Multivariate Financial Time Series and Volatility Models with applications to Tactical Asset Allocation

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October 11, 2015

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Abstract

The financial markets have a complex structure and the modelling techniques have recently been more and more complicated. So for a portfolio manager it is very important to find better and more sophisticated modelling techniques especially after the 2007-2008 banking crisis. The idea in this thesis is to find the connection between the components in macroeconomic environment and portfolios consisting of assets from OMX Stockholm 30 and use these relationships to perform Tactical Asset Allocation (TAA). The more specific aim of the project is to prove that dynamic modelling techniques outperform static models in portfolio theory.

Keywords: Multivariate Financial Time Series, Multivariate Volatility Models, Modern Portfolio Theory (MPT), Tactical Asset Allocation (TAA)

Multivariata finansiella tidsserier och volatilitetsmodeller med tillämpningar för Taktisk tillgångsallokering

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Abstract

Den finansiella marknaden är av en väldigt komplex struktur och modelleringsteknikerna har under senare tid blivit allt mer komplicerade. För en portföljförvaltare är det av yttersta vikt att finna mer sofistikerade modelleringstekniker, speciellt efter finanskrisen 2007-2008. Idéen i den här uppsatsen är att finna ett samband mellan makroekonomiska faktorer och aktieportföljer innehållande tillgångar från OMX Stockholm 30 och använda dessa för att utföra Tactial Asset Allocation (TAA). Mer specifikt är målsättningen att visa att dynamiska modelleringstekniker har ett bättre utfall än mer statiska modeller i portföljteori.

Nyckelord: Multivariata finansiella tidsserier, Multivariata volatilitets modeller, Modern portföljteori (MPT), Taktisk tillgångsallokering (TAA)

Acknowledgements

I would like to thank my supervisor Tatjana Pavlenko, Associate Professor of Mathematical Statistics at KTH, Royal Institute of Technology for encouraging me and for great advise during the whole project. In addition, I would like to thank Karl Steiner, Chief Quantitative FX Strategist at SEB Merchant Banking for inputs regarding Tactical Asset Allocation and the choice of macroeconomic factors used in the model.

Stockholm, October 2015

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Chapter 1

Introduction

In the case of mutual-traded funds there are two forms of portfolio management: passive and active. Passive management usually tracks a market index, but can also be considered as a equally weighted buy and hold strategy which is the case in this thesis. Active management involves a manager or a team who attempt to beat the passive strategy return.

In Modern Portfolio Theory (MPT) expected returns and the variace-covariance matrix has to be estimated in order to compute portfolio weights. Traditionally the expected returns have been calculated as the mean of historical returns and the variance-covarance matrix has been calculated in its time-invariant form. The idea in this thesis is to put effort in building a time series model, i.e. a VAR(1)-model and compute the expected returns with a one-step-ahead forecast and estimate the time-varying variance-covariance matrix with a multivariate volatility model i.e. the EWMA-model. The purpose within this framework is to try to outperform a time-invariant modelling technique with a time-varying one. The impact of macroeconomic factors i.e. Purchaser Managers Index (PMI) and Consumer Price Index (CPI) are also studied. The main idea for building the models are from the papers by Flavin and Wickens (1998) and (2001) but applied to Swedish stock data and macroeconomic factors.

The stock data to form different portfolios are collected from OMX Stockholm, the PMI data comes from Swedbank and CPI from SCB. All the datasets are in monthly frequency and the stock data is from January 2004 to June 2015, i.e 138 months. The PMI data is from January 2004 to May 2015, i.e. 137 months, finally the CPI data is from February 2004 to May 2015, i.e. 136 months. The stock data and PMI are transformed into log returns while CPI is already a return series, the only transformation in this case is to divide the series by 100 in order to get the percentage returns into decimal form.

The idea in this thesis is to use a backtesting technique for model valuation, consisting of time series data with a moving window consisting of 127 monthly returns. In each time window, the expected returns $\hat{\boldsymbol{\mu}}$ and the time-invariant variance-covariance matrix $\hat{\boldsymbol{\Sigma}}$ or the time-varying one $\hat{\boldsymbol{\Sigma}}_t$ are computed in order to get the portfolio weight vector $\mathbf{w}$. In each time step the weights are rebalanced.

The setup of the thesis is as follows. In chapter 2, the time series and volatility models are described. In chapter 3 Modern Portfolio Theory and computations related to this is presented. Further Tactical Asset allocation is also described in chapter 3. The modelling technique and outputs from R regarding the time series part are presented in chapter 4. Then the results are shown in chapter 5. Finally the conclusions and further developments of the thesis are discussed in chapter 6.

Chapter 2

Multivariate Time Series and Volatility Models

In this chapter all theoretical concepts regarding the time series approach when modelling portfolio weights are considered. First, let us start with more specific notations and basic concepts used in this chapter. Let $\mathbf{z}_t$ be an arbitrary k-dimensional time series, i.e a matrix of size $k \times l$ where l is the length of the series.

- A k-dimensional time series $\mathbf{z}_t$ is said to be weakly stationary if $E[\mathbf{z}_t] = \boldsymbol{\mu}$ is constant a k-dimensional vector, and $Cov(\mathbf{z}_t) = E[(\mathbf{z}_t - \boldsymbol{\mu})(\mathbf{z}_t - \boldsymbol{\mu})'] = \boldsymbol{\Sigma}_z$ is a constant $k \times k$ positive-definite matrix.

- $\boldsymbol{\Gamma}_l$ is the lag l cross-covariance matrix for a stationary time series $\mathbf{z}_t$ of length k, defined as

$$\boldsymbol{\Gamma}_l = Cov(\mathbf{z}_t, \mathbf{z}_{t-l}) = E[(\mathbf{z}_t - \boldsymbol{\mu})(\mathbf{z}_{t-l} - \boldsymbol{\mu})'] \tag{2.1}$$

- $\boldsymbol{\rho}_l$ is the lag-l cross-correlation matrix (CCM), we define it as

$$\boldsymbol{\rho}_l = \mathbf{D}^{-1}\boldsymbol{\Gamma}_l\mathbf{D}^{-1} \tag{2.2}$$

  where $\mathbf{D} = diag(\sigma_1, ..., \sigma_k)$ i.e. the diagonal matrix of the standard deviations of the components of $\mathbf{z}_t$.

- Given the sample $\{\mathbf{z}_t\}_{t=1}^{T}$, the sample mean vector is defined as

$$\hat{\boldsymbol{\mu}}_z = \frac{1}{T}\sum_{t=1}^{T}\mathbf{z}_t \tag{2.3}$$

  and the and lag 0 sample variance-covariance matrix as

$$\hat{\boldsymbol{\Gamma}}_0 = \frac{1}{T-1}\sum_{t=1}^{T}(\mathbf{z}_t - \hat{\boldsymbol{\mu}}_z)(\mathbf{z}_t - \hat{\boldsymbol{\mu}}_z)' \tag{2.4}$$

further the lag l sample cross-covariance matrix is defined as

$$\hat{\mathbf{\Gamma}}_l = \frac{1}{T-1}\sum_{t=l+1}^{T}(\mathbf{z}_t - \hat{\boldsymbol{\mu}}_z)(\mathbf{z}_{t-l} - \hat{\boldsymbol{\mu}}_z)' \tag{2.5}$$

finally the lag l sample CCM is

$$\hat{\boldsymbol{\rho}}_l = \hat{\mathbf{D}}^{-1}\hat{\mathbf{\Gamma}}_l\hat{\mathbf{D}}^{-1} \tag{2.6}$$

where $\hat{\mathbf{D}} = diag(\hat{\gamma}_{0,11}^{1/2}, ...\hat{\gamma}_{0,kk}^{1/2})$, in which $\hat{\gamma}_{0,ii}$ is the (i,i)th element of $\hat{\mathbf{\Gamma}}_0$.

- $vec(\mathbf{A})$ is the vectorized form of a matrix $\mathbf{A}$. As an example for the 2×2 matrix

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the vectorized form is

$$vec(\mathbf{A}) = \begin{bmatrix} a \\ c \\ b \\ d \end{bmatrix}$$

- Kronecker product $\otimes$ is defined as an operation on two matrices resulting in a block matrix. If $\mathbf{A}$ is a $m \times n$ and $\mathbf{B}$ is a $p \times q$, then the Kronecker product $\mathbf{A} \otimes \mathbf{B}$ is a $mp \times nq$ block matrix, example

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \otimes \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a\cdot e & a\cdot f & b\cdot e & b\cdot f \\ a\cdot g & a\cdot h & b\cdot g & b\cdot h \\ c\cdot e & c\cdot f & d\cdot e & d\cdot f \\ c\cdot g & c\cdot h & d\cdot g & d\cdot h \end{bmatrix}$$

- $tr(\mathbf{A})$ is the trace of matrix $\mathbf{A}$, that is the sum of the components on the main diagonal.

2.1 Multivariate Time Series Analysis

As first step in evaluating a multivariate time series model is to test if there is zero cross-correlation in the data, i.e. testing the null hypothesis $H_0 : \boldsymbol{\rho}_1 = ... = \boldsymbol{\rho}_m = \mathbf{0}$ against the alternative hypothesis $H_a : \boldsymbol{\rho}_i \neq \mathbf{0}$ for some i, where $1 \leq i \leq m$ and where $\boldsymbol{\rho}_i$ is the lag-i cross-correlation matrix of $\mathbf{r}_t$. A generalized multivariate *Portmanteau test* for zero cross correlation has been formed by Ljung-Box with the following test statistic

$$Q_k(m) = T^2\sum_{l=1}^{m}\frac{1}{T-l}tr(\hat{\mathbf{\Gamma}}_l'\hat{\mathbf{\Gamma}}_0^{-1}\hat{\mathbf{\Gamma}}_l\hat{\mathbf{\Gamma}}_0^{-1}) \tag{2.7}$$

where $tr(\mathbf{A})$ is the trace of matrix $\mathbf{A}$ and T is the sample size. k is simple to denote the specific test statistic. Rejecting H_0 means that there is evidence for no autocorrelation. If the null hypothesis cannot be rejected there is evidence for that a multivariate time series model has to be considered, for instance the Vector autoregressive (VAR) model. In this thesis, we focus on the $VAR(1)$ model when computing expected returns of equity portfolios.

2.1.1 Vector AR(1) Model

A simple model for modelling asset returns $\mathbf{r}_t$ is the Vector autoregressive model of order 1 i.e. $VAR(1)$, defined as (Tsay, 2010):

$$\mathbf{r}_t = \phi_0 + \phi_1 \mathbf{r}_{t-1} + \mathbf{a}_t \tag{2.8}$$

where ϕ_0 is a k-dimensional vector of constants, ϕ_1 is a time-invariant $k \times k$ matrix and $\mathbf{a}_t$ is a sequence of serially uncorrelated random vectors with zero mean and covariance matrix Σ_a, which is positive-definite.

2.1.1.1 Forecasting VAR(1)

For the $VAR(1)$ model the one-step ahead forecast is quite trivial.

$$\mathbf{r}_t(1) = E[\mathbf{r}_{t+1}|F_t] = \phi_0 + \phi_1 \mathbf{r}_t \tag{2.9}$$

where F_t is the information known at time t.

2.2 Multivariate Volatility Models

Multivariate Volatility Models are of huge importance in financial application especially in portfolio selection and asset allocation strategies. With a multivariate return series:

$$\mathbf{r}_t = \boldsymbol{\mu}_t + \mathbf{a}_t \tag{2.10}$$

where $\boldsymbol{\mu}_t = E[\mathbf{r}_t|F_{t-1}]$ i.e. the expected return given the information known at time $t-1$ and $\mathbf{a}_t$ is the innovation of the series at time t. The conditional variance-covariance matrix of $\mathbf{a}_t$ is defined as $\Sigma_t = Cov[\mathbf{a}_t|F_{t-1}]$ which can be modelled with different techniques, a few of them mentioned below.

2.2.1 Testing Conditional Heteroscedasticity

There are many different tests for testing conditional heteroscedasticity, in this thesis two of these are considered. In these tests $\mathbf{a}_t$ is the noise process. Since volatility is concerned with the second moment $\mathbf{a}_t$, the tests are considered to employ the $\mathbf{a}_t^2$ process.

2.2.1.1 Portmanteau Test 2

If there is no conditional hetroscedasticity in the noise process $\mathbf{a}_t$, then $\mathbf{\Sigma}_t$ is time invariant. This implies that $\mathbf{a}_t^2$ does not depend on $\mathbf{a}_{t-1}^2$. So, the hypothesis which is tested within this framework is $H_0 : \boldsymbol{\rho}_1^{(a)} = \boldsymbol{\rho}_2^{(a)} = ... = \boldsymbol{\rho}_m^{(a)} = \mathbf{0}$ against the alternative hypothesis $H_a : \boldsymbol{\rho}_i^{(a)}$ for some $i(1 \leq i \leq m)$, where $\rho_i^{(a)}$ is the lag-i cross-correlation matrix of $\mathbf{a}_t^2$. The test statistic for this approach is the Ljung-Box statistics defined as:

$$Q_k^*(m) = T^2 \sum_{i=1}^{m} \frac{1}{T-i} \mathbf{b}_i'(\hat{\boldsymbol{\rho}}_0^{(a)-1} \otimes \hat{\boldsymbol{\rho}}_0^{(a)-1})\mathbf{b}_i \qquad (2.11)$$

where T denotes the sample size, k is the dimension of $\mathbf{a}_t$, and $\mathbf{b}_i = vec(\hat{\rho}_i')$. Note that this test is similar to the *Portmanteau Test* for zero cross correlation, the only difference between these two tests are that the input of cross-correlation matrices differs, therefore the star in $Q_k^*(m)$

2.2.1.2 Rank-Based Test

Since asset returns often has heavy tails, extreme outcomes can effect the portmanteau statistics Q_k^*. This test is considered to be more robust than the Portmanteau test for conditional heteroscedasticity. With this approach the standardized series

$$e_t = \mathbf{a}_t'\mathbf{\Sigma}^{-1}\mathbf{a}_t - k \qquad (2.12)$$

is considered, where $\mathbf{\Sigma}^{-1}$ is the inverse of the time-invariant variance-covariance matrix. Further, let R_t be the rank of e_t. The lag-l rank autocorrelation of e_t can be defines as

$$\tilde{\rho}_l = \frac{\sum_{t=l+1}^{T}(r_t - \bar{r})(r_{t-l} - \bar{r})}{\sum_{t=1}^{T}(r_t - \bar{r})^2} \qquad (2.13)$$

for $l = 1, 2, ...,$ where

$$\bar{r} = \sum_{t=1}^{T} r_t/T = (T+1)/2,$$

$$\sum_{t=1}^{T}(r_t - \bar{r})^2 = T(T^2 - 1)/12.$$

Further it can be shown that

$$E(\tilde{\rho}_l) = -(T-l)/[T(T-1)]$$

$$Var(\tilde{\rho}_l) = \frac{5T^4 - (5l+9)T^3 + 9(l-2)T^2 + 2l(5l+8)T + 16l^2}{5(T-1)^2T^2(T+1)}$$

The test statistic for this model is

$$Q_R(m) = \sum_{i=1}^{m} \frac{[\tilde{\rho}_i - E(\tilde{\rho}_i)]^2}{Var(\tilde{\rho}_i)} \tag{2.14}$$

There the subscript R is just to denote this specific test. Note that this test is just considered as a comparison to the Portmanteau test in this thesis and to illustrate that when taking the occurrence of heavy tails in consideration the rejection rate becomes slightly larger.

