+1}|B_t=b_t}(b_{t+1}) \\ \quad - \sum\limits_{d_{t+1}} d_{t+1}\hat{P}_{D_{t+1}|D_t=d_t}(d_{t+1}), & \text{if } w > 1, \\ a_t, & \text{if } w = 1, \end{cases} \end{aligned} \tag{3.5}$$

which corresponds to the predictor of birth and death of assets, where $\hat{H}_{b,d}^{t,t+1}$ is the predicted number of active assets in the portfolio of trading day $t+1$ when the number of active assets, birth and death of assets are given at trading day t. This is rounded in the same manner as before in order to obtain integer values from the estimates. Here, it holds that the outcome of the random variable A_{t+1} can be described by $A_t + B_{t+1} - D_{t+1}$. Since A_t is known at trading day t, the only unknown variables are B_{t+1} and D_{t+1} which are assumed to be independent of each other and independent of A_t from which equation (3.5) holds.

These are the two different 1-day predictors that will be implemented as a leverage in the optimization model, which will be described in the next section below.

3.2 Portfolio Optimization Approach

The structure of matrices (2.1) and (2.2) forces the optimization routine to be programmed accordingly in order to generate optimal portfolio weights. The optimal portfolio weights are generated each trading day t by maximizing the modified Sharpe ratio measure, where modified in this context refers to the expected return of an asset. Furthermore, transaction costs are supposed to be penalized between trading days and this is also implemented in the optimization model.

3.2.1 Risk/Return Framework

The three measures that we use to describe the universe of assets are the mean, standard deviation and correlation between assets' returns. These quantifying measures are applied on the assets' historical time series to calculate the statistics from it, whereas these statistics are interpreted as the true estimates of the future behavior of the assets. This framework is applied to matrix (2.2) each trading day t, by first monitoring which assets that are active and then secondly by extracting their corresponding historical return series from matrix (2.1). The location, i.e. the mean of the active assets are given by

$$\mu_i = \frac{1}{T}\sum_{k=1}^{T} r_i^{t-k}, \qquad \text{for } i = 1, \ldots, N_t, \tag{3.6}$$

which corresponds to the unbiased estimate of the sample mean of the observed returns of asset i. The variable T is the sample size, N_t is the total number of active assets at trading day t and r_m^n is the 1-day return of asset m at trading day n in matrix (2.1). The dispersion, i.e. the variance of the active assets, are given by

$$\sigma_i^2 = \frac{1}{T-1}\sum_{k=1}^{T} (r_i^{t-k} - \mu_i)^2, \qquad \text{for } i = 1, \ldots, N_t, \tag{3.7}$$

which is the unbiased estimate of the sample variance of the observed returns of asset i. The covariance between the active assets' returns are given by

$$\sigma_{ij} = \frac{1}{T-1}\sum_{k=1}^{T} (r_i^{t-k} - \mu_i)(r_j^{t-k} - \mu_j) = \sigma_{ji}, \qquad \text{for } i = 1, \ldots, N_t \text{ and } i \neq j, \tag{3.8}$$

and this is the unbiased estimate of the sample covariance of the observed returns between assets i and j. However, when building a portfolio the importance lies in the combination of assets and their combined effect on the whole portfolio. At the end of each trading day t the portfolio return will be given by

$$r_p = \sum_{i=1}^{N_t} w_i r_i, \qquad \text{for } t \geq 0, \tag{3.9}$$

which can be written in vector form as $\mathbf{w}^{\mathrm{T}}\mathbf{r}$, where $\mathbf{w} = (w_1, \dots, w_{N_t})^{\mathrm{T}}$ and $\mathbf{r} = (r_1, \dots, r_{N_t})^{\mathrm{T}}$ is the weight and return vector respectively of the active assets at trading day t. The expected return of the portfolio is then constructed by taking expectations on both sides of equation (3.9), which yields

$$\mu_p = \sum_{i=1}^{N_t} w_i \mu_i, \qquad \text{for } t \geq 0. \tag{3.10}$$

As earlier, this is given in vector form by $\mathbf{w}^{\mathrm{T}}\boldsymbol{\mu}$, where $\boldsymbol{\mu} = (\mu_1, \dots, \mu_{N_t})^{\mathrm{T}}$ is the expected return vector of the active assets at trading day t. From this, the portfolio variance is given by

$$\begin{aligned} \sigma_p^2 &= \mathrm{E}\big[(r_p - \mu_p)^2\big] \\ &= \mathrm{E}\Bigg[\Big(\sum_{i=1}^{N_t} w_i(r_i - \mu_i)\Big)^2\Bigg] \\ &= \mathrm{E}\Bigg[\sum_{i=1}^{N_t}\sum_{j=1}^{N_t} w_i w_j (r_i - \mu_i)(r_j - \mu_j)\Bigg] \\ &= \sum_{i=1}^{N_t} (w_i\sigma_i)^2 + \sum_{i=1}^{N_t}\sum_{j=1}^{N_t} w_i w_j \sigma_{ij}, \qquad \text{for } i \neq j \text{ and } t \geq 0, \end{aligned} \tag{3.11}$$

which is given in vector form as $\mathbf{w}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{w}$, where $\boldsymbol{\Sigma} = (\sigma_{ij})_{ij}$ is the sample covariance matrix of the assets at trading day t. The expected portfolio return $\mathbf{w}^{\mathrm{T}}\boldsymbol{\mu}$ and the portfolio variance $\mathbf{w}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{w}$ are the two measures spanning the universe of performance and risk in the mean-variance framework. However, the expected return of an asset, i.e. the mean, does not agree with TII's view on how their assets are expected to preform because of their short investment horizon. Frequently, when assets are activated and added to the trading portfolio their expected returns are negative, see Figure 2.1 and Figure 2.2. This could result in that capital will not be invested to those assets in as great extent as wished unless the variance of the portfolio is lowered. This contradicts the trading strategy of TII since all assets returns are assigned the same belief. Another performance measure is instead needed in order to find optimal allocation weights and this is best modeled by letting the expected return of an asset correspond to its standard deviation, i.e.

$$\beta_i = \mathrm{E}[r_i] = \sqrt{\mathrm{Var}(r_i)}, \qquad \text{for } i = 1, \ldots, N_t. \tag{3.12}$$

Thus, the expected return of the whole portfolio is modified and now given by

$$\beta_p = \sum_{i=1}^{N_t} w_i \beta_i, \qquad \text{for } t \geq 0, \tag{3.13}$$

which is given in vector form as $\mathbf{w}^{\mathrm{T}}\boldsymbol{\beta}$, where $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_{N_t})^{\mathrm{T}}$ is the modified expected return vector of the active assets at each trading day t.

3.2.2 Transaction Costs

In a multi period framework where investors are allowed to modify their portfolio composition, transaction costs between trading days arise, and these transaction costs need to be accounted for when developing the optimization model. In [4], it is stated that the transaction cost of buying or selling an instrument is defined as

$$tc = commission + \Big(\frac{bid}{ask} - spread\Big) + \theta\sqrt{\frac{trade\ volume}{daily\ volume}}, \tag{3.14}$$

where tc is the total percentage fee for buying or selling an instrument. The *commission* is the percentage fee charged by the broker in order to make a trade on the market. The size of the *bid/ask-spread* of an instrument is one measure of the liquidity of the market and of the size of the transaction cost which is calculated as (*ask-bid*)/*ask*. The variable θ is a constant that needs to be estimated since it is dependent on the instrument that is traded and can not be determined in any other way. The *trade volume*/*daily volume* is the ratio of the actual trade size of an instrument and its daily traded volume which is decided from history. However, the time spent by a trader to invest in illiquid instruments should also be accounted for and incorporated in (3.14). For this study it suffice to run the optimization model for different levels of transaction costs tc. How this affects the trading portfolio will be revealed when the numerical results are presented to the reader later on.

Trading Portfolio Influenced by Transaction Costs

The difference in portfolio composition between trading day $t-1$ and trading day t together with the transaction cost tc are the driving factors that determines how large the total cost of the trading portfolio will be on trading day t. Mathematically, it is defined as $tc \cdot |\Delta\mathbf{w}^{\mathrm{T}}|\mathbf{1}$, where $|\Delta\mathbf{w}| = |\mathbf{w}_t - \mathbf{w}_{t-1}|$ is the absolute difference of the asset weights between trading day $t-1$ and trading day t and $\mathbf{1}$ is the vector of ones. Notice that the set of active assets between trading days does not necessarily have to be the same since assets are activated and deactivated at varying frequency.

High exchange of assets between trading days leads to increased transaction costs and vice versa. The transaction cost of buying and selling an asset is assumed to be equal, i.e $tc = tc_i^+ = tc_i^-$, and they are paid at the beginning of each trading day. In [4], it is stated that transaction cost estimates are likely to be of the size 0.01%-0.05% when 0-50 instruments are traded among the largest markets. This could be used as a guidance to determine tc, but in this study higher values of transaction costs are tested as well. This framework will be implemented in the optimization model, where a penalty parameter is applied in order to lower the transaction costs between trading days.

3.2.3 Modified Sharpe Ratio Maximization

The choice of optimization model varies between investors, and for TII it is a matter of investing in the set of active assets at trading day t such that the risk-adjusted return of the portfolio is maximized at all times. A natural optimization model would be to maximize the Sharpe ratio which is defined in [5] as

$$\mathrm{SR}(\mathbf{w}) = \frac{\mathbf{w}^{\mathrm{T}}\boldsymbol{\mu} - r_f}{\sqrt{\mathbf{w}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{w}}}, \quad \mathbf{w} \in \mathbb{R}^{N_t}, \tag{3.15}$$

where r_f denotes the return rate of a risk free asset, e.g. a zero coupon bond. The maximized value of equation (3.15) is obtained in the trivial case, for constraints $\mathbf{w}^{\mathrm{T}}\mathbf{1} = 1$ and $w_i \geq 0$ for $i = 1, \ldots, N_t$, by the standard strategy of optimizing the trade-off problem given by

$$\begin{aligned} \underset{\mathbf{w}\in\mathbb{R}^{N_t}}{\text{maximize}} \quad & \mathbf{w}^{\mathrm{T}}\boldsymbol{\mu} - \lambda \cdot \mathbf{w}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{w} \\ \text{subject to} \quad & \mathbf{w}^{\mathrm{T}}\mathbf{1} = 1, && (1) \\ & w_i \geq 0, \quad \text{for } i = 1, \ldots, N_t, && (2) \end{aligned} \tag{3.16}$$

where λ is the penalty parameter. The optimization model (3.16) generates optimal allocation weights $\mathbf{w}^{opt}$ for each λ from which the efficient frontier is constructed. The efficient frontier is composed of pairs $(\sigma_p(\lambda), \mu_p(\lambda))$ of standard deviations and expected returns of the future optimal portfolio values. This is illustrated for $r_f = 0$ in Figure 3.2 below.

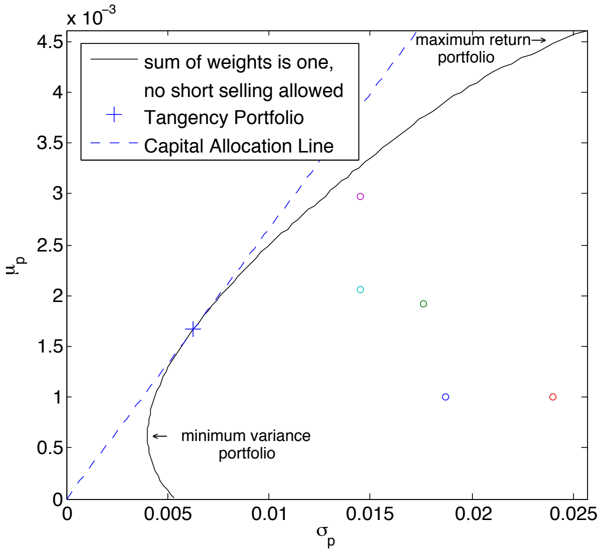


Figure 3.2: Efficient frontier of optimal portfolio compositions.

This graph indicates that for a correct specification of λ, the maximal Sharpe ratio is obtained as the tangency portfolio to the efficient frontier. Notice that the tangency portfolio will move higher along the efficient frontier if r_f is increased. The colored circles below the efficient frontier are non-optimal portfolio compositions in this mean-variance framework.

Changing the Performance Measure

Since the original performance measure of expected portfolio return μ_p does not apply to this framework, it is necessary to adjust the Sharpe ratio measure accordingly. This is done by replacing $\boldsymbol{\mu}$ with $\boldsymbol{\beta}$, which yields that

$$\mathrm{MSR}(\mathbf{w}) = \frac{\mathbf{w}^{\mathrm{T}}\boldsymbol{\beta} - r_f}{\sqrt{\mathbf{w}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{w}}}, \quad \text{for } \mathbf{w} \in \mathbb{R}^{N_t} \tag{3.17}$$

This is called the modified Sharpe ratio measure. Now, the maximized value of (3.17) is obtained in the trivial case, for constraints $\mathbf{w}^{\mathrm{T}}\mathbf{1} = 1$ and $w_i \geq 0$ for $i = 1, \ldots, N_t$, by the standard strategy of optimizing the trade-off problem which is given by

$$\begin{aligned} \underset{\mathbf{w}\in\mathbb{R}^{N_t}}{\text{maximize}} \quad & \mathbf{w}^{\mathrm{T}}\boldsymbol{\beta} - \lambda \cdot \mathbf{w}^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{w} & \\ \text{subject to} \quad & \mathbf{w}^{\mathrm{T}}\mathbf{1} = 1, & (1) \\ & w_i \geq 0, \quad \text{for } i = 1, \ldots, N_t, & (2) \\ & \mathrm{E}[r_i] = \sqrt{\mathrm{var}(r_i)}, \quad \text{for } i = 1, \ldots, N_t, & (3) \end{aligned} \tag{3.18}$$

where the correct parameter value of λ generates optimal allocation weights $\mathbf{w}^{opt}$ and maximizes the modified Sharpe ratio of the portfolio.

