Royal Institute of Technology

Master Thesis in Financial Mathematics

Internal Market Risk Modelling for Power Trading Companies

Markus Ahlgren

mahlg@kth.se

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Abstract

Since the financial crisis of 2008, the risk awareness has increased in the financial sector. Companies are regulated with regards to risk exposure. These regulations are driven by the Basel Committee that formulates broad supervisory standards, guidelines and recommends statements of best practice in banking supervision. In these regulations companies are regulated with own funds requirements for market risks.

This thesis constructs an internal model for risk management that, according to the "Capital Requirements Regulation" (CRR) respectively the "Fundamental Review of the Trading Book" (FRTB), computes the regulatory capital requirements for market risks. The capital requirements according to CRR and FRTB are compared to show how the suggested move to an expected shortfall (ES) based model in FRTB will affect the capital requirements. All computations are performed with data that have been provided from a power trading company to make the results fit reality. In the results, when comparing the risk capital requirements according to CRR and FRTB for a power portfolio with only linear assets, it shows that the risk capital is higher using the value-at-risk (VaR) based model. This study shows that the changes in risk capital mainly depend on the different methods of calculating the risk capital according to CRR and FRTB respectively and minor on the change of risk measure.

Keywords: Power Market, Electricity, Forward Curve, Market Risk, VaR, ES, Basel, CRR, FRTB, Risk Management

Intern Marknadsrisk Modellering för Energihandelsföretag

Sammanfattning

I samband med finanskrisen 2008 har riskmedvetenheten ökat i den finansiella sektorn. Företag regleras mot riskexponering av föreskrifter som drivs av Baselkommittén, de utformar tillsynsstandarder och riktlinjer samt rekommenderar åtgärder av bästa praxis. I dessa föreskrifter regleras företag av kapitalbaskrav mot marknadsrisker.

I det här examensarbetet beskrivs processen för att ta fram en intern riskmodell, enligt "Capital Requirements Regulation"(CRR) respektive Fundamental Review of the Trading Book"(FRTB), för att beräkna de lagstadgade kapitalkraven mot marknadsrisker. Kapitalbaskraven enligt regelverken jämförs för att förstå hur det föreslagna bytet till en expected shortfall (ES) baserad modell i FRTB kommer att påverka kapitalbaskraven. I alla beräkningar används data från ett elhandelsföretag för att göra resultaten mer intressanta och verklighetsanpassade. I resultatdelen, vid jämförelse av riskkapitalkraven enligt CRR och FRTB för en energiportfölj med endast linjära tillgångar kan det ses att riskkapitalet blir högre med en value-at-risk (VaR) baserad modell. Den viktigaste upptäckten med detta är att skillnaden i riskkapitalkraven inte främst beror på de olika riskmåtten utan snarare de olika metoderna för att beräkna riskkapitalet enligt CRR och FRTB.

Nyckelord: Elmarknad, Elektricitet, Forwardkurva, Marknadsrisk, VaR, ES, Basel, CRR, FRTB, Riskhantering

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Markus Ahlgren

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Chapter 1

Introduction

Since the global financial crisis of 2008, the risk awareness in the financial sector has increased. Companies are regulated with regards to risk exposure. These regulations are driven by the Basel Committee that formulates broad supervisory standards, guidelines and recommends statements of best practice in banking supervision. In expectation those member authorities and other national authorities will take action to implement the regulations through their own national systems. In Sweden this is regulated on national level by Swedish Financial Supervisory Authority (FI), which in turn is governed by EU rules. The attitude towards risks has changed since the last crisis. In the past risk management was often seen as a regulatory need, but now companies has started to realise the benefits of using risk measures as guidelines for decision making.

Companies are regulated by own funds requirements for market risks to protect them. Otherwise, when the market conditions change, the companies could suffer greatly. To avoid these situations, risk management is important and has received considerable attention since the financial crisis. In the financial crisis of 2008, weaknesses in the current regulation for capitalising trading activities was detected, value-at-risk (VaR) as risk measure for capturing market risks was one of them. In response to this, the Basel Committee initiated a fundamental review of the trading book regime. Companies are regulated by capital requirements against market risks, these capital requirements are currently based on a VaR model in the "Capital Requirements Regulation" (CRR), but there is a proposal in the "Fundamental Review of the Trading Book" (FRTB) to move from this VaR model to an expected shortfall (ES) model.

At present, FI has a standardised VaR calibrated model that companies are allowed to use for capital requirements calculations. Institutions under supervision of FI may also apply for permission to use an internal model for certain risk categories to calculate own funds requirements for market risks. If consent shall be given the institution to use an internal model they must fulfil a number of requirements prescribed in the regulations. Today both the standardised and internal models are VaR based, but will according to FRTB be replaced with ES based models. In FI's guidelines for implementing an internal risk model they have suggested one of the following methods for calculating VaR, "Historical Simulations", "Variance-Covariance" or "Monte Carlo". This is the reason why one of these methods will be used in this thesis, even though it's clear that there exists other superior methods.

Implementing an internal risk model for a power portfolio differs from doing it in other markets, since electricity as a commodity has special characteristics. Electricity is a non-storable and highly volatile commodity. The power market is complex and hard to predict by the nature of the underlying commodity. Electricity is non-storable and hence difficult to move forward in time. This implies that electricity in the Nordics can't be produced in the summer and used in the winter when the demand is higher. This makes seasonal trends occur in the electricity price, with lower prices in the summer and higher prices in the winter. Hence a power portfolio can't be hedged with physical electricity, instead hedging in this market is carried out by using futures and forward contracts in long or short positions. The complexity of the power market and the underlying commodity electricity makes it difficult to model from a risk management point of view. The high volatility and the large fluctuations in the market implies exposures to large risks and hence proper risk management is important for companies that are active in this market.

1.1 Purpose of the Thesis

The purpose of this thesis is two-folded: Firstly, to show how the changes from the present regulation CRR to the new proposals in FRTB will affect the capital requirements. Secondly, be a guideline for implementing a regulatory approved internal risk model for power portfolios. The aim with this thesis is to show all the quantitative steps that are involved in implementing an internal risk model. There is a lot of literature available about modelling the energy market, but usually only one step of the modelling chain takes into consideration, e.g. "forward curve construction" or "factor model". This makes it difficult to see the big picture. The purpose here is to combine all important parts that are required to build an internal risk model, rather than concentrating on one part. Hopefully this thesis will fill these gaps and contribute to future work.

The theoretical background will be explained since that's the foundation of all different mathematical methods, but the attention will be paid to implementations. Different mathematical models will be implemented and combined to achieve the final results. More specific, an internal risk model, fulfilling CRR and FRTB, to calculate capital requirements for market risks will be developed. The capital requirements according to each regulation will be compared to understand how the suggested move to an ES model in FRTB will affect the capital requirements. An internal risk model is a model developed to analyse the overall risk position and to quantify risks in monetary units to determine the economic capital required to meet those risks. The main purpose of using an internal, instead of the standardised, risk model would be to fully integrate processes of risk and capital management within the company. Another reason to use an internal risk model would be to possibly lower the capital requirement for the company. This thesis will also motivate the choice of risk estimation method and identify the advantages with using ES as a risk measure instead of VaR.

Finally, to perform this investigation, an internal risk model, calculating VaR and ES, will be implemented for a power portfolio that has been provided from a power trading company. All calculations and simulations will be carried out in MATLAB.

1.2 Outline of the Thesis

In Chapter 2, the background of the Nordic power market will be described, which will give the reader a deeper understanding of the remaining paper. It helps the unfamiliar reader to get a brief overview and understanding of the Nordic power market that later will be modelled. Chapter 3 will summarise the most important parts of CRR and FRTB that will be used in this thesis. Chapter 4 discusses risk measures and different risk estimation methods and selects the most suitable method for power portfolios. Chapter 5 shows the methodology of building an internal risk model with all the steps that are involved, and some partial results. Chapter 6 states the results that have been achieved, illustrated with charts and numbers. Chapter 7 discusses the obtained results, expresses the main conclusions in the thesis and proposes future work.

Chapter 2

Background

The power market is a highly volatile market, which implies large risks. To investigate those risks it's important to understand the nature of the power market. This chapter will briefly describe the Nordic power market and get the reader familiar with the market, which will be helpful to get a deeper understanding of the following chapters.

2.1 Power Market

Electricity is an essential part of our modern lives, both for households and industries. A stable power market is a foundation for our modern society. The Nordic power market is divided into four main parts according to [1]: Day-ahead market (Nord Pool Spot), Intraday market (Nord Pool Spot), Financial market (NASDAQ OMX) and Balancing market (TSO), as illustrated in Figure 2.1.

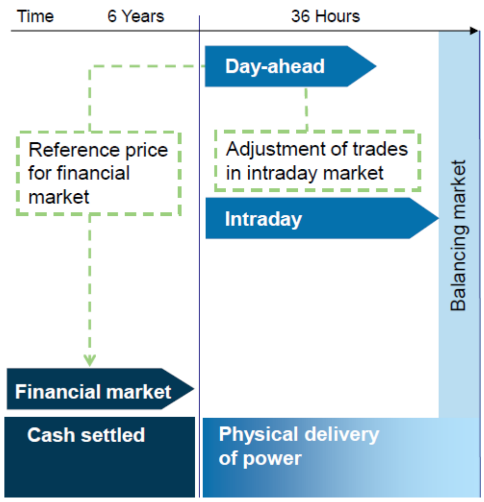

Figure 2.1: The Nordic power market model.

2.1.1 The Nordic power market

The Nordic countries have a common, deregulated power market, where members can trade physical power at Nord Pool Spot. Power production, transmission capacity and the transmission of power between countries has been extended over the years. This has resulted in a dynamic power market, where power can be traded across areas and different countries. The electricity price on this market is set by supply and demand. The Nordic countries deregulated their power markets in the early 1990s and brought their individual markets together into one common Nordic market (Nord Pool Spot) [2]. The Nordic countries deregulated their power market to make it more dynamic and increase the trade of energy between countries, which makes it easier to use the full capacity of the produced electricity. In a deregulated power market the market is no longer controlled by the state and free competition is introduced, but in contrast to the deregulated power market the distribution of power in the networks is controlled by monopolies. The national grids are owned and managed by each country's transmission system operator (TSO). They have the responsibility for securing the supply and the high-voltage grid to ensure that the power is delivered to the users.

The power in the Nordic grid is generated from various energy sources, e.g. hydro, thermal, nuclear, wind and solar. This variety of energy sources ensures a more "liquid" market and a more stable power supply. Electricity is a commodity, but the unique thing with electricity compared to other commodities is that it's a non-storable commodity. Electricity is classified as a non-storable commodity since it's not possible to buy electricity at a certain time point and then use it later for a reasonable price, and this feature makes the power market complex. In complement to the physical market at Nord Pool Spot there also exists a financial power market, where different financial contracts are used for risk management and price hedging. The electricity price for physical delivery from Nord Pool Spot is used as the reference price when pricing different contracts at the financial market. In the Nordic region financial contracts are traded through NASDAQ OMX Commodities.

2.1.2 Electricity price

The price of electricity is a key feature of the power market. The power price is determined by the balance between supply and demand, i.e. the intersection between the supply and demand curves. The electricity price is affected by multiple factors, e.g. the weather conditions or by power plants not producing at their full capacity. Everything that affects how much power that is produced (supply) and how much power that is used (demand) also affects the electricity price [3]. Weather conditions not only control the supply of water-, wind-, and solar power it also control the demand of power. Nuclear power is not affected by weather conditions, but the supply from nuclear power plants also varies since they are surrounded by strict safety regulations and sometimes need to be shut down for maintenance and repairs. In addition to the cost of producing and distributing electricity, the electricity price is also affected by the electricity certificate system and the Emission Trading Scheme (ETS).

The seasonal effects have the largest impact on the electricity price in the Nordic market. Temperature and water levels in the reservoirs are the most important factors of the seasonal effects. These factors make the electricity price increase in the winter and decrease in the summer. The power market sometimes is exposed to spikes, which are sharp short-term price increases. The most common reason for spikes is when power producers are producing at maximum capacity, but demand is nevertheless still higher. In these situations producers are forced to start up coal plants or generators that usually are not in operation. This is an expensive way to produce extra power but the supply increases, which means that the supply curve points steeply upwards and therefore the electricity price rises quickly. These characteristics of the electricity spot price mentioned above can be studied in figure 2.2, where historical daily system prices in Sweden are plotted.

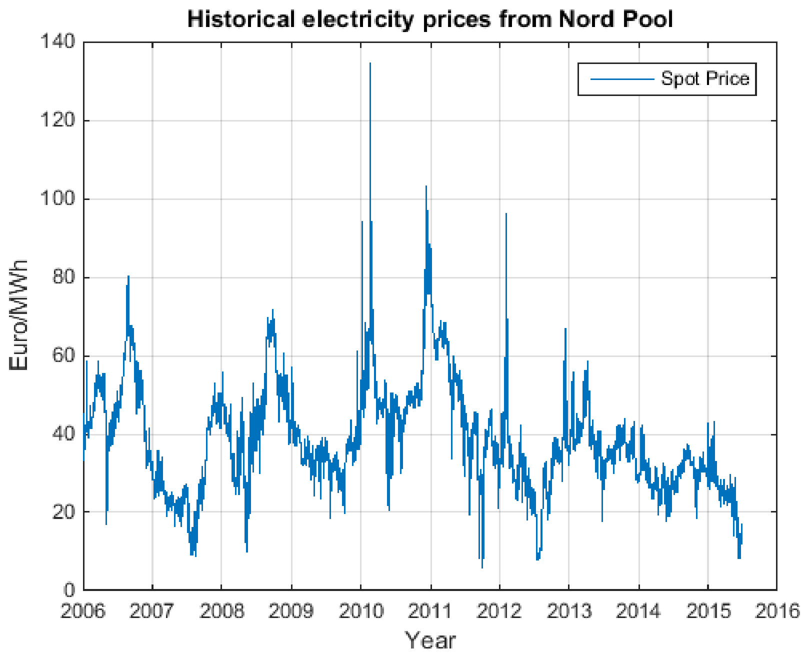

Figure 2.2: Historical daily system prices in Sweden between 2006-2015.

2.2 Nord Pool Spot

Nord Pool Spot is the Nordic market for trading with power for physical delivery. Nord Pool Spot offers standard agreements that simplify business between the market participants. Nord Pool Spot has a physical market (spot market) for trading with electricity every hour up to the day before delivery. Nord Pool Spot is the leading power market with physical delivery in Europe and offers both "day-ahead" and "intraday" markets within nine countries to their members. There are in total 380 companies from 20 countries trading on Nord Pool Spot according to [1]. Nord Pool Spot is owned by the Nordic transmission system operators (TSO): Svenska Kraftnät (Sweden), Statnett SF (Norway), Fingrid (Finland), Energinet.dk (Denmark) and the Baltic TSO's Elering (Estonia), Litgrid (Lithuania) and AST (Latvia) [4]. Power grid fees and taxes are regulated by the governments in each country, but the power cost on the other hand is the part of consumer price that is competitive. Power trading companies determine the price themselves, but usually this is done based on the market price on Nord Pool Spot. Producers of electricity can choose if they want to sell their produced electricity directly to the electricity exchange, to major users or to power trading companies. A major part of all electricity generated in the Nordics is sold at Nord Pool Spot.

2.2.1 Day-ahead market, Elspot

Nord Pool's day-ahead market Elspot is the world's largest day-ahead market for power trading and the main arena for power trading in the Nordic and Baltic regions [5]. On the day-ahead market contracts are made between sellers and buyers for power delivery during the next day. Trading on Elspot is based on three different types of orders, single hourly orders, block orders and flexible hourly orders. The members can use any one or a combination of all three order types to meet their requirements. Supply and demand are the key factors determining the hourly market price, but transmission capacity is also an essential feature. Bottlenecks can occur where power connections are linked to each other. If large volumes need to be transmitted to meet demand, this is managed by using different area prices.

Bidding areas

Elspot is divided in bidding areas and there are two different types of prices:

- **System price:** The system price is calculated disregarding the available transmission capacity between the bidding areas in the Nordic market and is only based on the sale and purchase orders. The system price is used as the reference price for the financial market.

• **Area price:** The available transmission capacity may vary in different areas and congest the flow of electricity between the bidding areas, and hence different area prices are established at Elspot [6].

When all members have submitted their orders at Elspot, then for all bidding areas equilibrium between the aggregated supply and demand curves is established and area prices are calculated. For each Nordic country, the local TSO decides which bidding areas the country is divided in. Sweden has four bidding areas, see Figure 2.3. The different bidding areas help indicate constraints in the transmission systems, and ensures that regional market conditions are reflected in the price.

Figure 2.3: Map showing different bidding areas for Elspot.

2.2.2 Intraday market, Elbas

Nord Pool's intraday market Elbas is a complement market to Elspot. It helps secure the balance between supply and demand in the power market for Northern Europe [7]. Of all trading handled by Nord Pool Spot, the majority of the volume is traded at Elspot, but unforeseen events may take place between the closing of Elspot and delivery the next day. At Elbas, buyers and sellers are able to trade volumes close to real time to bring the market back in balance. With an increasing amount of wind power entering the grid the intraday market is becoming increasingly important. Wind power is unpredictable by nature, hence the number of day-ahead contracts and produced volume often need to be adjusted. The intraday market Elbas will be a key enabler in the future to increase the share of renewable energy in Europe.

2.3 PPA Contracts

Producers can choose whether they want to sell their produced electricity directly to the electricity exchange, to electricity trading companies or to major users. This trade of electricity between two parties can be managed with a power purchase agreement (PPA). PPA it's an agreement between two parties, a seller who generates electricity and the buyer who is looking to purchase electricity. In the PPA contract all of the commercial terms for the sale of electricity between the two parties are defined, e.g. schedule for delivery of electricity, penalties for under delivery, payment terms, and termination [8]. PPA is a broad term and there exists several forms of PPA contracts and they vary a lot according to the needs of buyer, seller, and financing counterparties. PPA contracts can be assumed to follow the same price mechanism as forward contracts on the financial market.

2.4 Financial Market

NASDAQ OMX Commodities offers a financial electricity market [9]. Financial contracts are used for price hedging and risk management. Nord Pool Spot offers a spot market with physical trading in electricity each hour up to the day before delivery. In addition to this a financial market exists. In the Nordic region financial electricity contracts are traded through NASDAQ OMX Commodities. They offer a forward market for long-term trade. The contracts have a time horizon up to six years, covering daily, weekly, monthly, quarterly and annual contracts (in special cases other periods). For the financial market NASDAQ OMX Commodities, the system price calculated by Nord Pool Spot is used as reference price. There is no physical delivery for financial contracts. Financial contracts are entered without regards to technical conditions, such as capacity, grid capacity and other technical restrictions. The financial market primarily consists of trading with futures and forward contracts, which can be entered both as long and short positions.