2.2.2 BEKK Model

One basic but useful Multivariate Volatility Model for financial applications is the Baba-Engle-Kraft-Kroner (BEKK) model:

$$\hat{\mathbf{\Sigma}}_t = \mathbf{A}\mathbf{A}' + \sum_{i=1}^{m} \mathbf{A}_i(\hat{\mathbf{a}}_{t-i}\hat{\mathbf{a}}'_{t-i})\mathbf{A}'_i + \sum_{j=1}^{s} \mathbf{B}_j \hat{\mathbf{\Sigma}}_{t-j}\mathbf{B}'_j \tag{2.15}$$

where $\mathbf{A}$ is a lower triangular matrix, $\mathbf{A}_i$ and $\mathbf{B}_j$ are $k \times k$ matrices and $\mathbf{\Sigma}_t$ is almost surely positive definite. Even though the BEKK model is a nice and user-friendly approach it has its drawbacks. As an example, it contains of too many parameters, for instance if $k = 3$ the model is consisting of 24 parameters. For $k > 3$ the BEKK(1,1) model is hard to estimate.

2.2.3 Exponentially Weighted Moving Average (EWMA)

A common volatility model in financial applications is the EWMA method. This model provides positive-definite volatility matrices. Let $\hat{\mathbf{a}}_t$ be the residuals of the mean equation. The EWMA model for volatility is

$$\hat{\mathbf{\Sigma}}_t = \lambda\hat{\mathbf{\Sigma}}_{t-1} + (1-\lambda)\hat{\mathbf{a}}_{t-1}\hat{\mathbf{a}}'_{t-1} \tag{2.16}$$

where $0 < \lambda < 1$ is the decaying rate. The parameter λ can be estimated by QMLE or be fixed. In many financial applications the estimate of $\hat{\lambda} \approx 0.96$ which is default value in the *EMWAvol* function in the *MTS* package in R. In this thesis $\lambda = 0.96$ is used.

Chapter 3

Modern Portfolio Theory (MPT) combined with Tactical Asset Allocation

In 1952 Harry Markowitz introduced portfolio theory in a Journal of Finance article. A few years later James Tobin (Yale) and William Sharpe (Stanford) made important extensions to Markowitz model and hence won the Nobel Price for their work in 1990.

3.1 Data Input requirements to a MPT model

The following estimates for every security has to be considered in a MPT model:
1. The expected returns $E[R_i]$
2. The variance of returns σ_i^2
3. The covariance between all securities $\rho\sigma_i\sigma_j$ for $i \neq j$

As discussed in (MPT 2012, introduction) expected returns in a mean-variance framework can be estimated by a one-period forecast. The idea in this thesis is to use time-varying estimates in the model. The estimates of the expected returns are computed by a one-period forecast of a $VAR(1)$ model and the variance-covariance matrix is estimated by a $EWMA$ model. The reason for this is that financial data are most often non-stationary and there is hetroscedaticity in the residuals.

3.2 Portfolio weights

In a MPT framework the proportion of each security has to be considered, these proportions or weights denoted by w_i are the fractions of the total value of the portfolio that should be invested in security i. The following constraint has to hold for all portfolios:

$$\sum_{i=1}^{n} w_i = 1 \tag{3.1}$$

A feasible set of portfolios weights can be computed by Monte-Carlo simulation and the optimal weights in a MPT problem can be computed by an optimization algorithm. Hence the portfolio expected return is:

$$E[R_p] = E[\sum_{i=1}^{n} w_i R_i] = \sum_{i=1}^{n} w_i E[R_i] \tag{3.2}$$

and the time invariant variance-covariance matrix of the portfolio is:

$$\Sigma_p = \mathbf{w}^{\mathbf{T}} \Sigma \mathbf{w} \tag{3.3}$$

where p denoted the portfolio, $\mathbf{w}$ is the vector of portfolio returns.

3.3 Diversification

In portfolio analysis Markowitz diversification plays a significant role (MPT p.38). The idea is to reduce risk (volatility) without sacrificing any of the portfolio return. Markowitz explains his theory in the following way:

"Not only does (portfolio analysis) imply diversification, it implies the "right kind" of diversification for the "right reason". The adequacy of diversification is not thought by investors to depend on the number of different securities held. A portfolio with sixty different railway securities, for example, would not be as well diversified as the same size portfolio with some railroad, some public utility, various sorts of manufacturing etc. The reason is that it is more likely for firms within the same industry to do poorly at the same time than for firms in dissimilar industries. Similarly, in trying to make variance (of returns) small it is enough to invest in many securities. It is necessary to avoid to invest in securities with high covariances (or correlations) among themselves."

The conclusion of this framework is that a portfolio manager has to pick assets carefully to be able to reduce the risk. To not "put all eggs in the same basket" is of importance.

3.4 Sharpe's Ratio

A linear risk-return modelling technique has been formed by William Sharpe. This portfolio performance model has won the Nobel Prize too. the model S_p consists of the excess return $\bar{R}_p - R_f$ and the volatility of the portfolio:

$$S_p = \frac{\bar{R}_p - R_f}{\sigma_p} \tag{3.4}$$

where $\bar{R}_p$ is the mean return of the portfolio, R_f is the risk-free rate and σ_p is the volatility of the portfolio. In this thesis, the risk-free rate is set to zero, which is not a bad assumption when the interest rates are extremely low and in some cases even negative. So the interpretation of this model is to measure the excess return per unit of risk. In forward looking portfolio analysis the mean return $\bar{R}_p$ and the historical volatility σ_p can be substituted by $E[R_p]$ and $\hat{\sigma}_p = \sqrt{\mathbf{w}^{\mathbf{T}} \hat{\Sigma} \mathbf{w}}$. This extension of the model is considered in this thesis.

3.5 Mean-Variance Portfolio

An individual has Constant Relative Risk Aversion (CRRA) utility if the relative risk aversion is the same at all wealth levels. Under some simplified assumptions i.e. that asset returns follows a multivariate normal distribution and that the investor has CRRA the expected utility of wealth is expressed as (Lee 2000):

$$E[U(W)] = -exp\big(-\gamma(E[R_p] - \frac{\gamma}{2}\sigma_p^2)\big) \tag{3.5}$$

where γ is the CRRA coefficient, $E[R_p]$ and σ_p^2 are the expected return and variance of the portfolio, given by

$$E[R_p] = \mathbf{w^T} E[\mathbf{R}] \tag{3.6}$$

and

$$\sigma_p^2 = \mathbf{w^T}\Sigma\mathbf{w} \tag{3.7}$$

where $\mathbf{w}$ is the vector of portfolio weights.

Maximizing the expected utility in equation (3.5) is equivalent to solve:

$$\max_{\mathbf{w}} \ \mathbf{w^T} E[\mathbf{R}] - \frac{\gamma}{2}\mathbf{w^T}\Sigma\mathbf{w} \tag{3.8}$$

$$\text{s.t} \quad \mathbf{w^T 1} = 1 \tag{3.9}$$

Then, the Lagrangian of the problem is

$$L = \mathbf{w^T} E[\mathbf{R}] - \frac{\gamma}{2}\mathbf{w^T}\Sigma\mathbf{w} - \lambda(\mathbf{w^T 1} - 1) \tag{3.10}$$

Then, the first-order conditions are as follows, first for $\mathbf{w}$

$$\frac{\partial L}{\partial \mathbf{w}} = E[\mathbf{R}] - \gamma\mathbf{\Sigma w} - \lambda\mathbf{1} = 0 \tag{3.11}$$

$\Rightarrow$

$$\mathbf{w}^* = \frac{\mathbf{\Sigma}^{-1}}{\gamma}(E[\mathbf{R}] - \lambda\mathbf{1}) \tag{3.12}$$

then, for λ

$$\frac{\partial L}{\partial \lambda} = -(\mathbf{w^T 1} - 1) = 0 \tag{3.13}$$

$\Rightarrow$

$$\mathbf{w^T 1} = 1 \tag{3.14}$$

Substituting equation (3.11) into (3.14) yields for λ

$$\lambda = \frac{\mathbf{1^T}\mathbf{\Sigma}^{-1}E[\mathbf{R}]}{\mathbf{1^T}\mathbf{\Sigma}^{-1}\mathbf{1}} - \frac{\gamma}{\mathbf{1^T}\mathbf{\Sigma}^{-1}\mathbf{1}} \tag{3.15}$$

Finally the optimal portfolio weights are solved by substituting (3.15) into (3.11) as

$$\mathbf{w}^* = \left(1 - \frac{\mathbf{1^T}\mathbf{\Sigma}^{-1}E[\mathbf{R}]}{\gamma}\right)\frac{\mathbf{\Sigma}^{-1}\mathbf{1}}{\mathbf{1^T}\mathbf{\Sigma}^{-1}\mathbf{1}} + \left(\frac{\mathbf{1^T}\mathbf{\Sigma}^{-1}E[\mathbf{R}]}{\gamma}\right)\frac{\mathbf{\Sigma}^{-1}E[\mathbf{R}]}{\mathbf{1^T}\mathbf{\Sigma}^{-1}E[\mathbf{R}]} \tag{3.16}$$

Equation (3.16) is the well known *Mutual Fund Separation Problem*, which is the optimal portfolio under the mean-variance framework. Note that within this framework, short sales are allowed, i.e. that portfolio weights can be negative.

3.6 Tactical Asset Allocation

The definition of TAA made by Philips, Rogers and Capali (1996) is:

"A TAA manager's investment objective is to obtain better-than-benchmark returns with (possibly) lower-than-benchmark volatility by forecasting the returns of two or more asset classes, and varying asset class exposure accordingly, in a systematic manner"

3.6.1 Performance Measures

In practical purposes the TAA portfolio is measured against a passive benchmark portfolio and if the return of the TAA portfolio is higher than the benchmark the manager is said to delivered a positive "alpha", which is defined as

$$\alpha_t = R_{TAA,t} - R_t \tag{3.17}$$

where R_{TAA_t} is the return of the TAA model and R_t is the return of the benchmark portfolio. The number of out-performances are also measured to deliver more consistent results, measured by the volatility of alpha and known as the "tracking error" that is:

$$TE_t = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left(\alpha_t - \frac{1}{T}\sum_{t=1}^{T}\alpha_t\right)^2} \tag{3.18}$$

The performance of TAA managers are measured by the information ratio, defined as the ratio between alpha and the tracking error. The higher the information error the better (Lee 2000).

3.6.2 Tactical Asset Allocation based on macroeconomic factors

In Flavins and Wickens (2001) it is shown that macroeconomic information can be used to improve asset allocation. In their setup they use a VAR-model with a M-GARCH structure to compute the joint distribution of financial asset returns with macroeconomic variables. In their paper they are using three risky UK assets and inflation as a macroeconomic factor. Their main subject of study is to investigate how macroeconomic volatility can help to predict the volatility of asset returns, then this can be used to improve tactical asset allocation.

The authors conclusion of the paper is that compared to their analysis in 1998 where a tactical asset allocation strategy that continuously re-balanced the portfolio weights with a time-varying variance-covariance matrix, the model with a macroeconomic factor gave further significant gains in risk reduction. Their model in 1998 was compared to a traditional MPT model with constant variance-covariance matrix.

Chapter 4

Modelling procedure of the portfolios

This section covers the modelling part of the project. First in section 4.1 with a basic example to illustrate the backtesting technique. Futher on the full model will interpreted. The main target is to investigate whether macroeconomic factors, i.e. Inflation (CPI) and Purchaser Managers Index (PMI) will give positive impact on the portfolio dynamics. The idea is to pick a bunch of different portfolios consisting of five risky assets. Portfolio manager A builds his model under a mean-variance framework, i.e uses the time invariant estimates of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$. Portfolio manager B uses the time varying estimates, i.e estimates $\boldsymbol{\mu}$ with a VAR(1) model and the $\boldsymbol{\Sigma}_t$ with a EWMA model. Portfolio manager C uses the same approach as B but also include the macroeconomic factors in the model. Then all three strategies are compared with a benchmark portfolio, with the same assets but with equal weights, i.e 0.2 in each asset. Note that all strategies are so called self-financing portfolios i.e no additional amount of cash is injected or withdrawn from the portfolio beside the invested capital at $t = 0$. In the model short-sales also are allowed. A reasonable number for the CRRA-coefficient for modelling purposes is 10, which is shown in the result part.

