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Modelling of Private Infrastructure Debt in a Risk Factor Model

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Abstract

Allocation to private infrastructure debt investments has increased in the recent years [15]. For managers of multi-asset portfolios, it is important to be able to assess the risk of the total portfolio and the contribution to risk of the various holdings in the portfolio. This includes being able to explain the risk of having private infrastructure debt investments in the portfolio.

The modelling of private infrastructure debt face many challenges, such as the lack of private data and public indices for private infrastructure debt. In this thesis, two approaches for modelling private infrastructure debt in a parametric risk factor model are proposed. Both approaches aim to incorporate revenue risk, which is the risk occurring from the type of revenue model in the infrastructure project or company.

Revenue risk is categorised into three revenue models; merchant, contracted and regulated, as spread level differences can be distinguished for private infrastructure debt investments using this categorisation. The difference in spread levels between the categories are used to estimate β coefficients for the two modelling approaches. The spread levels are obtained from a data set and from a previous study.

In the first modelling approach, the systematic risk factor approach, three systematic risk factors are introduced where each factor represent infrastructure debt investments with a certain revenue model. The risk or the volatility for each of these factors is the volatility of a general infrastructure debt index adjusted with one of the β coefficients.

In the second modelling approach, the idiosyncratic risk term approach, three constant risk terms for the revenue models are added in order to capture the revenue risk for private infrastructure debt investments. These constant risk terms are estimated with the β coefficients and the historical volatility of a infrastructure debt index.

For each modelling approach, the commonly used risk measures stand-alone risk and risk contribution are presented for the entire block of the infrastructure debt specific factors and for each of the individual factors within this block.

Both modelling approaches should enable for better explanation of risk in private infrastructure debt investments by introducing revenue risk. However, the modelling approaches have not been backtested and therefore no conclusion can be made in regards to whether one of the proposed modelling approaches actually is better than current modelling approaches for private infrastructure debt.

Keywords: Private Infrastructure Debt, Value at Risk, Factor Models, Revenue Model Risk, Stand-Alone Risk, Risk Contribution

Sammanfattning

Investeringar i privat infrastrukturskuld har ökat de senaste åren [15]. För ägare av portföljer med investeringar i samtliga tillgångsslag är det viktigt att kunna urskilja risken från de olika innehaven i portföljen.

Det finns många utmaningar vad gäller modellering av privat infrastrukturskuld, så som den begränsade mängden privat data och publika index för privat infrastrukturskuld. I denna uppsats föreslås två tillvägagångssätt för att modellera privat infrastrukturskuld i en parametrisk riskfaktormodell. Båda tillvägagångssätten eftersträvar att inkorporera intäktsrisk, vilket är risken som beror på den underliggande intäktsmodellen i ett infrastrukturprojekt eller företag.

Intäksrisk delas in i intäksmodellerna "merchant", "contracted" och "regulated", då en skillnad i spreadnivå mellan privata infrastrukturskuldinvesteringar kan urskiljas med denna kategorisering. Skillnaden i spreadnivå mellan de olika kategorierna används för att estimera β-koefficienter som används i båda tillvägagångssätten. Spreadnivåerna erhålls från ett dataset och från en tidigare studie.

I det första tillvägagångssättet, den systematiska riskfaktor-ansatsen, introduceras tre systematiska riskfaktorer som representerar infrastrukturskuldinvesteringar med en viss intäktsmodell. Risken eller volatiliten för dessa faktorer är densamma som volatiliteten för ett index för infrastrukturskuld justerat med en av β-koefficienterna.

I det andra tillvägagångssättet, den idriosynktratiska riskterm-ansatsen, adderas tre konstanta risktermer för intäktsmodellerna för att fånga upp intäktsrisken i de privata infrastrukturinvesteringarna. De konstanta risktermerna är estimerade med β-koefficienterna och en historisk volatilitet för ett index för infrastrukturskuld.

För båda tillvägagångssätten presenteras riskmåtten stand-alone risk[1] och risk contribution[2]. Riskmåtten ges för ett block av samtliga faktorer för infrastrukturskuld och för varje enskild faktor inom detta block.