Optimal Trading Strategy

By combining the theory and models presented earlier in this chapter all the details are known in order to present the optimization model. This model will be used throughout this report when optimal allocation weights are generated when transaction costs are taken into account and penalized between trading days. Recall that the expected return vector $\boldsymbol{\mu}$ is replaced by $\boldsymbol{\beta}$, where $\beta_i = \mathrm{E}[r_i] = \sqrt{\mathrm{var}(r_i)}$, and transaction costs are subtracted from the expected portfolio return each trading day t. Thus, the model is defined as

$$
\begin{aligned}
\underset{\mathbf{w}_t \in \mathbb{R}^{N_t}}{\text{maximize}} \quad & \frac{\mathbf{w}_t^{\mathrm{T}}\boldsymbol{\beta}_t - (r_f + \gamma \cdot tc \cdot |\Delta \mathbf{w}^{\mathrm{T}}|\mathbf{1})}{\sqrt{\mathbf{w}_t^{\mathrm{T}}\boldsymbol{\Sigma}_t \mathbf{w}_t}}, \\
\text{subject to} \quad & \mathbf{w}_t^{\mathrm{T}}\mathbf{1} \leq \begin{cases} \frac{\hat{H}^{t,t+1}}{6}, & \text{if } \hat{H}^{t,t+1} < 6, \\ 1, & \text{if } \hat{H}^{t,t+1} \geq 6, \end{cases} \qquad (1) \\
& \mathbf{w}_t = \mathbf{w}_{t-1} + \Delta \mathbf{w}, \qquad (2) \\
& \mathbf{w}_t \geq 0, \qquad (3) \\
& |\Delta \mathbf{w}| \leq 1. \qquad (4)
\end{aligned}
\tag{3.19}
$$

This is called the optimal trading strategy when transaction costs are taken into account and penalized between trading days. This nonlinear model will be solved in MatLab using *fmincon* and a built-in *Sequential Quadratic Programming* algorithm solver (*SQP*-solver [5]). The variable γ is the penalty parameter which sets the limit on how much the total transaction cost is punished between trading days. Increased values on γ forces the optimal allocation weights at trading day t to be chosen such that the absolute allocation weight difference between trading day $t-1$ and t is lowered.

Constraint (1) is a budget constraint, where the right hand side is the threshold of the whole investment at trading day t. This threshold is determined by the 1-day prediction on the number of assets in the portfolio of trading day $t+1$. The predictor $\hat{H}^{t,t+1}$ is either $\hat{H}_a^{t,t+1}$ or $\hat{H}_{b,d}^{t,t+1}$, depending on which is suited better in terms of statistical performance. If the number of active assets at trading day $t+1$ is below six assets, then the total investable amount is the predicted number of assets divided by six, otherwise the total amount is allowed to be invested.

Constraint (2) is another budget constraint, stating that the allocation weight of each asset at trading day t is given by the allocation weight of each asset at trading day $t-1$ plus the change in allocation weight of each asset between trading day $t-1$ and trading day t.

Constraint (3) is the minimum allocation weight of each asset at trading day t which indicates that no short selling of any asset is allowed.

Constraint (4) is the maximum change of allocation weights between trading day $t-1$ and trading day t.

3.3 Risk Measure

Holding a portfolio over time is connected with risk due to reigning market conditions. An unexpected market movement can have great impact on the portfolio, which in worst case can result in a major trading loss. Therefore, it is of great importance to understand the riskiness of holding a portfolio. In order for TII to understand the riskiness of their trading portfolio, it is necessary to use a risk measure to model the probable loss distribution.

At the beginning of each trading day t the optimal allocation weights are generated from model (3.19), and from these weights the optimal portfolio r_p is constructed. The outcome of r_p is not known until the end of trading day t, and r_p is therefore seen as a random variable with unspecified distribution at the beginning of each trading day t. Therefore, Value-at-Risk (VaR), which perhaps is the most commonly known risk measure, developed by financial engineers at J.P. Morgan, can be used to model the loss distribution of the portfolio r_p. VaR is a measure related to percentiles of loss distributions, and represents the predicted maximum loss at a specified probability level $\alpha \in (0,1)$. The mathematical definition of VaR is given at the confidence level α by

$$\begin{aligned} \text{VaR}_\alpha(X) &= \min\{x \in \mathbb{R} : \mathbf{P}(x + X < 0) \leq \alpha\} \\ &= \min\{x \in \mathbb{R} : 1 - \mathbf{P}(-X \leq x) \leq \alpha\} \\ &= \min\{x \in \mathbb{R} : \mathbf{P}(L \leq x) \geq 1 - \alpha\} \\ &= F_L^{-1}(1-\alpha), \end{aligned} \tag{3.20}$$

where $X = V_1 - V_0 = V_0 r_p$ is the change in value generated from the random portfolio return r_p at the end of trading day t. Without loss of generality the invested capital V_0 is set to €1. The variable $L = -X$ is interpreted as the portfolio loss, where negative values of L indicate gains and positive values indicate losses. A formal definition of (3.20) is given by [6]: "VaR summarizes the expected maximum loss (or worst loss) over a target horizon within a given confidence interval." For example, if a portfolio has a one day 1% VaR of €1 million, this means that there is a probability of 1% that the portfolio will fall in value by more than €1 million over a one day period. Informally, a loss of €1 million or more on this portfolio is expected to happen on 1 day in 100. In order to fully understand the implication of this definition an illustration is made in Figure 3.3 below.

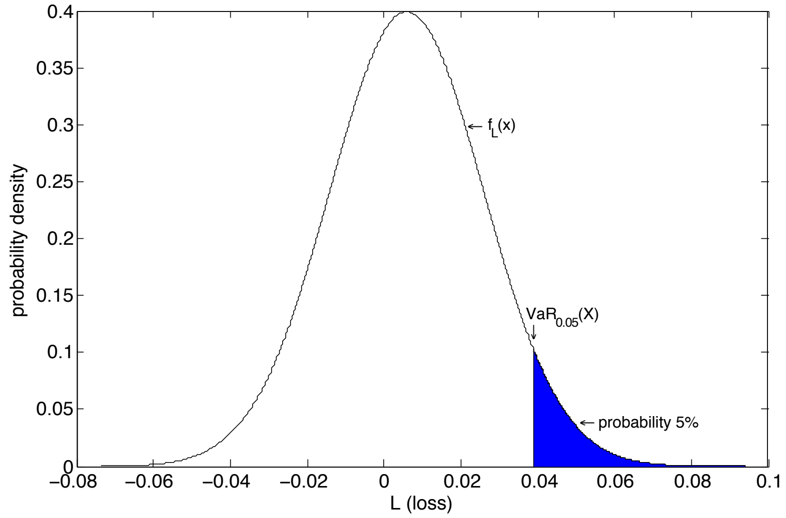


Figure 3.3: VaR of a portfolio loss distribution at confidence level $\alpha = 0.05$.

This graph illustrates a possible loss distribution of the portfolio $X = V_0 r_p$, where $\text{VaR}_{0.05}(X)$ and the total probability of the allowed maximum loss are depicted in the figure.

Equation (3.20) is the mathematical definition of VaR at confidence level α and in order for this to apply to the framework which is under study here, a model of this definition has to be established. The chosen candidate for this task is the empirical Value-at-Risk which is introduced below.

3.3.1 Empirical Value-at-Risk (E-VaR)

This is the nonparametric form of VaR and instead of trying to specify the distribution of the random portfolio return r_p at the beginning of each trading day t, e.g. Normal or Student's t, the portfolio return r_p can instead be specified by historical data to simulate its loss distribution. Given a sufficient amount of historical data, this method can give a realistic loss distribution of the portfolio since it accounts for times when major market movements such as market crashes has occurred. However, the downside of using historical simulations is that the model assumes that the distribution of returns in the future is similar of those in the past, which might not necessarily be true. A possible way of minimizing this problem is by updating the historical data to reflect the difference between the historical volatility of the market variable and its current volatility. The latter will unfortunately not be studied here since it is beyond the scope of this thesis. The database of historical value changes is generated at the beginning of each trading day t when the portfolio r_p is constructed according to the optimal allocation weights $\mathbf{w}_t^{opt}$ and the equal allocation weights

$\mathbf{w}_t^{eq}$. At this stage it is possible to apply the theory in [7], which states that the empirical estimate of $\mathrm{VaR}_\alpha(X)$ now is given by

$$\widehat{\mathrm{VaR}}_\alpha(X) = L_{\lfloor (t-1)\alpha \rfloor + 1, t-1}, \tag{3.21}$$

where $L_{1,t-1} \geq \cdots \geq L_{t-1,t-1}$ is the ordered sample of historical value changes between trading days, interpreted as earlier. Furthermore, $t-1$ is the sample size of the historical database which grows in size as t increases. The problem that arises when t is small is that the database contains too few values in order for the estimate of $\mathrm{VaR}_\alpha(X)$ to be accurate, and it is only possible to simulate more values by applying Monte Carlo simulation which once again is beyond the scope of this thesis. An illustration of the estimate of $\mathrm{VaR}_\alpha(X)$ is given in Table 3.2.

k	1	2	3	$\cdots$	$t-3$	$t-2$	$t-1$
$L_{k,t-1}$	$L_{1,t-1}$	$L_{2,t-1}$	$L_{3,t-1}$	$\cdots$	$L_{t-3,t-1}$	$L_{t-2,t-1}$	$L_{t-1,t-1}$

$$\widehat{\mathrm{VaR}}_{0.05}(X) = L_{\lfloor 50 \cdot 0.05 \rfloor + 1, t-1} = L_{\lfloor 2.5 \rfloor + 1, t-1} = L_{3,t-1}$$

for $\alpha = 0.05$ at trading day $t = 51$

Table 3.2: Estimate of $\mathrm{VaR}_\alpha(X)$ from ordered sample of value changes

This table shows explicitly how the value change of $L = -X$ is ordered and how $\mathrm{VaR}_\alpha(X)$ is estimated from the database each trading day t. The probability of losing more than or equal to €300 000 is determined by the company to be less than 5%. If the trading portfolio is supported to take positions which correspond to its accepted $\mathrm{VaR}_\alpha(X)$, then theoretically it holds that $V_0 \leq €300000/\widehat{\mathrm{VaR}}_{0.05}(X)$. V_0 is the capital that could be invested in the portfolio each trading day t if the investor wants to be as close to the risk tolerance level as possible. However, this is absolutely not standard procedure and investors do not invest according to this because of the risk of producing wrong estimates from the model, which could then have catastrophic impact on the portfolio.

Chapter 4

Numerical Results

4.1 Autocorrelation in Count Process Data

Autocorrelation refers to the correlation of a time series, in this case a count process, with its own past and future values. Positive autocorrelation might be considered a specific form of "persistence", i.e. the tendency of a system to remain in the same state from one observation to the next. Four figures will be presented below to illustrate the autocorrelation from the times series of counts corresponding to active assets, surviving assets, birth and death of assets respectively.

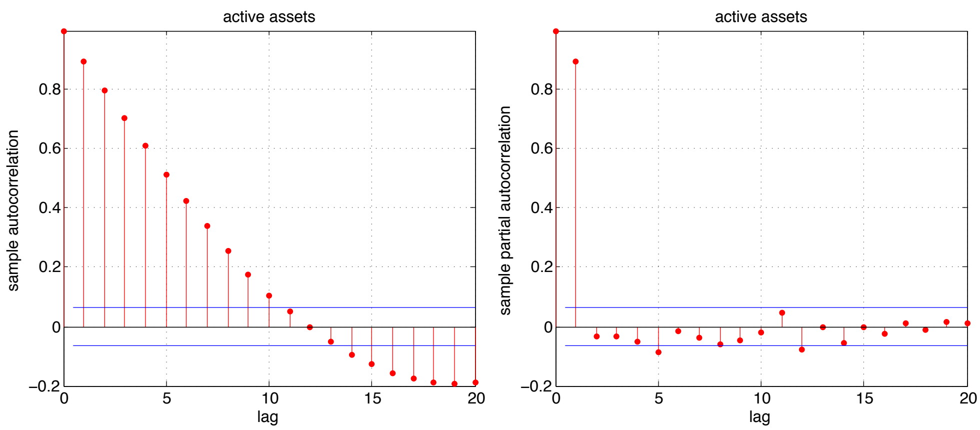


Figure 4.1: Autocorrelation of active assets.

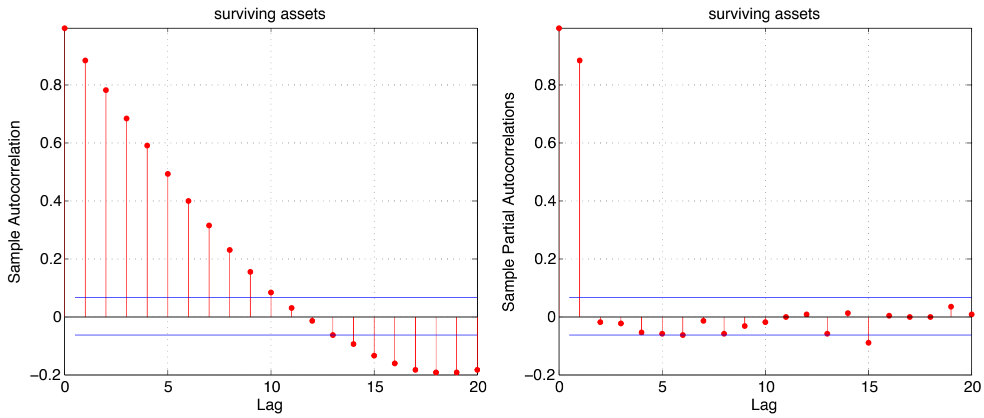


Figure 4.2: Autocorrelation of surviving assets.

These graphs indicate that the count process of the active and surviving assets has a trend since the sample autocorrelation in the left hand graph of both Figure 4.1 and Figure 4.2 exhibit a slow decay as the lag increases. The right hand graph of both Figure 4.1 and Figure 4.2 indicate that the data is highly persistent, i.e. positive movements from the mean tend to be followed by positive movements in the next step and vice versa.