4.1 Basic model with two risky assets

In a basic setup of the model we set up a global minimum-variance portfolio consisting of two risky assets e.g. two Swedish stocks, Holmen A and Alfa Laval A.

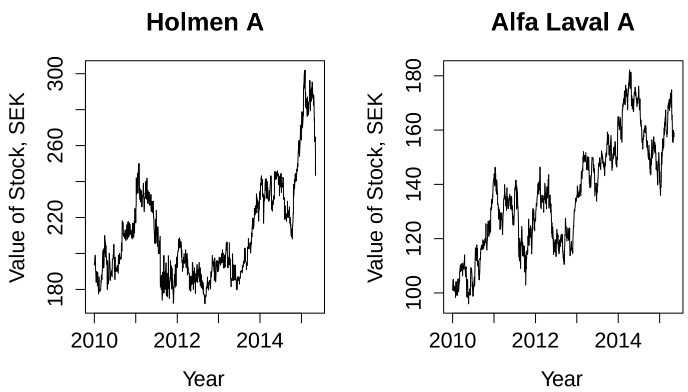

Figure 4.1: The stock prices of Holmen A and Alfa Laval A, from January 2010 to April 2015

The Variance-Covariance matrix is modelled by a BEKK(1,1) Multivariate GARCH model and the portfolio weights are balanced in each time step according to the estimates of this model. The portfolio weights in the global minimum-variance portfolio are:

$$w_1 = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \tag{4.1}$$

$$w_2 = \frac{\sigma_1^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} = 1 - w_1 \tag{4.2}$$

where σ_1^2 is the variance of the Holmen A stock, σ_2^2 is the variance of the Alfa Laval A stock and $\rho\sigma_1\sigma_2$ is the covariance between the two assets. Note that $w_1 + w_2 = 1$. The idea is to use a back testing model with a moving estimation window where each estimation window is of length 54 months. The first estimation window is from January 2010 to August 2014, the second is from February 2010 to September 2014 etc. The entire data sample is 63 months from January 2010 to April 2015. The total global minimum-variance/BEKK portfolio return from August 2014 to April 2015 is then computed by:

$$DP = V_0 \prod_{i=1}^{9} (w_1^{(i-1)} R_1^{(i)} + w_2^{(i-1)} R_2^{(i)}) \tag{4.3}$$

where V_0 is the invested amount of capital at $t = 0$ and $R_j^{(i)}$, $j = 1, 2$ are the simple returns of Holmen A and Alfa Laval respectively. Finally the goal is to compare this dynamic rebalancing strategy with a passive 50/50 strategy i.e. the portfolio weights are equally weighted and constant.

The passive portfolio is

$$PP = V_0 \prod_{i=1}^{9} (0.5R_1^{(i)} + 0.5R_2^{(i)}) \tag{4.4}$$

with the same returns as in the DP.

4.2 Full model with five risky assets

Five different diversified portfolios are formed and modelled with the same technique as described above, then the α, tracking errors and Sharpe ratios are computed to check the robustness and risk adjusted returns of the models. In all portfolios there are 5 different Swedish stocks. All inputs i.e. the five different stocks and PMI are in log returns while CPI is already a return series. The procedure of modelling for portfolio managers B and C in each time step are as follows:

- Testing Cross-Correlation in the multivariate time series
- Build a VAR(1)-model
- Testing Conditional Hetroscedasticity
- Estimate $\boldsymbol{\mu}$ by 1-step-ahead forecast of the VAR(1)-model
- Estimate $\boldsymbol{\Sigma}_t$ with the EWMA-model
- Model checking, i.e. check for adequacy by test statistics of the residuals in the volatility model.
- Compute portfolio weights by the Mutual Fund Separation Theorem at time t and multiply with the returns in $t+1$ and compute the total return of the portfolio strategy.

Note that for the model with macroeconomic factors i.e. the approach of portfolio manager C $\boldsymbol{\mu}$ is a 7×1 vector and $\boldsymbol{\Sigma}_t$ is a 7×7 matrix. Then a subvector of dimension 5×1 and a submatrix of dimension 5×5 is picked for computing portfolio weights. So, the information from the macroeconomic environment effects the portfolio dynamics, i.e. both mean and volatility, but is not included in the portfolio.

Note that we use $Q_k(m)$ and $Q_k^*(m)$ i.e. test statistics as main target within this framework because of the assumption of normally distributed returns in the MPT model. The Rank Based test is there for comparison. The returns of each portfolio is computed in the same way as in eq. (5.3) and the passive returns are calculated as in eq. (5.4) with the only difference that in the full model log returns are considered instead of simple returns.

4.2.1 Portfolio 1

The first portfolio is formed by the following assets:

- Alfa Laval

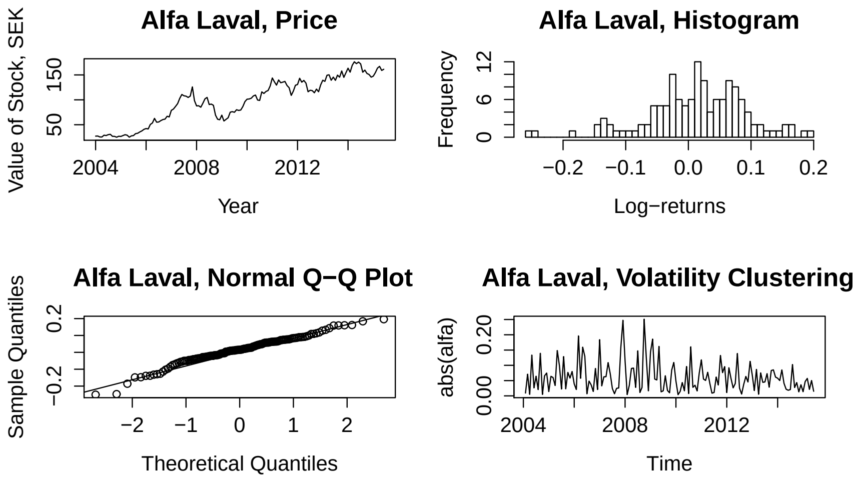

Figure 4.2: The stock price, Histogram, QQ-plot and Volatility Clustering of Alfa Laval, from January 2004 to May 2015

- Autoliv

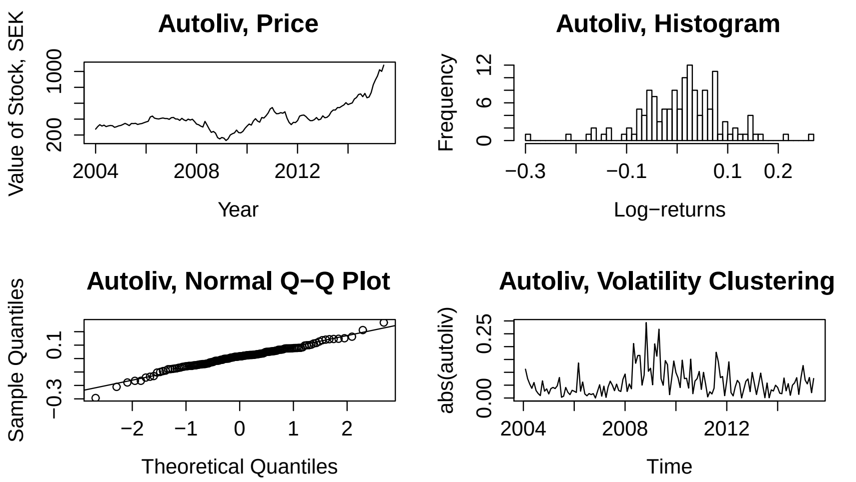

Figure 4.3: The stock price, Histogram, QQ-plot and Volatility Clustering of Autoliv, from January 2004 to May 2015

- Elekta B

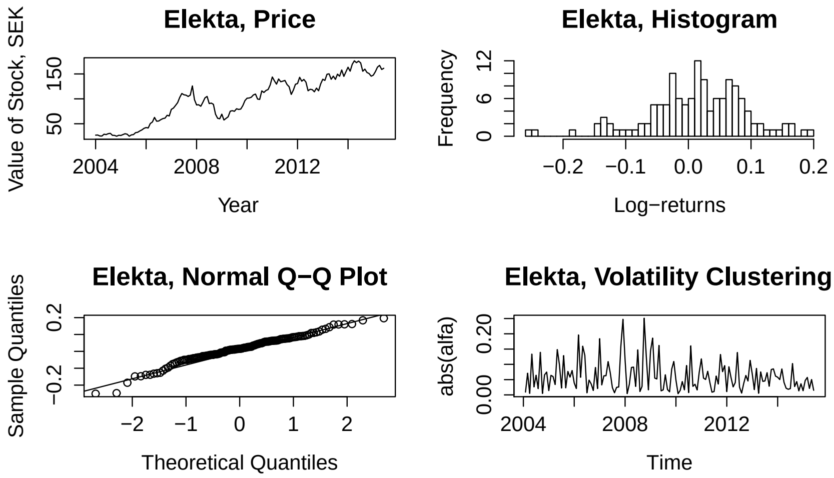

Figure 4.4: The stock price, Histogram, QQ-plot and Volatility Clustering of Elekta B, from January 2004 to May 2015

- Hennes & Mauritz B

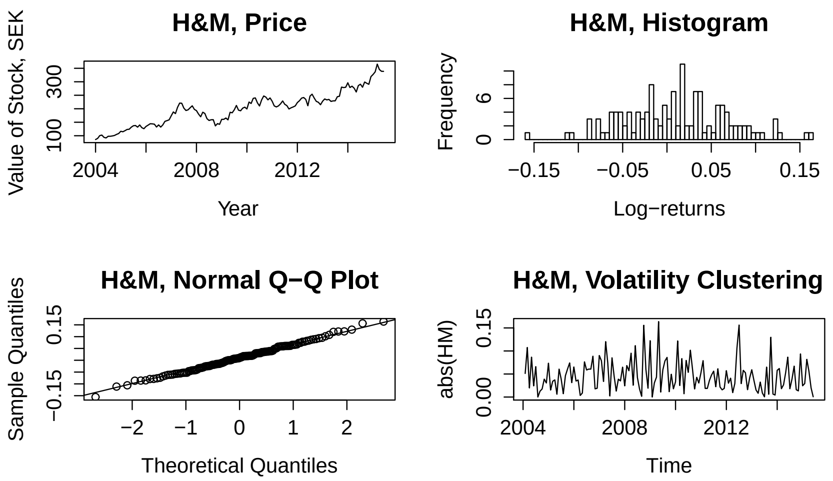

Figure 4.5: The stock price, Histogram, QQ-plot and Volatility Clustering of H&M, from January 2004 to May 2015

- Industrivarden C

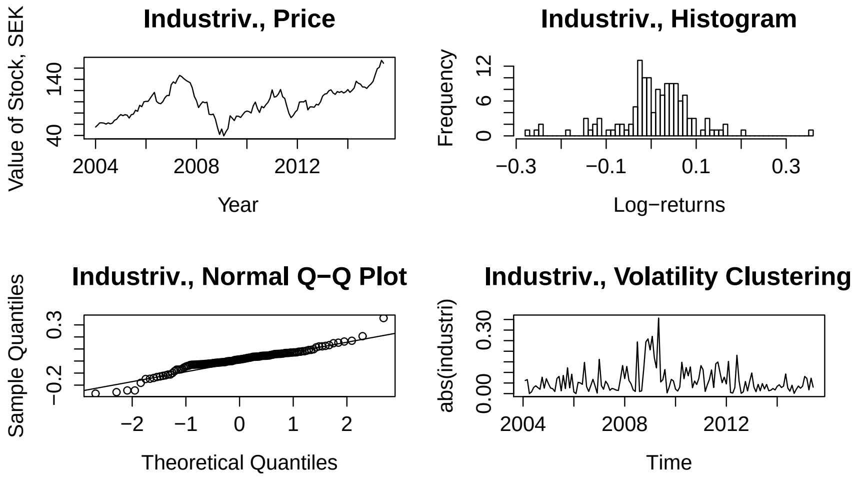

Figure 4.6: The stock price, Histogram, QQ-plot and Volatility Clustering of Industrivarden C, from January 2004 to May 2015

4.2.1.1 Portfolio Manager A

The setup for this approach is to compute the mean and time-invariant covariance matrix in each time step. Then the portfolio weights are computed, see results in next chapter.

4.2.1.2 Portfolio Manager B

The first step in the modelling technique is the perform the Portmanteau test for zeros cross correlation. The test is obtained from the *mq* function found in the *MTS* package in *R*. Note that all other functions used in this chapter are also found in the *MTS* package. The next step is to find evidence for conditional heteroscedasticity in the residuals of the VAR(1) model, by using the function *MarchTest*.

Listing 4.1: R output, testing zero cross-correlation

```
> mq(rtn[1:127,],lag=10)
Ljung-Box Statistics:
        m      Q(m)     df    p-value
 [1,]    1      170      49         0
 [2,]    2      334      98         0
 [3,]    3      469     147         0
 [4,]    4      587     196         0
 [5,]    5      716     245         0
 [6,]    6      818     294         0
 [7,]    7      896     343         0
 [8,]    8      948     392         0
 [9,]    9     1012     441         0
[10,]   10     1075     490         0
```

It is obvious that the null hypothesis of zero cross-correlation is rejected, i.e a VAR(1)-model is formed.
Finally the EWMA model is formed by using the function *EWMAvol* and the adequacy of the model is tested with the function *MCHdiag*.

Listing 4.2: R output, testing Conditional Heteroscedasticity

```
> at1=m1$residuals
> MarchTest(at1)
Q(m) of squared series(LM test):
Test statistic:  103.8102  p-value:  0
Rank-based Test:
Test statistic:  34.85775  p-value:  0.0001320395
Q_k(m) of squared series:
Test statistic:  525.9823  p-value:  0
Robust Test(5%) :  336.3953  p-value:  0.0002155404
```

Here the null hypothesis of Zero Conditional Heteroscedasticity is rejected, i.e. a Multivariate EMWA volatility model is considered. Note that *Q_k(m)* denotes the Portmanteau test for zero cross-correlation and the Rank-based test is self-explanatory. The other two test statistics are not considered here.