Båda tillvägagångssätten borde möjliggöra bättre förklaring av risken för privata infrastrukturskuldinvesteringar i en större portfölj genom att ta hänsyn till intäktsrisken. De två tillvägagångssätten för modelleringen har dock ej testats. Därför kan ingen slutsats dras med hänsyn till huruvida ett av tillvägagångssätten är bättre än de som används för närvärande för modellering av privat infrastrukturskuld.

Nyckelord: Privat Infrastrukturskuld, Value at Risk, Faktormodeller, Intäktsmodeller, Stand-Alone Risk, Risk Contribution

[1] Fristående risk

[2] Inverkan på risk

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Chapter 1

Introduction

1.1 Background

Asset Management refers to the activity of overseeing a client's financial portfolio. These services are, among others, provided by asset managers. The top 400 asset managers together manage approximately €56.3trn[1][14].

The majority of the asset managers have a risk model which they use for providing risk analytics of their portfolios. This risk model can either be sourced from another company or be constructed in-house by the asset manager. A well used model for portfolio risk analysis is the risk factor model which commonly is based on Analytical Value at Risk. Value at Risk (VaR) is the maximum loss of a portfolio over a specified time horizon and a set probability level. Analytical, parametric or variance-covariance VaR is based on the assumption that the financial instruments in a portfolio can be mapped to a set of simpler market instruments and factors. The factors are specific to each model. Some models have thousands of factors within areas such as equity, fixed income and alternatives.

The distribution for the market factors or returns are often assumed to be normal and by making this assumption, standard statistical methods can be used to calculate the volatility and the covariance matrices for the instruments in the portfolio. From this, VaR can easily be obtained [10]. The mapping to factors allows for a intuitive explanation of risk and sources of risk in a portfolio which justifies the popularity of the risk factor model.

The comprehensiveness of the risk models vary and not all risk models cover alternatives. Compared to equity and fixed income, alternatives pose specific modelling challenges. These challenges include lack of information, few data points, smoothing of returns, seasonality and specific factors that can not be represented by the commonly used risk factors. Moreover, the alternative models need to be developed continuously in order to satisfy on-going changes in the alternatives market.

[1] Data as of 12/31/2015.

Infrastructure is one of the asset classes that belong to alternatives. Infrastructure has been provided with various definitions. Allianz Global Investors [3], defined infrastructure as an asset class that refers to physical and technical structures that helps accommodate the operation, development and growth of societies and economies. This definition also encompasses functional services that enable economic development and maintain social structure. Financing of infrastructure can take place through various forms and instruments. Private infrastructure debt is the financing of infrastructure with debt by the private side.

As for the other asset classes within alternatives, private infrastructure debt face risk modelling challenges. This is mainly due to the limited amount of data on private infrastructure debt investments. Furthermore, the lack of good public benchmarks and indices as well as factors for the instrument makes it difficult to model private infrastructure debt in a risk factor model.

Asset allocation to infrastructure funds and direct investments by institutional investors has increased since the mid-2000s. In Europe, infrastructure investments have previously been very dependent on bank loans. Recapitalisation of banks and stricter regulations will however force banks to reduce their risk which could potentially mean a reduction of financing by banks [12]. Although fund managers have increased allocation to infrastructure debt, banks and investment banks still remain the largest investors with 19% [15]. Furthermore, in recent decades, governments have entirely or partially privatised toll-roads, utility companies, tunnels, airports, and bridges, which is a trend that is expected to accelerate [9]. This will spur private-sector investment in infrastructure going forward.

1.2 Problem Statement

There are many challenges when it comes to the risk modelling of private infrastructure debt investments. The increased allocation to infrastructure debt by the private sector puts greater importance on the risk modelling of the asset. Especially institutional investors with multi-asset portfolios, need to be able to assess the risk from allocating part of their capital to private infrastructure debt.

There is no consensus regarding how private infrastructure debt should be modelled in a risk factor model. As private infrastructure debt face unique challenges when it comes to risk modelling, it is of high interest for the asset management industry and risk model constructors, to examine if there are alternative risk modelling approaches for private infrastructure debt that could reflect the risk from the investments in a multi-asset portfolio better.