2.4.1 Futures and forward contracts

Futures and forward contracts are the simplest form of derivatives (linear). Futures and forward contracts give buyers and sellers the opportunity to protect themselves against unexpected price changes. Electricity futures and forward contracts are agreements between two parties, buyer and seller, in which they commit, to during a specific future period of time, exchange a specified quantity of electricity for a predefined price. Futures and forward contracts are usually agreed for products delivered at a certain time, but since electricity is a non-storable commodity it makes no sense to deliver all electricity at a specific time. For that reason electricity futures and forward contracts provide a delivery of a specified constant (or deterministically time-varying) power level over a period of time rather than at a specific point in time. This period of time may be an hour, a week or a year. Electricity futures and forward contracts are thereby defined by price and delivery period and not a delivery date.

The holder of these contracts does not get any physical electricity delivered. Instead, the buyer get the difference between the price of the contract and the spot price during the same period paid at time T_2, see Figure 2.4 for illustration of forward contracts. The cash-flow that the buyer will receive at time T_2 is equal to the positive cash-flow (light grey area) minus the negative cash-flow (dark grey area) during the time period of the contract.

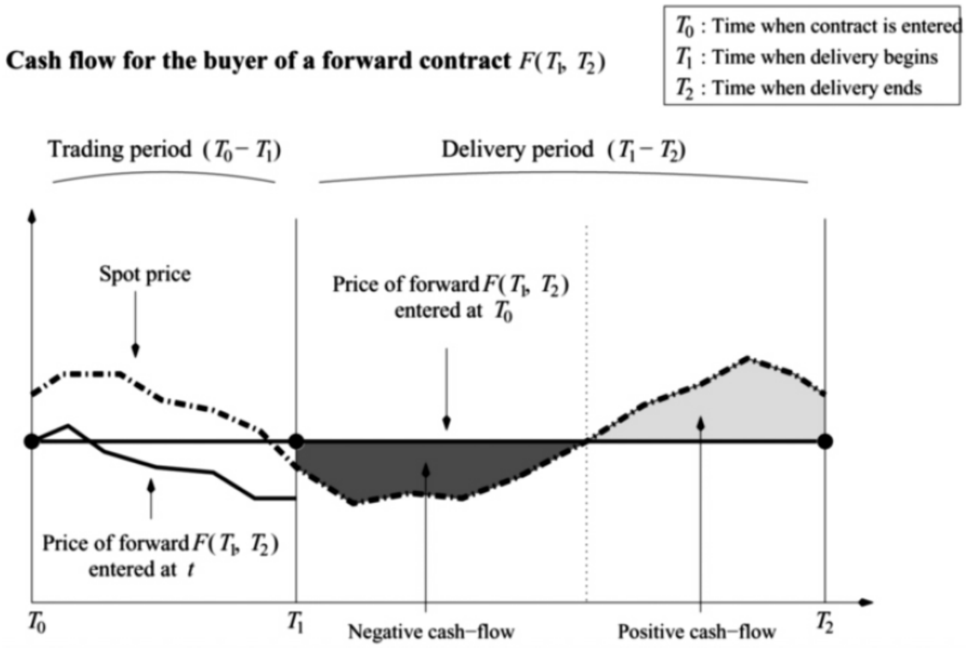

Figure 2.4: Illustration of forward contracts at NASDAQ OMX Commodities, from time T_0 to T_2.

Futures contracts

NASDAQ OMX Commodities offers two types of standard futures contracts with different length of delivery periods: daily and weekly contracts. Each contract corresponds to 1 MWh of electricity supply. Futures contracts are traded until the working day before the start of delivery. If all interest rates are deterministic, then futures contracts are equal to forward contracts and can be treated as similar products for modelling purposes.

Forward contracts

NASDAQ OMX Commodities offers three types of standard forward contracts with different length of delivery periods: monthly, quarterly and annual contracts (other special delivery periods may occur). Each contract corresponds to 1 MWh of electricity supply. Forward contracts are traded until the beginning of the delivery period [10].

2.4.2 Forward price curve

A forward price curve is a continuous curve where the market price of a forward contract can be read at an arbitrary time, even for those delivery periods that are not represented by an existing forward contract that day. At NASDAQ OMX Commodities, standardised forward contracts are traded with firm delivery periods, and by looking at a specific contract it can be seen what the market thinks is a reasonable price for this delivery period. However, one cannot see what the market thinks is a reasonable price for a delivery period that is not represented by a specific contract. But if modelling a continuous curve between the periods of all existing contracts that day, a forward curve can be constructed. Using the price information given from the forward curve, buyers and sellers can see at what price a forward contract should be traded, even if it does not exist in the present situation. The forward curve can be useful in several ways, e.g. pricing of new contracts, valuation of financial derivatives, budget planning and for investment calculations. Forward curves can be constructed by various methods which will be paid more attention later.

Chapter 3

Regulatory Frameworks

In order to prevent financial institutions from making riskier investments than they can handle, there exists regulatory frameworks. These regulatory frameworks contain standardised guidelines about the capital buffer that institutions have to set aside to protect themselves from market risks. The Basel Committee on Banking Supervision provides a forum for regular cooperation on banking supervisory matters. The objective is to enhance understanding of key supervisory issues and improve the quality of banking supervision worldwide [11]. The Basel Committee sets the lowest standard and each country are able to set stricter requirements. Different Basel frameworks have existed since the 1988. It started with Basel I and today Basel III is the current framework. Basel frameworks exists in order to strengthen the global capital markets, since with stronger capital and liquidity rules the banking sector will be more likely to absorb financial stress and crises better.

Regulatory frameworks:

- **Basel III:** It's a comprehensive set of reform measures, developed by the Basel Committee on Banking Supervision, to strengthen the regulation, supervision and risk management of the banking sector [12]. The Capital Requirements Regulation, CRR, is a regulation based on Basel III. CRR aim to stabilise and strengthen the banking system by making banks set aside more and higher quality capital as a buffer against crisis.

- **Basel 3.5:** It can be seen as the significant steps being taken by the Basel Committee to move beyond "Basel III", i.e. it's an initial move towards a "Basel IV" regulatory framework. The Fundamental Review of the Trading Book, FRTB, is a proposal of a new updated CRR regulation based on Basel 3.5. FRTB contains revisions of the capital framework in CRR and aim to contribute to a more stable banking sector by strengthening capital buffers for market risks.

3.1 Capital Requirements Regulation - CRR

This section will contain a summary of those parts in CRR that are important for this thesis. Institutions under supervision of the Swedish Financial Supervisory Authority (FI) may, in accordance with article 363 "Permission to use internal models" in CRR, apply for permission to use an internal VaR model to calculate own funds requirements for market risks for one or more risk categories. If consent shall be given the institution to use an internal VaR model they must meet a number of requirements specified in article 364 "Own funds requirements when using internal models" in CRR. Institutions using the internal model for one or more risk categories must also fulfil the standardised model for own funds requirements, for those risk categories which permission to use an internal VaR model has not been given.

3.1.1 Own funds requirements for commodity risks using internal models - CRR

This section will summarise the parts in CRR that determine the calculations of own funds requirements for the risk category commodity risks (see CRR [13] for complete details).

The use of an internal VaR model to calculate own funds requirements for market risks is comprised in the Internal Models Approach (IMA). The internal VaR model needs to be granted by competent authorities to be used to calculate capital requirements. Furthermore for the model to be granted the calculations must follow the requirements in Article 364 in CRR. Where VaR is required to be calculated according to Article 365(1) in CRR, specified in the first grey box. Additionally the stressed value-at-risk ($sVaR$) is required to be calculated according to Article 365(2) in CRR, specified in the second grey box.

Quantitative standards

According to Article 364 in CRR the own funds requirements, VaR_C, for risk categories approved using an internal VaR model is equal to the sum of points **a)** and **b)**:

a) The higher of the following values:

> **(i)** Previous days VaR (VaR_{t-1}).
> **(ii)** An average of the preceding sixty business day's VaR (VaR_{avg}) multiplied by the multiplication factor m_c.

The calculation of VaR shall be subject to the following minimum standards:
• At least daily calculation of the VaR measure.
• A 99th percentile, one-tailed confidence interval.
• A 10-day holding period (may use holding periods shorter than 10 days scaled-up to 10 days by an appropriate methodology).
• VaR model inputs calibrated to historical data from a period of at least one year except where a shorter observation period is justified by a significant upsurge in price volatility.
• At least monthly data set updates.

b) The higher of the following values:

(i) Latest available $sVaR$ ($sVaR_{t-1}$).
(ii) An average of the preceding sixty business day's $sVaR$ ($sVaR_{avg}$) multiplied by the multiplication factor m_s.

The calculation of $sVaR$ shall be subject to the following minimum standards:
• At least weekly calculation of $sVaR$ measure.
• A 99th percentile, one-tailed confidence interval.
• A 10-day holding period (may use holding periods shorter than 10 days scaled-up to 10 days by an appropriate methodology).
• VaR model inputs calibrated to historical data from a continuous 12-month period of significant financial stress relevant to the institutions portfolio.
• At least yearly data set updates.

The multiplication factors m_c respectively m_s shall be the sum of at least 3 and an addend obtained from table 3.1, i.e. $m_c = 3 + addend$ and $m_s = 3 + addend$. The addend takes a value between $0-1$ and depends on the number of overshootings for the most recent 250 business days. An overshooting is defined as when the actual daily portfolio loss exceeds the corresponding days 1-day VaR value.

Number of overshootings:	**addend:**
Fewer than 5	0,00
5	0,40
6	0,50
7	0,65
8	0,75
9	0,85
10 or more	1,00

Table 3.1: Table with number of overshootings and corresponding addends used to calculate the multiplication factors $m_c = 3 + addend$ and $m_s = 3 + addend$.

3.2 Fundamental Review of the Trading Book - FRTB

FRTB is a proposal for a new updated version of the current regulation CRR that will be a part of the regulatory framework Basel 3.5. The ambitions with the proposals in FRTB are to strengthen capital standards for market risk, and thereby contribute to a more stable banking sector. In FRTB there is a proposal to move from a VaR model to an ES model for capital requirement calculations, since a number of weaknesses have been identified with using a VaR model for regulatory capital requirements, mainly $VaR's$ inability to capture "tail risk".

3.2.1 Own funds requirements for commodity risks using internal models - FRTB

This section will summarise the changes and the parts in FRTB that determine the calculations of own funds requirements for the risk category commodity risks (see FRTB [14] for complete details).

The Basel Committee has agreed to use ES at confidence level 97.5% for the internal model approach, which also has been used to calibrate the revised standardised model for market risks. The proposed change by moving to a single stressed metric for the internal model approach in FRTB represents a rationalisation of the current regulation. In FRTB the Committee proposes to introduce "liquidity horizons" in the market risk metric. A liquidity horizon is defined as "the time required executing transactions that extinguish an exposure to a risk factor, without moving the price of the hedging instruments, in stressed market conditions". Five different "liquidity horizons" will be assigned for different categories of risk factors, with lengths between 10 days and one year.

Main changes

The Basel Committee has outlined a number of issues with CRR that are based on using VaR as a quantitative risk measure [14]. It has been noticed that the existing VaR based model raises a number of issues and the most notably one is the inability to capture the "tail risk" of the loss distribution. Hence the Committee has decided to use ES as a quantitative risk measure for market risks, since measuring the "tail risk" by using ES takes both the size and likelihood of losses into account. Based on the more complete capture of tail risks using an ES model, the Committee believes that moving to ES with a confidence level of 97.5% is an appropriate move. This confidence level will provide a broadly similar level of risk capture as the existing VaR with a confidence level of 99% while providing a number of benefits. ES is usually less sensitivity to extreme outliers in the observations and has in general a more stable model output.

A key weakness of the trading book regime before the financial crisis of 2008 was its reliance on risk metrics that were calibrated to current market conditions, which resulted in undercapitalised trading book exposures in the crisis. To overcome this weakness an additional capital charge based on a stressed VaR was introduced, but the Committee has recognised that basing regulatory capital on both current VaR and stressed VaR calculations, as in CRR, may be unnecessarily duplicative. To simplify the calculations the proposals in FRTB will simplify CRR by moving to a single ES calculation that is calibrated to a period of significant financial stress. A period of significant financial stress is defined as a 12-month continuously historical period that would maximise the risk metric for a given portfolio. In addition, to ensure that the reduced set of risk factors is sufficiently complete to allow the accurate identification of stressed periods, these factors must explain at least 75% of the variation of the full ES model.

Quantitative standards

• ES for risk capital purposes, ES_C, is equal to the maximum of the most recent observation, ES_{t-1}, and a weighted average of the previous 60 business days, ES_{avg}, scaled by a multiplier m_c. ES_C is calculated as

$$ES_C = max\{ES_{t-1},\, m_c \cdot ES_{avg}\}. \tag{3.1}$$

• ES of the most recent observation, ES_{t-1}, is calculated as

$$ES_{t-1} = ES_{R,S} \cdot \frac{ES_{F,C}}{ES_{R,C}}, \tag{3.2}$$

where $ES_{R,S}$ is equal to the expected shortfall based on a stressed observation period using a reduced set of risk factors multiplied by the ratio of the expected shortfall measure based on the most recent 12-month observation period with a full set of risk factors $ES_{F,C}$ and the expected shortfall measure based on the current period with a reduced set of risk factors $ES_{R,C}$.

• ES for a weighted average of the previous 60 business days, ES_{avg}, is calculated as

$$ES_{avg} = ES_{R,S}^{avg} \cdot \frac{ES_{F,C}^{avg}}{ES_{R,C}^{avg}}, \tag{3.3}$$

where $ES_{R,S}^{avg}$, $ES_{F,C}^{avg}$ and $ES_{R,C}^{avg}$ follows the same explanations used in equation 3.2, but calculated with a weighted average of the previous 60 business days.

• The multiplier, m_c, is calculated in the same way as in CRR according to table 3.1, where m_c is a multiplier which reflects the backtesting of daily VaR at 99% confidence level and based on current observations on the full set of risk factors.

• ES must be computed on a daily basis in the internal model for regulatory capital purposes.

• ES must be calculated using a 97.5% one-tailed confidence interval.

• For ES calculations according to [15] the liquidity horizons should be reflected by scaling the ES value calculated with a base liquidity horizon. For the scaling of ES to the liquidity horizon of the relevant risk factor, ES should first be calculated at a base liquidity horizon of 10 days with full revaluation. Then scaled up to the liquidity horizon of the risk factor category according to equation 3.4.

$$ES = \sqrt{(ES_T(P,Q))^2 + \sum_{j \geq 2} \left(ES_T(P,Q_j) \sqrt{\frac{(LH_j - LH_{j-1})}{T}} \right)^2} \tag{3.4}$$

Where ES is the regulatory liquidity horizon adjusted expected shortfall. T is the length of the base horizon, 10 days. $ES_T(P)$ is expected shortfall at horizon T of a portfolio with positions $P = (p_i)$ with respect to shocks to all risk factors that the positions P are exposed to. $ES_T(P,j)$ is expected shortfall at horizon T of a portfolio with positions P with respect to shocks for each position p_i in the subset of risk factors $Q(p_i,j)$, with all other risk factors held constant. Expected shortfall at horizon T, $ES_T(P)$ and $ES_T(P,j)$ must be calculated for changes in risk factors over the time interval T with full revaluation. LH_j is the liquidity horizon j, with lengths according to table 3.2.

j	LH_j
1	10
2	20
3	60
4	120
5	250

Table 3.2: Table with liquidity horizons (LH).

• Liquidity horizon adjusted expected shortfall should be calculated based on the liquidity horizon n. Risk factor categories and corresponding liquidity horizon can be seen in appendix B.

• The performance of the risk management models will be evaluated through daily backtesting. Backtesting requirements need to be based on comparing a 1-day VaR measure at both 97.5% and 99% confidence level to actual profit and loss ($P\&L$) outcomes, using at least one year of current observations of 1-day actual and theoretical $P\&L$.

3.3 Summary of Changes

Change from VaR to ES
Changing risk measure, moving from VaR at 99% confidence level in CRR to ES at 97,5% confidence level in FRTB. ES at 97,5% confidence level is expected to provide a broadly similar level of risk capture in the most cases as the existing VaR with a confidence level of 99%. The main reason for this change of quantitative risk measure is the inability of VaR to capture the "tail risk" of the loss distribution.

Use of different liquidity horizons
FRTB proposes varying "liquidity horizons" for different risk factor categories in the market risk metric. Five different "liquidity horizons" will be assigned for risk factor categories, ranging between 10 days and one year. This is a difference compared to CRR, where all VaR values using a 10-day liquidity horizon.

Move to a single stressed risk metric
Basing regulatory capital on both current VaR and stressed VaR as in CRR has been recognised as unnecessarily duplicative and FRTB will simplify the regulatory capital calculations by moving to a single stressed ES calculation that is calibrated to a period of significant financial stress.

Chapter 4

Risk Estimation Methods

Today's competitive and deregulated power market is characterised by high volatility in electricity prices, hence proper risk quantification is important. Risks can be quantified in monetary units by using different risk measures. VaR has become the most commonly used risk measure for quantifying market risks, ES is another commonly used risk measure. Tail risks are generally hard to measure by the nature of the event, since the event could change unexpected, very sudden and have a large impact. If the market conditions suddenly changes, companies may lose everything and go bankrupt (e.g. Lehman Brothers). To avoid situations like these, proper risk measures in monetary units are important and has devoted a lot of attention since the last crisis of 2008. The strength of both VaR and ES lies in their generality, they are general risk measures and based on the probability distribution for portfolios' market values.

Internal VaR models, according to CRR, has been suggested by FI's guidelines [17] to be calculated with one of the following methods, "Historical Simulations", "Variance-Covariance" or "Monte Carlo". This is the reason why one of these methods will be used, even though it's clear that there exists other superior methods (known from taking the course "SF2980, Risk Management" at KTH). FI has not stated any guidelines for implementation of the new proposals in FRTB yet, but in FRTB no particular type of ES model is prescribed. To make the comparison between CRR and FRTB as fair as possible the same method for calculating capital requirements according to CRR will also be used for FRTB. The reason that FI is suggesting one of these methods, despite their weaknesses, is probably due to the simplicity of the methods.

In the beginning of this chapter VaR and ES will be described, pros and cons will be discussed, which will be useful later when comparing capital requirements according to CRR and FRTB. The main focus in this chapter will be on the three different risk metrics suggested by FI. Each of them will be explained and the conclusions will be used to motivate the selection of risk metric. The reason for this investigation is that FI has written "Describe the models briefly and justify the choice of model" in their guidelines dealing with the application process for using an internal risk model approach.

4.1 Risk Measures

4.1.1 Value-at-Risk

VaR is a commonly used method for quantifying market risks. It has a great appeal since it can summarise all market risks of the company's entire portfolio across physical and derivatives positions and represent that as one number in monetary units. VaR is not a consistent risk measure, different models will give different VaR results [18]. VaR does not measure liquidity risk, political risk or regulatory risk, since it only measures quantifiable risks. VaR measures the worst expected loss on a portfolio over a given period of time with a given confidence level p, i.e. the maximum amount of money that may be lost during that period of time [19]. VaR models are simple to understand for all levels of staffs in the organisation and this is probably one of the main reasons why VaR has been adopted so rapidly as the most commonly used risk measure. VaR calculations should be based on a long historical time series, hence it's not always very inductive of the current level of market volatility. An issue with VaR is that historical time series misstate the current level of risk and this could potentially lead to an inappropriate level of risk in a portfolio. In periods when the market is volatile, VaR will not flag the high level of risk since it's based on an older time series from more stable market conditions. This could result in that investors could hold more risk than they should during these market conditions. In order to correct this effect, CRR uses stressed value-at-risk ($sVaR$) as a complement to VaR. The computations for $sVaR$ is the same as for VaR but applied to historical data from a period with significant financial stress on the market. $sVaR$ captures how stressed market conditions effects the portfolio, which occurs occasionally at the market. The stressed calculations are due to the importance of ensuring that regulatory capital will be sufficient in periods of significant market stress. As the financial crisis of 2008 showed, it's during stress periods the capital is most critical to absorb losses.