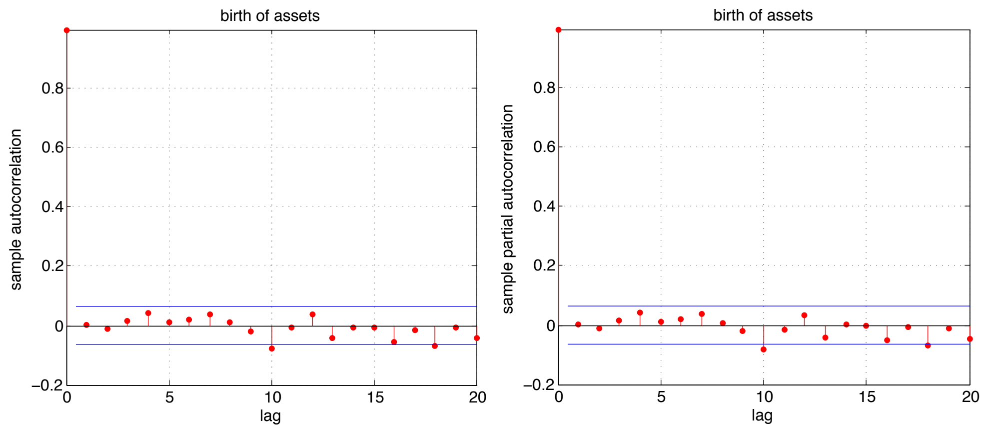


Figure 4.3: Autocorrelation of birth of assets.

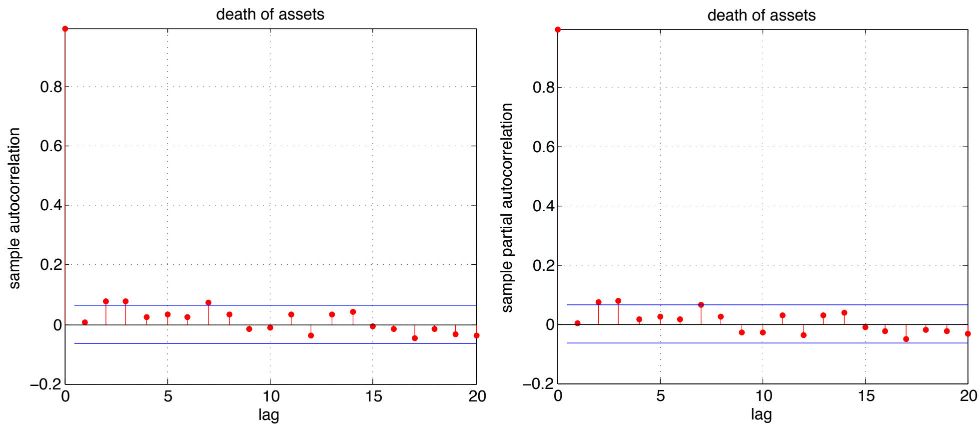


Figure 4.4: Autocorrelation of death of assets.

The graphs show no indication on a trend in the data and furthermore that the count process of births and deaths are highly random. However, this supports the dependence structure model constructed in Chapter 3 earlier to be applicable. Below are some statistics of the count processes gathered in order to illustrate their properties.

statistic	active assets	surviving assets	birth of assets	death of assets
min. count	0	0	0	0
max. count	21	20	9	5
median	7	6	0	1
mode	6	5	0	0
mean	7.767	6.836	0.931	0.925
variance	13.168	11.481	1.658	0.946

Table 4.1: Statistical properties of count processes.

The graphs above together with these statistics indicate that an appropriate AR(1)-process is applicable, which by all means must posses properties of modeling a count process. In [8], it is stated that predictions are based on the integer valued AR(1)-process, which models the counts to evolve as a birth and death (survival) process. This means that the count at trading day t is considered to be the sum of new arrivals at time t and survivors from time $t-1$. In contrast to the usual applications of this model, which assumes that the arrival process is Poisson distributed, it is allowed for the arrivals to follow any distribution within a specified finite set of distributions in the integer class. Three distributions are used to model the arrival process of the count data of this paper and they are the Poisson distribution, Binomial distribution and Negative Binomial distribution. They are respectively appropriate for arrivals that are equi-dispersed (mean and variance equal), under-dispersed (variance less than mean) and over-dispersed (variance greater than mean). In Table 4.1, the variance-mean ratio of the birth process is approximately 1.78 which indicates that a Negative Binomial distribution should be most appropriate as a model of the arriving assets in this context. Unfortunately has this model not been able to be applied in this thesis due to the complex nature of the theories associated with this model and because of the time frame of this thesis.

4.2 Backtest of 1-Day Predictors

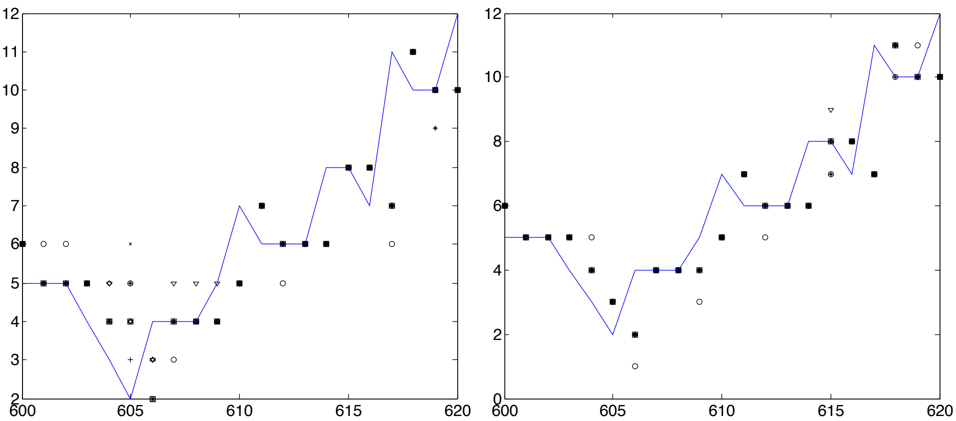


Figure 4.5: Sample one of 1-day predictions from $\hat{H}_a^{t,t+1}$ and $\hat{H}_{b,d}^{t,t+1}$.

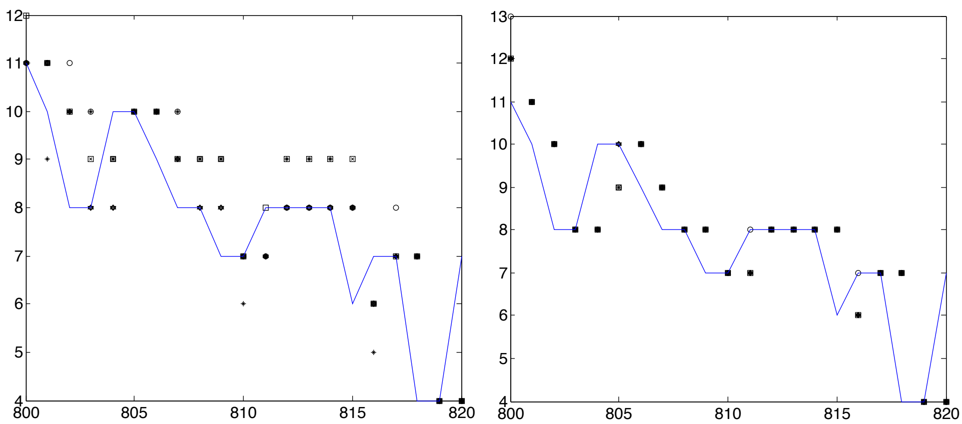


Figure 4.6: Sample two of 1-day predictions from $\hat{H}_a^{t,t+1}$ and $\hat{H}_{b,d}^{t,t+1}$.

These graphs illustrate the accuracy of the predictors $\hat{H}_a^{t,t+1}$ (left hand graph of Figure 4.5 and Figure 4.6) and $\hat{H}_{b,d}^{t,t+1}$ (right hand graph of Figure 4.5 and Figure 4.6) for $w = \{1, 25, 50, 75, 100, 200, 300, 400, 500\}$. The 1-day prediction of the number of active assets are here studied for trading day $t = [600, 620]$ and trading day $t = [800, 820]$. The blue solid line corresponds to the actual number of active assets at each trading day t and the markers $+, \circ, *, \cdot, \times, \square, \diamond, \triangle, \triangledown$ correspond to each w respectively that have been used as an input parameter to the predictors. As can be seen, a large amount of the 1-day predictions are clustered which indicate that the predictors tend to generate similar values. However, there are two qualitative differences between the predictors that can be seen in the figure above. The cluster effect is both greater and more accurate for the $\hat{H}_{b,d}^{t,t+1}$ predictor compared to the $\hat{H}_a^{t,t+1}$ predictor and this seems to hold for all sizes of w. The drawback of these predictors is that they do not manage to be accurate when large jumps occur between trading day t and trading day $t+1$. This is most likely due to the "smoothing effect" of the conditional expected value applied in equations (3.4) and (3.5).

From these graphs above, it is difficult to state which size of w that is optimal in order to generate the most accurate 1-day prediction, but a qualified guess would be $w = 1$. That is, the 1-day prediction on the number of active assets in the portfolio at trading day $t + 1$ is a_t. To gain support to this assumption, we evaluate the 1-day prediction models by the use of statistical measures to reveal their overall performance. The result from this study is summarized in Table 4.2 below.

w	$\hat{H}_a^{t,t+1}$					$\hat{H}_{b,d}^{t,t+1}$				
	normalized abs. error	normalized squ. error	normalized rel. error	normalized 1st score	normalized 2nd score	normalized abs. error	normalized squ. error	normalized rel. error	normalized 1st score	normalized 2nd score
1 (+)	1.1411	2.6406	0.1701 (17.01%)	0.3253 (32.53%)	0.3814 (38.14%)	1.1411	2.6406	0.1701 (17.01%)	0.3253 (32.53%)	0.3814 (38.14%)
25 (∘)	1.4164	3.7364	0.2160 (21.60%)	0.2513 (25.13%)	0.3621 (36.12%)	1.3446	3.4267	0.1991 (19.91%)	0.2718 (27.18%)	0.3641 (36.41%)
50 (∗)	1.3979	3.6063	0.2172 (21.72%)	0.2421 (24.21%)	0.3842 (38.42%)	1.2600	3.1084	0.1860 (18.60%)	0.2832 (28.32%)	0.3958 (39.58%)
75 (·)	1.3243	3.2941	0.2081 (20.81%)	0.2627 (26.27%)	0.3881(38.81%)	1.2238	2.9449	0.1806 (18.06%)	0.2919 (29.19%)	0.3968 (39.68%)
100 (×)	1.2978	3.1689	0.2041 (20.41%)	0.2700 (27.00%)	0.3822 (38.22%)	1.2167	2.9522	0.1794 (17.94%)	0.3000 (30.00%)	0.3911 (39.11%)
200 (□)	1.2537	2.9863	0.2002 (20.02%)	0.2737 (27.37%)	0.4037 (40.37%)	1.1963	2.8338	0.1757 (17.57%)	0.3088 (30.88%)	0.3812 (38.12%)
300 (⋄)	1.2586	2.9214	0.2099 (20.99%)	0.2657 (26.57%)	0.4057 (40.57%)	1.2000	2.8257	0.1810 (18.10%)	0.3086 (30.86%)	0.3757 (37.57%)
400 (△)	1.2900	3.0767	0.2130 (21.30%)	0.2583 (25.83%)	0.4100 (41.00%)	1.2333	2.9833	0.1822 (18.22%)	0.3033 (30.33%)	0.3700 (37.00%)
500 (▽)	1.2640	2.9840	0.2130 (21.30%)	0.2780 (27.80%)	0.3860 (38.60%)	1.2200	2.9160	0.1817 (18.17%)	0.3020 (30.20%)	0.3820 (38.20%)

Table 4.2: Statistics of backtested 1-day predictors.

This table summarizes the statistical properties of the two predictors for different sizes of w. The measures are *normalized* by the total number of prediction steps in order to compare their individual properties. The *1st score* measure is a dummy variable for the number of times the 1-day prediction coincides with the actual number of active assets. The *2nd score* measure is a dummy variable for the number of times the absolute error of the 1-day prediction and the actual number of active assets is one. The table does indeed support the assumption earlier that $w = 1$, which not surprisingly reveals that both predictors perform equally. Furthermore, the $\hat{H}_{b,d}^{t,t+1}$ predictor performs overall statistically better than the $\hat{H}_a^{t,t+1}$ predictor for each w. Thus, since $w = 1$ and $T = 62$ the optimization starts at trading day $t = 63$ and ends at trading day $t = 1000$.

4.3 Sharpe Ratios and Portfolio Performances

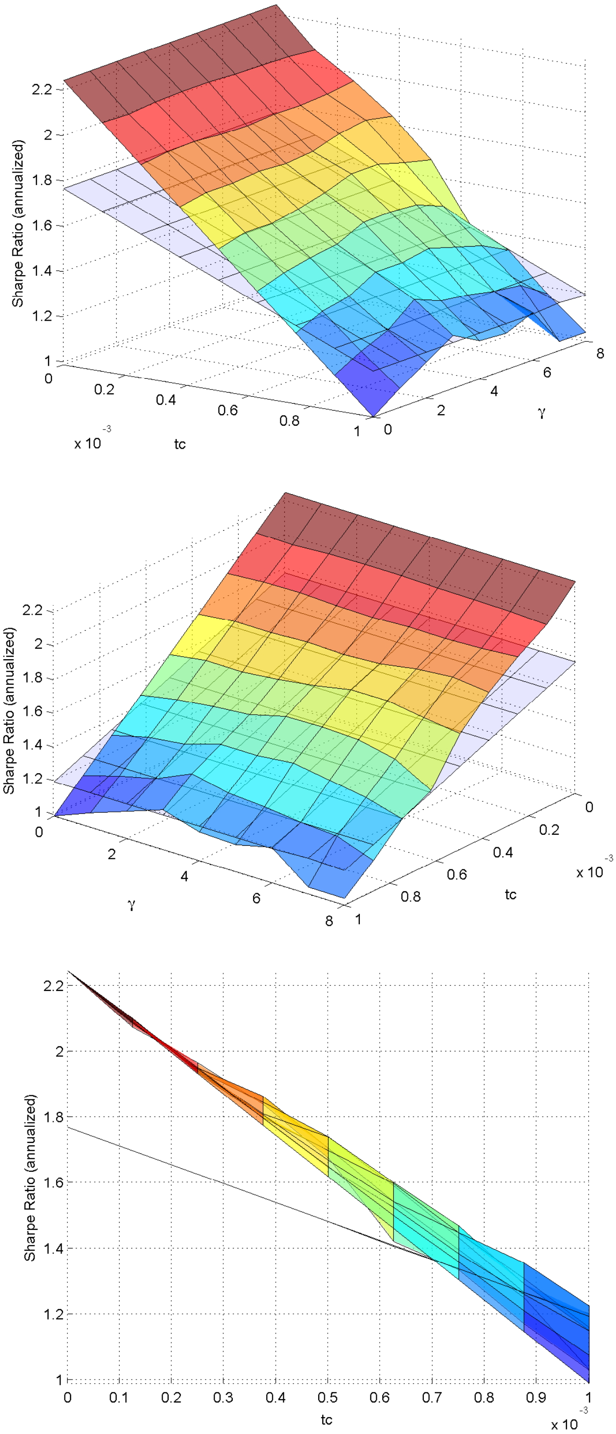


Figure 4.7: Annualized Sharpe ratios of optimal/benchmark trading strategy after transaction costs are subtracted from the portfolio return each trading day t.