1.3 Purpose

The purpose of this study is to propose an alternative modelling approach for private infrastructure debt which is suitable for a parametric risk factor model.

1.4 Research Questions

In order to fulfil the purpose of the study, the aim is to answer the following research question:

- How can an alternative approach for modelling private infrastructure debt in a parametric risk factor model be constructed?

In order two answer the main research question, the following sub-questions aim to be answered:

- Which are the main risk drivers for private infrastructure debt?
- How can the most important risk drivers be incorporated into the modelling of private infrastructure debt?

1.5 Delimitations and Requisites

Infrastructure can be financed by equity, debt and various mixes of these two. This can further be split into private/unlisted or public/listed instruments. The thesis will solely cover the modelling of private infrastructure debt.

A key requirement of the suggested private infrastructure debt modelling approach is that it can be integrated and combined with a multi-asset parametric risk factor model. This implies that all existing factors for the model, both in the alternative space and for other asset classes, can be utilised for the modelling approach.

The parametric risk factor model aims at providing an overview of the risk and risk contribution in a multi-asset portfolio where private infrastructure debt investments only represent a part of the portfolio's allocation. The risk model is assumed to be scalable and scalability is therefore also a requirement of the suggested modelling approach for private infrastructure debt. In other words, the modelling approach should be easy to implement and backtest.

This thesis will solely provide possible modelling approaches for private infrastructure debt. The implementation and backtesting of the models are not in the scope of this thesis. However, there is a short section which brings up possible ways of evaluating the modelling approaches.

Chapter 2

Infrastructure Debt

This chapter aims to provide a deeper knowledge of infrastructure investments and particularly investments in private infrastructure debt. The purpose is to identify risk drivers for private infrastructure debt that could be incorporated into the risk modelling of the instrument in a multi-asset portfolio.

2.1 Infrastructure

2.1.1 Introduction

The term infrastructure has been provided various definitions, from very broad to much more specified. Yet there is no universal definition of infrastructure. Furthermore, other terms relating to infrastructure are often used incorrectly, which brings further confusion. The financial industry has provided a more narrow definition of infrastructure. The authors Weber, Staub-Bisang and Alfen [16], defined material infrastructure as "all physical assets, equipment and facilities of interrelated systems and their necessary service providers offering related commodities and services to the individual economic entities or the wider public with the aim of enabling, sustaining or enhancing societal living conditions".

An infrastructure "facility" or "asset" refer to the physical objects of the material infrastructure, where the term "asset" is commonly used by the finance industry. Infrastructure investors will to different degrees be owners of the infrastructure asset. In the case of infrastructure projects, the investor will rather support the provision of the asset and get compensated with revenue from the project, alternatively receive regular payments [16].

Infrastructure investments typically offer long-term and predictable income streams, low correlation to other assets as well as lower default and better recovery rates compared to other debt instruments [16]. Infrastructure assets are vital to the functioning of societies and therefore have the benefit of being isolated from economic cycles [9]. Today, institutional investing in infrastructure usually take place in dedicated private investment vehicles. These include closed-end funds, direct investments in infrastructure projects and companies, and co-investments in various forms.

Although infrastructure have many features that appeal to investors, the investments also come with disadvantages. Direct investments in infrastructure include large initial capital requirements, contractual obligations, long lock-up periods and fees. Furthermore, direct investments are also subject to concentration risk in illiquid, leveraged investments. In contrast, infrastructure funds are regulated, liquid, diversified, easy to invest in and comparatively cheap to direct investments. The funds offer the exposure to the publicly traded shares of owners and operators of infrastructure assets [9].

The characteristics of infrastructure debt has traditionally been very attractive to institutional investors with long-term liabilities and annual cash flows. The changing regulatory environment has however made it more difficult to invest long-term in relatively illiquid assets [12].

2.1.2 Financing of Infrastructure

Infrastructure can be financed by public or private provision of capital. Central, regional, local and other government institutions are referred to as public providers of capital. Private capital is provided through project finance or corporate finance. Project finance is further divided into PPP and Non-PPP[1], corporate finance is split into public companies and private companies [12].