4.1.2 Expected Shortfall

The biggest weakness with VaR is that it possibly could hide catastrophic risks in the left tail since it's only a quantile value and a remedy for this is to use ES instead. ES is in some literature called Average VaR, Conditional VaR, Tail VaR or Tail Conditional Expectation. ES is calculated as the average VaR value beyond confidence level p, and hence it captures catastrophic loss events with small probabilities located in the left tail. ES is often proposed to be a superior risk measure compared to VaR. The main advantages of ES is that it considers all values located in the left tail of the probability distribution and it's a coherent measure of risk. If a risk measure is not coherent it could (possibly) discourage portfolio diversification [18]. ES at confidence level p could be explained as the expected return of the portfolio in the worst $1 - p$ percentage of all cases. Similar to VaR, ES represent risks as one number in monetary units, it's not a consistent risk measure and does not measure liquidity risk, political risk or regulatory risk. The advantage that ES cover extreme losses better than VaR is the main reason why the Basel committee has decided to change to ES in FRTB as the standardised risk measure for market risks. As with everything else, the advantages of ES over VaR do not come without some disadvantages. For ES calculated with a fat-tailed underlying distribution the estimation errors are greater than for a corresponding VaR measure. This estimation error can be reduced by increasing the sample size of the simulation, hence ES is more costly to compute when considering tail risk with fat-tailed distribution. Another disadvantage with ES is that it has more complicated backtesting than VaR, hence the backtesting according to FRTB is still performed using VaR. Stressed expected shortfall (sES) can be seen as a complement to ES. The computations for sES is the same as for ES but applied to a historical period with significant financial stress on the market.

4.2 Simulation Methods for Risk Measures

Risk associated with financial instruments in a portfolio arises because of changes in risk factor values over future time periods. These changes in risk factor values can be simulated by using various methods. This section will focus on market risks, i.e. the exposure to losses in the market place through physical and derivative positions. There are several methods for calculating VaR and ES, the methods can be either parametric or non-parametric. Parametric methods are based on statistical parameters of the risk factor distribution and non-parametric methods are based on simulations or historical models [20]. There are three different methods suggested in FI's guidelines for implementing an internal risk model:

1) Historical simulations (non-parametric)
2) Variance–Covariance method (parametric)
3) Monte Carlo simulations (non-parametric)

It's easy to estimate VaR and ES once the portfolio $P\&L$ distribution has been constructed. The difference between these methods are due to the manner in which this $P\&L$ distribution is constructed. Each of these methods for VaR and ES calculations will be explained and compared against each other in the following sections.

4.2.1 Historical simulations

Historical simulation is the easiest non-parametric method to implement. The idea is simply to use only historical market data in calculation of VaR and ES for the current portfolio. Historical simulation is a full valuation method, i.e. it estimates the probability distribution by generating a number of scenarios and revaluates a portfolio under these scenarios. Historical simulation doesn't require any statistical assumption about the distribution of returns or their volatility. Using the historical simulation approach, a set of historical data is needed to model the value at a future time $T > 0$ of a portfolio. The key assumption made for historical simulation is that the information in the samples is representative of future values and that no additional probability beliefs of the modeller are relevant, i.e. in this approach the set of possibly future scenarios are fully represented by events in the historical observation period [21].

Historical simulations in three steps:
1) Identify the instruments in the portfolio and obtain time series for each of these instruments over some defined historical period.
2) Use the historical data in the current portfolio to obtain the portfolio $P\&L$ distribution.
3) VaR and ES estimates can then be determined from histogram of the portfolio $P\&L$.

This is a subjective approach, but it's non-parametric and reasonable under the assumption that the mechanisms that produced the returns in the past are the same as those that will produce the returns in the future. For the historical simulation to be useful the sample preparation is very important, since if the generated sample of returns, or value changes, can be viewed as samples from IID R.V., then standard statistical techniques can be used to investigate the probability distribution of future portfolio values expressed as known functions of future returns or value changes [22].

Advantages:
+ It's simple to understand, intuitive and straightforward to implement.

+ It takes fat-tails into account in the $P\&L$ distribution.

+ It can be applied virtually to any type of instrument, all market risk types and uses full valuations.

+ It doesn't make any assumptions about the statistical distribution.

Disadvantages:
- It requires sufficient history of the relevant market variables, to get enough data for simulations.

- It assumes that the history will repeat itself. Even though this assumption is often reasonable, it may lead to underestimations of extreme losses, since future losses may be worse than past losses.

- It determines the distribution of the portfolio completely by the distribution of the underlying market variables over the selected time period. This can lead to abrupt changes in the risk measure estimates when different periods of historical data are used.

4.2.2 Variance–Covariance method

The variance-covariance method is a parametric method. It's based on the assumption that changes in market parameters and portfolio values are normally distributed. The simplicity of this method and the assumption of normality makes it ideal for simple portfolios with only linear instruments and without fat-tailed distribution. For normally distributed profit and loss distributions, VaR and ES are scalar multiples of each other, since both are scalar multiples of the standard deviation σ. Therefore, in this case with the variance-covariance method, VaR provides the same information about the tail loss as ES. This implies that for the normality assumption, ES has no advantage over VaR [23].

Variance-covariance method in three steps:
1) Map individual investments into a set of simple and standardised market instruments. Each instrument is then stated as a set of positions in these standardised market instruments.
2) Estimate the variances and covariances of these instruments. The statistics are usually obtained by looking at the historical data.
3) Calculating VaR and ES for the portfolio by using the estimated variances and covariances (covariance matrix) and the weights on the standardised positions.

When using the variance-covariance method, options are handled by representing them in terms of a delta equivalent position in the underlying asset. The assumption of normally distributed returns and the delta approximation leads to normally distributed portfolio returns. The variance-covariance approach is widely used and accepted as the basic method for evaluating VaR

and ES for portfolios without a large component of options [22].

Advantages:
+ It's simple to understand and computationally efficient, i.e. calculations only involving simple vector and matrix multiplication.

Disadvantages:
- It assumes that portfolio values are normally distributed, which is not always realistic. This assumption is not valid for markets which are characterised by fat-tailed return distributions, i.e. in reality extreme outcomes are more likely than normal distribution would suggest. In this cases VaR and ES may be underestimated by using the variance–covariance method.

- It's difficult to improve the model while retaining the simple delta VaR and ES calculations because the simplicity relies on the normality assumption.

- It's not really appropriate for derivatives portfolios, since the delta equivalent approximation throws away all the options information, whose returns are non-linear functions of risk variables, which can lead to a major underestimation of VaR and ES.

4.2.3 Monte Carlo simulations

Monte Carlo simulations is another non-parametric method. It's the most popular of the three approaches when there is a need for a sophisticated and powerful risk metric system, but it's also the most challenging one to implement. The Monte Carlo method is based on simulations of the joint behavior of all relevant market variables and uses this simulation to generate possible future values of the portfolio. Monte Carlo simulations is a full valuation method, i.e. it estimates the probability distribution by generating a number of scenarios and revaluates a portfolio under these scenarios [22].

Monte Carlo simulation method in two steps:
1) Stochastic processes for financial variables are specified and correlations and volatilities are estimated on the basis of market or historical data.
2) Price paths for all financial variables are simulated (thousands of times). The portfolio is revaluated with these price realisations and then compiled to a joint $P\&L$ distribution, from which VaR and ES estimates can be calculated.

One of the largest strengths with Monte Carlo simulations is the ability of pricing non-linear derivatives on the market variables, since option pricing models are used to compute the changes in the option prices for each of the simulated states of the underlying forward curve. Monte Carlo simulation techniques are flexible and very powerful. It takes all nonlinearities of the portfolio value with respect to its underlying risk factor into account as well as all desirable distributional properties, e.g. time varying volatilities and heavy-tails. The problem with this approach is that it's by far the most computationally consuming since you need to revalue the portfolio many times. But with today's powerful computers this is not a major problem. More significant disadvantages are that the method is more complex and harder to understand and implement than other more basic methods.

Advantages:
+ It incorporates all desirable distributional properties, such as fat-tails and time varying volatilities.

+ It prices non-linear derivatives on the market variables.

Disadvantages:
- It's complex to implement and understand.

- It's computationally demanding.

4.3 The Choice of Method

As seen in the previous sections, all methods have different advantages and disadvantages. Below, the main differences between the methods are summarised.

Main differences:
• The ability to capture the risk of non-linear instruments (e.g. options).

• The simplicity of implementation and ease of understanding for all levels of staff in the organisation.

• Flexibility in the method, to be able to incorporate alternative assumptions.

• Most importantly, the reliability of the results; e.g. capturing fat-tails and using time varying volatilities.

Historical simulations captures the fat-tails of the portfolio $P\&L$ distribution. However historical simulations only gives an accurate picture if history repeats itself, which may lead to underestimations of extreme losses. The historical method is rejected due to the paradox of the future worst case scenarios, since it doesn't allow future values to be worse than in the past.

The covariance-variance method doesn't capture heavy-tails and is not appropriate for the power market, since it's characterised by fat-tailed distributions. It's not a good approach for portfolios with options either, but it's suitable for simple portfolios that includes only linear instruments and no fat-tails. The variance-covariance approach is rejected due to its underestimation of risk, since fat-tails are significant for the power market.

With the Monte Carlo method, knowledge of future changes in the market can be incorporated into the simulations, which can be especially useful for the power market. The Monte Carlo method seems to be the most suitable method for calculating VaR and ES for a power portfolio, since it both captures heavy-tailed distributions and allows time varying volatilities. In fact the Monte Carlo method is the only one of the different methods that fulfils three out of four criteria above. Another reason for the choice of this approach is the need for large amounts of simulated data in order to cover as many scenarios as possible, which is an advantage compared to the historical simulations method. The Monte Carlo method is considered to have the greatest advantages in this case and the computationally consuming disadvantages is not a major problem, since there is access to powerful computers today which makes it easier to handle large amounts of data. Another advantage by using the Monte Carlo method is the generality. An internal model using Monte Carlo simulations can later be extended to cover portfolios containing all kinds of non-linear instruments.

Finally, the investigation of different risk metrics has resulted in choosing the Monte Carlo method. It's considered to be the most suitable of FI's three suggested methods for calculating market risk in power portfolios. This motivation of model selection is requested by FI when applying for permission to use an internal model for calculating own funds requirements for market risks.

Chapter 5

Methodology

This chapter will describe the methodology of the thesis and show partial results achieved during the different steps. Risk management in the power market requires knowledge about the electricity forward price curve, i.e. market values that can be applied to forward positions in the portfolio. Representing forward prices by one continuous term structure curve is regarded as an efficient way of representing market prices. Smooth forward curves have been constructed with a method that uses the maximum smoothness criteria. From the term structure of these forward price curves, volatilities for different times to delivery can easily be determined. Future movements of the electricity forward curve have been studied by using a non-arbitrage term structure three factor–model. Finally, Monte Carlo simulations has been used to determine the $P\&L$ probability distribution of the portfolio to be able to calculate VaR and ES. The procedure can be summarised in the following six steps.

General steps:
1) Process data
2) Model continuous forward curves
3) Determine volatility term structure
4) Use a forward market model to determine future movements of the electricity forward price curve
5) Run Monte Carlo simulations to determine portfolio $P\&L$ probability distribution
6) Calculate VaR and ES with the results from Monte Carlo Simulations

These six steps above are general guidelines covering how VaR and ES can be calculated for power portfolios. All steps can be performed in different ways. In the following sections each step will be explained and described how it has been performed in this thesis. But first we will start with making some important assumptions.

5.1 Assumptions

Throughout this thesis, to enable use of mathematical financial theory, the following three assumptions are made about the market:

- The value additivity principle applies
- The absence of arbitrage in the market
- The market is rational and competitive

These assumptions are prerequisites for being able to use some financial models. Another assumption that is assumed throughout this thesis is that all interest rates are deterministic. For stochastic processes, the forward price is a martingale under the forward measure and the futures price is a martingale under the risk-neutral measure. When interest rates are deterministic, the forward measure and the risk neutral measure are the same, hence futures and forward contract prices will evolves in the same manner and can be treated as equal products for modelling purposes.

5.2 Data Set

Historical electricity system spot prices on an hourly granularity between 1 January 2006 until 7 July 2015 from Nord Pool Spot are collected from Energinet.dk [24]. This downloaded data set is processed, to get daily spot prices, by taking the average value of the 24 hourly spot prices for each day during the period. By studying the electricity spot price in figure 2.2 it can be seen that electricity spot prices exhibit seasonality trends, mean-reversion, high volatility and large jumps. The seasonal component is due to the shifting demand during different seasons, though also the supply is affected. We will focus on the seasonality component. The daily spot price time series will be used to calibrate the seasonality part of the electricity price, which later will be used as a prior-function when calculating the forward curves by using the maximum smoothness criteria.

A historical data set containing price information about all futures and forward contracts that have been available on NASDAQ OMX Commodities [25] during a five years period between July 2010 and July 2015 has been provided by NASDAQ. The futures and forward contracts in this data set concern delivery of 1 MWh during every hour (i.e. base load) of the delivery period and all prices in the data set are closing prices.

Preparation of data:
1) The NASDAQ data set is sorted into different worksheets, one worksheet for each date, which results in a total of 1282 worksheets.

2) All irrelevant contracts and contracts without available closing price are removed.

3) All contracts for each day are sorted in descending order and overlapping contracts are deleted. When deleting overlapping contracts, contracts for shorter time periods are prioritised, i.e. if there are weekly contracts available for some month and also a monthly contract, the weakly contracts are kept and the monthly contract is deleted.

4) All available daily contracts are removed from each day. This is done for two reasons. First of all the very short end (daily contracts) of the electricity forward curve are not analysed here because daily contracts exhibit volatility nearly as high as the spot price and significantly greater than volatilities of weekly and longer-term contracts. Secondly, daily contracts are removed to avoid having an almost singular matrix in the forward curve calculations, since these contracts often correspond to almost dependent rows in the system of equations.

After that the NASDAQ data set is rearranged and divided into different worksheets, containing contracts available each day, an example of the information that each worksheet contains is plotted and shown in figure 5.1. Notice that the contracts in the left part of this figure fluctuates a lot in price and this is due to seasonal effects.

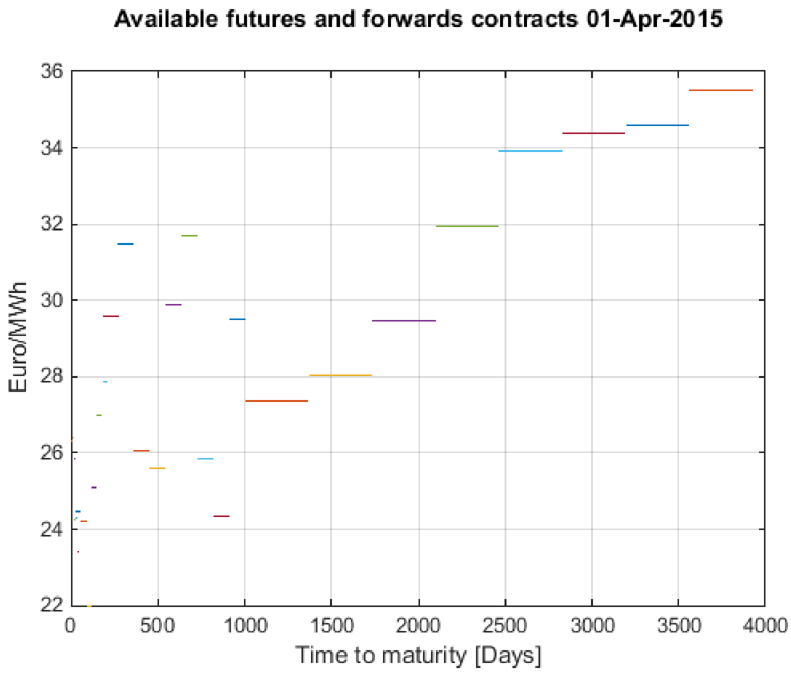

Figure 5.1: An example from the NASDAQ data set showing all weekly, monthly, quarterly and yearly contracts available as of first of April 2015.

This processed data will later be used to calculate a smooth forward price curve for each day in the time period and calculate the volatility term structure in the market for different times to maturity. The volatility in the market for different times to maturity has to be known to estimate the future movements of the forward price curve when calculating the value of a contract at a future date.

5.3 Power Portfolio

According to section 1 in article 362 "Permission to use internal models" in CRR (similar for FRTB) institutions could apply for permissions to use their internal models for a specific risk category and the external for the other risk categories. This model will be built as an internal model for risk category (f), commodities risk.

Portfolios containing the following types of assets will be considered:
- Forward contracts
- Futures contracts (Treated as forward contracts, since deterministic interest rates assumption)
- PPA contracts

As earlier mentioned, electricity contracts are written on a commodity flow rather than on a bulk delivery. The forward price curve will give us today's price of a unit (1 MWh) of electricity delivered at a specific instant of time in the future, but all electricity contracts are written on a future average, i.e. the delivery during a time period. Hence the relation between the forward price function and the average based contracts need to be used. Let $F(t, T_1, T_2)$ be today's contract price of an average based future contract delivering one unit of electricity at a rate $1/(T_2 - T_1)$ in the time period T_1 to T_2, where $t \leq T_1 < T_2$. Assume that the contract is paid as a constant cash flow during the delivery period, then the average contract price in [26] is written as

$$F(t, T_1, T_2) = \int_{T_1}^{T_2} w(u, r) f(t, u) du \tag{5.1}$$

where

$$w(u, r) = \frac{e^{-ru}}{\int_{T_1}^{T_2} e^{-rv} dv}. \tag{5.2}$$

This can be approximated (assuming a zero interest rate, $r \approx 0$) as

$$F(t, T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f(t, u) du, \tag{5.3}$$

which is a very good approximation for reasonable levels of the interest rate.

5.4 Forward Curve Model

Determining the electricity forward curve is a non-trivial task and requires special methods. These continuous term structure curves are also used in other fields, but these methods cannot be applied directly to the power market. The power market differs from other commodity markets in how electricity futures and forward contracts are designed with delivery during a time interval rather than as a bulk delivery. Unlike the yield curve, the electricity forward curve has seasonal patterns, is weather dependent and is extremely volatile at the short end. Electricity futures and forward contracts concern delivery of electricity during a given time period (day, week, month, quarter or year) in the future, not a single hour or day and hence the electricity forward curve cannot be constructed simply by interpolating between points in the price-maturity space. Consequently, the methods developed for fixed income markets cannot be applied directly to electricity price data.