The graphs in Figure 4.7 illustrates the three dimensional shape of the annualized Sharpe ratio of the two trading portfolios for different tc and γ values. It is clear that the optimal trading strategy (multicolored plane) outperforms the benchmark trading strategy (purple colored plane) for low tc values. As can be seen, penalizing transaction costs between trading days often result in increased annualized Sharpe ratio values. However, high γ values combined with high tc values has a negative impact . The benchmark trading strategy is independent of γ and thereby the shape of the annualized Sharpe ratio is flat, solely dependent on the transaction cost variable tc. The high amount of activated and deactivated assets, see Figure 2.9, impact the annualized Sharpe ratio negatively since turnovers increases and thereby transaction costs increases when allocation weights are chosen optimally each trading day.

$\text{SR}^{optimal}$ (annualized)

	tc (%)								
γ	0	0.0125	0.025	0.0375	0.050	0.0625	0.075	0.0875	0.1
0	2.243	2.087	1.93	1.772	1.615	1.459	1.302	1.145	0.9885
1	2.243	2.082	1.944	1.794	1.646	1.497	1.357	1.208	1.073
2	2.243	2.094	1.948	1.794	1.644	1.522	1.382	1.259	1.148
3	2.243	2.098	1.937	1.804	1.666	1.54	1.446	1.354	1.223
4	2.243	2.097	1.933	1.796	1.696	1.594	1.445	1.306	1.158
5	2.243	2.09	1.942	1.808	1.737	1.598	1.466	1.29	1.149
6	2.243	2.087	1.94	1.843	1.737	1.588	1.43	1.264	1.199
7	2.243	2.083	1.947	1.861	1.703	1.54	1.394	1.248	1.029
8	2.243	2.071	1.963	1.837	1.673	1.419	1.332	1.17	1.028

$\text{SR}^{benchmark}$ (annualized)

	tc (%)								
γ	0	0.0125	0.025	0.0375	0.050	0.0625	0.075	0.0875	0.1
ind.	1.767	1.695	1.623	1.551	1.48	1.408	1.336	1.264	1.192

Table 4.3: Numerical values of annualized Sharpe ratios of optimal/benchmark trading strategy.

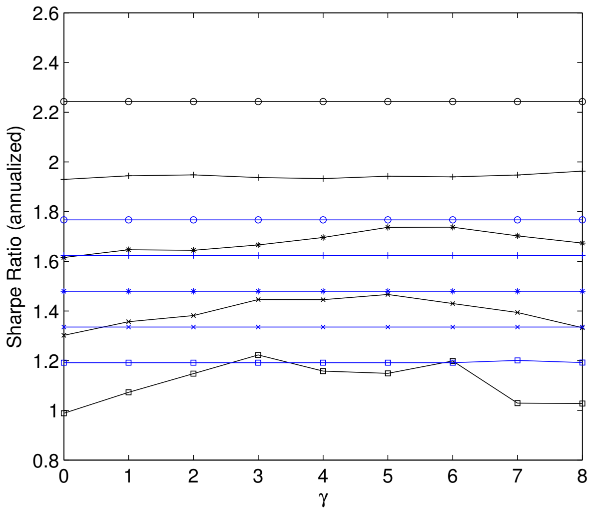


Figure 4.8: Two dimensional illustration of Figure 4.7 for $tc = 0, 0.00025, 0.00050, 0.00075, 0.001$.

This graph illustrates the two dimensional aspect of how the annualized Sharpe ratios of the optimal trading strategy (black colored line) and the benchmark trading strategy (blue colored line) vary for increased γ values. From top to bottom, the black and blue colored lines correspond to $tc = 0, 0.00025, 0.00050, 0.00075, 0.001$ respectively and it can be seen for $tc \geq 0.001$ there is no longer any positive effect from choosing weights optimally, except for $\gamma = 3$ and $\gamma = 6$.

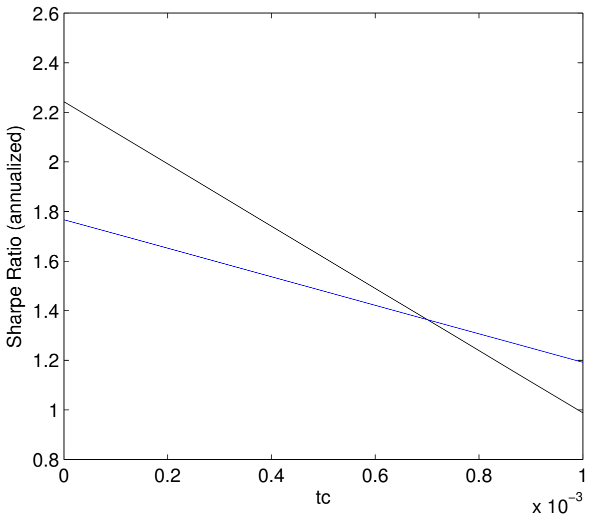


Figure 4.9: Two dimensional illustration of Figure 4.7 for $\gamma = 0$.

This graph illustrates the annualized Sharpe ratio of the optimal trading strategy (black colored line) and the benchmark trading strategy (blue colored line) when $\gamma = 0$, i.e no emphasis is put on reducing transaction costs between trading days when optimal weights are computed. It can be seen that the annualized Sharpe ratio of the optimal trading strategy is linear and for $tc \approx 0.00070$, the two trading strategies are equal in performance.

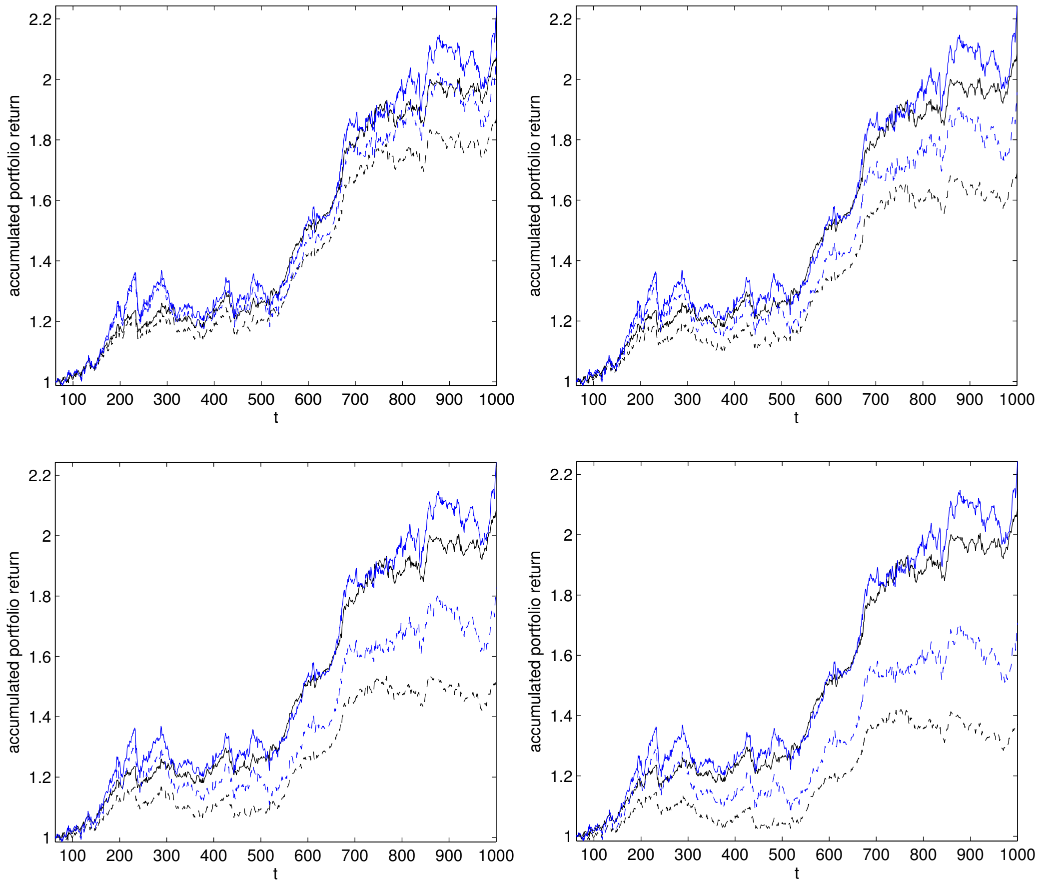


Figure 4.10: Accumulated portfolio returns of optimal/benchmark trading strategy for $tc = 0, 0.00025, 0.00050, 0.00075, 0.001$ and $\gamma = 0$.

The graphs in Figure 4.10 illustrate the accumulated portfolio returns generated by the optimal trading strategy (black color) and the benchmark trading strategy (blue color) when transaction costs are subtracted each trading day t. The black and blue solid lines correspond to $tc = 0$ and the black and blue dashed lines correspond to $tc = 0.00025$ (top left hand graph), $tc = 0.00050$ (top right hand graph), $tc = 0.00075$ (bottom left hand graph) and $tc = 0.001$ (bottom right hand graph).

4.4 Transaction Costs

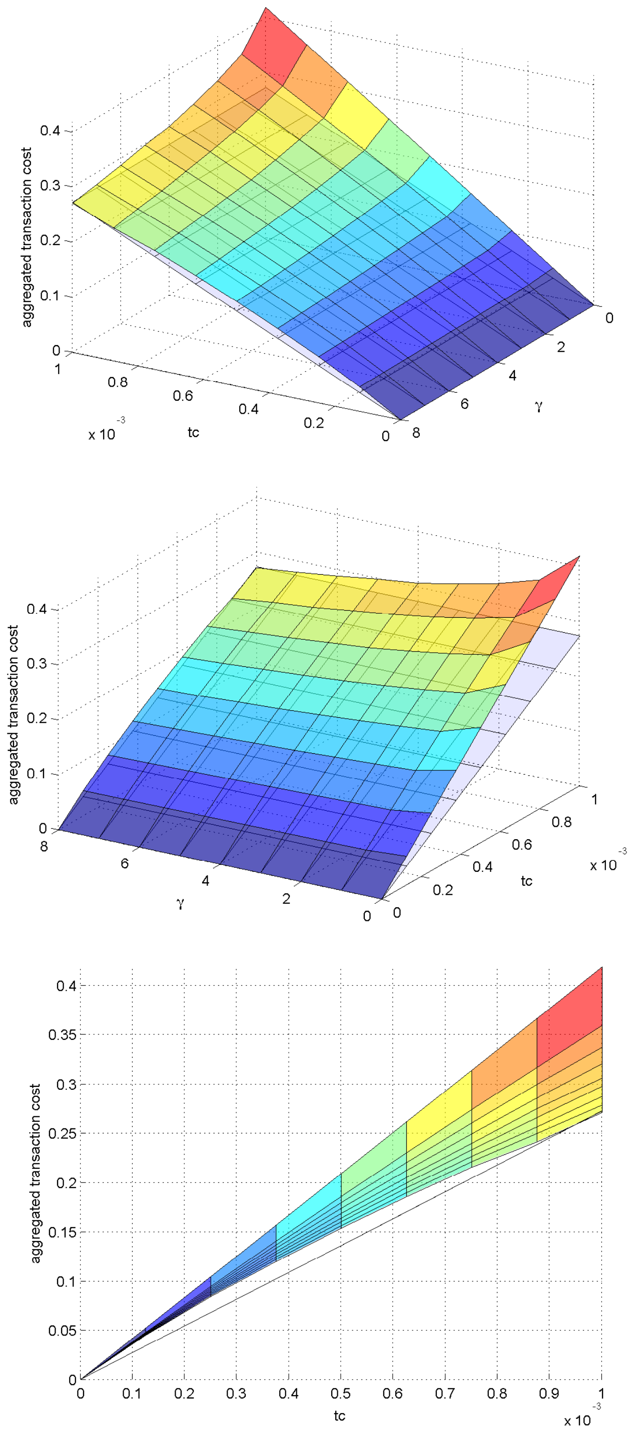


Figure 4.11: Aggregated transaction costs of optimal/benchmark trading strategy.

The graphs in Figure 4.11 illustrate the shape of the aggregated transaction cost and how it depends on tc and γ, where the multicolored plane and purple colored plane once again correspond to the optimal and benchmark trading strategy respectively. When the γ values increase the aggregated transaction cost of the optimal trading strategy decreases, and as γ grows large the aggregated transaction cost tend to converge to its corresponding minimum limit, for each tc. For $\gamma = 0$ and $tc = 0.001$, it can be seen from Table 4.4 below that 41.8% of the invested capital from trading day $t = 63$ to trading day $t = 1000$ is paid as transaction costs when the optimal trading strategy is used. However, for $\gamma = 8$ it can be seen that the aggregated transaction cost of the optimal trading strategy is lowered to 27.1%, compared to the benchmark trading strategy where the aggregated transaction cost is 27.3%.