Infrastructure financing is realised through a range of financing instruments and investment vehicles. The most common financing instruments are equity, debt and mezzanine. There is also a range of infrastructure investment products that are a mix of or are based on the mentioned instruments. The investments vehicles could either be publicly traded or privately traded and the investments in infrastructure are either direct or indirect via funds [12]. Each of these financing alternatives have different advantages and disadvantages and the most suitable financing is dependent on the maturity of the asset, the time of the financing requirements and the type of contract, which in turn is dependent on the infrastructure asset itself [16]. From the investor's point of view, the optimal solution is based on the investor's individual perspective and strategy. Infrastructure financing gap is defined as the difference between the the investment needs and the available investments [12].

[1]PPP stands for Public-Private Partnerships

2.2 Infrastructure Debt

Debt is the major source of financing for infrastructure projects and assets, contributing with 60% to 90% of the capital [16]. Infrastructure debt includes both loans and bonds, which in turn could be broken down further into subcategories [12].

2.2.1 Senior Infrastructure Loans

Senior loans are used for all types of financing, including financing of infrastructure. Senior loans can either by provided by one player or several banks and/or financial investors and is then referred to as syndicated loans. The loan terms and conditions are constructed to fit the specific asset or project and the interest and final principal payment can be tailored to reflect the underlying cash flows of the borrowing company or project. The average length of a senior bank loans is 7-12 years but some projects require loan durations up to 30 years. The longer term loans are typically partly financed by development banks. Independent of the term length of the loan, lenders usually require the loan to be repaid a couple of years before maturity. This is referred to as the tail of the loan and gives the borrower time to repay the lenders in the case of debt restructuring or late payments. The ultimate goal of a lender is a fully repaid loan [16].

The interest rate or return of the debt instrument is based on a reference interest rate and a specific margin for the infrastructure project or asset. The margin is determined by several factors such as current market and industry standards, the risk profile of the asset or project, and the lenders yield expectation. The debt's interest rate can either be fixed, variable or set to be in a specific interval [16].

The risk of a senior loan is typically low as they are secured by standard collateral and is senior to other financing by debt. The standard collateral include amongst others; present and future claims of the company from material contracts, pledge of the shares held by equity owners, pledge on account balances, ensuring sufficient capitalisation, maintaining liquidity and cash reserves as well as achieving financial covenants. Furthermore, the risk of the instrument is lowered by the fact that it is not subject to business risk, such as lower profit than expected. However, if the project company fails to repay the debt, the lender will need to take part in the renegotiations [16]. In the case of large loan transactions, the debt will be split into tranches where junior debt is subordinated to the senior debt. As the risk and default probability tend to be higher for the junior debt, the lender is compensated with a higher return [16].

Senior Syndicated Infrastructure Loans

The traditional senior loan is provided by one bank or lender. The senior syndicated loan on the other hand, is provided by a group of banks, referred to as the syndicate, and then placed on debt market which makes the loan available to a larger group of investors. The incentives for syndication is the lower credit risk for the lender as well as opportunities to gain arrangement fees. The financing of the transaction is structured and arranged by one or sometimes several banks, called the "lead manager" or "lead arranger". The lead manager decides upon the size of the loan and the amount that is to be kept at the balance sheet after the final take of the syndication, which is usually around 10%. Other lenders are then invited to participate and the commitment of these are referred to as underwriting [16].

The underwriting participation is determined by factors such as attractiveness of the transaction, the financing structure and the fees offered for participating in the syndication. Subsequently to the underwriting, the syndicate offer tranches of the loan to other lenders and participants. Financial long-term investors have in the recent years also started to buy syndicated tranches and participate in the syndication from the start, which previously was dominated by banks. There are extensive contractual agreements concerning syndicated loans in which the topics covenants, representations, warranties and the events of default are of great importance [16].

2.2.2 Infrastructure Bonds

Bonds may be issued instead of, before, or after a loan. Bonds are typically issued for long durations and transaction volumes exceeding approximately £200m. The longest terms stretch to nearly 50 years. In comparison to loans, the bonds are not tailored to the financing needs of the borrower. The term and interest rate will instead be based on other factors such as current capital markets and the creditworthiness of the project company. Interest conditions of the bond is therefore superior to the ones of the loan. However, as there is a large number of bond holders, there is little or no ability of restructuring of the bond when borrowers face repayment challenges [16].