Listing 4.3: R output, Model checking, Volatility model

```
> m21=EWMAvol(at1,lambda=0.96)
> Sigma.t1=m21$Sigma.t
> m31=MCHdiag(at1,Sigma.t1)
Test results:
Q(m) of et:
Test and p-value:  71.87529 1.923628e-11
Rank-based test:
Test and p-value:  20.4799 0.02502678
Qk(m) of epsilon_t:
Test and p-value:  457.8394 2.176037e-14
Robust Qk(m):
Test and p-value:  327.4891 0.0007115821
```

We also find evidence for that the volatility model is adequate both under the Portmanteau test and the Rank Based Test. Here, *Qk(m)* is the Portmanteau test for conditional heteroscedasticity.

4.2.1.3 Portfolio Manager C

Listing 4.4: R output, testing zero cross-correlation

```
> mq(rtn[1:127,],lag=10)
Ljung-Box Statistics:
        m      Q(m)     df    p-value
 [1,]     1      170     49        0
 [2,]     2      334     98        0
 [3,]     3      469    147        0
 [4,]     4      587    196        0
 [5,]     5      716    245        0
 [6,]     6      818    294        0
 [7,]     7      896    343        0
 [8,]     8      948    392        0
 [9,]     9     1012    441        0
```

```
[10,]     10       1075      490         0
```

It is obvious that the null hypothesis of zero cross-correlation is rejected, i.e a VAR(1)-model is formed.

Listing 4.5: R output, testing Conditional Heteroscedasticity

```
> at1=scale(rtn[1:127,],center=T,scale=F)
> MarchTest(at1)
Q(m) of squared series(LM test):
Test statistic:  246.0123  p-value:  0
Rank-based Test:
Test statistic:  98.97975  p-value:  1.110223e-16
Q_k(m) of squared series:
Test statistic:  1343.991  p-value:  0
Robust Test(5%) :  915.8655  p-value:  0
```

Here the null hypothesis of Zero Conditional Heteroscedasticity is rejected, i.e. a Multivariate EMWA volatility model is considered.

Listing 4.6: R output, Model checking, Volatility model

```
> m21=EWMAvol(at1,lambda=0.96)
> Sigma.t1=m21$Sigma.t
> m31=MCHdiag(at1,Sigma.t1)
Test results:
Q(m) of et:
Test and p-value:  201.8445 0
Rank-based test:
Test and p-value:  93.95286 8.881784e-16
Qk(m) of epsilon_t:
Test and p-value:  1040.678 0
Robust Qk(m):
Test and p-value:  755.5982 1.182388e-13
```

We also find evidence for that the volatility model is adequate.

4.2.2 Portfolio 2

The second portfolio is formed by the following assets:

- ABB
- Astra Zeneca
- Investor B
- Lundin Petroleum
- Nordea

All relevant plots are found in appendix.

4.2.2.1 Portfolio Manager B

Listing 4.7: R output, testing zero cross-correlation

```
> mq(rtn[1:127,],lag=10)
Ljung-Box Statistics:
        m      Q(m)     df    p-value
 [1,]   1.0     28.6    25.0     0.28
 [2,]   2.0     44.2    50.0     0.71
 [3,]   3.0     72.2    75.0     0.57
 [4,]   4.0     95.9   100.0     0.60
 [5,]   5.0    134.2   125.0     0.27
 [6,]   6.0    162.4   150.0     0.23
 [7,]   7.0    177.7   175.0     0.43
 [8,]   8.0    202.7   200.0     0.43
 [9,]   9.0    238.8   225.0     0.25
[10,]  10.0    267.0   250.0     0.22
```

Since the hypotheis of zero-cross correlation can not be rejected in this case, a time series model is not considered. Hence, Portfolio Manager A and B uses the same approach for portfolio 2.

4.2.2.2 Portfolio Manager C

Listing 4.8: R output, testing zero cross-correlation

```
> mq(rtn[1:127,],lag=10)
Ljung-Box Statistics:
        m      Q(m)     df    p-value
 [1,]     1      164      49        0
 [2,]     2      307      98        0
 [3,]     3      452     147        0
 [4,]     4      565     196        0
 [5,]     5      682     245        0
 [6,]     6      768     294        0
 [7,]     7      835     343        0
 [8,]     8      902     392        0
 [9,]     9      968     441        0
[10,]    10     1034     490        0
```

It is obvious that the null hypothesis of zero cross-correlation is rejected, i.e a VAR(1)-model is formed.

Listing 4.9: R output, testing Conditional Heteroscedasticity

```
> at1=m1$residuals
> MarchTest(at1)
Q(m) of squared series(LM test):
Test statistic:  76.48064  p-value:  2.448486e-12
Rank-based Test:
Test statistic:  33.0848  p-value:  0.0002635897
Q_k(m) of squared series:
Test statistic:  960.6741  p-value:  0
Robust Test(5%) :  653.6542  p-value:  9.735502e-07
```

Here the null hypothesis of Zero Conditional Heteroscedasticity is rejected, i.e. a Multivariate EMWA volatility model is considered.

Listing 4.10: R output, Model checking, Volatility model

```
> m21=EWMAvol(at1,lambda=0.96)
> Sigma.t1=m21$Sigma.t
> m31=MCHdiag(at1,Sigma.t1)
Test results:
Q(m) of et:
Test and p-value:  50.12866 2.527533e-07
Rank-based test:
Test and p-value:  24.90879 0.005521107
Qk(m) of epsilon_t:
Test and p-value:  716.0721 1.014047e-10
Robust Qk(m):
Test and p-value:  563.6266 0.01177117
```

We also find evidence for that the volatility model is adequate.

4.2.3 Portfolio 3

The third portfolio is formed by the following assets:

- Assa Abloy B
- Elektrolux B
- Kinnevik B
- SEB C
- Tele 2 B

All relevant plots are found in appendix.

4.2.3.1 Portfolio Manager B

Listing 4.11: R output, testing zero cross-correlation

```
> mq(rtn[1:127,],lag=10)
Ljung-Box Statistics:
        m       Q(m)     df    p-value
 [1,]   1.0      48.0    25.0     0.00
 [2,]   2.0      67.7    50.0     0.05
 [3,]   3.0     103.8    75.0     0.02
 [4,]   4.0     133.7   100.0     0.01
 [5,]   5.0     159.6   125.0     0.02
 [6,]   6.0     188.6   150.0     0.02
 [7,]   7.0     224.9   175.0     0.01
 [8,]   8.0     256.5   200.0     0.00
 [9,]   9.0     281.2   225.0     0.01
[10,]  10.0     326.2   250.0     0.00
```

Here the null hypothesis of zero cross-correlation is rejected, i.e a VAR(1)-model is formed.

Listing 4.12: R output, testing Conditional Heteroscedasticity

```
> at1=m1$residuals
> MarchTest(at1)
Q(m) of squared series(LM test):
Test statistic:  29.13984  p-value:  0.00118271
Rank-based Test:
Test statistic:  21.30273  p-value:  0.01907876
Q_k(m) of squared series:
Test statistic:  387.6748  p-value:  5.188785e-08
Robust Test(5%) :  356.3169  p-value:  1.121463e-05
```

Here the null hypothesis of Zero Conditional Heteroscedasticity is rejected, i.e. a Multivariate EMWA volatility model is considered.

Listing 4.13: R output, Model checking, Volatility model

```
> m21=EWMAvol(at1,lambda=0.96)
> Sigma.t1=m21$Sigma.t
> m31=MCHdiag(at1,Sigma.t1)
Test results:
Q(m) of et:
Test and p-value:  22.64045 0.01215441
Rank-based test:
Test and p-value:  28.01945 0.00179235
Qk(m) of epsilon_t:
Test and p-value:  294.2669 0.02852071
Robust Qk(m):
Test and p-value:  341.6782 0.0001021771
```

We also find evidence for that the volatility model is adequate.

4.2.3.2 Portfolio Manager C

Listing 4.14: R output, testing zero cross-correlation

```
> mq(rtn[1:127,],lag=10)
Ljung-Box Statistics:
        m       Q(m)     df    p-value
 [1,]     1       186      49         0
 [2,]     2       335      98         0
 [3,]     3       485     147         0
 [4,]     4       614     196         0
 [5,]     5       717     245         0
 [6,]     6       813     294         0
 [7,]     7       898     343         0
 [8,]     8       976     392         0
 [9,]     9      1043     441         0
[10,]    10      1123     490         0
```

It is obvious that the null hypothesis of zero cross-correlation is rejected, i.e a VAR(1)-model is formed.

Listing 4.15: R output, testing Conditional Heteroscedasticity

```
> at1=m1$residuals
> MarchTest(at1)
Q(m) of squared series(LM test):
Test statistic:  67.71659  p-value:  1.221143e-10
Rank-based Test:
Test statistic:  32.96659  p-value:  0.0002759291
Q_k(m) of squared series:
Test statistic:  754.3744  p-value:  1.471046e-13
Robust Test(5%) :  581.2579  p-value:  0.002780797
```

Here the null hypothesis of Zero Conditional Heteroscedasticity is rejected, i.e. a Multivariate EMWA volatility model is considered.

Listing 4.16: R output, Model checking, Volatility model

```
> m21=EWMAvol(at1,lambda=0.96)
> Sigma.t1=m21$Sigma.t
> m31=MCHdiag(at1,Sigma.t1)
Test results:
Q(m) of et:
Test and p-value:  57.90781 8.997223e-09
Rank-based test:
Test and p-value:  36.47819 6.964344e-05
Qk(m) of epsilon_t:
Test and p-value:  683.1129 1.638541e-08
Robust Qk(m):
Test and p-value:  647.2248 2.232855e-06
```

We also find evidence for that the volatility model is adequate.

4.2.4 Portfolio 4

The fourth portfolio is formed by the following assets:

- Atlas Copco B
- Skanska B
- Swedbank A
- Telia Sonera
- Modern Times Group B (MTG)

All relevant plots are found in appendix.

4.2.4.1 Portfolio Manager B

Listing 4.17: R output, testing zero cross-correlation

```
> mq(rtn[1:127,],lag=10)
Ljung-Box Statistics:
         m       Q(m)     df    p-value
 [1,]   1.0      41.0    25.0     0.02
 [2,]   2.0      85.7    50.0     0.00
 [3,]   3.0     119.4    75.0     0.00
 [4,]   4.0     183.1   100.0     0.00
 [5,]   5.0     210.6   125.0     0.00
 [6,]   6.0     251.6   150.0     0.00
 [7,]   7.0     290.9   175.0     0.00
 [8,]   8.0     315.7   200.0     0.00
 [9,]   9.0     337.3   225.0     0.00
[10,]  10.0     370.1   250.0     0.00
```

It is obvious that the null hypothesis of zero cross-correlation is rejected, i.e a VAR(1)-model is formed.

Listing 4.18: R output, testing Conditional Heteroscedasticity

```
> at1=m1$residuals
> MarchTest(at1)
Q(m) of squared series(LM test):
Test statistic:  54.61833  p-value:  3.721121e-08
Rank-based Test:
Test statistic:  38.78617  p-value:  2.767312e-05
Q_k(m) of squared series:
Test statistic:  553.679  p-value:  0
Robust Test(5%) :  414.9823  p-value:  2.546421e-10
```

Here the null hypothesis of Zero Conditional Heteroscedasticity is rejected, i.e. a Multivariate EMWA volatility model is considered.

Listing 4.19: R output, Model checking, Volatility model

```
> m21=EWMAvol(at1,lambda=0.96)
> Sigma.t1=m21$Sigma.t
> m31=MCHdiag(at1,Sigma.t1)
Test results:
Q(m) of et:
Test and p-value:  6.087996 0.8078168
Rank-based test:
Test and p-value:  20.33978 0.02619788
Qk(m) of epsilon_t:
Test and p-value:  311.4454 0.004944768
Robust Qk(m):
Test and p-value:  311.3056 0.005022721
```

We also find evidence for that the volatility model is adequate.

4.2.4.2 Portfolio Manager C

Listing 4.20: R output, testing zero cross-correlation

```
> mq(rtn[1:127,],lag=10)
Ljung-Box Statistics:
        m       Q(m)     df    p-value
 [1,]     1       176      49         0
 [2,]     2       350      98         0
 [3,]     3       501     147         0
 [4,]     4       651     196         0
 [5,]     5       766     245         0
 [6,]     6       877     294         0
 [7,]     7       976     343         0
 [8,]     8      1046     392         0
 [9,]     9      1100     441         0
[10,]    10      1160     490         0
```

It is obvious that the null hypothesis of zero cross-correlation is rejected, i.e a VAR-model is formed.

Listing 4.21: R output, testing Conditional Heteroscedasticity

```
> at1=m1$residuals
> MarchTest(at1)
Q(m) of squared series(LM test):
Test statistic:  10.0609  p-value:  0.4351663
Rank-based Test:
Test statistic:  11.41437  p-value:  0.3261583
Q_k(m) of squared series:
Test statistic:  615.6382  p-value:  9.390825e-05
Robust Test(5%) :  489.5691  p-value:  0.4969953
```

Here the null hypothesis of Zero Conditional Heteroscedasticity is rejected for the *Portmanteau Test*, i.e. a Multivariate EMWA volatility model is considered. Note that the Rank-Based test fails to reject the null hypothesis in this case.

Listing 4.22: R output, Model checking, Volatility model

```
> m21=EWMAvol(at1,lambda=0.96)
> Sigma.t1=m21$Sigma.t
> m31=MCHdiag(at1,Sigma.t1)
Test results:
Q(m) of et:
Test and p-value:  11.20502 0.3417701
Rank-based test:
Test and p-value:  14.48115 0.1521541
Qk(m) of epsilon_t:
Test and p-value:  579.1562 0.003338438
Robust Qk(m):
Test and p-value:  542.8098 0.04936901
```

We also find evidence for that the volatility model is adequate for the *Portmanteau Test* while it is not for the *Rank Based Test*.