	$\text{TC}^{optimal}$ (aggregated)								
	tc (%)								
γ	0	0.0125	0.025	0.0375	0.050	0.0625	0.075	0.0875	0.1
0	0	0.0523	0.1046	0.1569	0.2092	0.2615	0.3138	0.3661	0.4184
1	0	0.0479	0.0946	0.1408	0.1858	0.2301	0.2738	0.3166	0.3590
2	0	0.0473	0.0930	0.1369	0.1796	0.2203	0.2602	0.2994	0.3370
3	0	0.0469	0.0913	0.1336	0.1739	0.2122	0.2492	0.2856	0.3204
4	0	0.0465	0.0898	0.1301	0.1687	0.2050	0.2396	0.2747	0.3058
5	0	0.0460	0.0883	0.1275	0.1640	0.1994	0.2331	0.2657	0.2975
6	0	0.0456	0.0869	0.1246	0.1604	0.1945	0.2265	0.2562	0.2871
7	0	0.0452	0.0855	0.1223	0.1563	0.1895	0.2212	0.2496	0.2783
8	0	0.0449	0.0843	0.1199	0.1538	0.1854	0.2154	0.2410	0.2707
	$\text{TC}^{benchmark}$ (aggregated)								
	tc (%)								
γ	0	0.0125	0.025	0.0375	0.050	0.0625	0.075	0.0875	0.1
ind.	0	0.0341	0.0681	0.1022	0.1363	0.1703	0.2044	0.2385	0.2726

Table 4.4: Numerical values of aggregated transaction costs of optimal/benchmark trading strategy.

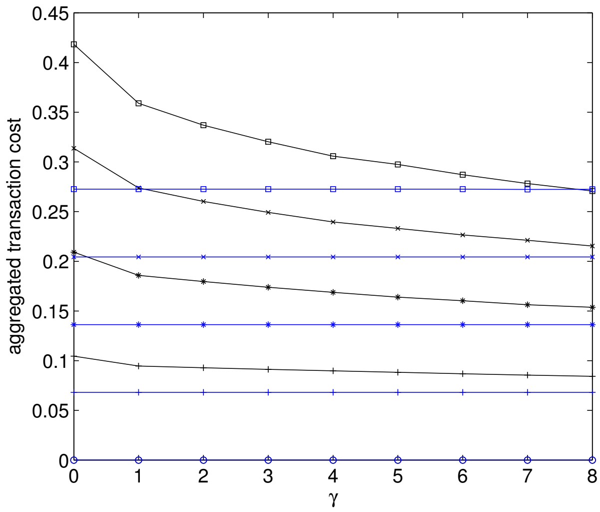


Figure 4.12: Two dimensional illustration of Figure 4.11 for $tc = 0, 0.00025, 0.00050,$ $0.00075, 0.001$.

This graph illustrates the two dimensional view of the aggregated transaction cost generated by the optimal trading strategy (black colored line) and the benchmark trading strategy (blue colored line). The lines in Figure 4.13 above correspond, from bottom to top, to $tc = 0, 0.00025, 0.00050, 0.00075, 0.001$ respectively. It can be seen that the effect of penalizing transaction costs between trading days is greater for large values compared to small values of tc.

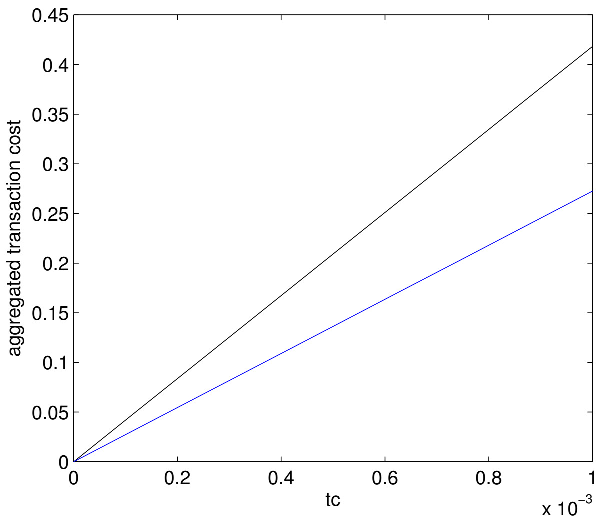


Figure 4.13: Two dimensional illustration of Figure 4.11 for $\gamma = 0$.

This graph illustrates for $\gamma = 0$ the linear evolvement between the aggregated transaction costs of the optimal trading strategy (black colored line) and the benchmark trading strategy (blue colored line), i.e. when no emphasis is put on minimizing transaction costs between trading days.

4.5 Allocation Weights

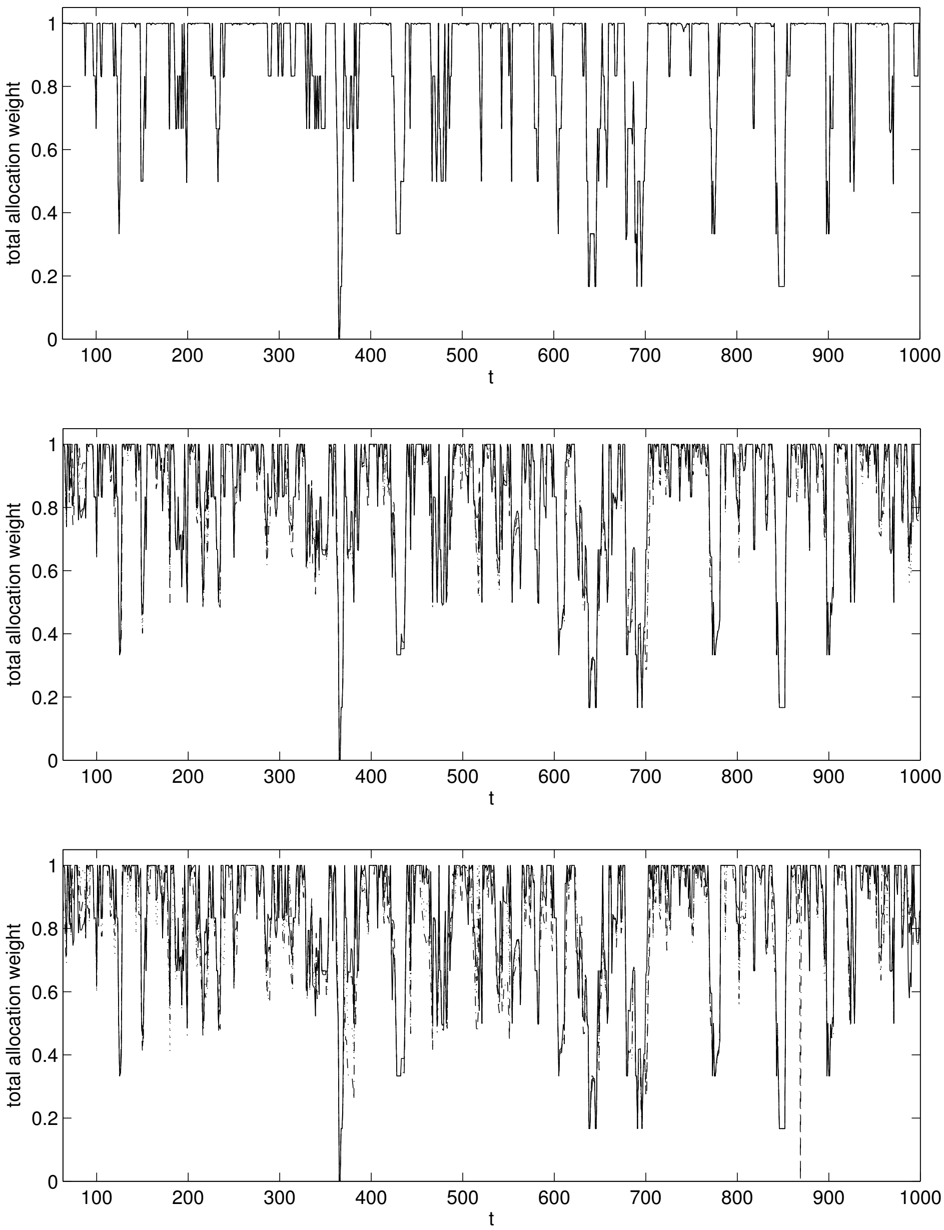


Figure 4.14: Total allocation weight of optimal trading strategy for $tc = 0.00025$, $0.00050, 0.00075, 0.001$ and $\gamma = 0, 4, 8$ (top to bottom).

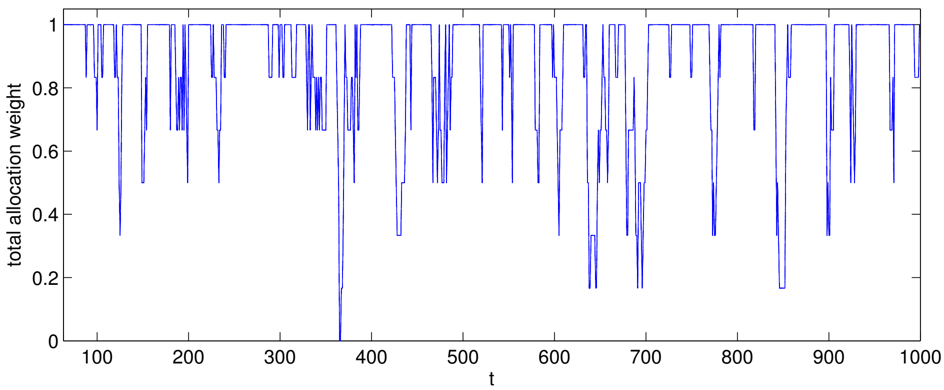


Figure 4.15: Total allocation weight of benchmark trading strategy for $tc = 0.00025$, $0.00050, 0.00075, 0.001$.

Figure 4.14 and Figure 4.15 illustrate, for different γ values, the sum of the allocation weights each trading day t of the optimal and benchmark trading strategy. The optimal-weighted portfolio (black solid line, black dashed line, black dotted line, black dashed-dotted line) and the equal-weighted portfolio (blue solid line, blue dashed line, blue dotted line, blue dashed-dotted line) correspond to $tc = 0.00025$, 0.00050, 0.00075, 0.001 respectively. For $\gamma = 0$ it can be seen for which trading days t the predictor has been connected and lowered the leverage of the investment of the optimal and benchmark trading startegy. When γ increases, it can be seen in Figure 4.14 how the sum of allocation weights have been changed in order for the optimization model (3.19) to find optimal allocation weights. These optimal weights are chosen such that the goal function of (3.19) is maximized when transaction costs are minimized at the same time. These pictures are studied in more detail below, where a smaller time frame is under observation.

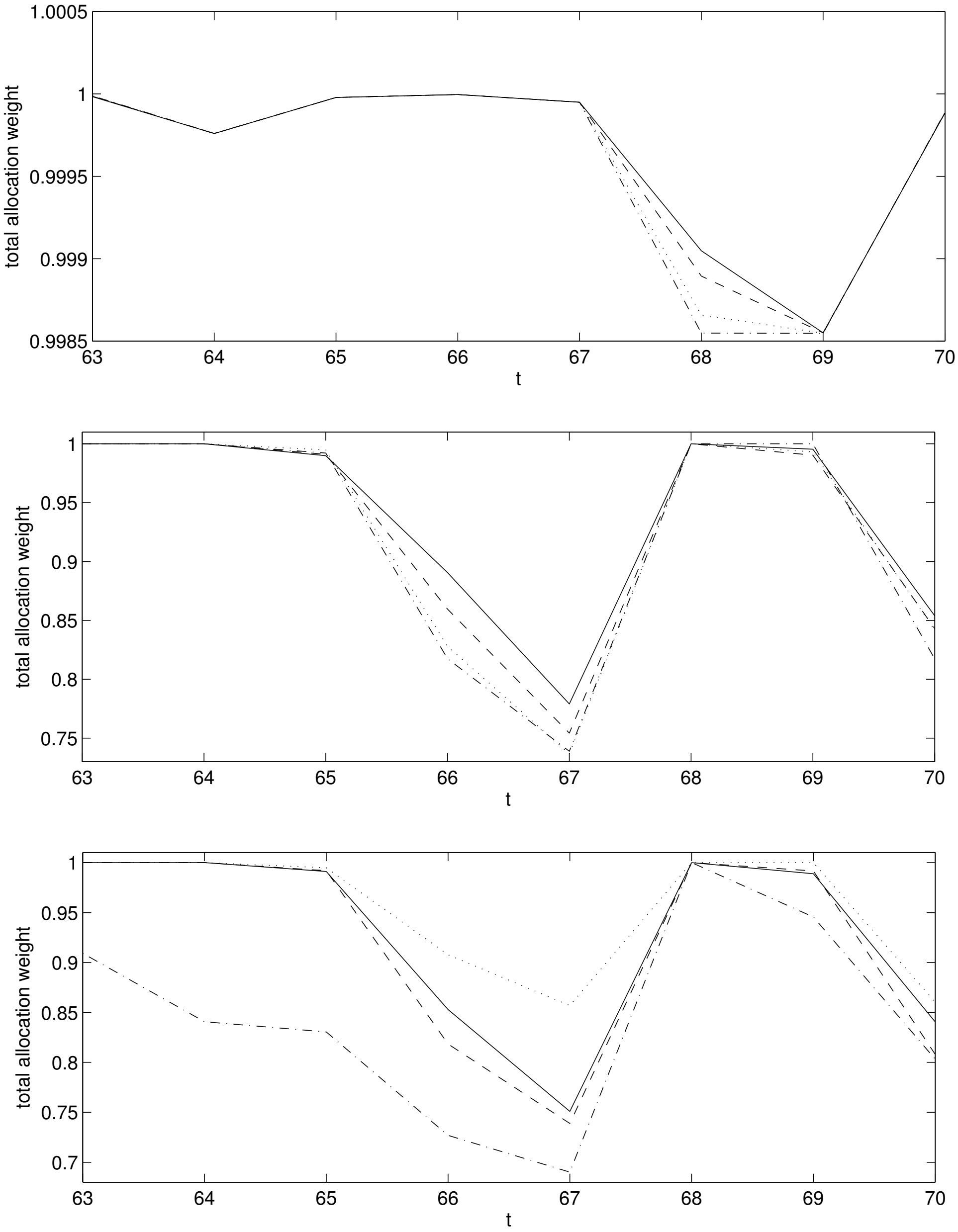


Figure 4.16: Inspection of total allocation weight of optimal trading strategy for $tc = 0.00025, 0.00050, 0.00075, 0.001$ and $\gamma = 0, 4, 8$ (top to bottom).