Bonds have either a floating or fixed interest rate, where the floating rate is the most common. Fixed rate bonds with annual payments are attractive to insurance groups and other investors with similar cash flow structures. Pension funds with inflation linked liabilities also have an interest in floating inflation linked bonds as they are not as sensitive to changes in the interest rate as for example insurance companies [16].

The bond may either be placed privately or offered to the public. Private placements have many advantages compared to public as they are not as costly to issue, less time consuming and generate funds more quickly. Bond issues in connection to infrastructure projects and assets as well as PPP's, are most common in the U.K, Canada and Australia. In the rest of world, the market for PPP bonds is very illiquid, however, it is expected to pick up as a result of increased infrastructure volumes and transactions going forward [16].

A common financing instrument is the corporate bond which serves as a financing instrument for infrastructure companies active in the capital markets. The U.S. corporate bond market is much more developed than the European, due to the European markets relying heavily on bank debt in the past. U.S. infrastructure financing is largely occurring from municipal bonds with the feature of being tax-exempt [12]. An infrastructure bond could be seen as a corporate bond with focus on infrastructure investments.

Loan financing has the major share of infrastructure financing but project bonds have increased in volume post financial crisis. However, in Europe there is almost no financing by project bonds post financial crisis [12]. The projects bonds are issued by project finance companies and institutional investors and other financial institutions then have the opportunity to invest in these. The projects could either be private placements or more commonly, traded in a secondary market.

In addition to spending by governments and private investments of infrastructure companies, PPP can facilitate public infrastructure investments. This form is a type of project finance which include an agreement between a public authority and a private party to invest and provide a particular public project or service. A Specific Purpose Vehicle (SPV) is usually set up in order to develop, build, maintain and operate the asset under the period that it is contracted [12]. A private investor in a PPP project usually become shareholder of the project company responsible for the provision of the asset, while the actual ownership of the asset still belongs to the public partner. The projects are time limited and as the private investor do not own the asset, the asset can not be sold at termination of the contract [16].

2.3 Private Infrastructure Debt

Private infrastructure debt refers to loans and bonds that have been financed by the private side. Private infrastructure investments can be attractive to institutional investors that seek returns from allocations to illiquid alternatives. A decision to make a specific allocation to infrastructure implies that the risk profile of the asset is unique. Private infrastructure debt is today considered to have high risk by regulators. This is mainly because infrastructure debt is a long-term and illiquid asset with lack of track record [6].

Due to its unique risk profile, Blanc-Brude, Hasan and Whittaker [6] argued that infrastructure should have its own bucket or sub-bucket in a broader group of assets. Private infrastructure investments could benefit of being referred to as financial instruments rather than industrial sectors, as the latter has a limited role in explaining and predicting the performance of the asset. The authors further argued that private infrastructure investments should not be conceptualised as real assets. Contractual and legal aspects as well as the business model of the infrastructure project largely determines the value of the investor's claims [6].

Infrastructure project finance debt is typically priced as a floating rate instrument with a benchmark rate and a credit spread. The all-in spread, i.e. the spread inclusive of fees, for project finance loans tend to be lower than for comparable corporate loans. This could possibly be due to the the fact that project financing solves some of the agency issues that are typically involved in a creditor and borrower relationship. On the other hand, political and regulatory risk should be taken into account in long-term investments like infrastructure projects. The risk of deterioration of a public sector's commitment in the long term contracts is not consistently priced in by investors. Hence, political risk protection and guarantees contributes to lowering spreads of long term loans [7].

Blanc-Brude and Ismail [7] examined the relationship between loan characteristics and credit spreads as well as the the impact by macro-level factors and project-level risk factors on the credit spread of infrastructure project finance debt. Their conclusion was that infrastructure debt has two pricing dimensions. On a cross-sectional basis, project risk factors can explain spread levels between loans for projects that have different contractual structures. On a longitudinal basis, each project loan can be attributed a decreasing path reflecting continuous deleverage and change in risk profile as time passes. Therefore there is a difference in credit risk both between loans and within loans in a portfolio. The level of credit risk faced by an investor is both determined by the type of project and the contractual agreement, as well as at the time point in the loan's life cycle [7].