4.2.5 Portfolio 5

The fifth portfolio is formed by the following assets:

- Sandvik
- SKF B
- SCA B
- Handelsbanken B
- Volvo B

All relevant plots are found in appendix.

4.2.5.1 Portfolio Manager B

Listing 4.23: R output, testing zero cross-correlation

```
> mq(rtn[1:127,],lag=10)
Ljung-Box Statistics:
        m       Q(m)     df    p-value
 [1,]   1.0     38.1     25.0     0.05
 [2,]   2.0     79.0     50.0     0.01
 [3,]   3.0    122.5     75.0     0.00
 [4,]   4.0    159.2    100.0     0.00
 [5,]   5.0    188.6    125.0     0.00
 [6,]   6.0    227.6    150.0     0.00
 [7,]   7.0    278.5    175.0     0.00
 [8,]   8.0    325.3    200.0     0.00
 [9,]   9.0    364.0    225.0     0.00
[10,]  10.0    407.3    250.0     0.00
```

It is obvious that the null hypothesis of zero cross-correlation is clearly rejected, i.e a VAR(1)-model is formed.

Listing 4.24: R output, testing Conditional Heteroscedasticity

```
> at1=m1$residuals
> MarchTest(at1)
Q(m) of squared series(LM test):
Test statistic:  42.76007  p-value:  5.489007e-06
Rank-based Test:
Test statistic:  50.21036  p-value:  2.441541e-07
Q_k(m) of squared series:
Test statistic:  781.5635  p-value:  0
Robust Test(5%) :  339.8707  p-value:  0.0001323179
```

Here the null hypothesis of Zero Conditional Heteroscedasticity is rejected, i.e. a Multivariate EMWA volatility model is considered.

Listing 4.25: R output, Model checking, Volatility model

```
> m21=EWMAvol(at1,lambda=0.96)
> Sigma.t1=m21$Sigma.t
> m31=MCHdiag(at1,Sigma.t1)
Test results:
Q(m) of et:
Test and p-value:  30.0822 0.0008305176
Rank-based test:
Test and p-value:  27.91302 0.001864042
Qk(m) of epsilon_t:
Test and p-value:  662.4454 0
Robust Qk(m):
Test and p-value:  269.4625 0.1898617
```

We also find evidence for that the volatility model is adequate.

4.2.5.2 Portfolio Manager C

Listing 4.26: R output, testing zero cross-correlation

```
> mq(rtn[1:127,],lag=10)
Ljung-Box Statistics:
        m      Q(m)     df    p-value
 [1,]     1       169      49        0
 [2,]     2       335      98        0
 [3,]     3       493     147        0
 [4,]     4       619     196        0
 [5,]     5       724     245        0
 [6,]     6       827     294        0
 [7,]     7       933     343        0
 [8,]     8      1022     392        0
 [9,]     9      1100     441        0
[10,]    10      1182     490        0
```

It is obvious that the null hypothesis of zero cross-correlation is rejected, i.e a VAR(1)-model is formed.

Listing 4.27: R output, testing Conditional Heteroscedasticity

```
> at1=m1$residuals
> MarchTest(at1)
Q(m) of squared series(LM test):
Test statistic:  82.96707  p-value:  1.312284e-13
Rank-based Test:
Test statistic:  75.95574  p-value:  3.099188e-12
Q_k(m) of squared series:
Test statistic:  1190.6  p-value:  0
Robust Test(5%) :  590.399  p-value:  0.001214725
```

Here the null hypothesis of Zero Conditional Heteroscedasticity is rejected, i.e. a Multivariate EMWA volatility model is considered.

Listing 4.28: R output, Model checking, Volatility model

```
> m21=EWMAvol(at1,lambda=0.96)
> Sigma.t1=m21$Sigma.t
> m31=MCHdiag(at1,Sigma.t1)
Test results:
Q(m) of et:
Test and p-value:  59.56521 4.379784e-09
Rank-based test:
Test and p-value:  61.54479 1.846686e-09
Qk(m) of epsilon_t:
Test and p-value:  1236.399 0
Robust Qk(m):
Test and p-value:  631.956 1.462015e-05
```

We also find evidence for that the volatility model is adequate.

Chapter 5

Results

5.1 Basic model with two risky assets

Time	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9
Weights Holmen	0.591	0.668	0.562	0.803	0.646	0.496	0.639	0.797	0.899
Returns Holmen	0.934	0.996	0.993	1.109	1.073	1.077	1.012	1.004	0.992
Weights Alfa Laval	0.409	0.332	0.438	0.197	0.354	0.504	0.369	0.203	0.101
Returns Alfa Laval	0.963	0.973	0.970	1.027	0.952	0.995	1.103	1.031	1.005

Table 5.1: The weights of the Global Minimum-Variance Portfolio modelled with a BEKK(1,1) Volatility Model

With these weights and returns the total return of the Global Minimum-Variance Portfolio is

$$GVMP = 1.123$$

and the passive 50/50 portfolio gives the total return

$$PP = 1.103$$

These returns gives an alpha

$$\alpha_t = 1.123 - 1.103 = 0.02$$

i.e, the strategy is successful in this case.

The volatility equation of the fitted BEKK(1,1) model in April 2015

$$\begin{bmatrix}\sigma_{11,t} & \sigma_{12,t}\\ \sigma_{21,t} & \sigma_{22,t}\end{bmatrix} = \begin{bmatrix}0.032 & 0\\ 0.007 & 0.009\end{bmatrix}\begin{bmatrix}0.032 & 0.007\\ 0 & 0.009\end{bmatrix} +$$
$$\begin{bmatrix}0.037 & 0.500\\ -0.500 & 0.536\end{bmatrix}\begin{bmatrix}a^2_{1,t-1} & a_{1,t-1}a_{2,t-1}\\ a_{2,t-1}a_{1,t-1} & a^2_{2,t-1}\end{bmatrix}\begin{bmatrix}0.037 & -0.500\\ 0.500 & 0.536\end{bmatrix} +$$
$$\begin{bmatrix}0.234 & 0.058\\ -0.376 & 0.852\end{bmatrix}\begin{bmatrix}\sigma_{11,t-1} & \sigma_{12,t-1}\\ \sigma 21, t-1 a_{1,t-1} & \sigma 22, t-1\end{bmatrix}\begin{bmatrix}0.234 & -0.376\\ 0.058 & 0.852\end{bmatrix}$$

5.2 Full model with five risky assets

5.2.1 Portfolio 1

Time	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9
Weights A Alfa	0.275	0.285	0.313	0.304	0.249	0.242	0.239	0.233	0.299
Weights B Alfa	0.792	0.296	0.816	0.213	0.645	-0.123	-0.006	0.388	-0.315
Weights C Alfa	0.563	0.208	0.717	0.073	0.607	-0.278	-0.181	0.281	-0.483
Returns Alfa	1.027	0.956	0.987	0.964	1.014	1.047	1.057	1.021	0.950
Weights A Autoliv	0.022	0.025	0.002	0.012	0.011	0.041	0.043	0.042	0.073
Weights B Autoliv	-0.682	0.113	0.184	-0.077	-0.148	-0.365	0.404	0.842	-0.482
Weights C Autoliv	0.054	0.453	0.776	0.300	-0.031	-0.003	0.795	1.068	-0.032
Returns Autoliv	1.059	0.921	1.014	1.078	1.127	1.070	1.055	1.080	0.979
Weights A Elekta	0.368	0.353	0.354	0.363	0.363	0.345	0.346	0.336	0.313
Weights B Elekta	0.264	0.523	0.370	0.433	0.417	0.474	0.517	0.598	0.642
Weights C Elekta	-0.139	0.209	-0.045	0.138	0.245	0.232	0.271	0.412	0.264
Returns Elekta	0.916	0.905	1.068	1.045	1.010	1.105	0.989	0.867	1.026
Weights A H& M	0.509	0.524	0.517	0.501	0.541	0.542	0.537	0.545	0.506
Weights B H& M	0.944	0.388	0.923	0.490	0.176	0.341	-0.013	-0.002	0.964
Weights C H& M	0.984	-0.004	0.655	0.268	-0.130	0.305	0.093	-0.124	0.602
Returns H& M	1.067	0.984	0.987	1.094	1.024	1.029	1.082	0.944	0.980
Weights A Industri	-0.174	-0.187	-0.186	-0.180	-0.164	-0.170	-0.165	-0.156	-0.192
Weights B Industri	-0.319	-0.321	-1.292	-0.059	-0.090	0.674	0.099	-0.825	0.191
Weights C Industri	-0.462	0.133	-1.103	0.221	0.309	0.745	0.023	-0.636	0.649
Returns Industri	0.999	0.980	1.036	1.025	1.035	1.081	1.072	1.017	1.073

Table 5.2: The weights of the strategies for Portfolio managers A, B and C and the returns in each time step

The passive 50/50 portfolio gives the total return

$$PP = 1.176$$

With these weights and returns the total return of the Mutual Fund Portfolio 1 for Manager A is

$$P_A^1 = 1.017$$

These returns gives an alpha

$$\alpha_A^1 = 1.017 - 1.176 = -0.159$$

For Manager B

$$P_B^1 = 1.072$$

These returns gives an alpha

$$\alpha_B^1 = 1.072 - 1.176 = -0.104$$

i.e, the strategy is successful in this case. And for portfolio manager C:

$$P_C^1 = 1.314V_0$$

These returns gives an alpha

$$\alpha_C^1 = 1.314 - 1.176 = 0.114$$

i.e, the strategy is successful in this case.

In the same kind of way, alphas for portfolio 2-5 are computed.

Manager	A	B	C
P_i^2	1.215	1.215	1.120
PP_i^2	1.194	1.194	1.194
α_i^2	0.021	0.021	-0.074
P_i^3	1.376	1.472	1.434
PP_i^3	1.273	1.273	1.273
α_i^3	0.103	0.198	0.160
P_i^4	1.149	1.135	1.206
PP_i^4	1.136	1.136	1.136
α_i^4	0.013	-0.001	0.070
P_i^5	1.214	1.539	1.443
PP_i^5	1.245	1.245	1.245
α_i^5	-0.032	0.294	0.197

Table 5.3: The alphas for each portfolio and manager, for $i = A, B, C$

Tables for weights and returns for portfolio 2-5 are found in Appendix.

The estimate of α_i is computed by

$$\hat{\alpha}_i = \frac{1}{N}\sum_{j=1}^{N}\alpha_i \tag{5.1}$$

where $N = 5$ i.e. the number of portfolios and the tracking error is computed by equation (3.18).

Manager	A	B	C
$\hat{\alpha}_i$	-0.011	0.082	0.093
TE_i	1.041e-17	1.388e-17	6.939e-18

Table 5.4: The mean of alphas and Tracking error for each portfolio and manager, for $i = A, B, C$

The estimates of Sharpe ratios are computed by

$$\widehat{SR}_i = \sqrt{9}\frac{1}{T}\sum_{i=1}^{T} SR_i \tag{5.2}$$

where $T = 9$, i.e. number of rebalancing time periods and the factor $\sqrt{9}$ comes from when transforming monthly Sharpe ratios into three quarters of a year.

Manager	A	B	C
$\widehat{SR}_1$	0.630	0.840	1.468
$\widehat{SR}_2$	0.530	0.530	0.814
$\widehat{SR}_3$	0.331	0.494	1.396
$\widehat{SR}_4$	0.352	0.343	1.331
$\widehat{SR}_5$	0.251	0.420	1.275

Table 5.5: The mean Sharpe ratios for each portfolio and manager

Then for justification of the choice of CRRA parameter γ different outcomes of α is plotted against γ.

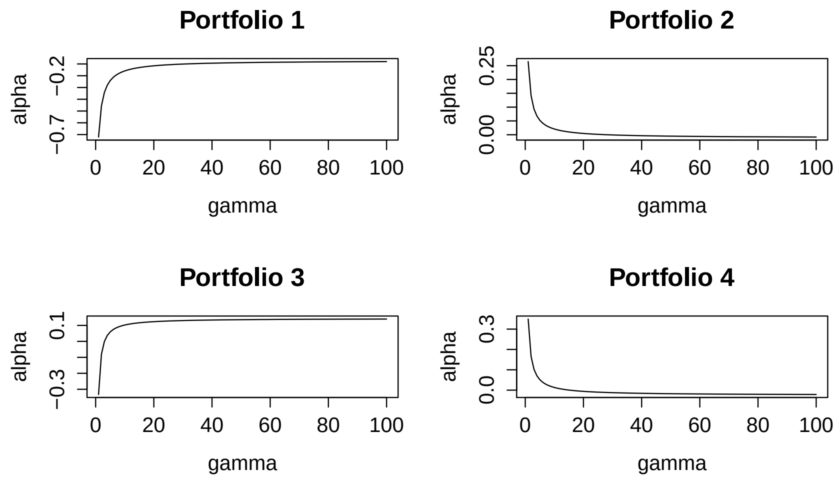

Figure 5.1: The outcome of alpha for portfolio 1-4 and manager A for $\gamma = 1 : 100$

Here we see that the results start to converge approximately around a $\gamma \approx 10$, for lower γ the risk or uncertainty of the outcome is higher. Note that this is a user input and not a statistical estimate, this is just a measure of the investors risk.

Chapter 6

Conclusions

As a first conclusion one can notice that the portfolio weights do not vary much over time for Portfolio manager A while they differs much between each time step for manager B and C. It seems at first glance that the dynamic procedure is more useful for manager B and C than for manager A, i.e. the models capture the time-varying dynamics better than manager A which seems to have only a little bit or non of time-variation in the weighting procedure. This can be explained by for instance by observing Table (5.2), Weights A does not vary much over time compared to Weights B and C.