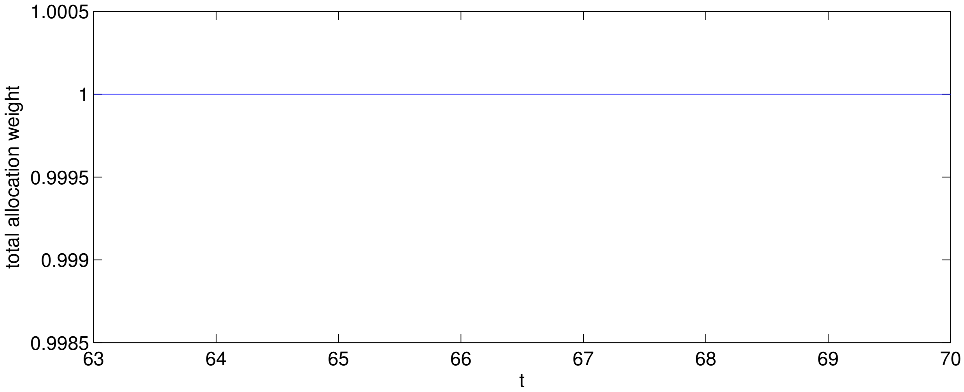


Figure 4.17: Inspection of total allocation weight of benchmark trading strategy for $tc = 0.00025, 0.00050,\ 0.00075, 0.001$.

These graphs illustrate a detailed view on how the sum of the allocation weights evolve between trading day $t = 63$ and trading day $t = 70$, where it can be seen in Figure 4.16 how the outcome depends on the variables tc and γ. Below are the weight distributions illustrated of the affected assets during this time frame, which compare the optimal-weighted asset composition with the equal-weighted asset composition between trading day $t = 63$ and trading day $t = 70$.

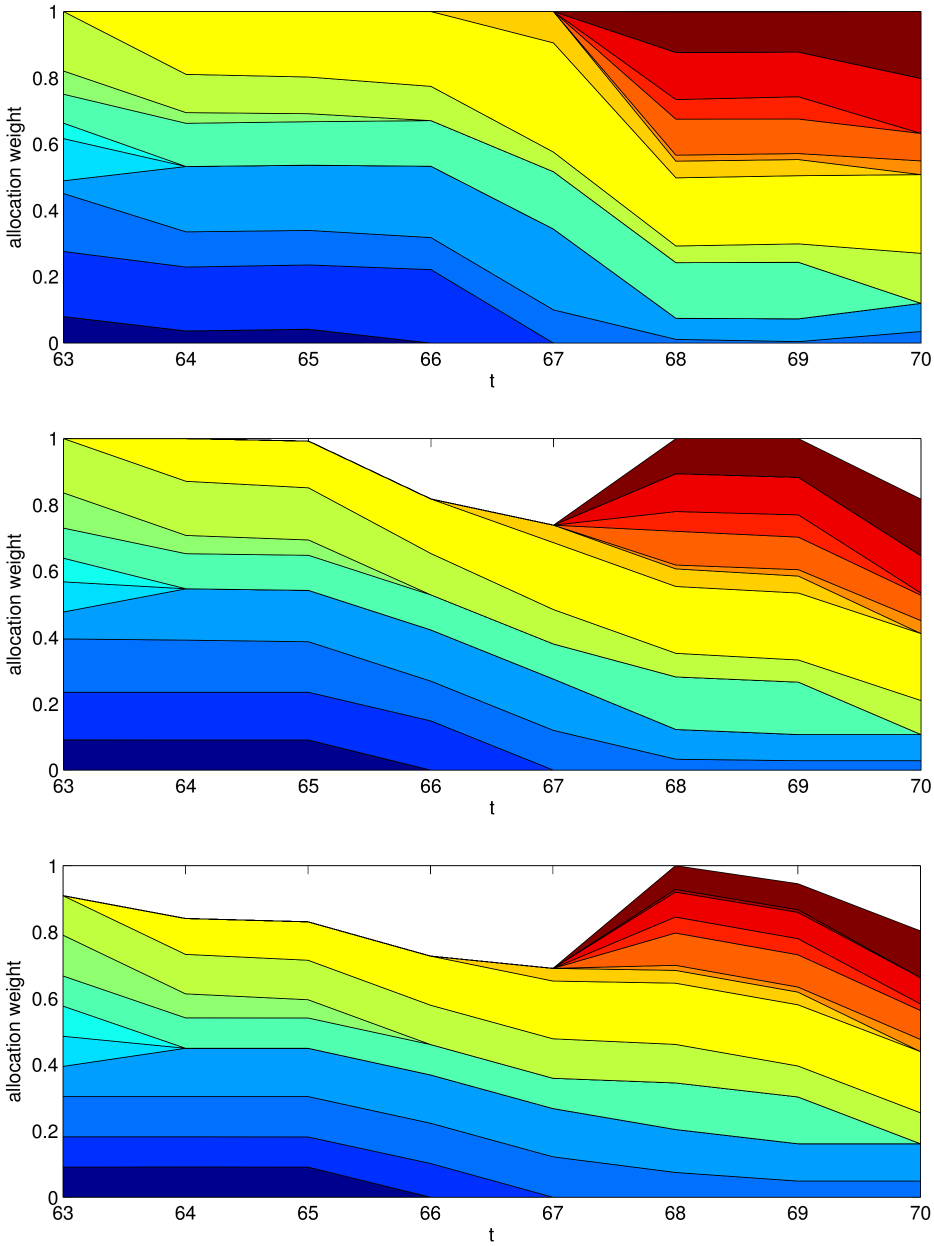


Figure 4.18: Allocation weight distribution of optimal trading strategy for $tc = 0.001$ and $\gamma = 0, 4, 8$ (top to bottom).

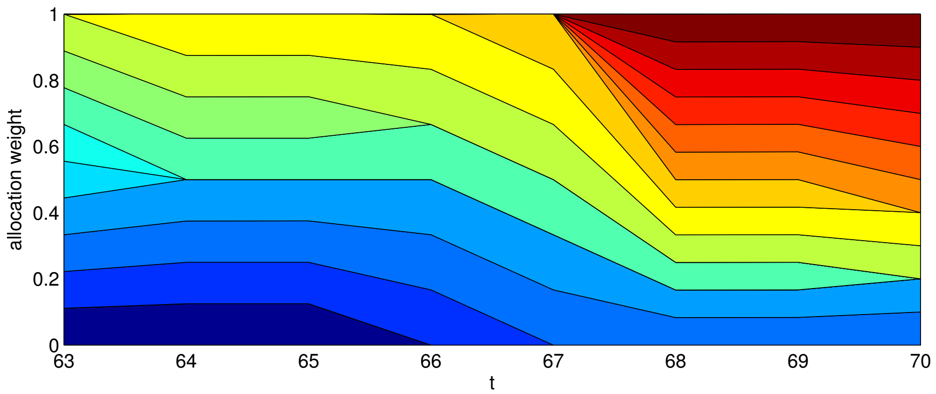


Figure 4.19: Allocation weight distribution of benchmark trading strategy for $tc = 0.001$.

These graphs illustrate how the allocation weights are distributed among the assets between trading day $t = 63$ and trading day $t = 70$. From the *information matrix* (2.2), it is found that the affected assets during this time period are assets $47, 50, 51, \ldots, 65$. They are invested as the optimal trading strategy suggests according to Figure 4.18 as γ is increased. This should be compared to the benchmark trading strategy in Figure 4.19. See Table 4.5 below for more details.

	Allocation Weights															
	optimal-weighted portfolio: $tc = 0.001, \gamma = 0$								optimal-weighted portfolio: $tc = 0.001, \gamma = 4$							
asset/t	63	64	65	66	67	68	69	70	63	64	65	66	67	68	69	70
47	0.0797	0.0365	0.0416	0	0	0	0	0	0.0909	0.0909	0.0909	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0.1963	0.1925	0.1941	0.2215	0	0	0	0	0.1438	0.1438	0.1438	0.1481	0	0	0	0
51	0.1749	0.1062	0.1041	0.0967	0.0998	0.0109	0.0043	0.0349	0.1609	0.1574	0.1526	0.1199	0.1197	0.0328	0.0282	0.0282
52	0.0385	0.1972	0.1968	0.2149	0.2437	0.0637	0.0689	0.0847	0.0813	0.1548	0.1548	0.1548	0.1549	0.0898	0.0790	0.0790
53	0.1273	0	0	0	0	0	0	0	0.0909	0	0	0	0	0	0	0
54	0.0465	0	0	0	0	0	0	0	0.0713	0	0	0	0	0	0	0
55	0.0874	0.1304	0.1311	0.1376	0.1733	0.1677	0.1704	0	0.0909	0.1057	0.1057	0.1057	0.1057	0.1584	0.1582	0
56	0.0698	0.0322	0.0242	0	0	0	0	0	0.1061	0.0555	0.0464	0	0	0	0	0
57	0.1796	0.1152	0.1104	0.1035	0.0588	0.0502	0.0553	0.1510	0.1639	0.1629	0.1571	0.1245	0.1040	0.0713	0.0665	0.1024
58	0	0.1895	0.1977	0.2258	0.3298	0.2056	0.2056	0.2371	0	0.1291	0.1410	0.1645	0.2019	0.2016	0.2023	0.2023
59	0	0	0	0	0.0946	0.0505	0.0490	0	0	0	0	0	0.0528	0.0532	0.0508	0
60	0	0	0	0	0	0.0181	0.0180	0.0418	0	0	0	0	0	0.0114	0.0192	0.0388
61	0	0	0	0	0	0.1084	0.1039	0.0825	0	0	0	0	0	0.1020	0.0983	0.0765
62	0	0	0	0	0	0.0592	0.0674	0	0	0	0	0	0	0.0592	0.0673	0.0067
63	0	0	0	0	0	0.1420	0.1348	0.1657	0	0	0	0	0	0.1144	0.1127	0.1127
64	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
65	0	0	0	0	0	0.1222	0.1209	0.2022	0	0	0	0	0	0.1060	0.1174	0.1706
	optimal-weighted portfolio: $tc = 0.001, \gamma = 8$								equal-weighted portfolio: $tc = 0.001,$ independent of γ							
asset/t	63	64	65	66	67	68	69	70	63	64	65	66	67	68	69	70
47	0.0909	0.0909	0.0909	0	0	0	0	0	0.1111	0.1250	0.1250	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0.0909	0.0909	0.0909	0.1014	0	0	0	0	0.1111	0.1250	0.1250	0.1667	0	0	0	0
51	0.1218	0.1218	0.1218	0.1218	0.1218	0.0745	0.0485	0.0485	0.1111	0.1250	0.1250	0.1667	0.1667	0.0833	0.0833	0.1000
52	0.0909	0.1455	0.1455	0.1455	0.1455	0.1295	0.1126	0.1126	0.1111	0.1250	0.1250	0.1667	0.1667	0.0833	0.0833	0.1000
53	0.0909	0	0	0	0	0	0	0	0.1111	0	0	0	0	0	0	0
54	0.0909	0	0	0	0	0	0	0	0.1111	0	0	0	0	0	0	0
55	0.0909	0.0915	0.0915	0.0915	0.0915	0.1407	0.1410	0	0.1111	0.1250	0.1250	0.1667	0.1667	0.0833	0.0833	0
56	0.1229	0.0727	0.0551	0	0	0	0	0	0.1111	0.1250	0.1250	0	0	0	0	0
57	0.1190	0.1190	0.1190	0.1190	0.1190	0.1162	0.0934	0.0934	0.1111	0.1250	0.1250	0.1667	0.1667	0.0833	0.0833	0.1000
58	0	0.1083	0.1158	0.1478	0.1740	0.1846	0.1846	0.1846	0	0.1250	0.1250	0.1667	0.1667	0.0833	0.0833	0.1000
59	0	0	0	0	0.0384	0.0384	0.0385	0	0	0	0	0	0.1667	0.0833	0.0833	0
60	0	0	0	0	0	0.0153	0.0153	0.0365	0	0	0	0	0	0.0833	0.0833	0.1000
61	0	0	0	0	0	0.0974	0.0974	0.0875	0	0	0	0	0	0.0833	0.0833	0.1000
62	0	0	0	0	0	0.0483	0.0483	0.0194	0	0	0	0	0	0.0833	0.0833	0.1000
63	0	0	0	0	0	0.0753	0.0794	0.0794	0	0	0	0	0	0.0833	0.0833	0.1000
64	0	0	0	0	0	0.0082	0.0082	0	0	0	0	0	0	0.0833	0.0833	0.1000
65	0	0	0	0	0	0.0716	0.0783	0.1413	0	0	0	0	0	0.0833	0.0833	0.1000

Table 4.5: Numerical values of Figure 4.18 and Figure 4.19.