2.3.1 Project Finance

Project finance could serve as a well defined form of investment structuring for infrastructure projects. Project financing do not represent all investable infrastructure. However, it is a long-term investment that is solely dedicated to repay creditors and investors over the project life cycle [6].

The borrower in project finance is usually a special purpose entity (SPE), which only has permissions to develop, own, and operate the installation. As a consequence, the project's cash flow as well as the collateral value of the project's assets are the main determinants of repayment ability. However, as the collateral is typically worth nothing outside the contract between the parties, it could be argued that the cash flows of the project is the most important for determining repayment ability. Infrastructure project finance debt is different from corporate debt due to the setup of the SPE. The SPE only invest in the initial phase of a project and will continue to deleverage as time passes. Furthermore lenders have the ability to structure aspects of the credit risk. Project finance thus have a dynamic credit risk profile which is reflected on the credit spread changes with the passage of time [7].

Corporate governance in project finance is different from that in traditional corporates. In project finance, lenders have an important role from the investment decision stage where they are involved in determining the parameters that are usually only controlled by the firm. Furthermore they can minimise the credit risk by taking use of covenants as well as having control right over the free cash flow in the project. The structuring of project debt could therefore be seen as an optimisation exercise between borrowers and lender and therefore certain risk profiles for a level of yield can be targeted. On average, the level of credit risk in project debt is between Baa1/BBB+ and Ba2/BB [7].

Delegation allows for the creation of investable infrastructure assets. When financing a new venture, the companies or governments can choose to invest themselves or they can enter into a contractual agreement. The contractual agreement involves purchasing a product or service from a third part after they have invested in the project. Contracts therefore become vital in order to create enforceable and valuable claims. The relationship-specific infrastructure investments have little or no value outside the contractual agreement and therefore the investment's characteristics could be best described by its contractual characteristics [7].

2.3.2 Loan Characteristics and Macro-Level Factors

Loan characteristics and macro-level risk factors could possibly explain credit spreads of infrastructure debt. Macro-level factors include country specific risk, the credit cycle and the business cycle amongst others factors. These factors have however shown to have a limited impact on the credit-spread of infrastructure debt [7].

In terms of specific loan characteristics, default rates are low and recovery levels are very high in project finance. As time passes, the project loans systematically receives a lower default probability. As it is systematic and predictable, it could be argued that it is necessary to take into account when building and looking at portfolio with infrastructure debt investments. Contrary to intuition, maturity has been shown to not have a positive relationship to project loan pricing. In one study it was shown that longer projects tend to have lower credit spread and in another study it was found that longer loans have lower spreads beyond a certain maturity. Furthermore, loan size and syndicate size have been shown to have a limited impact on spreads [7].

2.3.3 Project-Level Risk Factors

Project-level risk factors include leverage, construction risk and revenue risk. Risk factors that can't be managed, such as political or revenue risk are significant and renumerated risk factors that can explain the cost of infrastructure project finance debt [7].

Leverage and Construction Risk

Blanc-Brude and Ismail [7] came to the conclusion that the impact of deleveraging may be more relevant than the initial leverage itself for the credit dynamics which suggest that infrastructure project finance should be moved to lower risk categories.

Many institutional investors have feared funding new projects because of construction risk, an idea that new projects are much more risky than existing ones. Although construction risk exists, investors often overestimate the construction risk in private infrastructure investments. Blanc-Brude and Ismail [7] argued that it should rather be welcomed as a credit risk diversifier in infrastructure debt portfolios.

Project financing structures for infrastructure requires construction and operating risks to be managed through a network of contracts. This allows for transfer of main parts of the uncertainty from the SPE to subcontractors that have committed to a fixed-date and fixed-price requirement. Construction risk that the company can manage through its network of contracts therefore only impact the risk pricing at the margin. Although it has been shown in earlier studies that the risk of cost overruns and delays are well-managed, the completion of the construction phase is still important in project finance. Construction risk, just as leverage, is constantly changing during the life-cycle of the project. In the initial phases of the project, the outturn of the costs are unknown. After this period has passed, its risk profile changes [7].