Electricity futures and forward contracts consist of load pattern, start date and end date for the delivery period. A load pattern is a deterministic function of time that specifies how the amount of electricity should be divided during the delivery period. There are different types of load pattern, base load and peak load are two types of load patterns. Base load has constant delivery and for peak load the amount of electricity that should be delivered varying in time, in this thesis only base load will be used, but peak load could be implemented in the used models. As a clarification the peak load is used during the peak period, which occurs during the opening hours of the industries when the demand for electricity is highest. The literature about electricity forward curve modelling is not as rich as for yield curve modelling. Representing forward prices by one continuous term structure curve is required for implementing a non-arbitrage term structure model for risk management. After investigating several methods, the maximum smoothness criteria with a seasonal prior-function has been decided to be used to fit the smoothest possible forward curve to closing prices.

This method proposed in [27] uses the maximum smoothness criteria to construct a smooth forward curve that consist of a prior function and an adjustment function. The main advantages with this method are calculation speed, closed form solution and the ability of handling overlapping contracts.

5.4.1 Maximum smoothness model

From the NASDAQ data set, start and end dates $((T_1^s, T_1^e),(T_2^s, T_2^e),...,(T_m^s, T_m^e))$ for the delivery period for all contracts available that day are known. This list of dates needs to be transformed to the form $(t_0, t_1,..., t_n)$, where overlapping contracts are divided into sub periods. This new list is almost a copy of the first list but duplicated dates are removed and the list is sorted in ascending order. In the data set from NASDAQ, all prices are given as closing prices, which will be denoted as F_i^C, where i represent all different contracts available that day. The forward function in this model is defined as

$$f(t) = h(t) + g(t) \quad t \in [t_0, t_n], \tag{5.4}$$

where $h(t)$ is an exogenous prior function and $g(t)$ is an adjustment function. The prior function can be seen as a subjective forward curve adjusted to the market price.

Maximum smoothness criteria

The smoothest possible forward curve on the interval $[t_0, t_n]$ is defined as one that minimises the following expression

$$\min_{\mathbf{x}} \int_{t_0}^{t_n} [g''(t; \mathbf{x})]^2 dt, \tag{5.5}$$

where smoothness of a function is defined as the mean square value of its second derivative. As can be seen in equation 5.5 it minimises the mean square value of the adjustments function's $(g(t))$ second derivative, this is due to better reflect the seasonal patterns in the prior function ($h(t)$). The adjustment function, $g(t)$, will not only be the smoothest possible function it also has to fulfil the following four criteria:

1) Twice continuously differentiable.
2) Horizontal at time t_n.
3) Smoothest possible in the sense of equation 5.5.
4) The average value of the forward price function $f(t) = g(t) + h(t)$ for contract i is equal to the closing price F_i^C.

Adjustment function

The present value of the forward price function, $f(t)$, has been approximated with the average value according to equation 5.3. This seems to be a valid approximation since the interest rate effect is less than both the effect of the prior and the smoothing functions. The smoothest adjustment function, $g(t)$, with properties that fulfil all the above mentioned criteria is a polynomial spline of order five. This has been proved in [28]. The proof is left outside the scope of this thesis and the curious reader is referred to the additional literature. With this clarified the adjustment function can be written as

$$\text{g(t)=} \begin{cases} a_1t^4 + b_1t^3 + c_1t^2 + d_1t + e_1 & t \in [t_0, t_1] \\ a_2t^4 + b_2t^3 + c_2t^2 + d_2t + e_2 & t \in [t_1, t_2] \\ \qquad\vdots & \\ a_nt^4 + b_nt^3 + c_nt^2 + d_nt + e_n & t \in [t_{n-1}, t_n]. \end{cases}$$

Constraints

The parameter $\mathbf{x}$ to the adjustment function can be determined by solving the equality constrained convex quadratic programming problem corresponding to equation 5.5 subject to the following five (**C1**,...,**C5**) natural constraints in the connectivity and derivatives smoothness at the knots, j=1,. . . , n-1.

C1:

$$(a_{j+1}-a_j)t_j^4+(b_{j+1}-b_j)t_j^3+(c_{j+1}-c_j)t_j^2+(d_{j+1}-d_j)t_j+e_{j+1}-e_j = 0 \quad (5.6)$$

C2:

$$4(a_{j+1} - a_j)t_j^3 + 3(b_{j+1} - b_j)t_j^2 + 2(c_{j+1} - c_j)t_j + d_{j+1} - d_j = 0 \quad (5.7)$$

C3:

$$12(a_{j+1} - a_j)t_j^2 + 6(b_{j+1} - b_j)t_j + 2(c_{j+1} - c_j) = 0 \quad (5.8)$$

C4:

$$g'(t_n; \mathbf{x}) = 0 \quad (5.9)$$

$$\int_{T_i^s}^{T_i^e} e^{-rt}(g(t)+h(t))dt = \int_{T_i^s}^{T_i^e} e^{-rt}F_i^C dt \tag{5.10}$$

for $i = 1, ..., m$.
C5:

$$F_i^C \approx \frac{1}{T_i^e - T_i^s}\int_{T_i^s}^{T_i^e}(g(t)+h(t))dt \tag{5.11}$$

where the approximation from equation 5.3 are used. This minimisation problem has a total of $[3n+m-2]$ constraints and it can be written on matrix form, matrix notations used for implementations can be seen in appendix A.

Prior function

Different prior-functions can be used, but for electricity contracts a prior-function with seasonality seems to give the best result. If no prior-function is used, the curve will almost be totally smooth in the end when only yearly contracts are available and that doesn't seems to be a good solution, since electricity prices has seasonal variations. But by using a prior-function that captures these seasonality effects, we get a curve that seems to describe forward prices better. The impact of the prior-function is small in the beginning of the smooth forward curve, when shorter contracts are available, after that the prior-function influences the smoothed forward function more and more. In the end of the smoothed forward curve when there are only yearly contracts available the effect of the prior-function is largest, hence this long settlement period gives the adjustment function less constraints and thus the prior-function influence the forward curve more. This can be summarised as that the prior function has a small effect on the smoothed forward curve when contracts with settlement periods shorter than a year are available and a large effect in the right end of the curve when only yearly contracts are available.

The prior function selected in this thesis is derived from the seasonality effect of the spot prices on Nord Pool Spot. The logarithm of the electricity price (spot price) S_t at time t can be modelled as the sum of a deterministic part D_t and a stochastic part X_t as follows:

$$log(S_t) = D_t + X_t. \tag{5.12}$$

The deterministic (seasonal) part D_t in equation 5.12 is assumed to be totally predictable and is presented by a known deterministic function of time. A crucial part of forward curve modelling is to select a suitable deterministic part (seasonal). As seen in figure 2.2, daily spot prices are subject to seasonality trends. Hence the deterministic seasonal part D_t in equation 5.12 is modelled by using trigonometric functions suggested in [29] as follows:

$$D_t = c_1 sin(2\pi t) + c_2 cos(2\pi t) + c_3 sin(4\pi t) + c_4 cos(4\pi t) + c_5. \tag{5.13}$$

$$\begin{cases} c_1, ..., c_5 = \text{Constant parameters} \\ t = \text{The annualised time factors} \end{cases}$$

In this report we will not pay attention to the stochastic part of the spot price, but it's important to have in mind that this part also affects the electricity price. We are only interested in determining the seasonality effect of the electricity price, which corresponds to the deterministic part D_t in equation 5.12. This is done by estimating the parameters in equation 5.13 and the results will be used as a prior function when calculating forward price curves by using maximum smoothness criteria. The model parameters, for the deterministic part D_t of the model, are estimated by using the method of least squares as follows:

$$\min_{c_1,..,c_5} \quad \sum_{t=1}^{N} (\log(S_t) - D_t)^2. \tag{5.14}$$

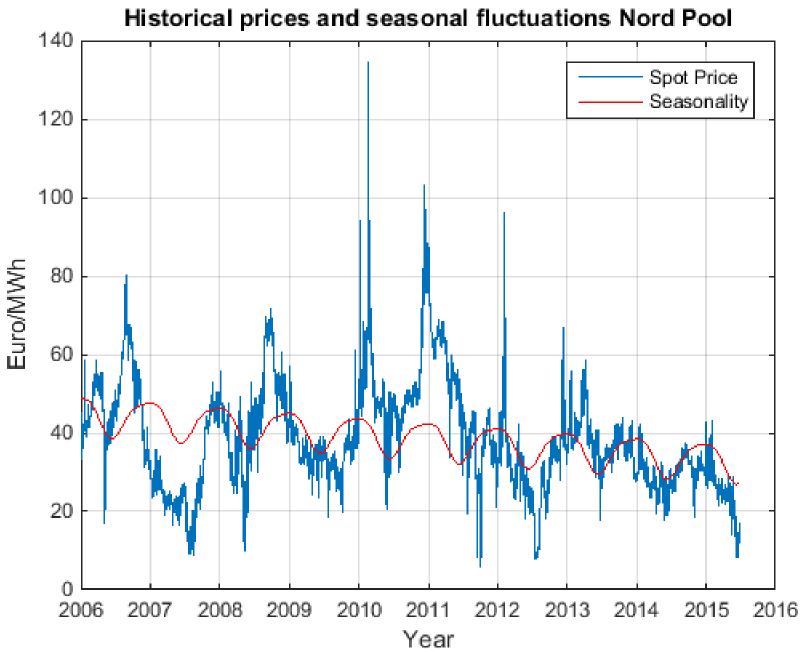

Figure 5.2: Historical electricity spot prices from Nord Pool Spot and seasonality between 2006 and 2015.

In figure 5.2 historical spot prices and estimated seasonality effect are plotted, this seasonality curve has been used as prior-function. As can be seen in this figure, since the seasonality has been estimated with the least square method, the resulting seasonality function can be seen as the average seasonality during this time period. This implies that the red seasonality curve gets lower amplitude than the blue spot price curve and this seems reasonable to use as a prior-function (seasonal) for smooth forward curves, since forward contracts are less volatile than the spot price. The red seasonality function has been extrapolated to get the prior-function for future dates.

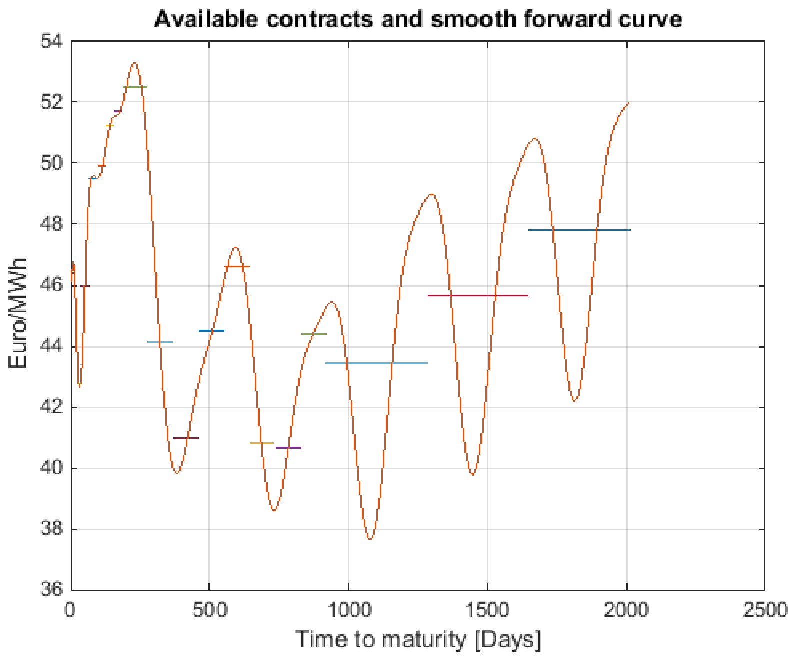

Figure 5.3: Available contracts and the smoothed forward curve 2010-06-25. The horizontal line segments represent the actual market prices for contracts and the length of these lines corresponds to the delivery period on which the contracts are written. The orange continuous line is the smoothed term structure.

Figure 5.3 shows the result of a continuous forward price curve calculated with maximum smoothness criteria and available contracts for the first day in the NASDAQ data set, i.e. 2010-06-25.

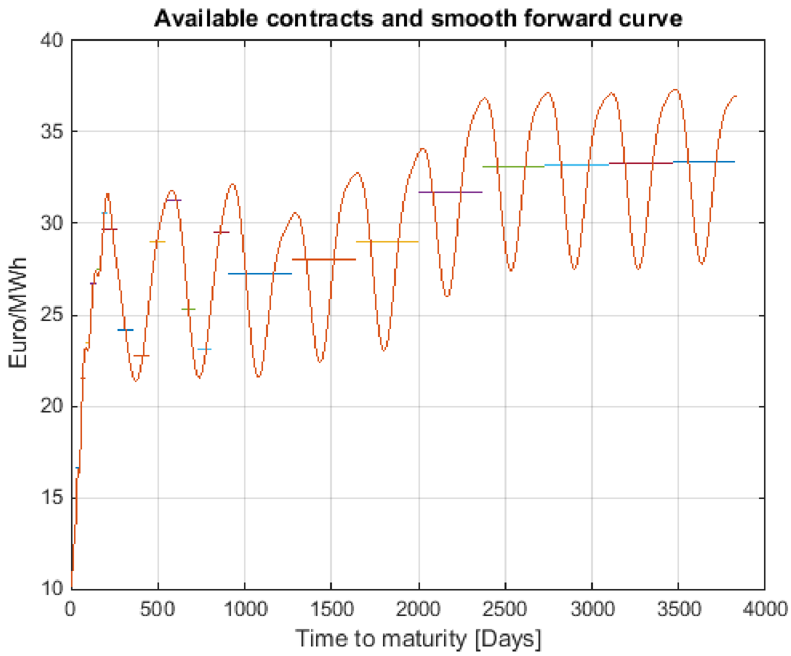

Figure 5.4: Available contracts and the smoothed forward curve 2015-07-06. The horizontal line segments represent the actual market prices for contracts and the length of these lines corresponds to the delivery period on which the contracts are written. The orange continuous line is the smoothed term structure.

Figure 5.4 shows the result of a continuous forward price curve calculated with maximum smoothness criteria and available contracts for the last day in the NASDAQ data set, i.e. 2015-07-06.

5.5 Volatility Term Structure

To simulate future movements of the forward price curve, the volatility term structure has to be investigated. The smooth forward curve, described in the previous section, has been simulated for all days in the data set from NASDAQ until 2015-04-01, in total 1215 smooth forward curves have been calculated. The results from these forward curve simulations are plotted in 3D in figures 5.5 to 5.10, divided in one plot for each year. Each of these forward curves has been calculated from the historical data of contracts that were available at the market on that specific day. The x-axis shows the date that the forward curve is calculated for, the y-axis shows number of days to maturity and the z-axis shows the forward price in Euro/MWh.

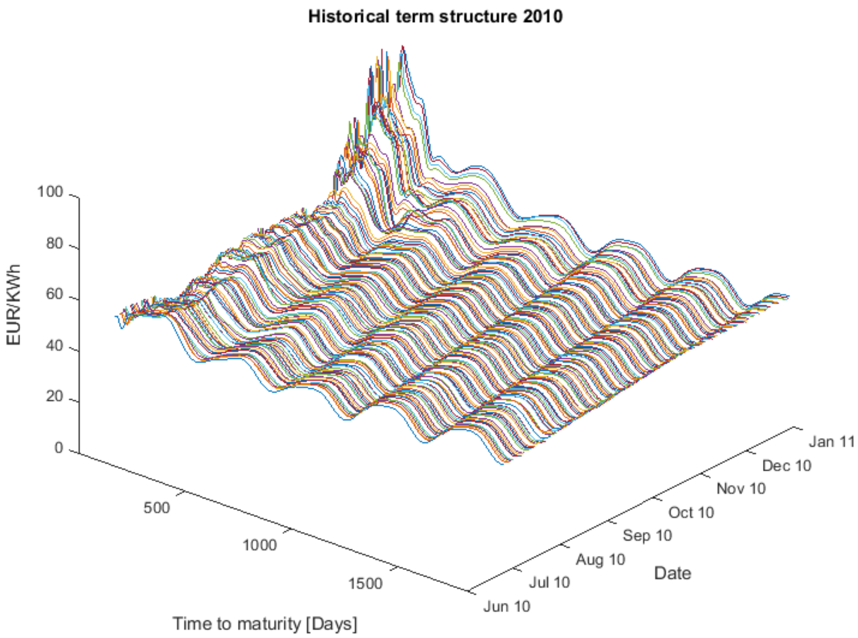

Figure 5.5: Smooth forward curves Jul-Dec 2010.

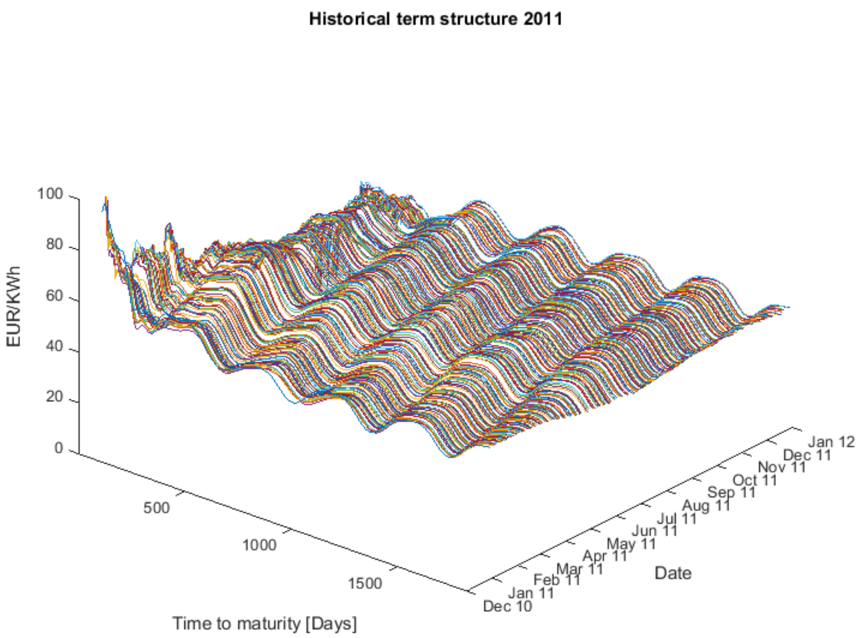

Figure 5.6: Smooth forward curves 2011.

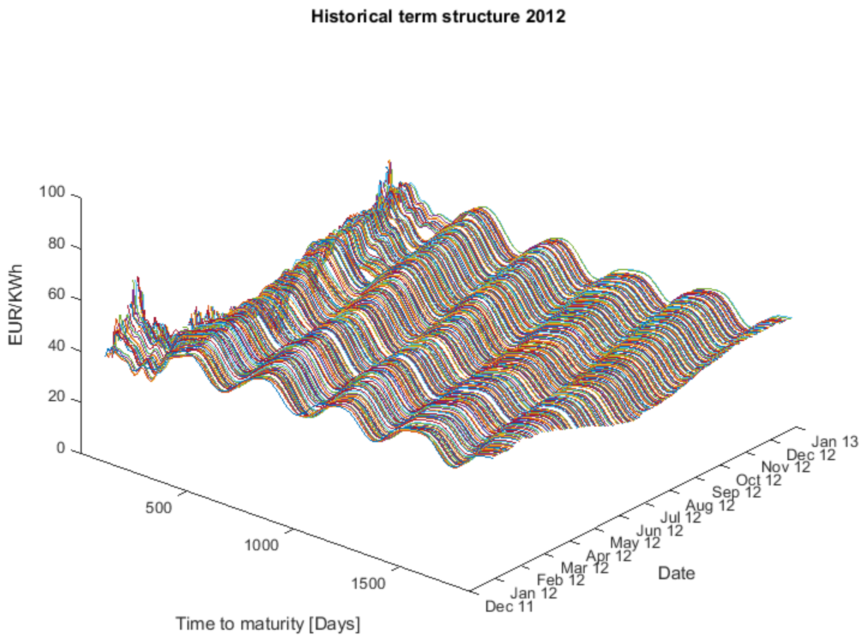

Figure 5.7: Smooth forward curves 2012.