4.6 VaR Estimates

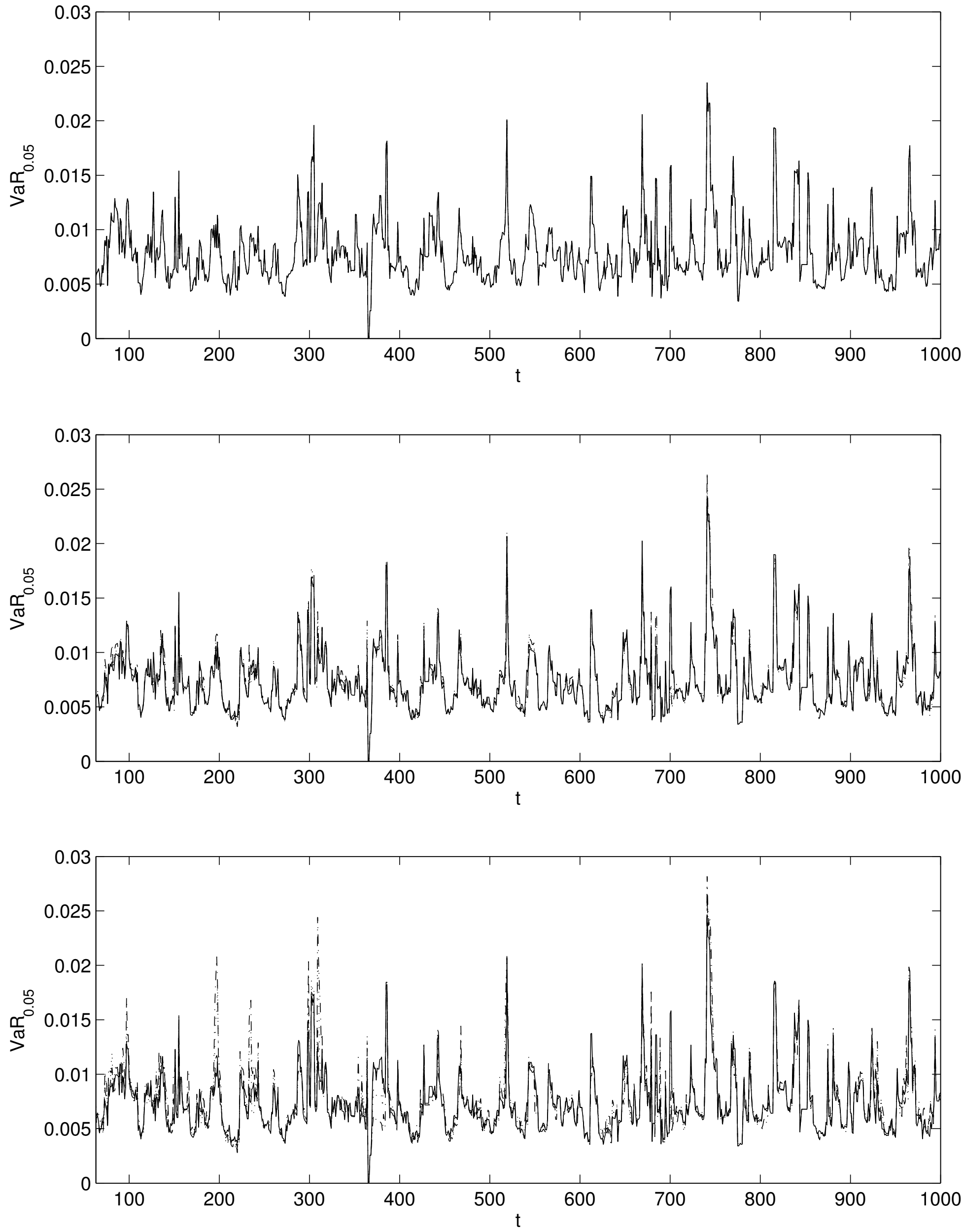


Figure 4.20: $\mathrm{VaR}_{0.05}$ estimates of optimal trading strategy for $tc = 0.00025$, 0.00050, 0.00075, 0.001 and $\gamma = 0, 4, 8$ (top to bottom).

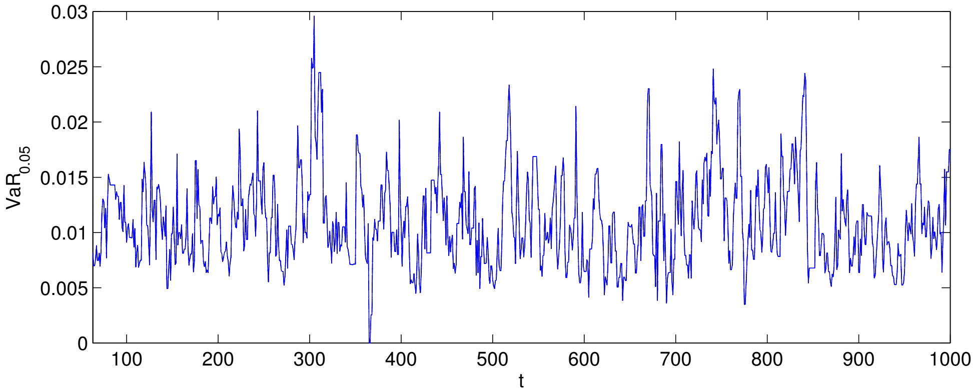


Figure 4.21: $\text{VaR}_{0.05}$ estimates of benchmark trading strategy for tc = 0.00025, 0.00050, 0.00075, 0.001.

These figures compares the $\text{VaR}_{0.05}$ estimates of the optimal trading strategy in Figure 4.20 and the benchmark trading strategy in Figure 4.21. By studying the pictures it is clear that the optimal trading strategy generates much more stable estimates for all tc and all γ compared to the benchmark trading strategy. Furthermore, are the spikes of the estimates less frequently occurring when optimal allocation weights are used to construct the portfolio in comparison to being equal-weighted. This is not surprising since the optimization model (3.19) generates optimal allocation weights that maximizes the ratio of expected portfolio return and portfolio variance. Thus, a lowered variance results in more stable VaR-estimates. The capital that is at most allowed to be invested each trading day is $V_0 \leq €300000/\widehat{\text{VaR}}_{0.05}$, where $\widehat{\text{VaR}}_{0.05}$ is the estimate on each trading day t in the graphs above.

4.7 Figures

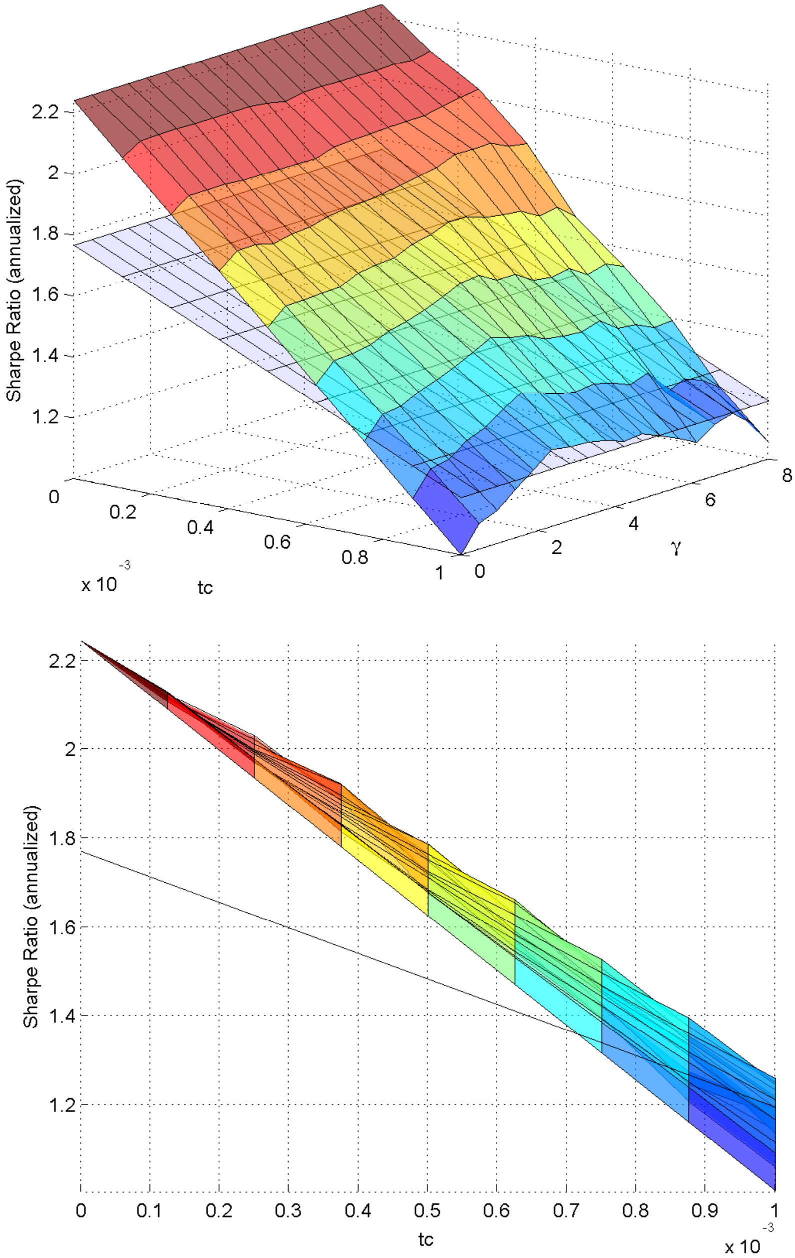


Figure 4.22: Annualized Sharpe ratio of optimal/benchmark trading strategy after transaction costs are subtracted from the portfolio return each trading day t.

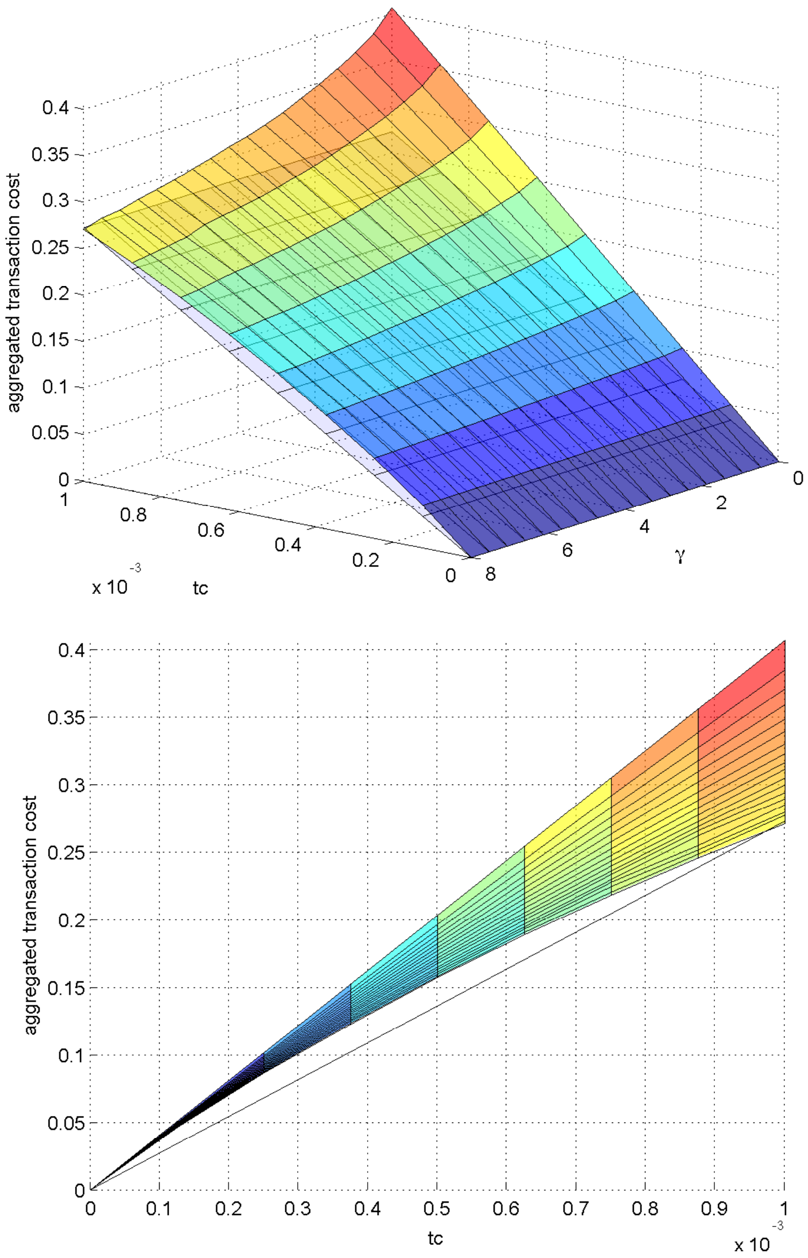


Figure 4.23: Aggregated transaction costs of optimal/benchmark trading strategy.

Figure 4.22 and Figure 4.23 illustrate the outcome when 25% of the capital is invested equally among the assets when using the optimal trading strategy compared to the benchmark trading strategy.

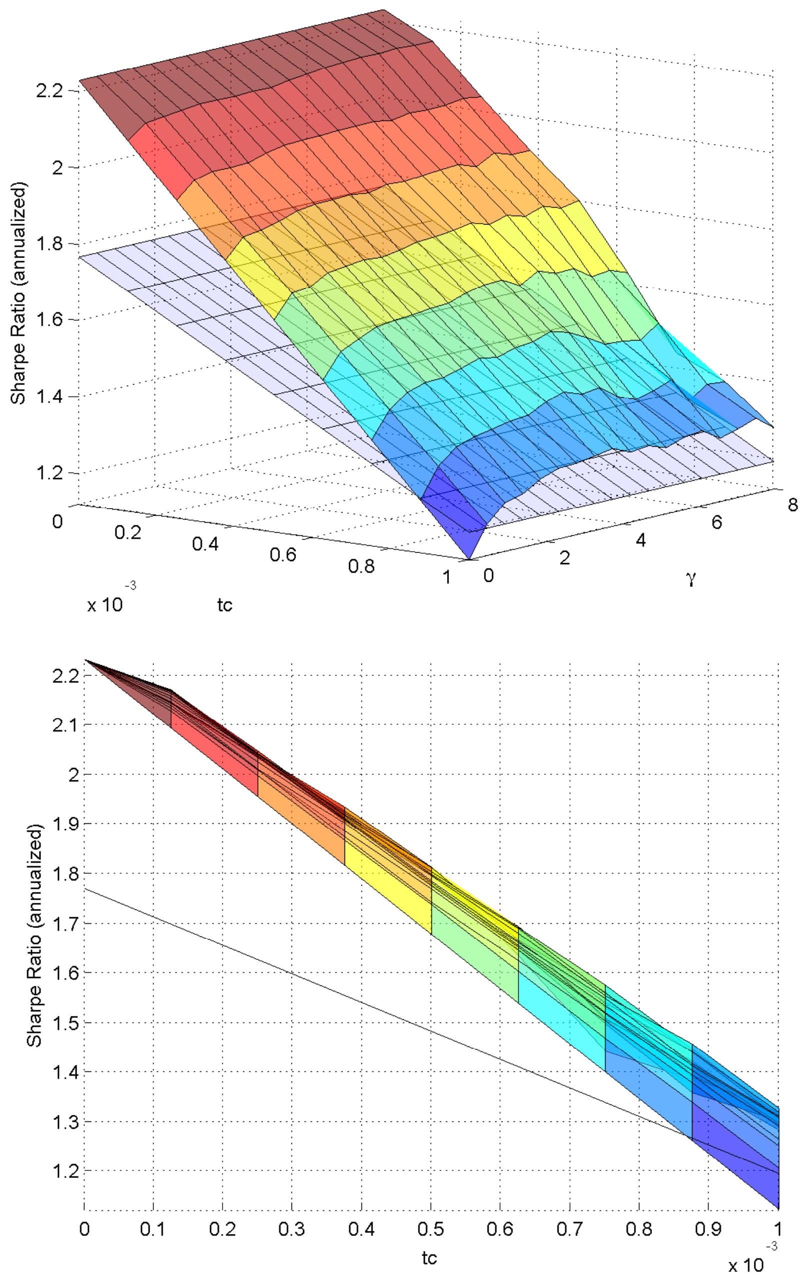


Figure 4.24: Annualized Sharpe ratio of optimal/benchmark trading strategy after transaction costs are subtracted from the portfolio return each trading day t.