On a cross-sectional basis, factors such as construction risk or leverage seem to be idiosyncratic and insufficient to explain loan spreads. However, over time, these factors appear to be systematic and able to explain changes in risk profile and the trend of decreasing infrastructure project finance loan spreads over time [7].

Revenue Risk

Risk-and return profiles for infrastructure projects could be grouped by revenue risk and by life-cycle stage [6]. Revenue risk is a significant risk factor driving the cost of debt in infrastructure project finance as it represents an unmanaged dimension. Traditionally, revenue risk has been associated with the type of industrial sector. This could be misleading and it should rather be seen as a contractual feature of each project as revenue risk in sectors can vary significantly. The same type of project may have a very high or low revenue risk which is more likely to affect the credit risk than the fact that the project belongs to a specific industrial sector. Government service projects like a PPP, has on average a leverage of 90% but the revenue risk in these type of projects is very low or non-existing. Telecom projects have an average leverage around 67% and transportation, energy and environmental services have an average leverage of 75-79%. Although these projects have a lower average leverage, the revenue risk tend to be higher [7].

Blanc-Brude and Ismail [7] suggested that revenue risk should be split into three categories depending on the revenue model. These categories are contracted (availability payment scheme), regulated (partial commercial scheme, partially contracted, shadow tolls) and merchant (commercial scheme, real tolls). The number of projects in each category is roughly equal. The commercial scheme does however make up a greater portion in terms of total investment [7].

Availability payment schemes refers to the case where a public sector agrees to pay a fixed income over a set period to the investor. In exchange, the investor has the responsibility for investment, operations, residual equity and debt service cash flows that are necessary for delivering the infrastructure project as in the pre-agree output specification. At the end of the contract, the value is set to zero and the asset is returned to the public sector. Contracted revenue models are often used for social infrastructure projects, e.g. schools, government buildings and hospitals [7]. The risk for this revenue model is typically lower than for the average infrastructure project.

Commercial schemes include a similar contract, but the big difference is the floating and variable income instead of a fixed income to the investor. The investor would typically have the right to collect tolls or tariffs from the users. The value at the end of the contract is often set to zero [7]. Merchant revenue risk models are expected to generate a higher return due to demand risk [5].

Capped commercial schemes is similar to the commercial scheme although the revenue from users is often shared between the investor and the public sector. The terminal value in these types of projects are not always set to zero as they often involve tangible assets and an implicit contract with the public sector which conditions the value of the investment [7].

In the study by Blanc-Brude and Strange [8], the authors found that revenue risk factors had a positive impact on spreads. The study was conducted on the European road sector over a specified period and it was found that real tolls increased the spread on average by 41.2 basis points, and shadow tolls by 33.6 basis points, ceteris paribus, in comparison to the spread of availability payment roads. The same study was conducted using a UK PFI and PPP sample. The results of the study was also that real tolls and shadow tolls in urban rail and roads increased the spread in comparison to availability payments in social infrastructure projects.

Blanc-Brude and Ismail [7] performed empirical analysis using two large data sets of project finance loans for cross-sectional as well as longitudinal analysis. The study included several dimensions, there-amongst the impact on credit spread by project-finance variables such as revenue risk. The contractual characteristics for each of the projects could not be observed and therefore proxies were used. The proxies were primarily based on the fact that some sectors typically have a certain revenue model. E.g., social infrastructure projects tend to be contracted as they receive pre-agreed fixed income from the public sector. The empirical study showed that project finance that has a revenue model corresponding to availability payment, has credit spreads that are 30-60 basis points lower than those that have a greater exposure to demand or traffic risk. Furthermore, projects that receive shadow tolls or has partially contracted revenues, has a spread that is 15-40 basis points lower than for projects with a merchant revenue model [7].