It is shown that the time-varying time series approach outperforms the traditional modelling technique. The impact of the macroeconomic factors seems to boost up the alphas compared to the portfolio model with only stocks. The tracking error is also significantly reduced when including PMI and CPI in the model. So the Sharpe ratios i.e. the risk adjusted returns are significantly higher for the modelling setup with macroeconomic factors compared to the two other techniques. We see that information about the macroeconomic environment is clearly a refinement of the modelling of portfolio weights in MPT. The main conclusion is therefore stated in Table (6.5), i.e. all semi-annual Sharpe ratios are above 1 (except for portfolio 2), all Sharpe ratios are highest for portfolio manager C comparing to the other techniques. It seems that this suggested strategy is very successful from a risk-return perspective.

These results reflects the study of Flavins and Wickens (2001) where they used four financial assets (three risky assets and a riskless) and one macroeconomic variable (inflation). The risky assets were UK equity, UK govenment bond and a short-term UK government bond. The risk-free asset was a 30-day treasury bill. It is interesting to draw the conclusion that the modelling approach work for Swedish stock data and macroeconomic variables and also that the model captures the recent 2007-2008 recession.

For the future it can be interesting to investigate how the models performs on different type of assets, for example stocks from small cap, the bond market or on a different market with more volatile assets that on OMX30. Other improvements of the modelling technique would have been to use a weighting procedure which takes in consideration that returns for financial data tend to have heavier tails than the case for normally distributed returns. The univariate t-distribution can sometimes be closer to reality when modelling financial asset returns, same for the portfolio but with multivariate t-distribution. Also it would have been interesting to use a more dynamic volatility modelling technique than the EWMA. The assumption that a constant $\lambda = 0.96$ capture all the variation in the residuals might not be fully realistic but good enough to illustrate the example of refinement of the MPT model by Markowitz. Another thing to consider is the view of the macroeconomic factors if they are considered as exogenous variables in the model, then a VARX model could be useful, for deeper investigation of the model for manager C this is a suggestion of improvement.

Appendices

Appendix A

Plots for Portfolio 2-5

A.1 Portfolio 2

Portfolio 2 is formed by the following assets

- ABB

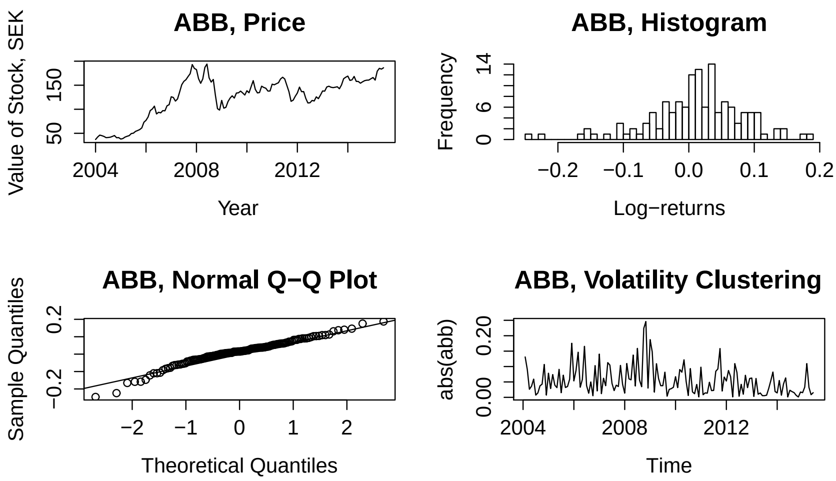

Figure A.1: The stock price, Histogram, QQ-plot and Volatility Clustering of ABB, from January 2004 to May 2015

- Astra Zeneca

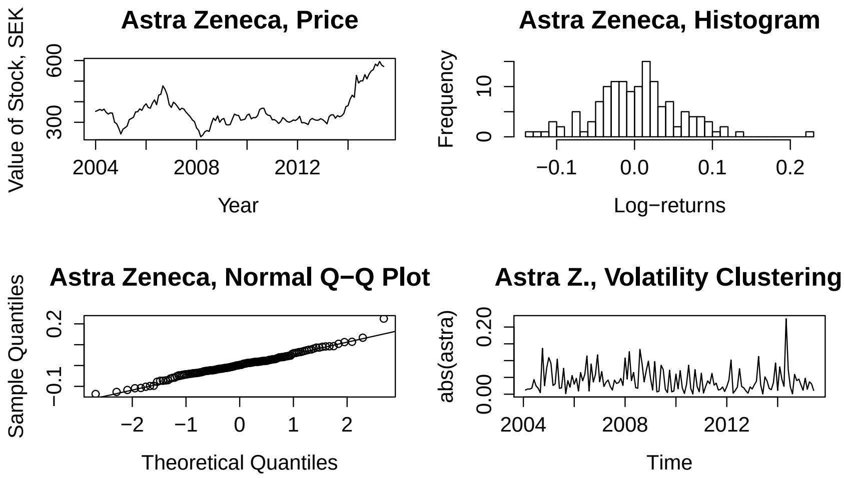

Figure A.2: The stock price, Histogram, QQ-plot and Volatility Clustering of Astra Zeneca, from January 2004 to May 2015

- Investor

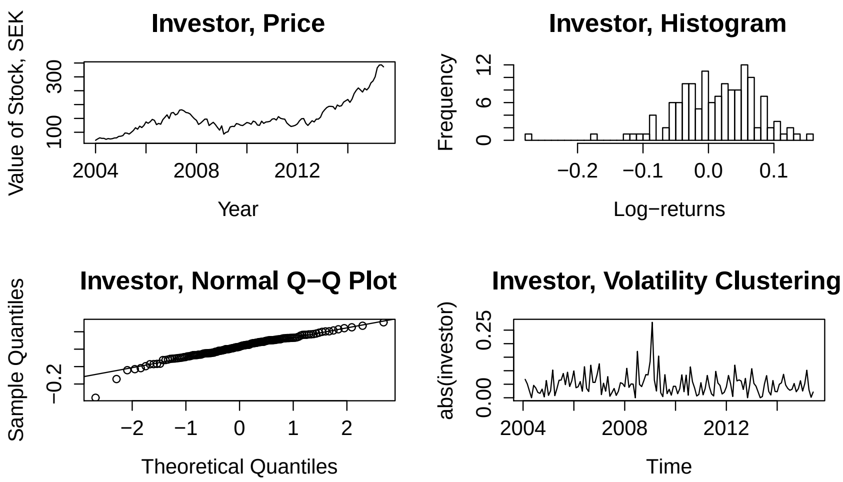

Figure A.3: The stock price, Histogram, QQ-plot and Volatility Clustering of Investor, from January 2004 to May 2015

- Lundin Petroleum

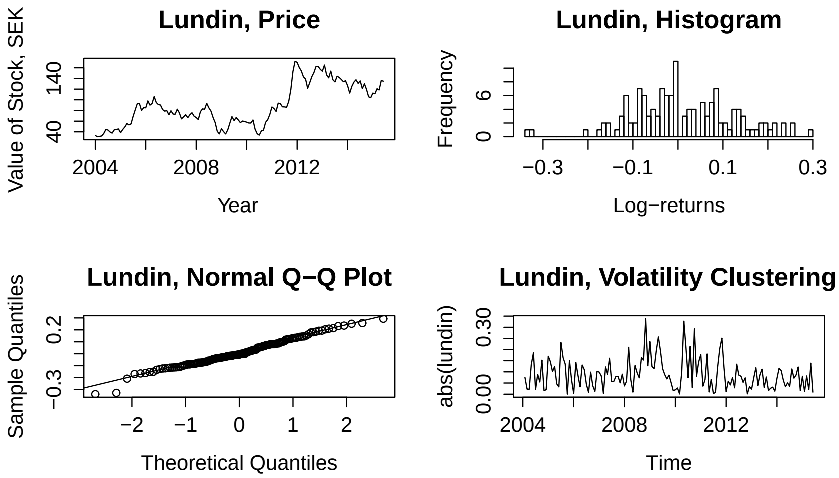

Figure A.4: The stock price, Histogram, QQ-plot and Volatility Clustering of Lundin Petroleum, from January 2004 to May 2015

- Nordea

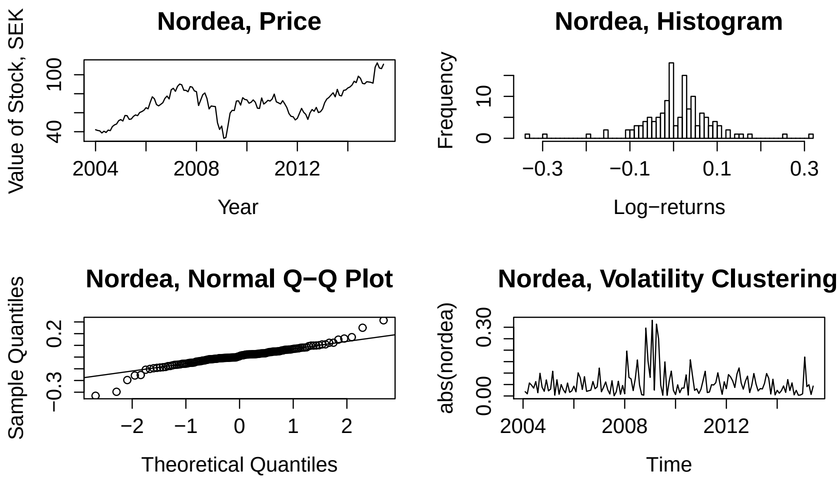

Figure A.5: The stock price, Histogram, QQ-plot and Volatility Clustering of Nordea, from January 2004 to May 2015

A.2 Portfolio 3

Portfolio 2 is formed by the following assets

- Assa Abloy

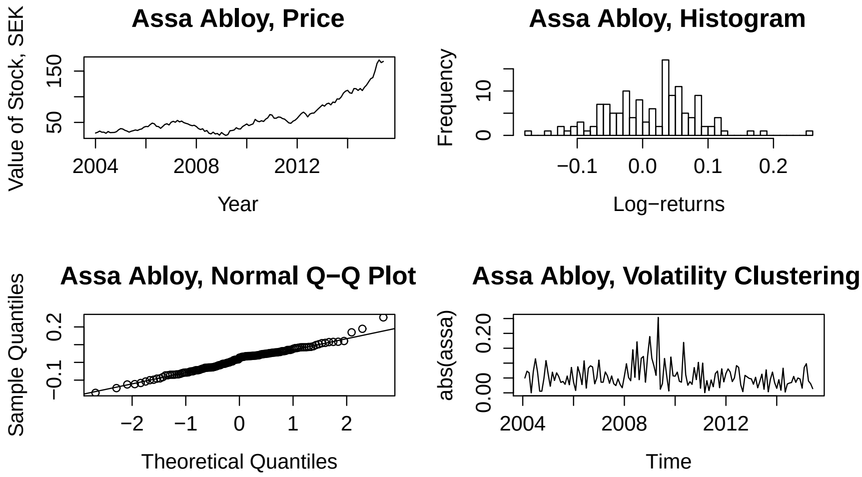

Figure A.6: The stock price, Histogram, QQ-plot and Volatility Clustering of Assa Abloy, from January 2004 to May 2015

- Elektolux

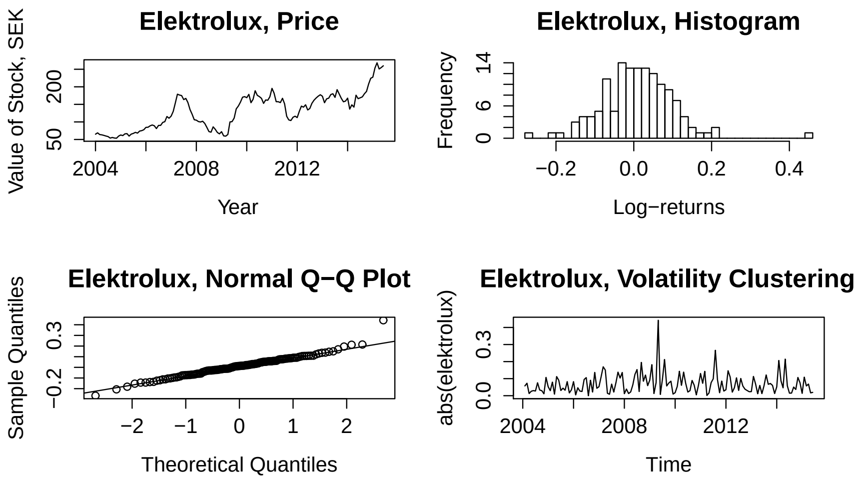

Figure A.7: The stock price, Histogram, QQ-plot and Volatility Clustering of Elektolux, from January 2004 to May 2015

- Kinnevik

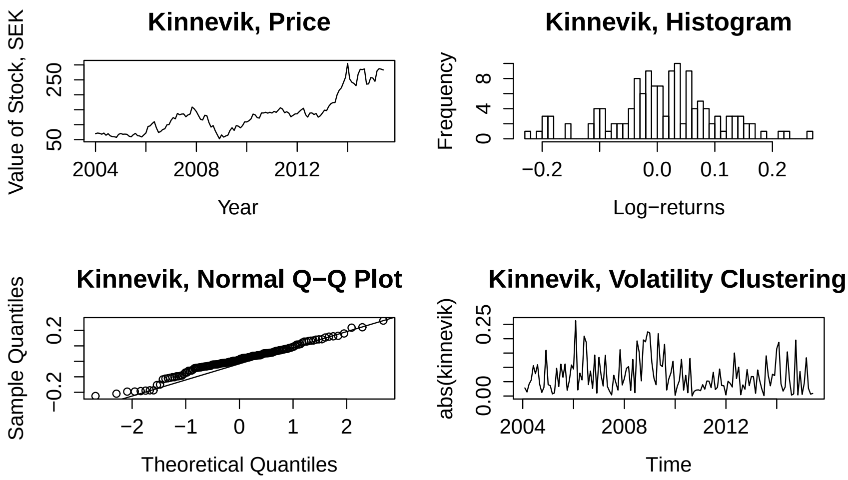

Figure A.8: The stock price, Histogram, QQ-plot and Volatility Clustering of Investor, from January 2004 to May 2015