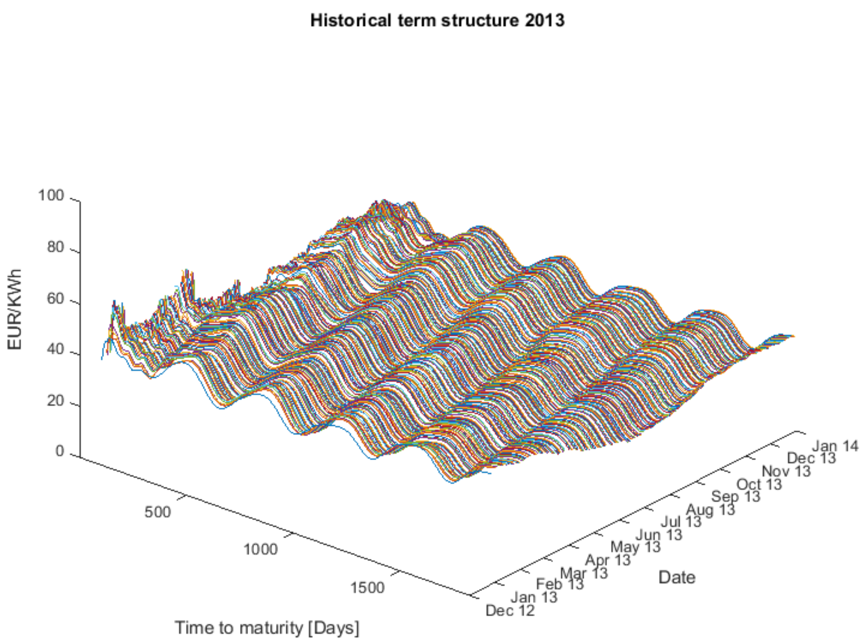

Figure 5.8: Smooth forward curves 2013.

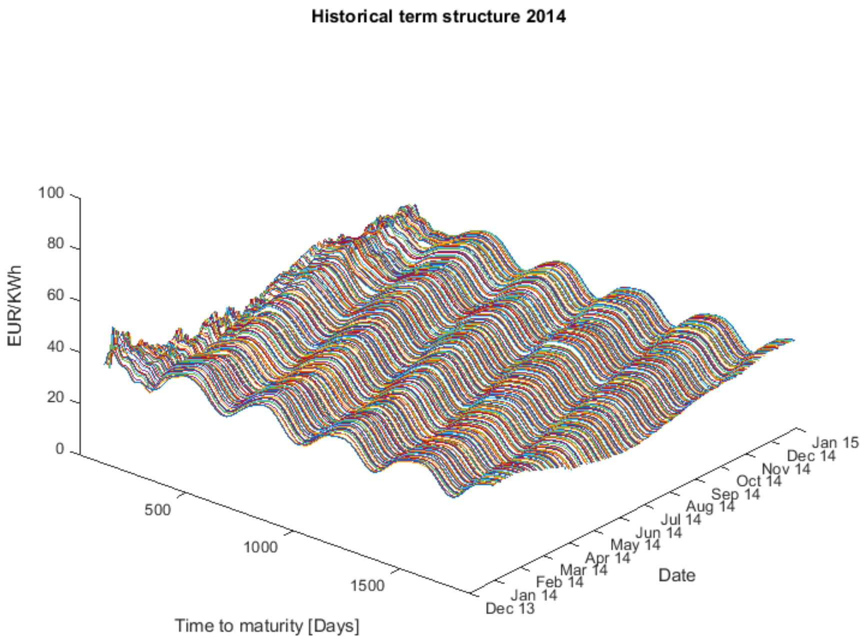

Figure 5.9: Smooth forward curves 2014.

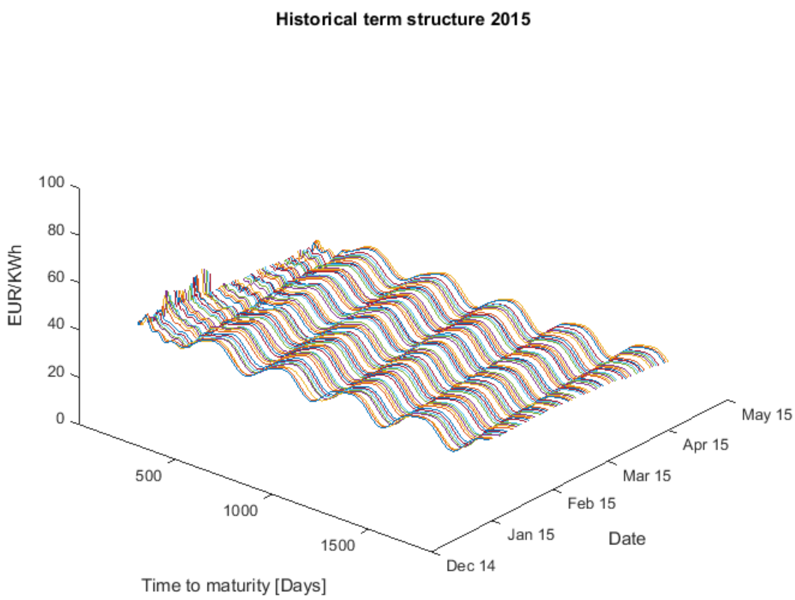

Figure 5.10: Smooth forward curves Jan-Mar 2015.

From studying figures 5.5 to 5.10 it can be seen that the highest volatility in the market that occurred during this almost five years timespan, was in the end of year 2010 in figure 5.5 and in the beginning of year 2011 in figure 5.6. From the volatility term structure showed in figures 5.5 to 5.10 the standard deviation (volatility) for all contracts with equal time to maturity has been calculated. To analyse the volatility curve all log-returns has to be calculated as

$$R_i^m = ln\left(\frac{P_{i+1}^m}{P_i^m}\right), \tag{5.15}$$

where $i = 1, ..., N_s$ and $m = 1, ..., N_m$, in this thesis $N_s = 1215$ business days and $N_m = 730$ days to maturity. Then the standard deviation for all log-returns with equal time to maturity are calculated as

$$\sigma_d^m = std(\mathbf{R}^m), \tag{5.16}$$

where $\mathbf{R}^m$ is a vector containing all log-returns with the same time to maturity. If daily prices, P, are inserted in equation 5.16 this results in daily standard deviation and to get the annual standard deviation the results has been multiplied by $\sqrt{252}$ according $square - root - of - time$ rule in [30], since one year has 252 business days. This will be calculated as

$$\sigma_y^m = \sqrt{252}(\sigma_d^m), \tag{5.17}$$

where σ_d^m is the daily standard deviation for a specific time to maturity, m, scaled up to annual standard deviation σ_y^m for the same time to maturity. The resulting volatility curve from these calculations is achieved by plotting σ_y^m for all different times to maturity, which is showed in figure 5.11.

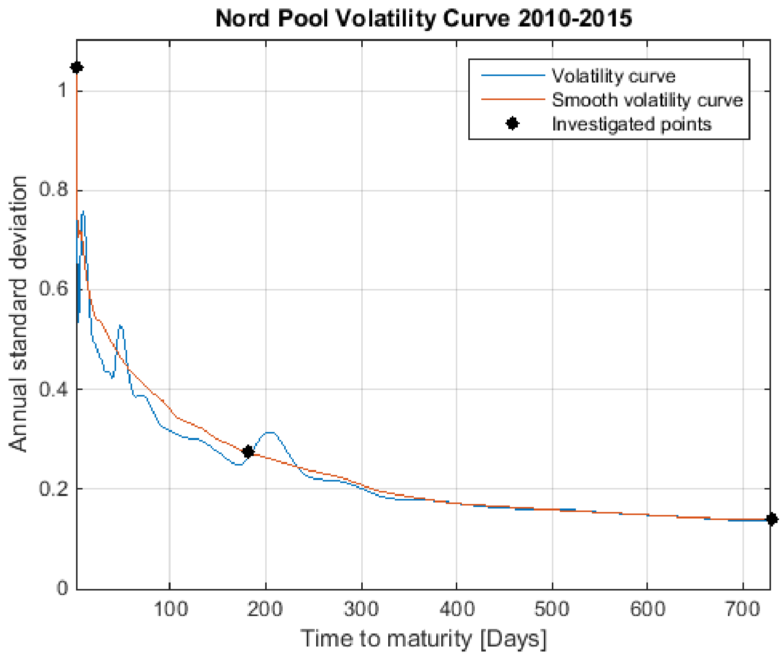

Figure 5.11: Volatility curve, smooth volatility curve and investigated points.

It can be seen in figure 5.11 that the volatility curve (blue line) achieved by plotting σ_y^m for all different times to maturity has some jumps. These jumps could possibly make the result unfair if the volatility is measured in such point, hence the curve has been smoothed as can be seen in the same figure (red line). From figure 5.11 it can be seen that the volatility function is a decreasing function, the volatility decreases for contracts with longer time to maturity. This is a typical characteristic for the electricity forward market. Short-term contracts have very high volatility and long-term contracts have significantly lower volatility. This is the main reason why it's common to hedge a power portfolio with contracts that have different time to maturity and not only buying short-term contracts, which would make the portfolio highly volatile and exposed to large risks.

To model the instantaneous volatility function, the forward market at date t is represented in [31] by a continuous forward price function, where $f(t,T)$ denotes the forward price at date t on a contract with time to delivery $T-t \geq 0$. Consider a forward contract with time to delivery $T-t$ and assume with respect to the risk-adjusted martingale probability measure the following forward price dynamics at date t:

$$\frac{df(t,T)}{f(t,T)} = \left(\frac{a}{T-t+b} + c\right) dW^*(t) \tag{5.18}$$

where a, b and c are positive constants and $dW^*(t)$ is the increment of a standard Brownian motion (often called Weiner process) with expected value 0 and variance dt. Set

$$\sigma(T-t) = \frac{a}{T-t+b} + c \tag{5.19}$$

where $T-t \geq 0$ is time to delivery. To determine the three parameters a, b and c the instantaneous volatility in equation 5.19 is calibrated to three different delivery points $0 \leq \tau_S \leq \tau_M \leq \tau_L$. Let σ_S denote short volatility, i.e. a point in the short end of the volatility curve. Let σ_M denote medium volatility, i.e. a point with medium volatility on the curve. Finally let σ_L denote long volatility, i.e. a point in the long end of the volatility curve. Where $\sigma_S \geq \sigma_M \geq \sigma_L$ translates into a decreasing and convex volatility structure. By using these definitions, the three different volatilities can be written as

$$\sigma_S = \sigma(0) = \frac{a}{b} + c \tag{5.20}$$

$$\sigma_M = \sigma(0.5) = \frac{a}{0.5+b} + c \tag{5.21}$$

$$\sigma_L = \sigma(\infty) = \lim_{\tau \to \infty} \left(\frac{a}{\tau + b} + c \right) \tag{5.22}$$

where $\sigma_S \geq \sigma_M \geq \sigma_L$. With $\tau_S = 0$, $\tau_M = 0.5$ and $\tau_L = 2$ years, this results in the following volatilities:

$$\begin{cases} \sigma_S = \sigma(\tau_S) = \sigma(0) = 1.0477 \\ \sigma_M = \sigma(\tau_M) = \sigma(0.5) = 0.2737 \\ \sigma_L = \sigma(\tau_L) = \sigma(2) = 0.1392 \end{cases}$$

Then the three parameters a, b and c can be determined from equations 5.20-5.22 as follows:

$$a = 0.5\,\frac{\sigma_M - \sigma_L}{\sigma_S - \sigma_M}(\sigma_S - \sigma_L) \tag{5.23}$$

$$b = 0.5\,\frac{\sigma_M - \sigma_L}{\sigma_S - \sigma_M} \tag{5.24}$$

$$c = \sigma_L. \tag{5.25}$$

With the calculated volatilities above inserted in equations 5.23-5.25 $a = 0.0789$, $b = 0.0869$ and $c = 0.1392$. One thing worth mentioning is that the volatility calculations in this section has been simplified, since it has been assumed that the volatility curve is the same throughout the year. This is probably a bit unfair since the electricity price follows seasonal trends. Seasonality effects can also be seen in the volatility, characterised with higher volatilities in the winter and lower volatilities in the summer for the Nordic market. An expansion would be to simulate the volatility curve with a volatility function that follows the seasonal pattern in the volatility, but this falls outside the scope for this thesis and is left for further development.

5.5.1 Stressed volatility term structure

To calculate $sVaR$ and sES the volatility structure has to be calibrated to a continuous 12-month period when the market has been in significant financial stress. From the volatility term structure showed in figures 5.5 to 5.10 it can be seen that the market was in significant financial stress the second half of 2010 and the first half of 2011, hence this period has been selected for calculation of $sVaR$ and sES. The selected period containing significant financial stress has been illustrated in figure 5.12.

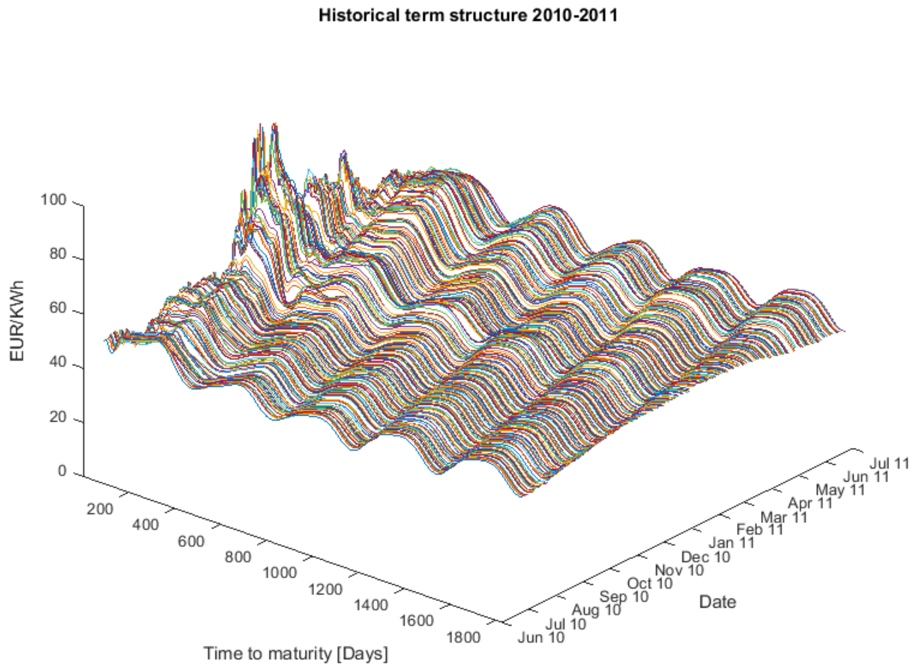

Figure 5.12: Smooth forward curves in a significant financial stressed period between the second half of 2010 and the first half of 2011.

In figure 5.13 the stressed volatility curve has been plotted, which has been calculated in the same way as the volatility curve in figure 5.11 with equations 5.15-5.17.

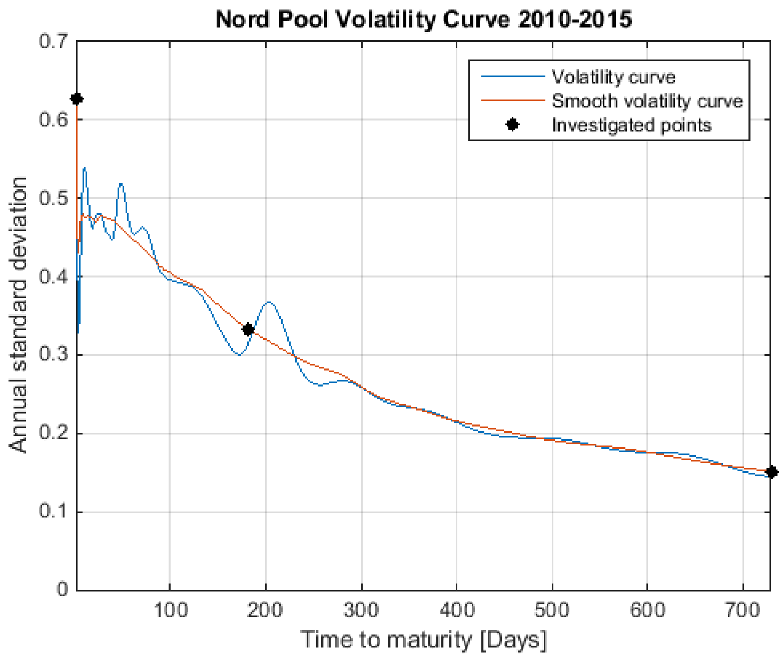

Figure 5.13: Stressed volatility curve, smooth stressed volatility curve and investigated points.

By comparing the volatility curves in figure 5.13 and 5.11, it can be seen that the volatility at τ_S is lower for stressed market conditions. It may initially be considered strange, but by inspection of figure 5.12 it can be seen that the volatilities for stressed market conditions are highest for contracts with slightly longer time to maturity ($> \tau_S$). The volatilities at time τ_M and τ_L are both higher for the stressed market conditions. With $\tau_S = 0$, $\tau_M = 0.5$ and $\tau_L = 2$ years and the same calculations as before, this results in the following stressed volatilities:

$$\begin{cases} \sigma_S = \sigma(\tau_S) = \sigma(0) = 0.6271 \\ \sigma_M = \sigma(\tau_M) = \sigma(0.5) = 0.3326 \\ \sigma_L = \sigma(\tau_L) = \sigma(2) = 0.1516 \end{cases}$$

The calculated stressed volatilities above gives the following parameters $a = 0.1461$, $b = 0.3073$ and $c = 0.1516$.