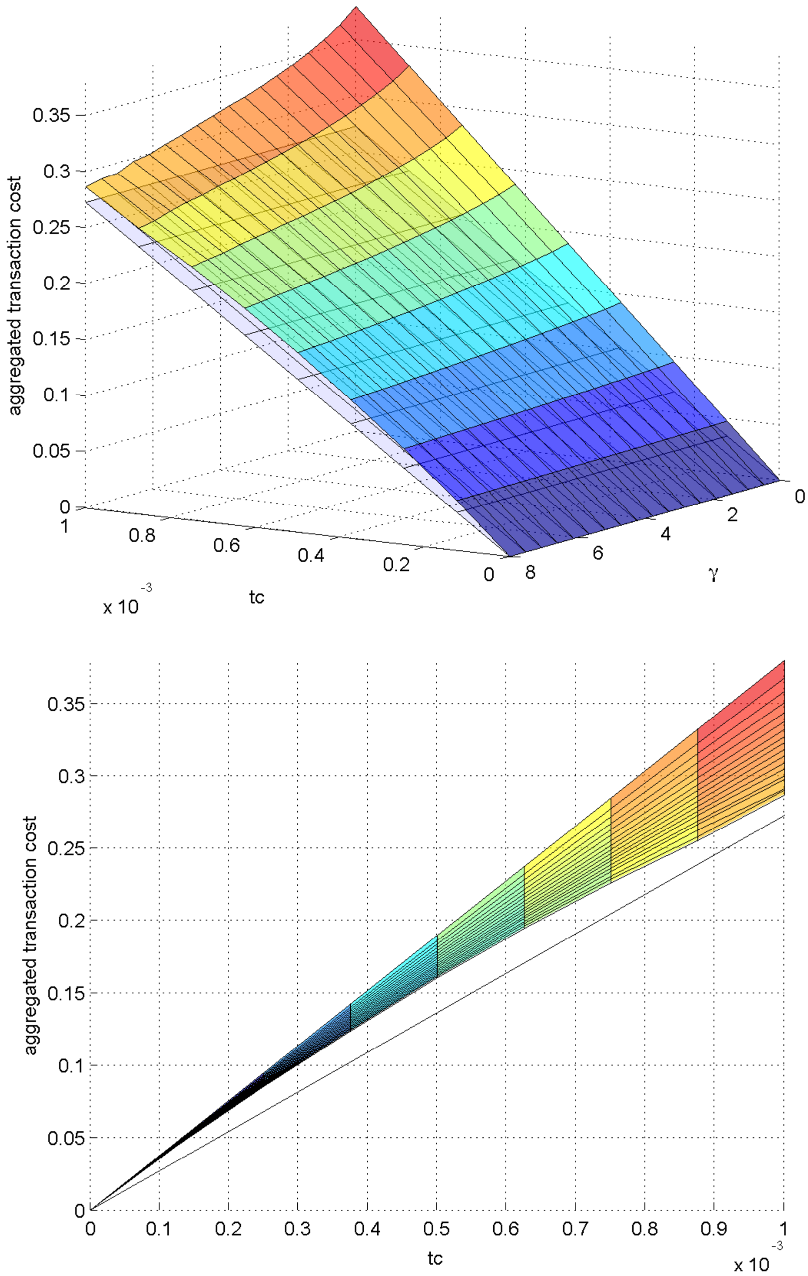


Figure 4.25: Aggregated transaction costs of optimal/benchmark trading strategy.

Figure 4.22 and Figure 4.23 illustrate the outcome when 50% of the capital is invested equally among the assets when using the optimal trading strategy compared to the benchmark trading strategy.

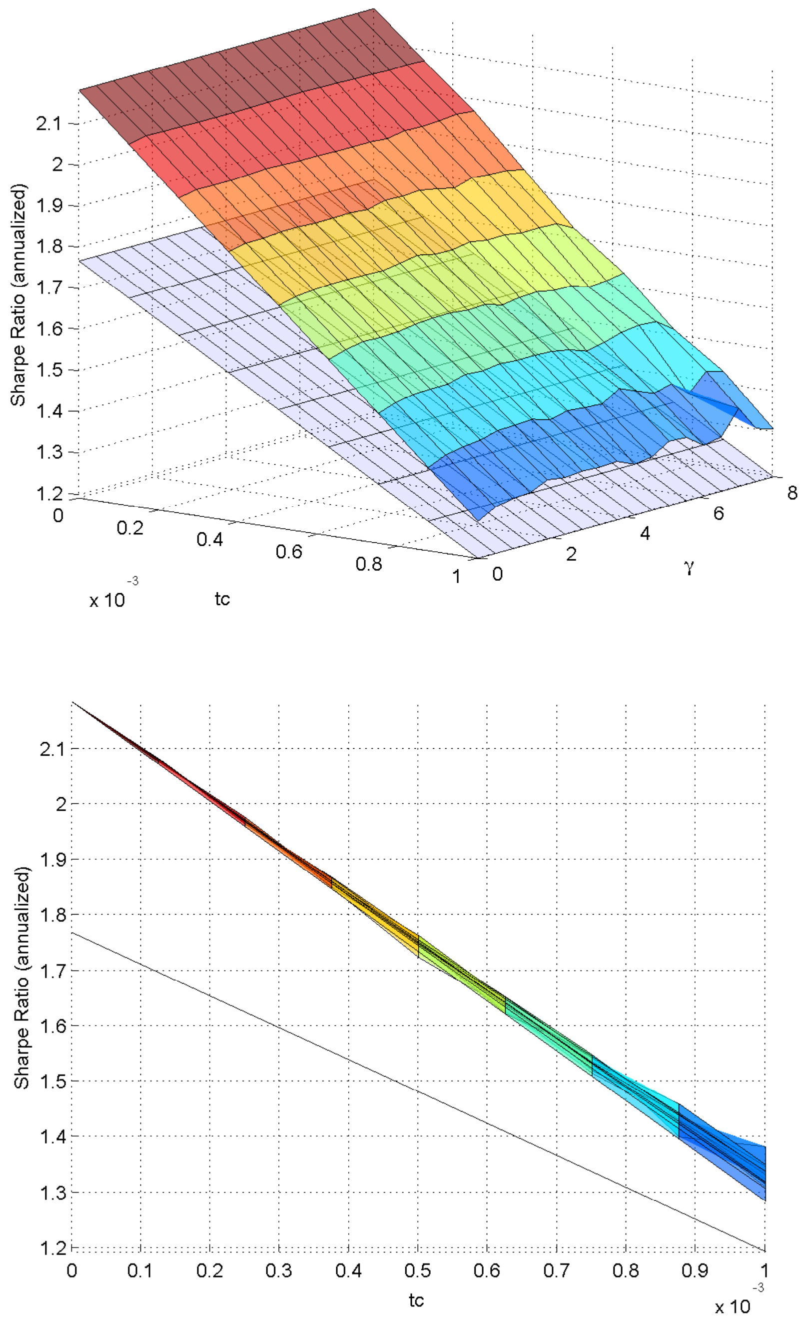
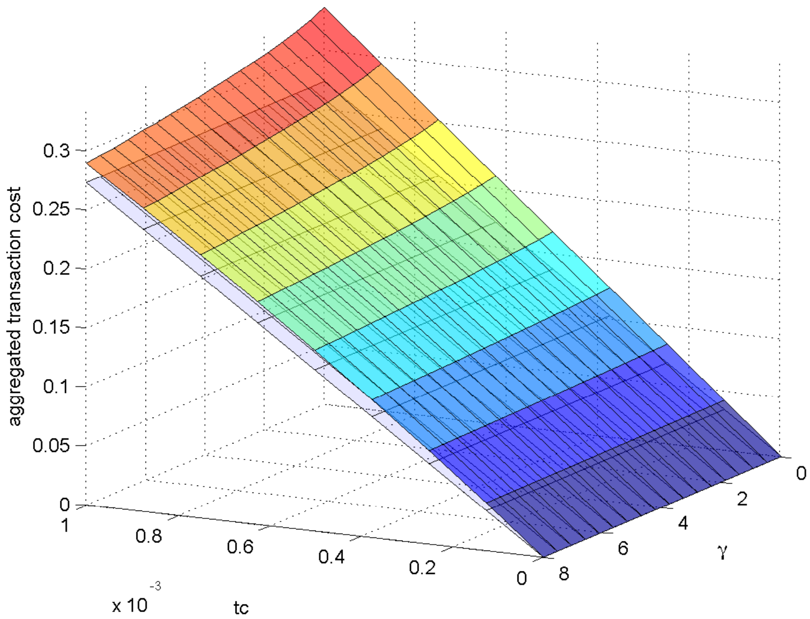


Figure 4.26: Annualized Sharpe ratio of optimal/benchmark trading strategy after transaction costs are subtracted from the portfolio return each trading day t.



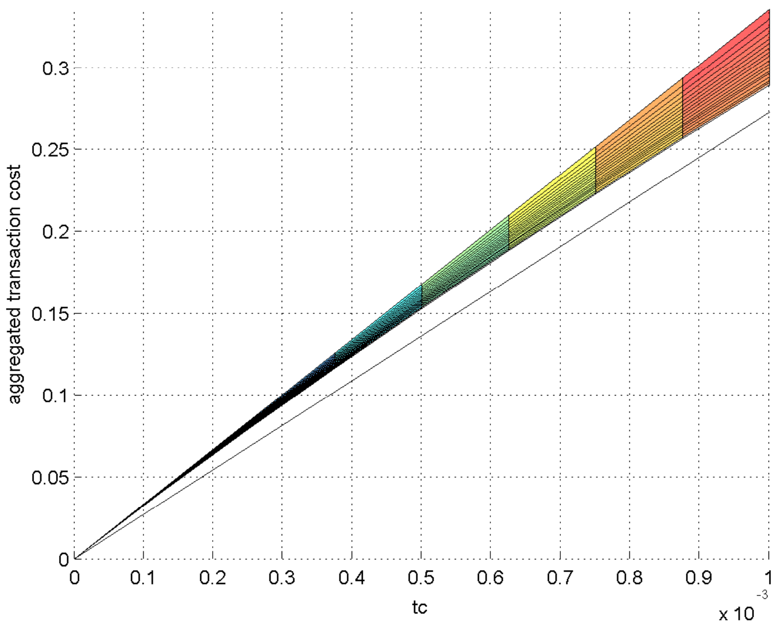


Figure 4.27: Aggregated transaction costs of optimal/benchmark trading strategy.

Figure 4.22 and Figure 4.23 illustrate the outcome when 75% of the capital is invested equally among the assets when using the optimal trading strategy compared to the benchmark trading strategy.

Chapter 5

Conclusions

In this thesis we give a thorough comparison between an optimal trading strategy and a benchmark trading strategy when transaction costs between trading days are taken into account. Optimal allocation weights are generated by maximizing the modified Sharpe ratio when transaction costs are penalized. The modification of the expected return of an asset is implemented in the optimization in order to reflect TII's belief on how assets are expected to perform in the future. These allocation weights are invested in the portfolio accordingly and the performance is compared to an equal-weighted portfolio approach. It is found that a mixture of being 50% to 75% equal-weighted and 50% to 25% optimal-weighted, respectively, each trading day increases the annualized Sharpe ratio of the portfolio, compared to being 100% equal-weighted. This portfolio composition is used to determine the capital allowed to be invested at each trading day and this is computed from the $\text{VaR}_{0.05}$ estimates of the portfolio. Furthermore, it is found that the effect of penalizing transaction costs between trading days is in fact a lower aggregated transaction cost of the optimal portfolio. When the penalty parameter γ is increased the aggregated transaction cost decreases. However, too high values of the penalty parameter γ and of the transaction costs tc can interfere with the feasibility region of the optimization mode, which could yield non-optimal solutions. This is an area that could be studied further.

Since the study in this thesis is based on recreated data reflecting the true confidential data, it is necessary for TII to apply the models obtained on true data, with the actual transaction costs they face from trading, in order to determine which trading model is best suited in their daily business.

As mentioned earlier, another way to predict the number of portfolio assets of the next coming day is to use an integer valued AR(1)-process as a foundation. This would produce values which agree with the properties of the data obtained with the model established in this thesis when the latter is rounded. The number of portfolio assets predicted by using an integer valued AR(1)-process do not need to be rounded, and is therefor a better prediction than the prediction generated from the model in this thesis. This should therefore be studied further in detail, and the models and theory that apply is given in [8].

Changing the performance measure, i.e. the expected return of an asset, could result in a large difference in the optimal-weighted portfolio performance. Thus, it is advisable to study the impact of this by searching for different approaches, and analyzing the result of these.

The amount of capital that is invested during this analyzed time frame is large. It would therefore be interesting and of great importance to see how the discounting effect on the invested capital affects the outcome of the portfolio performance when using the two trading strategies.

Bibliography

[1] Gut, A. (2009) *An Intermediate Course in Probability*, second edition. Springer, London, UK, p. 11:33-37

[2] Taylor, H. M. and Karlin, S. (1998) *An Introduction to Stochastic Modeling*, third edition. Academic Press, London, UK, p. 95-112

[3] Ross, S. M. (2010) *Introduction to Probability Models*, tenth edition. Academic Press, London, UK, p. 191-275

[4] Scherer, B. (2007) *Portfolio Construction and Risk Budgeting*, third edition. Risk Books, London, UK, p. 167-177

[5] Cornuejols, G. and Tütüncü, R. (2007) *Optimization Methods in Finance*, first edition. Cambridge University Press, Cambridge, UK, p. 111:142-145

[6] Jorion, P. (2003) *Financial Risk Manager Handbook*, second edition. John Wiley & Sons, New Jersey, USA, p. 246

[7] Hult, H., Lindskog, F., Hammarlid, O. and Rehn, C. J. (2012), *Risk and Portfolio Analysis: Principles and Methods*, first edition. Springer, London, UK, p. 183-186

[8] McCabe, B. P. M. and Martin, G. M. (2003) *Coherent Bayesian Predictions of Low Count Time Series*, `http://users.monash.edu.au/~gmartin/count_03-11-18.pdf`.