- SEB

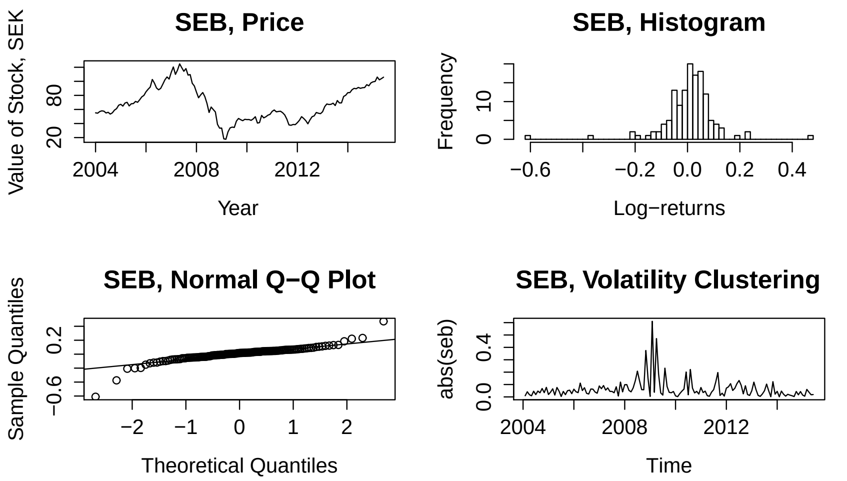

Figure A.9: The stock price, Histogram, QQ-plot and Volatility Clustering of SEB, from January 2004 to May 2015

- Tele 2

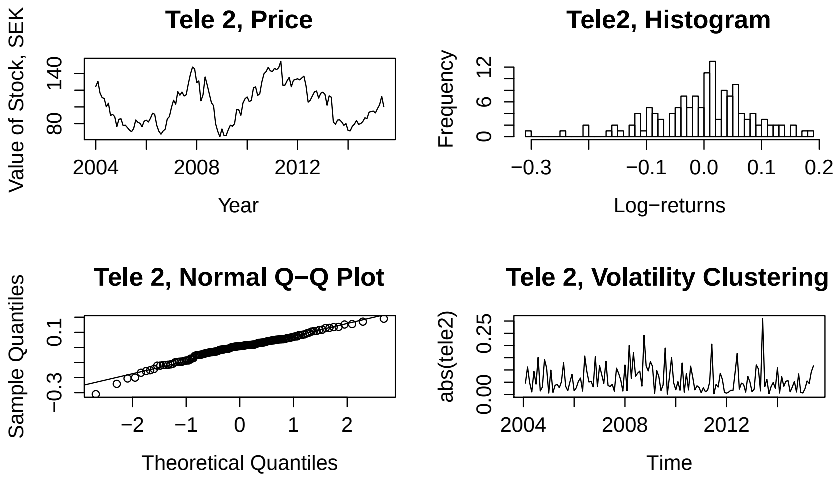

Figure A.10: The stock price, Histogram, QQ-plot and Volatility Clustering of Tele 2, from January 2004 to May 2015

A.3 Portfolio 4

Portfolio 2 is formed by the following assets

- Atlas Copco

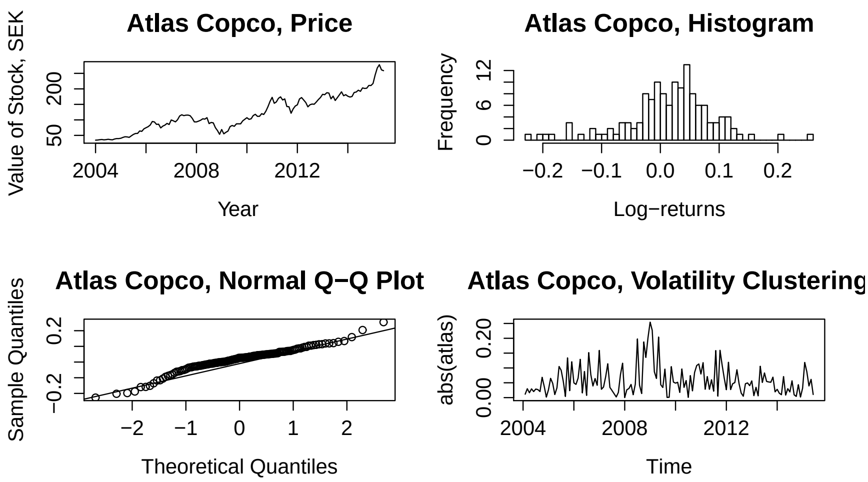

Figure A.11: The stock price, Histogram, QQ-plot and Volatility Clustering of Atlas Copco, from January 2004 to May 2015

- Skanska

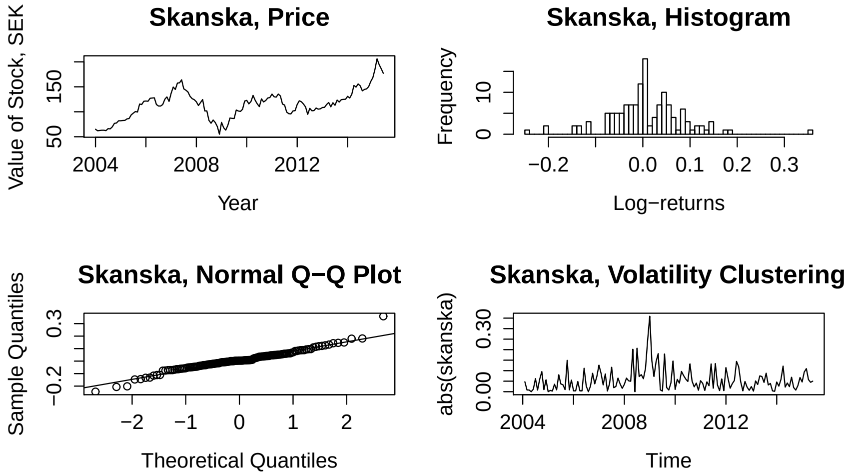

Figure A.12: The stock price, Histogram, QQ-plot and Volatility Clustering of Skanska, from January 2004 to May 2015

- Swedbank

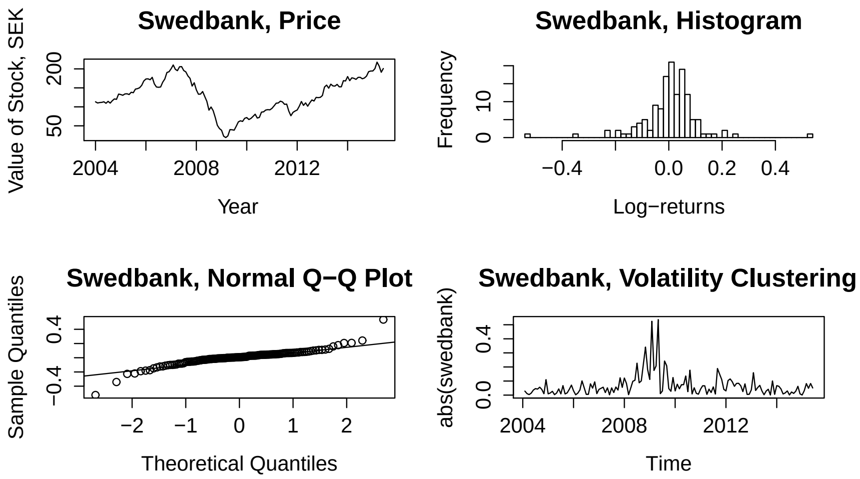

Figure A.13: The stock price, Histogram, QQ-plot and Volatility Clustering of Swedbank, from January 2004 to May 2015

- Telia

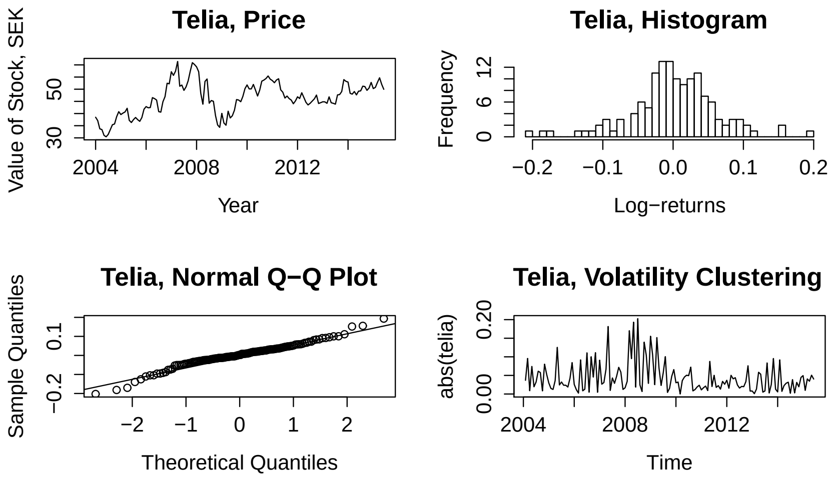

Figure A.14: The stock price, Histogram, QQ-plot and Volatility Clustering of Telia, from January 2004 to May 2015

- MTG

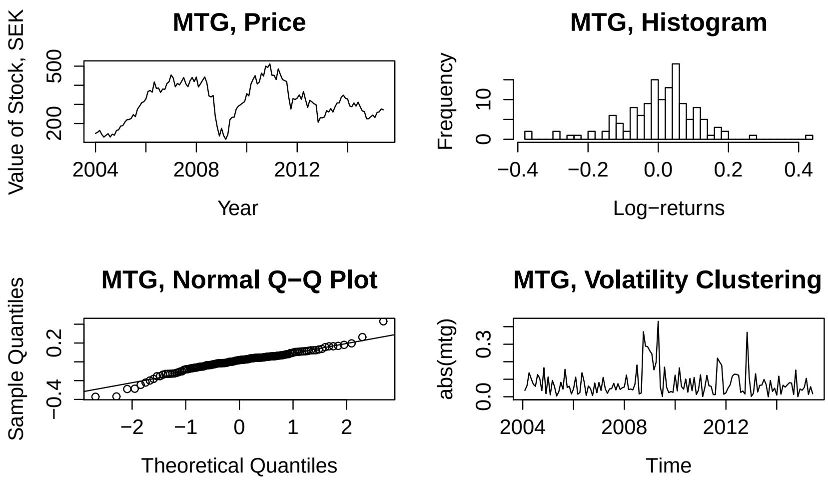

Figure A.15: The stock price, Histogram, QQ-plot and Volatility Clustering of MTG, from January 2004 to May 2015

A.4 Portfolio 5

Portfolio 5 is formed by the following assets

- Sandvik

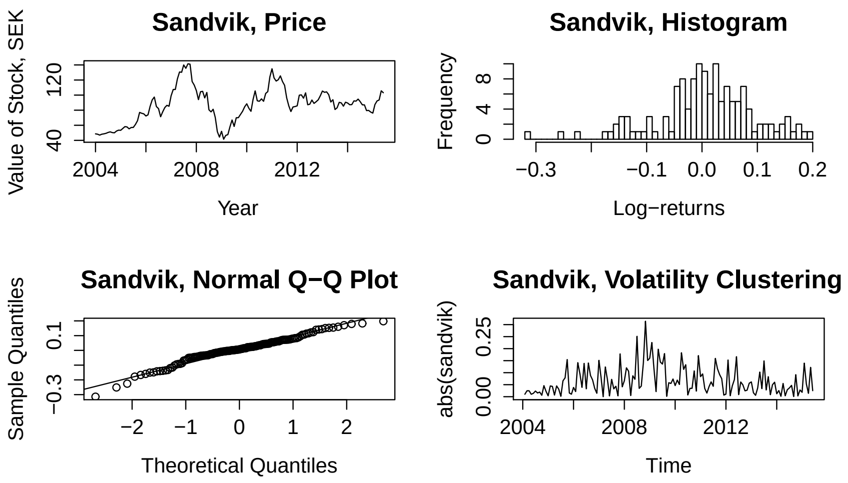

Figure A.16: The stock price, Histogram, QQ-plot and Volatility Clustering of Sandvik, from January 2004 to May 2015

- SKF

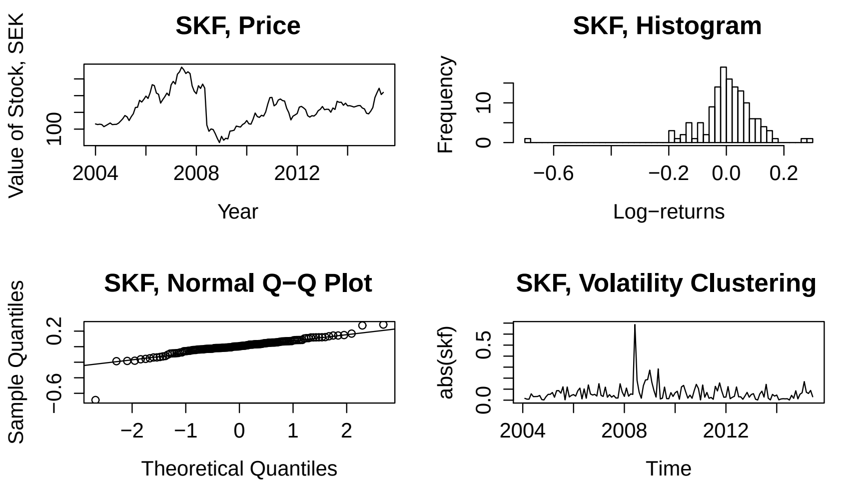

Figure A.17: The stock price, Histogram, QQ-plot and Volatility Clustering of SKF, from January 2004 to May 2015