5.6 Forward Market Model

The one-factor model in equation 5.19 could be used for risk management purpose as a representation of the forward price dynamics. But since that equation only represent a one-factor model the instantaneous forward price changes for different delivery dates will be perfectly correlated. If this one-factor model would be used as simulation model this would imply that the set of possible scenarios will be restricted to more or less parallel shifts in the forward curve. This would not be a good representation for the most markets and hence risks would be underestimated, which could lead to terrible unexpected losses. Instead of a one-factor model, in order to obtain a richer class of possible forward price functions (possible scenarios), a three-factor model is introduced in [32]. This three-factor model will be used in this thesis and it represent the forward market at date t by a continuous forward price function. In this model considering a forward contract with delivery at date $T \geq t$, the following forward price dynamics are assumed with respect to the martingale probability measure:

$$\frac{df(t,T)}{f(t,T)} = \frac{a}{T-t+b}dW_1^*(t) + \left(\frac{2ac}{T-t+b}\right)^{\frac{1}{2}} dW_2^*(t) + c dW_3^*(t), \quad (5.26)$$

where $a, b, and\ c$ are positive constants and $dW_1^*(t), dW_2^*(t)\ and\ dW_3^*(t)$ are increments of three uncorrelated standard Brownian motions with expectation $E_t^*[dW^*(t)] = 0$ and $Var_t^*[dW^*(t)] = dt$. For computer simulations the increments of the standard Brownian motions (often called Wiener increment) can be discretised with a timestep dt as

$$dW \sim \sqrt{dt}N(0,1), \quad (5.27)$$

according to [33]. The instantaneous dynamics of equation 5.26 is normal with expected value zero and variance:

$$Var_t^*\left[\frac{df(t,T)}{f(t,T)}\right] = \left\{\left(\frac{a}{T-t+b}\right)^2 + \frac{2ac}{T-t+b} + c^2\right\}ds, \quad (5.28)$$

which is consistent with the dynamics in equation 5.18. The positive constants $a, b, and\ c$ derived from equation 5.18 also applies for equation 5.26. It follows from equation 5.26 that the forward price function $f(\tau, T)$ at a future date $\tau \in [t, T]$ is given by the following stochastic integral equation:

$$\begin{aligned} f(\tau,T) = f(t,T) exp\left\{ \int_t^\tau \frac{a}{T-s+b}\, dW_1^*(s) - \frac{1}{2}\int_t^\tau \left(\frac{a}{T-s+b}\right)^2 ds \right\} \\ exp\left\{ \int_t^\tau \left(\frac{2ac}{T-s+b}\right)^{\frac{1}{2}} dW_2^*(s) - \frac{1}{2}\int_t^\tau \frac{2ac}{T-s+b}\, ds \right\} \\ exp\left\{ \int_t^\tau c\, dW_3^*(s) - \frac{1}{2}\int_t^\tau c^2\, ds \right\}, \end{aligned} \quad (5.29)$$

where $f(t,T)$ is the forward price for a contract at time t with delivery at date $T \geq t$ and the three exponential terms represents the future movement of that point during the future time period $\tau - t$. It can be confirmed that the forward price is a martingale with respect to the $*$-probability measure by observing that $E_t^*[1_\tau f(\tau,T)] = f(\tau,T)$. By using the discretisation from equation 5.27 and evaluating equation 5.29 in each point of the calculated smooth forward curve, a new curve showing possible future movements at time $\tau - t$ of the forward curve will be obtained. These possible realisations of the forward curve at a future time will later be used for calculating expected portfolio profit or loss.

5.7 Risk Management

The idea with both VaR and ES is to analyse the downside of the probability distribution of the future portfolio value. To obtain this probability distribution ($P\&L$) of the portfolio value at a future date, the Monte Carlo simulation method will be used. This Monte Carlo simulation procedure for determining the probability distribution of future portfolio profit and loss function can be described in three main steps.

1) Use a random generator and calculate possible outcomes for the forward price function in equation 5.29, possibly outcomes are shown in figure 5.14 below (red curves).
2) Evaluate each asset in the portfolio by using the forward curve for the current day and also evaluate each asset in the portfolio by using the simulated forward curve from equation 5.29 at a future time. Then calculate future profit or loss for each asset by taking the difference between the simulated forward curve and the values of the asset evaluated with the forward curve that day.
3) Sum up (value additivity) calculated profit and loss values for each position to get portfolio profit or loss.

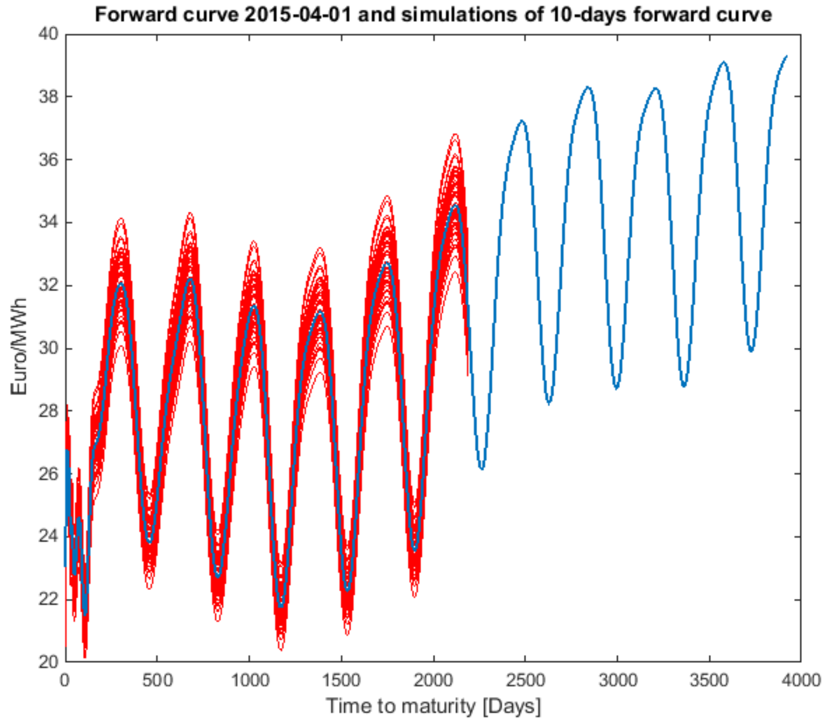

Figure 5.14: The smooth forward curve 2015-04-01 and examples of 50 simulated 10-day forward curves.

Repeat points **1**-**3** for a large number of iterations (thousands) to get a large number of profits and losses for the portfolio, and then the probability distribution of the future profits and losses can be approximated by the histogram following from the simulations. In the calculations, thousands of forward curves will be simulated and the portfolio will be revaluated for each of them to approximate the future portfolio profit and loss distribution. The portfolio $P\&L$ will later be used in risk management purposes to calculate VaR and ES for the portfolio.

5.8 Backtesting

In accordance with both CRR and FRTB shall VaR be used for backtesting of internal risk models and hence this section will focus on VaR backtesting. Backtesting refers to testing the accuracy of a model over a historical time period when the true outcome is known. Efficient market risk modelling requires a method to test the accuracy of the model. Even if the risk measure is estimated in detailed risk analysis, the measure itself does not tell anything about its accuracy. Hence the concept backtesting has been introduced to achieve a measure of accuracy regarding the risk measure. Backtesting is used to confirm that actual losses are in line with projected losses. The general approach for performing VaR backtesting for an asset, is to record the number of occasions, over a historical time period, when the actual loss exceeds the model predicted VaR and compare this number to the pre-specified confidence level of VaR. Backtesting can be used to verify whether the assumptions, parameters or methods in the model need to be further calibrated. From the backtesting results the user may be able to draw conclusions to improve the model if it does not achieve a certain standard or reject the choice of model if competing options is favored. The Basel committee has adopted backtesting when implementing internal risk models for capital requirements. Both in CRR and FRTB the calculated backtesting are performed by comparing 1-day VaR values with 1-day actual $P\&L$ and 1-day theoretical $P\&L$ using at least one year of historical observations.

• **Actual** $P\&L$**:** Actual $P\&L$, also called "dirty" $P\&L$, is the reported $P\&L$ for a portfolio by the accounting system. It's impacted by trades and fee income that takes place during the VaR horizon, i.e. trades that VaR cannot anticipate.

• **Theoretical** $P\&L$**:** Theoretical $P\&L$, also called "clean" $P\&L$, is the hypothetical $P\&L$ that would have been realised if no trading took place and no fee income were earned during the VaR horizon.

Since only a fixed portfolio held at 2015-04-01 has been provided, this thesis will focus on clean backtesting. Clean backtesting use a fixed portfolio and the clean $P\&L$ series are compared with the risk measure. Clean backtesting can be used to determine if the risk model has a systematic bias. If the results from the backtesting procedure are satisfactory, then it raises no issues regarding the quality of the internal risk model. However if the backtesting uncovers differences between the VaR and the theoretical (or actual) $P\&L$ it indicates that problems almost certainly must exist, either with the internal risk model or with the assumptions made in the selected backtesting procedure.

The backtesting results of an accurate VaR model should fulfil a number of criteria. Firstly, the number of exceedances should be as close as possible to the number implied by the used confidence level in the VaR model. If a VaR model at confidence level 99% is backtested for 250 business days, it should be as close as possible to 2-3 (2,5) exceedances. Secondly, for an accurate VaR model the exceedances should be randomly distributed over the sample. There shouldn't be any "clustering" of exceedances, since this would indicate that the model under/over estimates VaR in certain periods. This is an important criteria to have in mind when performing backtesting for power portfolios, since "clustered" exceedances are common for power portfolios. This is due to the seasonality of the electricity price. An inappropriate risk model can possible have difficulties capturing this cyclicality of the power market, which could lead to "clustering" of exceedances in some season.

Chapter 6

Results

6.1 Portfolio Hedging

The power portfolio, used in this thesis, is described in details in appendix C, where all positions in the portfolio are showed. This portfolio was held at 2015-04-01 and is considered for all calculations and results showed in this thesis. The portfolio contains PPA, futures and forward contracts. It has a mixture of monthly, quarterly and yearly contracts (some other special contracts exists). The portfolio takes both long and short positions respectively and is hedged for seasonal patterns in the electricity price, see figure 6.1. In this figure it's clear that the number of long and short positions are hedged by the seasonality of the electricity price, with a majority of short contracts in the summer when the electricity price is low and a majority of long contracts in the winter when the electricity price is high. In this figure the net sum of number of long and short contracts held in the portfolio has simply been divided to each day, i.e. the net exposure each day. For example, if the portfolio has 31 month long contracts in January they have been divided into 1 contract for each day (would have been -1 contract for a short position). This division of contracts into days is performed for all contracts in the portfolio, and the numbers of contracts each day are summed up to get the net exposure over number of contracts on a daily resolution. This is useful for seeing how the portfolio is hedged against seasonal price variation.

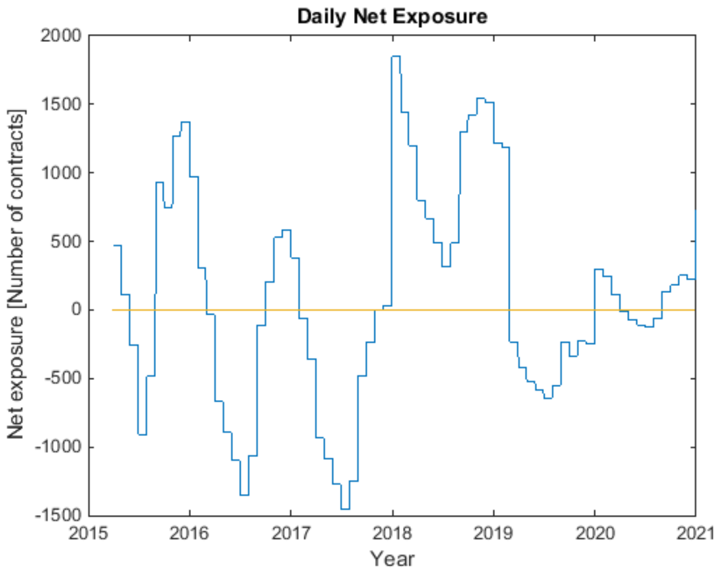

Figure 6.1: Daily net exposure of portfolio held 2015-04-01.

This portfolio has been provided from a power trading company, but could easily be changed to some other portfolio. This portfolio is only used as an example to show the results from the calculations with real numbers.

6.2 Risk Measures

VaR is calculated with a 10-day holding period and a 99% confidence level, in accordance with CRR. ES is also calculated with a 10-day holding period but with a 97.5% confidence level, in accordance with FRTB.

6.2.1 Value-at-Risk

Figure 6.2 shows a $P\&L$ histogram, with a red part for values below VaR and a grey part for values above. The blue line shows a PDF that is fitted to the $P\&L$ histogram.

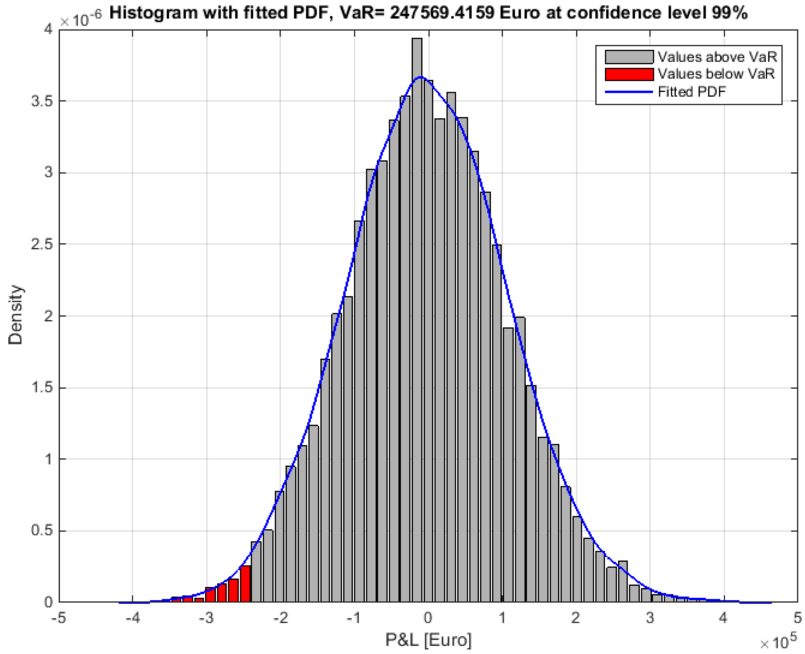

Figure 6.2: $P\&L$ histogram with fitted PDF and portfolio VaR at 99% confidence level.

6.2.2 Expected Shortfall

Figure 6.3 shows a $P\&L$ histogram, with a red part for values below the highest value in the ES calculations and a grey part for values above. The blue line shows a PDF that is fitted to the $P\&L$ histogram.

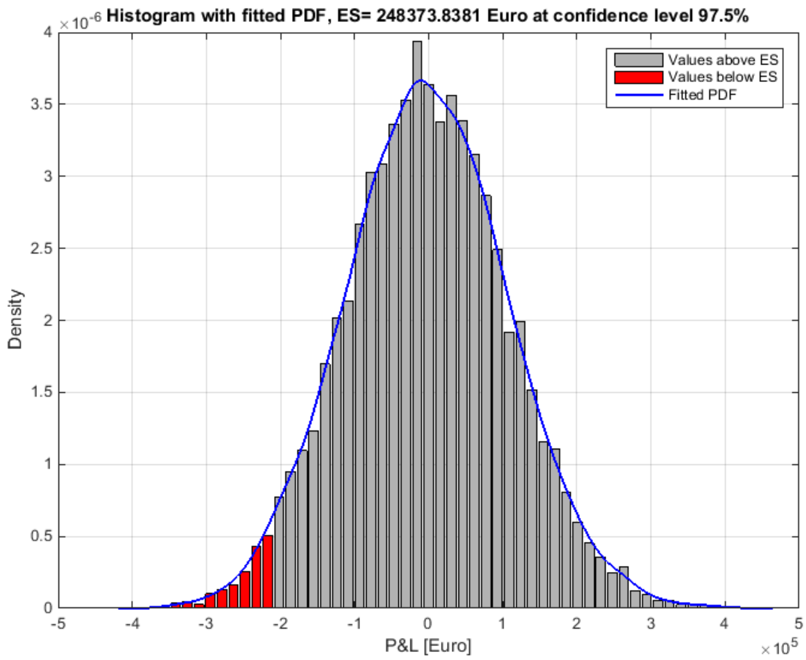

Figure 6.3: $P\&L$ histogram with fitted PDF and portfolio ES at 97.5% confidence level.

6.2.3 Risk report

The risk report in figure 6.4 is the output presented in MATLAB when performing the risk calculations at a certain date, in this case first of April 2015. It present some useful information about the considered portfolio but most importantly 10-day VaR and ES for the portfolio. Total number of contracts is the sum of long and short contracts in the portfolio. Net exposure for the portfolio is the sum of all exposures in long and short positions.

```
==================================================
        Portfolio Risk Report 01-Apr-2015
==================================================
Portfolio information
Total number of contracts:       8878677
Number of long contracts:        4467570
Number of short contracts:       4411107

Exposure long positions:         125993950.57   €
Exposure short positions:        -121750292.32  €
Portfolio net exposure:          4243658.25     €

10-day VaR estimation at 99.0% confidence level
Portfolio VaR:    247569.42€

10-day ES estimation at 97.5% confidence level
Portfolio ES:     248373.84€
--------------------------------------------------
```

Figure 6.4: Risk report output from MATLAB.

To comment the results in the risk report it can be seen that ES is only slightly larger than VaR, which was expected since the portfolio only contains linear assets. The size of VaR and ES respectively is around 6% of the total portfolio net exposure, which seems like a reasonable number.

6.3 Stressed Risk Measures

$sVaR$ is calculated during a 12-month period (July 10 to June 11) with significant financial stress for a 10-day holding period and a 99% confidence level, in accordance with CRR. sES is calculated during the same 12-month period with significant financial stress for a 10-day holding period but for a 97.5% confidence level, in accordance with FRTB.

6.3.1 Stressed Value-at-Risk

Figure 6.5 shows a stressed $P\&L$ histogram, with a red part for values below VaR and a grey part for values above. The blue line shows a PDF that is fitted to the stressed $P\&L$ histogram.

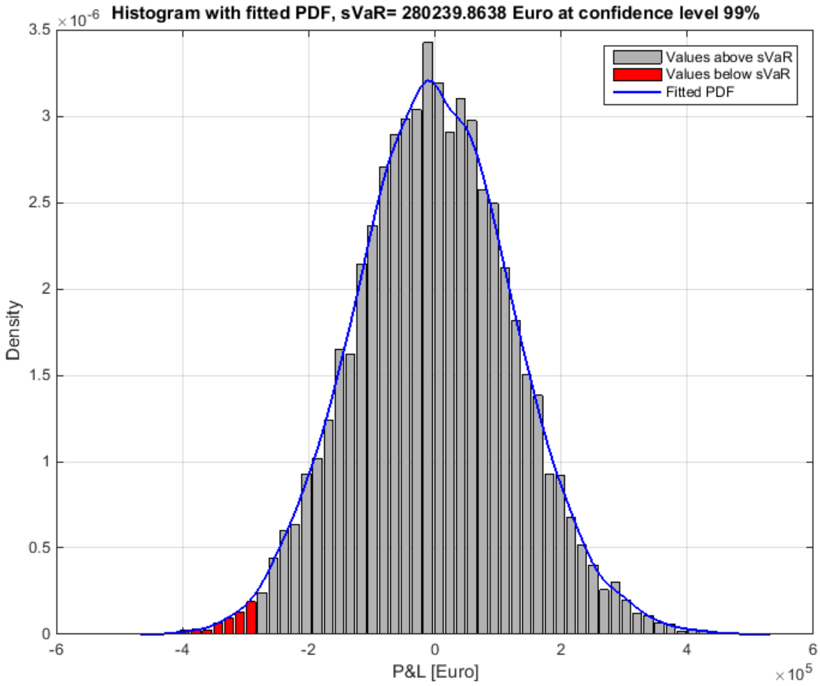

Figure 6.5: Stressed $P\&L$ histogram with fitted PDF and portfolio VaR at 99% confidence level.