- SCA

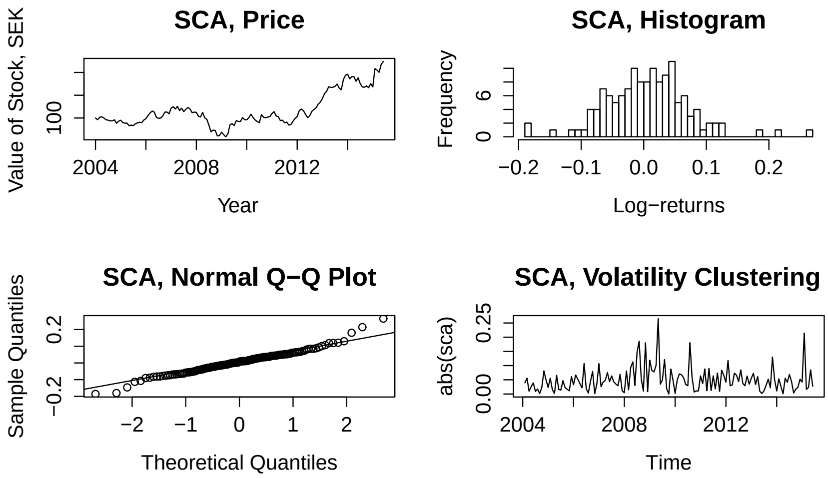

Figure A.18: The stock price, Histogram, QQ-plot and Volatility Clustering of SCA, from January 2004 to May 2015

- Handelsbanken

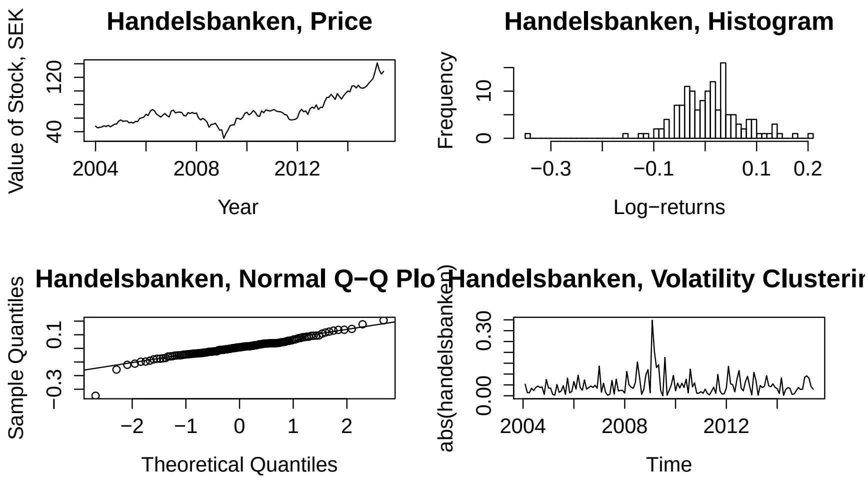

Figure A.19: The stock price, Histogram, QQ-plot and Volatility Clustering of Handelsbanken, from January 2004 to May 2015

- Volvo

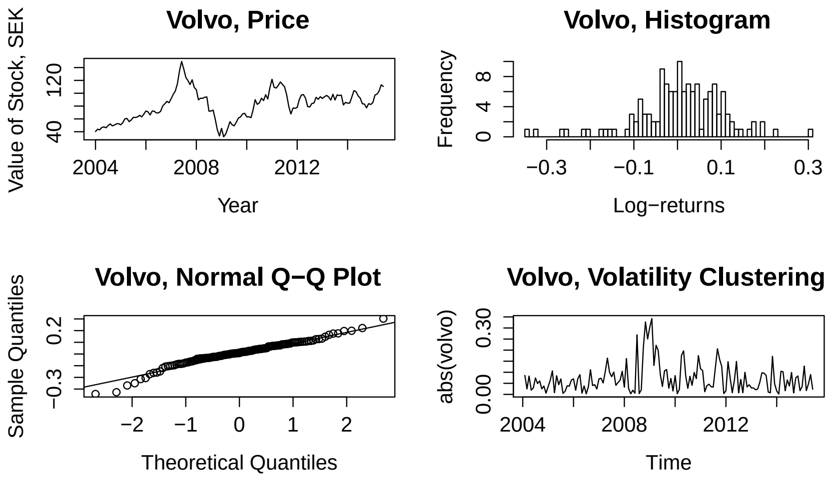

Figure A.20: The stock price, Histogram, QQ-plot and Volatility Clustering of Volvo, from January 2004 to May 2015

Appendix B

Weights and returns Portfolio 2-5

B.1 Portfolio 2

Time	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9
Weights A ABB	0.243	0.227	0.228	0.226	0.236	0.248	0.233	0.228	0.226
Weights B ABB	0.243	0.227	0.228	0.226	0.236	0.248	0.233	0.228	0.226
Weights C ABB	0.900	0.493	0.399	0.562	0.748	0.634	-0.543	1.466	0.833
Returns ABB	1.015	1.006	1.001	1.017	1.015	0.967	1.110	1.032	0.991
Weights A Astra	0.411	0.418	0.421	0.422	0.413	0.410	0.421	0.407	0.408
Weights B Astra	0.411	0.418	0.421	0.422	0.413	0.410	0.421	0.407	0.408
Weights C Astra	-0.026	0.245	-0.013	0.330	0.310	0.084	0.497	0.455	0.146
Returns Astra	1.059	0.960	1.045	1.028	1.012	1.047	0.985	1.037	0.969
Weights A Investor	0.332	0.331	0.304	0.311	0.344	0.363	0.343	0.369	0.402
Weights B Investor	0.332	0.331	0.304	0.311	0.344	0.363	0.343	0.369	0.402
Weights C Investor	1.064	-0.530	0.939	0.097	-0.971	0.293	2.498	-1.936	-0.496
Returns Investor	1.053	0.978	1.033	1.063	1.025	1.052	1.102	1.030	1.002
Weights A Lundin	0.083	0.090	0.082	0.068	0.057	0.037	0.036	0.043	0.050
Weights B Lundin	0.083	0.090	0.082	0.068	0.057	0.037	0.036	0.043	0.050
Weights C Lundin	-0.350	0.187	-0.303	-0.531	-0.137	0.156	-0.604	0.202	-0.093
Returns Lundin	1.072	0.914	0.878	0.984	1.080	0.988	1.082	0.980	1.139
Weights A Nordea	-0.070	-0.065	-0.034	-0.026	-0.050	-0.057	-0.034	-0.047	-0.086
Weights B Nordea	-0.070	-0.065	-0.034	-0.026	-0.050	-0.057	-0.034	-0.047	-0.086
Weights C Nordea	-0.588	0.604	-0.021	0.542	1.050	-0.167	-0.849	0.813	0.611
Returns Nordea	0.995	1.025	0.997	0.996	0.991	1.170	1.041	0.951	0.993

Table B.1: The weights of the strategies of Portfolio 2 for managers A, B and C and the returns in each time step

B.2 Portfolio 3

Time	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9
Weights A Assa	0.671	0.672	0.667	0.685	0.684	0.690	0.692	0.681	0.695
Weights B Assa	0.804	0.092	-0.148	-0.106	0.854	0.668	0.036	0.679	0.522
Weights C Assa	0.410	-0.153	-0.454	-0.276	0.783	0.404	-0.181	0.610	0.345
Returns Assa	1.055	1.035	1.050	1.047	1.016	1.086	1.098	1.040	0.969
Weights A Elektro	0.103	0.102	0.117	0.115	0.122	0.112	0.118	0.121	0.107
Weights B Elektro	-0.081	0.422	1.368	0.393	-0.097	0.262	0.876	0.067	0.314
Weights C Elektro	-0.073	0.370	1.398	0.375	-0.160	0.285	0.949	0.086	0.313
Returns Elektro	1.051	1.037	1.107	1.075	1.014	1.109	1.057	0.932	1.017
Weights A Kinnevik	0.182	0.179	0.137	0.132	0.128	0.134	0.128	0.132	0.142
Weights B Kinnevik	0.159	0.501	0.231	0.414	0.248	0.138	0.277	0.418	0.243
Weights C Kinnevik	0.516	0.411	0.313	0.351	0.084	0.131	0.380	0.401	0.107
Returns Kinnevik	1.007	0.805	1.004	1.085	0.995	0.955	1.134	1.026	0.993
Weights A SEB	-0.007	-0.006	-0.004	-0.019	-0.020	-0.023	-0.025	-0.035	-0.042
Weights B SEB	0.191	-0.200	-0.510	0.132	0.022	-0.129	-0.385	-0.348	-0.555
Weights C SEB	0.257	0.400	-0.246	0.496	0.483	0.187	-0.389	-0.239	-0.028
Returns SEB	1.003	1.044	0.983	1.043	1.015	1.006	1.061	0.962	1.017
Weights A Tele 2	0.052	0.053	0.083	0.087	0.086	0.088	0.086	0.101	0.099
Weights B Tele 2	-0.073	0.185	0.059	0.168	-0.028	0.061	0.196	0.184	0.476
Weights C Tele 2	-0.110	-0.029	-0.011	0.053	-0.190	-0.007	0.240	0.142	0.263
Returns Tele 2	1.053	0.990	1.084	1.008	1.006	0.977	1.055	1.045	1.093

Table B.2: The weights of the strategies of Portfolio 3 for managers A, B and C and the returns in each time step

B.3 Portfolio 4

Time	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9
Weights A Atlas	0.531	0.509	0.498	0.496	0.487	0.481	0.493	0.498	0.535
Weights B Atlas	-0.048	0.707	0.549	0.564	0.650	0.789	0.048	0.596	0.168
Weights C Atlas	-1.008	0.870	0.200	0.417	0.864	0.410	-0.504	0.492	0.072
Returns Atlas	0.991	1.004	1.044	0.998	1.032	1.119	1.087	1.039	0.938
Weights A Skanska	0.059	0.083	0.089	0.084	0.079	0.097	0.093	0.094	0.065
Weights B Skanska	0.717	-0.163	-0.278	-0.118	-0.042	-0.529	-0.158	-0.292	0.554
Weights C Skanska	1.182	0.090	-0.010	0.127	0.226	-0.230	0.092	-0.079	0.972
Returns Skanska	1.020	1.008	1.029	1.068	1.046	1.094	1.110	0.943	0.955
Weights A Swedbank	0.000	0.006	0.019	0.017	0.002	0.007	0.007	0.005	0.000
Weights B Swedbank	-0.427	0.097	-0.032	0.227	0.246	0.163	0.168	0.589	-0.434
Weights C Swedbank	0.039	0.210	0.213	0.410	0.365	0.453	0.442	0.673	-0.145
Returns Swedbank	1.012	1.028	1.063	1.009	1.001	1.031	1.082	0.952	0.918
Weights A Telia	0.506	0.510	0.533	0.532	0.544	0.534	0.532	0.517	0.518
Weights B Telia	0.370	0.424	0.362	0.167	0.346	0.933	0.626	0.090	0.801
Weights C Telia	0.720	-0.338	0.313	-0.103	-0.470	0.808	0.923	-0.059	0.087
Returns Telia	0.997	0.969	1.019	1.045	0.950	1.010	1.041	1.034	0.949
Weights A MTG	-0.096	-0.108	-0.139	-0.129	-0.111	-0.120	-0.125	-0.114	-0.118
Weights B MTG	0.389	-0.065	0.399	0.161	-0.199	-0.356	0.315	0.016	-0.089
Weights C MTG	0.067	0.168	0.284	0.149	0.016	-0.440	0.048	-0.027	0.014
Returns MTG	0.986	0.848	1.000	1.044	1.037	0.947	1.105	1.014	1.0573

Table B.3: The weights of the strategies of Portfolio 4 for managers A, B and C and the returns in each time step

B.4 Portfolio 5

Time	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9
Weights A Sandvik	0.531	0.509	0.498	0.496	0.487	0.481	0.493	0.498	0.535
Weights B Sandvik	-0.248	0.011	-0.132	0.254	0.652	0.685	0.982	-0.047	0.982
Weights C Sandvik	-1.367	0.084	-0.552	0.094	0.854	0.353	0.549	-0.256	0.817
Returns Sandvik	0.999	0.909	1.003	0.972	0.982	1.140	1.053	1.014	1.122
Weights A SKF	0.059	0.083	0.089	0.084	0.079	0.097	0.093	0.094	0.065
Weights B SKF	0.210	0.207	0.448	-0.406	0.283	-0.669	1.338	-0.308	-0.218
Weights C SKF	1.553	0.245	1.000	-0.171	0.111	-0.269	1.683	-0.081	0.023
Returns SKF	0.983	0.915	0.988	1.054	1.067	1.168	1.072	1.061	0.912
Weights A SCA	0.000	0.006	0.019	0.017	0.002	0.007	0.007	0.005	0.000
Weights B SCA	-0.300	0.223	-0.202	1.350	-0.137	0.979	-0.009	0.736	0.063
Weights C SCA	-0.635	-0.110	-0.486	1.078	-0.393	0.743	0.139	0.621	-0.142
Returns SCA	0.996	1.017	0.977	1.052	0.958	1.214	0.984	0.977	1.085
Weights A Handelsbanken	0.506	0.510	0.533	0.532	0.544	0.534	0.532	0.517	0.518
Weights B Handelsbanken	1.355	0.606	1.264	-0.542	0.583	-0.358	-0.466	-0.008	0.441
Weights C Handelsbanken	2.031	0.627	1.558	-0.422	0.605	-0.205	-0.267	0.066	0.395
Returns Handelsbanken	1.008	1.023	1.038	1.029	1.030	1.086	1.092	0.919	0.957
Weights A Volvo	-0.096	-0.108	-0.139	-0.129	-0.111	-0.120	-0.125	-0.114	-0.118
Weights B Volvo	-0.018	-0.048	-0.378	0.344	-0.381	0.363	-0.846	0.627	-0.268
Weights C Volvo	-0.582	0.155	-0.520	0.420	-0.178	0.378	-1.103	0.649	-0.093
Returns Volvo	0.993	0.925	1.083	0.983	1.035	1.128	1.017	1.047	1.090

Table B.4: The weights of the strategies of Portfolio 5 for managers A, B and C and the returns in each time step

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