6.3.2 Stressed Expected Shortfall

Figure 6.6 shows a stressed $P\&L$ histogram, with a red part for values below the highest value in the ES calculations and a grey part for values above that. The blue line shows a PDF that is fitted to the stressed $P\&L$ histogram.

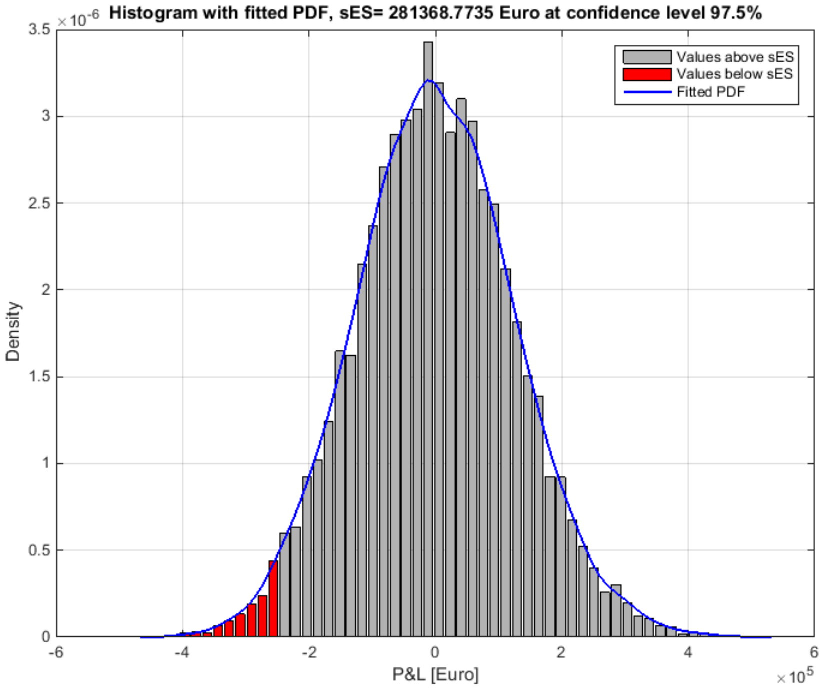

Figure 6.6: Stressed $P\&L$ histogram with fitted PDF and portfolio ES at 97.5% confidence level.

6.3.3 Stressed risk report

The stressed risk report in figure 6.7 is the output presented in MATLAB when performing the stressed risk calculations at a certain date, in this case first of April 2015. It present some useful information about the considered portfolio but most importantly 10-day $sVaR$ and sES for the portfolio. Total number of contracts is the sum of long and short contracts in the portfolio. Net exposure for the portfolio is the sum of all exposures in long and short positions.

```
==================================================
   Stressed Portfolio Risk Report 01-Apr-2015
==================================================
Portfolio information
Total number of contracts:        8878677
Number of long contracts:         4467570
Number of short contracts:        4411107

Exposure long positions:          125993950.57   €
Exposure short positions:         -121750292.32  €
Portfolio net exposure:           4243658.25     €

10-day sVaR estimation at 99.0% confidence level
Portfolio sVaR:   280239.86€

10-day sES estimation at 97.5% confidence level
Portfolio sES:    281368.77€
--------------------------------------------------
```

Figure 6.7: Stressed risk report output from MATLAB.

To comment on the results in the stressed risk report it can be seen that sES is only slightly larger than $sVaR$ in this case also, which was expected with the same motivation as before. The size of $sVaR$ and sES respectively is around 7% of the total portfolio net exposure, which seems like a reasonable number. By comparing the results in the stressed risk report in figure 6.7 and the risk report in figure 6.4 it can be seen that both value-at-risk and expected shortfall is higher using stressed market conditions, around 7% of the total portfolio net exposure instead of around 6%. This is a valid result, since if the risk would have been smaller for the stressed period then it would have indicated that the market wasn't in significant financial stress during the selected period of 2010-2011.

6.4 Capital Requirements

This section combines the regulatory calculation standards stated in chapter 3 with the results obtained previously in this chapter, to calculate capital requirements for market risks.

6.4.1 Capital requirement - internal VaR model

An average of the preceding 60 business day's $sVaR$ ($sVaR_{avg}$)) can't be calculated since the only portfolio data that is available is for a fix day. Electricity price has seasonal effects and it's not reasonable to assume that the portfolio looks the same as on 2015-04-01 the preceding 60 business day's. The seasonal effect in the portfolio can be seen in figure 6.1, which clearly shows that the portfolio is hedged against seasonal effects, by taking different weights between short or long positions. We have to assume that for 2015-04-01, VaR and $sVaR$ are the highest values and use them to calculate value-at-risk for risk capital purposes, VaR_C, hence

$$VaR_C = VaR_T + sVaR_T, \tag{6.1}$$

where VaR_T and $sVaR_T$ are our calculated values for VaR and respectively $sVaR$ with base liquidity horizon T, i.e. 10 days. From the risk report in figure 6.4 value-at-risk, VaR, on the first of April, 2015, with the 10-day base liquidity horizon and a 99% confidence level is given as $247,569$ Euro and corresponding stressed value-at-risk, $sVaR$, is given as $280,240$ Euro in figure 6.7. These values inserted in equation 6.1 gives the value-at-risk for risk capital purposes, $VaR_C = 527,809$ Euro, the second of April, 2015.

6.4.2 Capital requirement internal ES model

With the same motivation as in the VaR_C calculation, we have to assume that the most recent observation ES_{t-1} is the maximum value in equation 3.1 and use it to calculate expected shortfall for risk capital purposes, ES_C. Since our model only has one risk factor category, energy price, equation 3.4 can be simplified to

$$ES = \sqrt{(ES_T)^2 + \left(ES_T\sqrt{\frac{(LH_j - LH_{j-1})}{T}}\right)^2}, \tag{6.2}$$

where ES_T is our calculated value for ES with base liquidity horizon T, i.e. 10 days. The liquidity horizons LH_j and LH_{j-1} are given in table 3.2. From the table in appendix B it can be seen that risk factor category "Energy price" shall be scaled up to a 20-day liquidity horizon, i.e. $LH_j = 20$ and $LH_{j-1} = 10$. Hence in our case equation 6.2 can be written as

$$ES = \sqrt{2(ES_T)^2}, \tag{6.3}$$

where ES_T could be either ES or sES with base liquidity horizon T. With the same motivation as above (only one risk factor category) the expected shortfall for risk capital purposes, ES_C in equation 3.1 can be simplified to

$$ES_C = ES_{R,S} = sES = \sqrt{2(sES_T)^2}. \tag{6.4}$$

From the risk report in figure 6.7 stressed expected shortfall, sES, on the first of April, 2015, with the 10-day base liquidity horizon and a 97,5% confidence level is given as $281,369$ Euro. Stressed expected shortfall, $sES = 281,369$ Euro, inserted in equation 6.4 gives the expected shortfall for risk capital purposes, $ES_C = 397,916$ Euro, the second of April, 2015.

6.4.3 Capital requirements comparison

In table 6.1 risk capital requirements VaR_C and ES_C are presented for the second of April 2015. As can be seen, VaR_C is significantly higher than ES_C despite that VaR and ES from the calculations are almost equal. This shows that for portfolios only containing linear assets, the difference in capital requirements for market risks is not due to the risk measures but rather the different methods of calculating the risk capital in the respective regulations.

	Capital Requirement 2015-04-02
VaR_C	527,809 Euro
ES_C	397,916 Euro

Table 6.1: Risk capital requirements, VaR_C and ES_C.

6.5 Performed Backtesting

Basic "backtesting" has been performed for a period of 60 business days. The "backtesting" has been calculated for a time period that includes 2015-04-01, 30 business days before and 30 business days after. This "backtesting" has been performed with the fixed portfolio ("clean" backtesting) held at 2015-04-01 and fixed volatilities calculated for the same date. Results from the "backtesting" is showed in figure 6.8. In this figure a 1-day VaR is compared to a 1-day theoretical portfolio $P\&L$. The reason for that "backtesting" has been performed for only 30 business days before and after 2015-04-01 is due to the reason that the model uses fix volatilities. Before more rigorous backtesting, for a longer period, can be performed the model has to be extended by using time varying volatilities, but this is omitted in this thesis.

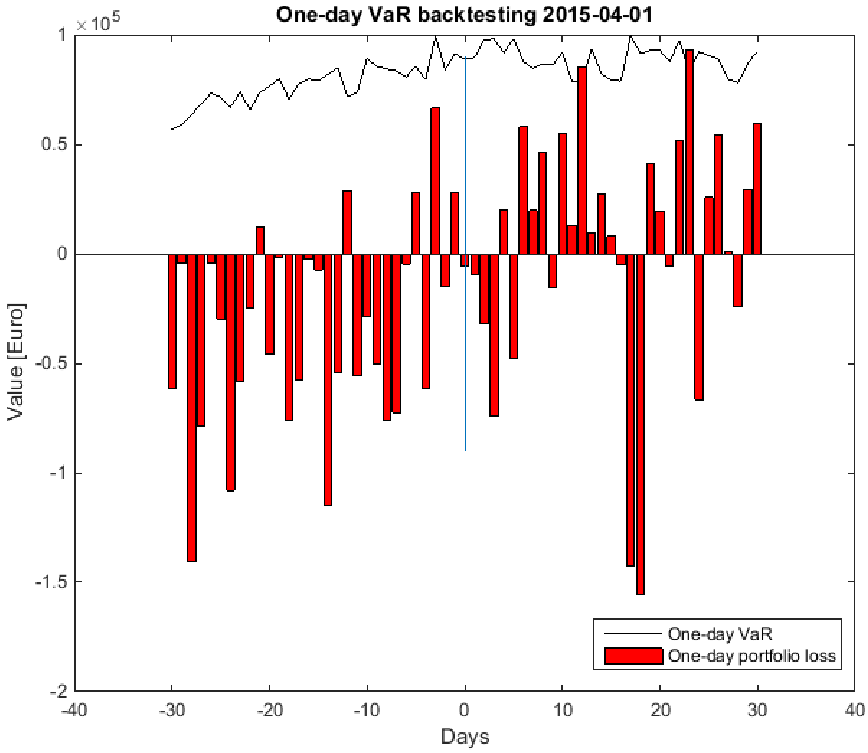

Figure 6.8: 1-day VaR and 1-day theoretical portfolio loss (positive loss) at 2015-04-01 and for the 30 latest/upcoming business days, calculated with the fixed portfolio held at 2015-04-01.

Studying this figure shows that around day 0 (2015-04-01) when this portfolio was held and properly hedged the actual portfolio return shifts between profit and loss. For the majority of days before 2015-04-01 the portfolio returns are profits and for the majority of days after the portfolio returns are losses. This figure doesn't say that much about the performance of the model, but it indicates that the numbers are in reasonable size, since VaR is larger than the theroetical $P\&L$ in the most cases. To investigate the performance of the model further, backtesting for a longer time period and for all different kinds off portfolios has to be performed. Otherwise the model could be suitable for one portfolio but not for another portfolio. This could be tested by using different portfolios e.g. portfolios with only short positions, only long positions, only contracts with short time to maturity, only contracts with long time to maturity, a mixture of these combinations etc.

Chapter 7

Summary and Conclusion

This chapter interprets the results, summarises the main conclusions and discusses further studies.

For the considered portfolio, containing only linear instruments (PPA, futures and forward contracts), the capital requirement with an internal VaR model in accordance with CRR has been calculated to $527,809$ Euro. Corresponding capital requirement with an internal ES model in accordance with FRTB has been calculated to $397,916$ Euro. It's observed for this portfolio that VaR at a 99% confidence level and ES at a 97.5% confidence level are in fact almost identical. The explanation to this is that the portfolio only contains linear instruments and for such portfolios ES has no major advantages over VaR. But still the capital requirements for the portfolio differs, the internal VaR model in accordance with CRR results in a larger capital requirements than the internal ES model in accordance with FRTB for the same portfolio. The reason for this difference in capital requirements is not, in this case, due to the choice of risk measure, but instead affected by the different procedures for calculating risk capital VaR_C and ES_C respectively in the models. If adding non-linear instruments (options) to the portfolio ES_C would probably increase more than VaR_C. The reason for this is that portfolios containing non-linear instruments often have larger risks and heavier tails, which are better captured by ES than VaR.

This is the first version of this internal risk model. It performs all modelling steps and the result seems reasonable, but before the model could be implemented for risk management it has to be extended with a better calibration to the market and more backtesting. The internal risk model developed in this thesis fulfils the requirements in CRR and FRTB to calculate capital requirements for market risks, but it's not ready to be used for risk management. First of all, different prior-functions should be tested when calculating the smooth forward curve and compared against each other to find the prior-function that fits reality best. The model should also be extended by using a seasonal driven volatility function. In the present version of the model the volatility has been considered unchanged throughout the year and this is an unfair simplification. Another source of error that should be investigated is the length of the historical period that the volatilities are calculated over. In this version they have been calculated over almost 5 years. This could possibly be a too long time period and the volatility of the market might have changed during it. If the model will be used for portfolios that contain non-linear instruments the model has to be extended.

For risk management use of the model it would have been good if it allowed overlapping contracts when calculating the forward price curve. How this could be implemented in the model is showed in equation A.14 in the end of appendix A. This would make the model more general and easier to use on a daily basis, since contracts on NASDAQ OMX commodities are available with overlapping delivery periods. In the list below, the main points that are suggested for further studies are summarised.

Suggestions for further studies:
- Compare different prior-functions and select the one that fits reality best.
- Use seasonal driven volatilities to reflect seasonal fluctuations in the volatilities.
- Extend the backtesting by considering a longer time period, different types of portfolios and recalculated volatilities.
- Extend the model to capture non-linear assets (options).
- Extend the model to allow contracts with overlapping delivery periods.

Bibliography

[1] Nord Pool Spot, *EUROPE'S LEADING POWER MARKETS*, 2014. Avaliable at http://www.nordpoolspot.com/globalassets/download-center/annual-report/nord-pool-spot_europes-leading-power-markets.pdf

[2] Nord Pool Spot, *THE POWER MARKET*, 2015. Available at http://nordpoolspot.com/How-does-it-work/

[3] Nord Pool Spot, *Elmarknader och elhandel*, 2015. Available at http://ei.se/sv/el/elmarknader-och-elhandel/

[4] Nord Pool Spot, *TRANSMISSION SYSTEM OPERATORS*, 2015. Available at http://nordpoolspot.com/How-does-it-work/Transmission-system-operators-TSOs/

[5] Nord Pool Spot, *DAY-AHEAD MARKET*, 2015. Available at http://nordpoolspot.com/How-does-it-work/Day-ahead-market-Elspot-/

[6] Nord Pool Spot, *BIDDING AREAS*, 2015. Available at http://nordpoolspot.com/How-does-it-work/Bidding-areas/

[7] Nord Pool Spot, *INTRADAY MARKET*, 2015. Available at http://nordpoolspot.com/How-does-it-work/Intraday-market/

[8] PPPIRC, *Power Purchase Agreements*, 2015. Available at http://ppp.worldbank.org/public-private-partnership/sector/energy/energy-power-agreements/power-purchase-agreements#where_appropriate

[9] Nord Pool Spot, *FINANCIAL MARKET*, 2015. Available at http://nordpoolspot.com/How-does-it-work/Financial-market/

[10] Nord Pool Spot, *The Financial Market*, 2008. Available at http://www.cmb.gov.tr/filesys/emissions_electricity/presentations/3_sk.pdf

[11] Basel, *About the Basel Committee*, 2014. Available at `http://www.bis.org/bcbs/about.htm?m=3|14|573`

[12] Basel, *International regulatory framework for banks (Basel III)*, 2015. Available at `http://www.bis.org/bcbs/basel3.htm?m=3|14|572`

[13] Basel Committee on Banking Supervision, *Capital Requirements Regulation*, 2011.

[14] Basel Committee on Banking Supervision, *Fundamental Review of the Trading Book: A revised market risk framework*, 2013.

[15] Basel Committee on Banking Supervision, *Fundamental Review of the Trading Book: Outstanding issues*, 2014.

[16] Clemens U. Frei, *Value at Risk for Linear and Non-Linear Derivatives*, 2003.

[17] FINANSINSPEKTIONEN, *Intern VaR-modell för beräkning av kapitalkrav för marknadsrisk*, 2004-04-26.

[18] Yasuhiro Yamai, Toshinao Yoshiba, *Value-at-risk versus expected shortfall: A practical perspective*, 2003, Journal of Banking & Finance 29 (2005) 997-1015.

[19] Hult, H., Lindskog, F., Hammarlid, O., and Rehn, C.J., *Risk and Portfolio Analysis*, 2012, Springer.

[20] Mehdi Sadeghi, Saeed Shavvalpour, *Energy risk management and value at risk modeling*, 2005, Energy Policy 34 (2006) 3367-3373.

[21] Giovanni Barone-Adesi, Kostas Giannopoulos, *Non-parametric VaR techniques. Myths and Realities.*, 2000.

[22] LES CLEWLOW, CHRIS STRICKLAND, VINCE KAMINSKI, *Which Var for energy derivatives?*, 2000.

[23] Yasuhiro Yamai, Toshinao Yoshiba, *On the Validity of Value-at-Risk: Comparative Analyses with Expected Shortfall*, 2002, MONETARY AND ECONOMIC STUDIES.

[24] Energinet, *Udtræk af markedsdata*, 2015. Avaliable at: http://www.energinet.dk/DA/El/Engrosmarked/Udtraek-af-markedsdata/Sider/default.aspx

[25] Nasdaq Commodities, *Historical data set*, 2015.

[26] Steen Koekebakker, Fridthjof Ollmar, *Forward curve dynamics in the Nordic electricity market*, 2005, Managerial Finance, Vol.31 Iss 6 pp. 73-94.

[27] Fred Espen Benth, Jurate Saltyte Benth, Steen Koekebakker, *Stochastic Modelling of Electricity and Related Markets*, 2008, World Scientific.

[28] Fridthjof Ollmar, *Smooth forward price curves in electricity markets*, 2003.

[29] Seifert, Jan, Uhrig-Homburg, Marliese, *Modelling Jumps in Electricity Prices: Theory and Empirical Evidence*, 2007, Review of Derivatives Research, Vol. 10, pp 59-85.

[30] Jón Daníelsson, Jean–Pierre Zigrand *On time-scaling of risk and the square–root–of–time rule*, 2003, Journal of Economic Literature classification numbers: G18, G20, D81.

[31] Gunnar Stensland, Petter Bjerksund, *Correlation in VaR simulations*, 2009, Viz Risk Management.

[32] P. Bjerksund, H. Rasmussen and G. Stensland, *VALUATION AND RISK MANAGEMENT IN THE NORWEGIAN ELECTRICITY MARKET*, 2000.

[33] Jeffrey M. Moehlis, *A Standard Wiener Process*, 2001. Available at `https://me.ucsb.edu/~moehlis/APC591/tutorials/tutorial7/node2.html`

Appendix A

Matrix Notations

A.1 Matrix notations used for implementation of smooth forward curves

It's showed by Ollmar in [28] that by inserting $g^{''}(t)$ in equation 5.5 and integrating the first part of the minimisation problem, it can be written in matrix form as

$$\min_{\mathbf{x}} \quad \mathbf{x}^T \mathbf{H} \mathbf{x}, \tag{A.1}$$

where

$$\mathbf{x}^T = [a_1\ b_1\ c_1\ d_1\ e_1\ a_2\ b_2\ c_2\ d_2\ e_2\ \ldots\ a_n\ b_n\ c_n\ d_n\ e_n], \tag{A.2}$$

$$\begin{bmatrix} h_1 & & 0 \\ & \ddots & \\ 0 & & h_n \end{bmatrix} and \tag{A.3}$$

$$\begin{bmatrix} \frac{144}{5}\Delta_j^5 & 18\Delta_j^4 & 8\Delta_j^3 & 0 & 0 \\ 18\Delta_j^4 & 12\Delta_j^3 & 6\Delta_j^2 & 0 & 0 \\ 8\Delta_j^3 & 6\Delta_j^2 & 4\Delta_j & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{A.4}$$

$\mathbf{x}$ is a column vector with dimensions [5n x 1], $\mathbf{H}$ is an asymmetric matrix with dimensions [5n x 5n] and $\Delta_j^l = t_{j+1}^l - t_j^l$ for $l = 1, \ldots, 5$. The constraints **C1-C5** in equations 5.6-5.11 can also be written in matrix form, $\mathbf{A}\mathbf{x} = \mathbf{B}$, where $\mathbf{B}$ is a column vector with dimensions [3n+m-2 x 1] and $\mathbf{A}$ is a matrix with dimensions [3n+m-2 x 5n]. An explicit solution to equation 5.5 can be obtained by using the Lagrange multiplier method, where $\lambda^T = [\lambda_1, \lambda_2, ..., \lambda_{3n+m-2}]$ is the corresponding Lagrange multiplier vector to the constraints **C1**-**C5**. By applying this to equation 5.5, it can be written as the following unconstrained minimisation problem:

$$\min_{\mathbf{x},\lambda} \quad \mathbf{x}^T\mathbf{H}\mathbf{x} + \lambda^T(\mathbf{A}\mathbf{x} - \mathbf{B}). \tag{A.5}$$

The solution $[\mathbf{x}^*, \lambda^*]$ to the unconstrained minimisation problem in equation A.5 could be determined by solving the following linear system of equations:

$$\begin{bmatrix} 2\mathbf{H} & \mathbf{A}^T \\ \mathbf{A} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{B} \end{bmatrix}, \tag{A.6}$$

where matrix **A** and vector **B** can be constructed by writing the conditions **C1-C5** in chapter 6 from equations 5.6-5.11 as a linear system of equations **Ax**=**B**. The dimensions of equation A.6 is as follows: left matrix has dimensions [8n+m-2 x 8n+m-2] and hence the two vectors must have the dimensions [8n+m-2 x 1]. This system of equation can easily be solved numerically by Gaussian elimination. First some variables are explained and then it's showed how the matrix **A** and the vector **B** are built-up from the constraints.

$$\begin{cases} n = \text{Number of polynomials} \\ m = \text{Number of forward contracts} \\ t_0, t_1, ..., t_n = \text{The knot points of the polynomial} \end{cases}$$

The continuity condition **C1** corresponds to the following elements in matrix **A**:

$$\begin{matrix} -t_1^4 & -t_1^3 & -t_1^2 & -t_1 & -1 & t_1^4 & t_1^3 & t_1^2 & t_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & -t_2^4 & -t_2^3 & -t_2^2 & -t_2 & -1 & t_2^4 & t_2^3 & t_2^2 & t_2 & 1 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -t_3^4 & -t_3^3 & -t_3^2 & -t_3 & -1 & t_3^4 & t_3^3 & t_3^2 & t_3 & 1 & \dots \\ \vdots \end{matrix} \tag{A.7}$$

which has the size [n-1 x 5n] and the corresponding elements in vector **B** are $n - 1$ zeros. Condition **C2** corresponds to the following elements in matrix **A**:

$$\begin{matrix} -4t_1^3 & -3t_1^2 & -2t_1 & -1 & 0 & 4t_1^3 & 3t_1^2 & 2t_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & -4t_2^3 & -3t_2^2 & -2t_2 & -1 & 0 & 4t_2^3 & 3t_2^2 & 2t_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4t_3^3 & -3t_3^2 & -2t_3 & -1 & 0 & 4t_3^3 & 3t_3^2 & 2t_3 & 1 & 0 & \dots \\ \vdots \end{matrix} \tag{A.8}$$

which has the size [n-1 x 5n] and the corresponding elements in vector **B** are $n - 1$ zeros. Condition **C3** corresponds to the following elements in matrix **A**:

$$\begin{matrix} -12t_1^2 & -6t_1 & -2 & 0 & 0 & 12t_1^2 & 6t_1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & -12t_2^2 & -6t_2 & -2 & 0 & 0 & 12t_2^2 & 6t_2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -12t_3^2 & -6t_3 & -2 & 0 & 0 & 12t_3^2 & 6t_3 & 2 & 0 & 0 & \dots \\ \vdots \end{matrix} \tag{A.9}$$

which has the size [n-1 x 5n] and the corresponding elements in vector **B** are $n-1$ zeros. The terminal condition **C4** corresponds to the following elements in matrix **A**:

$$\begin{array}{cccccccccccccccccccc} \ldots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4t_n^3 & 3t_n^2 & 2t_n & 1 & 0 \end{array} \tag{A.10}$$

which has the size [1 x 5n] and the corresponding element in vector **B** is zero. The last condition **C5** is equal to

$$\int_{T_j^s}^{T_j^e} g(t)dt = F_j^C(T_j^e - T_j^s) - \int_{T_j^s}^{T_j^e} h(t)dt \tag{A.11}$$

and to rewrite this condition as a linear system of equation the left-hand side of equation A.11 are inserted in matrix **A** and the right-hand side in vector **B**. Next step is to construct a set of knot points where each sub-period is equal to the domain of a polynomial. This set of knot points are defined as

$$\begin{aligned} \rho_i^1 &= [(T_j^e)^5 - (T_j^s)^5]/5 \\ \rho_i^2 &= [(T_j^e)^4 - (T_j^s)^4]/4 \\ \rho_i^3 &= [(T_j^e)^3 - (T_j^s)^3]/3 \\ \rho_i^4 &= [(T_j^e)^2 - (T_j^s)^2]/2 \\ \rho_i^5 &= T_j^e - T_j^s. \end{aligned} \tag{A.12}$$

Condition **C5** for non-overlapping contracts corresponds to the following elements in matrix **A**:

$$\begin{array}{cccccccccccccccccccccc} \rho_0^1 & \rho_0^2 & \rho_0^3 & \rho_0^4 & \rho_0^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & 0 & \rho_1^1 & \rho_1^2 & \rho_1^3 & \rho_1^4 & \rho_1^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_2^1 & \rho_2^2 & \rho_2^3 & \rho_2^4 & \rho_2^5 & 0 & 0 & 0 & 0 & 0 & \ldots \\ \vdots \end{array} \tag{A.13}$$

An example of condition **C5** for overlapping contracts, where the first contracts has a settlement period $[t_1, t_2]$, when we have divided the integral corresponds to the following elements in matrix **A**:

$$\begin{array}{cccccccccccccccccccccc} \rho_0^1 & \rho_0^2 & \rho_0^3 & \rho_0^4 & \rho_0^5 & \rho_1^1 & \rho_1^2 & \rho_1^3 & \rho_1^4 & \rho_1^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & 0 & \rho_1^1 & \rho_1^2 & \rho_1^3 & \rho_1^4 & \rho_1^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_2^1 & \rho_2^2 & \rho_2^3 & \rho_2^4 & \rho_2^5 & 0 & 0 & 0 & 0 & 0 & \ldots \\ \vdots \end{array} \tag{A.14}$$

In both the case overlapping and non-overlapping contracts the element in vector **B** that corresponds to contract j can be written as

$$F_j^C(T_j^e - T_j^s) - \int_{T_j^s}^{T_j^e} h(t)dt. \tag{A.15}$$

When all corresponding rows in matrix $\mathbf{A}$ are combined, it gives the following matrix:

$$\mathbf{A} = \begin{bmatrix}
-t_1^4 & -t_1^3 & -t_1^2 & -t_1 & -1 & t_1^4 & t_1^3 & t_1^2 & t_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & 0 & 0 & 0 & 0 & -t_2^4 & -t_2^3 & -t_2^2 & -t_2 & -1 & t_2^4 & t_2^3 & t_2^2 & t_2 & 1 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -t_3^4 & -t_3^3 & -t_3^2 & -t_3 & -1 & t_3^4 & t_3^3 & t_3^2 & t_3 & 1 & \dots \\
\vdots \\
-4t_1^3 & -3t_1^2 & -2t_1 & -1 & 0 & 4t_1^3 & 3t_1^2 & 2t_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & 0 & 0 & 0 & 0 & -4t_2^3 & -3t_2^2 & -2t_2 & -1 & 0 & 4t_2^3 & 3t_2^2 & 2t_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -4t_3^3 & -3t_3^2 & -2t_3 & -1 & 0 & 4t_3^3 & 3t_3^2 & 2t_3 & 1 & 0 & \dots \\
\vdots \\
-12t_1^2 & -6t_1 & -2 & 0 & 0 & 12t_1^2 & 6t_1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & 0 & 0 & 0 & 0 & -12t_2^2 & -6t_2 & -2 & 0 & 0 & 12t_2^2 & 6t_2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -12t_3^2 & -6t_3 & -2 & 0 & 0 & 12t_3^2 & 6t_3 & 2 & 0 & 0 & \dots \\
\vdots \\
\dots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4t_n^3 & 3t_n^2 & 2t_n & 1 & 0 \\
\\
\rho_0^1 & \rho_0^2 & \rho_0^3 & \rho_0^4 & \rho_0^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & 0 & 0 & 0 & 0 & \rho_1^1 & \rho_1^2 & \rho_1^3 & \rho_1^4 & \rho_1^5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_2^1 & \rho_2^2 & \rho_2^3 & \rho_2^4 & \rho_2^5 & 0 & 0 & 0 & 0 & 0 & \dots \\
\vdots
\end{bmatrix} \tag{A.16}$$

which has dimensions [3n+m-2 x 5n]. When all corresponding elements in vector $\mathbf{B}$ are combined, it gives the following column vector:

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \dots & F_j^C(T_j^e - T_j^s) - \int_{T_j^s}^{T_j^e} h(t)dt & \dots \end{bmatrix}' \tag{A.17}$$

which has dimensions [3n+m-2 x 1]. The first 3(n-1)+1 elements in vector B are zero and the last m elements corresponds to equation A.15.

Appendix B

Risk Factor Categories - FRTB

B.1 Table with risk factor categories and liquidity horizons

This table is given in [15].

Risk factor category	**n**	**Risk factor category**	**n**
Interest rate – domestic currency of a bank: EUR, USD, GBP, AUD, JPY, SEK and CAD	120	Equity price (small cap) volatility	120
Interest rate ATM volatility – other currencies	260	Equity (other)	120
Interest rate ATM volatility Interest rate (other)	60	FX rate – liquid currency pairs	120
Interest rate ATM volatility (other than yields and ATM volatility)	60	FX rate (other currency pairs)	20
Credit spread – sovereign (IG)	20	FX volatility	60
Credit spread – sovereign (HY)	60	FX (other)	60
Credit spread – corporate (IG)	60	Energy price	20
Credit spread – corporate (HY)	120	Precious metal price	20
Credit spread – structured (cash and CDS)	250	Other commodities price	60
Credit (other)	250	Energy price volatility	60
Equity price (large cap)	10	Precious metal price volatility	60
Equity price (small cap)	20	Other commodities price volatility	120
Equity price (large cap) volatility	20	Commodity (other)	120

Table B.1: Risk factor categories in FRTB.

Appendix C

Power Portfolio

C.1 PPA Contracts 2015-04-01

Contracts:	**Period:**	**Type:**	**Number of contracts:**
APR-15	Monthly	Buy	31823
MAY-15	Monthly	Buy	31560
JUN-15	Monthly	Buy	26708
JUL-15	Monthly	Buy	70691
AUG-15	Monthly	Buy	80057
SEP-15	Monthly	Buy	104904
OCT-15	Monthly	Buy	108474
NOV-15	Monthly	Buy	116831
DEC-15	Monthly	Buy	122476
JAN-16	Monthly	Buy	75072
FEB-16	Monthly	Buy	63303
MAR-16	Monthly	Buy	56923
APR-16	Monthly	Buy	44696
MAY-16	Monthly	Buy	40973
JUN-16	Monthly	Buy	37619
JUL-16	Monthly	Buy	34641
AUG-16	Monthly	Buy	40149
SEP-16	Monthly	Buy	52922
OCT-16	Monthly	Buy	59090
NOV-16	Monthly	Buy	63804
DEC-16	Monthly	Buy	66160
JAN-17	Monthly	Buy	45947
FEB-17	Monthly	Buy	39460
MAR-17	Monthly	Buy	35140
APR-17	Monthly	Buy	27472
MAY-17	Monthly	Buy	25128

JUN-17	Monthly	Buy	22654
JUL-17	Monthly	Buy	21125
AUG-17	Monthly	Buy	24346
SEP-17	Monthly	Buy	33128
OCT-17	Monthly	Buy	38640
NOV-17	Monthly	Buy	41827
DEC-17	Monthly	Buy	42484
JAN-18	Monthly	Buy	37100
FEB-18	Monthly	Buy	32575
MAR-18	Monthly	Buy	28664
APR-18	Monthly	Buy	22822
MAY-18	Monthly	Buy	20744
JUN-18	Monthly	Buy	18440
JUL-18	Monthly	Buy	17411
AUG-18	Monthly	Buy	19903
SEP-18	Monthly	Buy	27546
OCT-18	Monthly	Buy	31802
NOV-18	Monthly	Buy	34060
DEC-18	Monthly	Buy	34418
JAN-19	Monthly	Buy	37162
FEB-19	Monthly	Buy	32651
MAR-19	Monthly	Buy	28734
APR-19	Monthly	Buy	22911
MAY-19	Monthly	Buy	20846
JUN-19	Monthly	Buy	18535
JUL-19	Monthly	Buy	17479
AUG-19	Monthly	Buy	19952
SEP-19	Monthly	Buy	27576
OCT-19	Monthly	Buy	25440
NOV-19	Monthly	Buy	27323
DEC-19	Monthly	Buy	27792
JAN-20	Monthly	Buy	24121
FEB-20	Monthly	Buy	21053
MAR-20	Monthly	Buy	18522
APR-20	Monthly	Buy	14436
MAY-20	Monthly	Buy	13014
JUN-20	Monthly	Buy	11538
JUL-20	Monthly	Buy	11450
AUG-20	Monthly	Buy	13263
SEP-20	Monthly	Buy	18603
OCT-20	Monthly	Buy	20635

NOV-20	Monthly	Buy	21982
DEC-20	Monthly	Buy	21930

Table C.1: PPA contracts in portfolio at 2015-04-01.

C.2 Forward Contracts 2015-04-01

Contracts:	Period:	Type:	Number of contracts:
ENOMAPR-15	Monthly	Buy	31823
ENOMMAY-15	Monthly	Buy	31560
ENOMJUN-15	Monthly	Buy	26708
ENOMJUL-15	Monthly	Buy	24289
ENOMAUG-15	Monthly	Buy	27681
ENOMSEP-15	Monthly	Buy	40263
ENOMOCT-15	Monthly	Buy	45523
ENOMNOV-15	Monthly	Buy	47293
ENOMDEC-15	Monthly	Buy	50332
ENOMJAN-16	Monthly	Buy	52382
ENOMFEB-16	Monthly	Buy	36978
ENOMMAR-16	Monthly	Buy	40394
ENOMAPR-16	Monthly	Buy	31709
ENOMMAY-16	Monthly	Buy	31287
ENOMJUN-16	Monthly	Buy	26356
ENOMJUL-16	Monthly	Buy	23853
ENOMAUG-16	Monthly	Buy	26983
ENOMSEP-16	Monthly	Buy	39601
ENOMOCT-16	Monthly	Buy	44000
ENOMNOV-16	Monthly	Buy	45116
ENOMDEC-16	Monthly	Buy	48282
ENOMJAN-17	Monthly	Buy	49223
ENOMFEB-17	Monthly	Buy	34229
ENOMMAR-17	Monthly	Buy	37977
ENOMAPR-17	Monthly	Buy	29842
ENOMMAY-17	Monthly	Buy	29610
ENOMJUN-17	Monthly	Buy	24872
ENOMJUL-17	Monthly	Buy	22563
ENOMAUG-17	Monthly	Buy	25481
ENOMSEP-17	Monthly	Buy	37480
ENOMOCT-17	Monthly	Buy	41643
ENOMNOV-17	Monthly	Buy	42613
ENOMDEC-17	Monthly	Buy	45804

ENOMJAN-18	Monthly	Buy	47908
ENOMFEB-18	Monthly	Buy	33086
ENOMMAR-18	Monthly	Buy	36970
ENOMAPR-18	Monthly	Buy	29064
ENOMMAY-18	Monthly	Buy	28913
ENOMJUN-18	Monthly	Buy	24255
ENOMJUL-18	Monthly	Buy	21583
ENOMAUG-18	Monthly	Buy	24336
ENOMSEP-18	Monthly	Buy	4302
ENOMOCT-18	Monthly	Buy	4779
ENOMNOV-18	Monthly	Buy	5075
ENOMDEC-18	Monthly	Buy	5027
ENOMJAN-19	Monthly	Buy	3298
ENOMFEB-19	Monthly	Buy	2867
ENOMMAR-19	Monthly	Buy	2523
ENOMAPR-19	Monthly	Buy	1950
ENOMMAY-19	Monthly	Buy	1749
ENOMJUN-19	Monthly	Buy	1549
ENOMJUL-19	Monthly	Buy	1548
ENOMAUG-19	Monthly	Buy	1806
ENOMSEP-19	Monthly	Buy	2552
ENOMOCT-19	Monthly	Buy	2835
ENOMNOV-19	Monthly	Buy	3011
ENOMDEC-19	Monthly	Buy	2983
ENOQ2-15	Quarterly	Sell	10920
ENOQ3-15	Quarterly	Sell	-13248
ENOQ4-15	Quarterly	Sell	-41971
ENOQ1-16	Quarterly	Buy	2183
ENOQ4-16	Quarterly	Buy	6627
ENOQ1-17	Quarterly	Buy	10795
ENOYR-15	Yearly	Sell	-1443578
ENOYR-16	Yearly	Sell	-1205340
ENOYR-17	Yearly	Sell	-1059960
ENOYR-18	Yearly	Sell	-359160
ENOYR-19	Yearly	Sell	-464280
ENOYR-20	Yearly	Sell	-184464
JAN-14 - DEC-15	Other	Buy	17520
SEP-18 - DEC-18	Other	Buy	143521
JAN-19 - FEB-19	Other	Buy	66552

Table C.2: Forward contracts in portfolio at 2015-04-01.