2.3.4 Research and Data

The available research on private infrastructure investments is limited and the existing articles almost solely focus on equity infrastructure investments. As the research and investment knowledge is limited, private infrastructure debt has remained an area with many unanswered questions. This is mainly due to three reasons: the absence of trustworthy market proxies, the limitations of existing private databases and studies, and the focus on inadequate investment metrics in private investments [6].

There are currently no infrastructure investment benchmarks which is challenging for investors that want to benchmark their infrastructure investment's managers or their strategies. A few databases exist and have been used in studies of private equity investments in infrastructure. This data is not categorised in terms of factors and the data is mainly on cash flows and asset values of private equity funds, which is not really representative for underlying infrastructure investments [6].

Rating agencies hold some data on private infrastructure debt. They have collected data in order to rate both listed and private bonds and loans. The issues are ranked in relative to each other but are not considered on a portfolio level. Furthermore, the rating only tell about expected future performance but it is not actually monitored. The data collected by rating agencies is therefore also a poor infrastructure investment benchmark [6].

There is a clear demand of data on private infrastructure investments. The questions of which data, for what purpose, and how it should be collected still remains. Blanc-Brude, Hasan and Whittaker [6] have proposed a framework for collecting data and evaluating privately held infrastructure debt and equity. EDHEC Institute started the process of collecting, aggregating and classifying investment data and infrastructure cash flow in 2015 [6]. This database is however not available to the public.

Chapter 3

Method and Data

This chapter aims to give a brief overview of the method and the data used in the thesis.

3.1 Method

The master thesis has been conducted during the spring semester of 2017, spanning over a period of 5 months. The method can be summarised in the following overlapping phases:

> *Problem Formulation:* An initial problem formulation as well as expectations of the outcome was formulated in the initial phase of the thesis. The problem formulation was updated as further knowledge of private infrastructure debt investments and risk models were gained from a literature review and interviews.
>
> *Literature Review and Interviews:* A literature review covering infrastructure debt investments and possible modelling approaches for private infrastructure debt was conducted. Existing research on the modelling of private infrastructure debt is very limited and the goal of the literature review has therefore been to understand private infrastructure debt as an investment. In order to gain better insight into current and tested approaches for modelling of private infrastructure debt in risk factor models, interviews with risk professionals were conducted.
>
> *Data:* A data set with private infrastructure debt deals was received in the middle of the thesis period. The data set was used to test findings from research and to compute empirical risk estimates for one of the modelling approaches. Details of the data set and modifications of the data can be found under Section 3.2.
>
> *Results:* The modelling approaches were constructed after the literature review and the interviews had been performed. Computations of

estimates for the modelling approaches were made using the obtained data set and a sample from a previous study.

Analysis: Analysis of the results and the method was performed after the modelling approaches had been constructed and computations had been performed.

Report writing: The first sections of the report were initiated early in the process and have been amended throughout the process. The last sections in the report were completed after all the results were obtained.

3.2 Data

A small data set and public indices are used for the modelling approaches proposed in the thesis.

3.2.1 Data Set

The data set used in the study consists of approximately 200 private infrastructure debt deals where some of these deals are split into tranches with various risk profiles, sizes, maturities and spreads. Each deal has several attributes related to the transaction. The data set is not complete, meaning that information for attributes are missing for some transactions. The data set contain several attributes that are not relevant for the study. The attributes used in this study are mainly the name of the transaction, the sector, the credit spread and notes regarding the investment.

The data set was modified in the following way:

- A deal with several tranches was split so that each tranche is seen as a separate instrument or investment.
- Deals with no information regarding spread margin were removed.
- The spread margin for each deal was split into reference rate and credit spread as solely the credit spread is relevant for the study.
- For deals that had a spread interval instead for a fixed spread, the spread was set as the average of the spread interval.

The modifications resulted in a data set suitable for the study. The data was further split into categories for the modelling approach which is discussed in Section 5.7.3.

3.2.2 Time Series

The general infrastructure debt factor, discussed later in the modelling approach, is represented by a public index. The index is perceived as well diversified and representative for infrastructure debt deals with lower volatility and spread levels in comparison the broad market and other available infrastructure indices. As the index is public, the time series is accessed easily and can be used for various estimations and computations.

Chapter 4

Mathematical Background

This chapter aims to give an overview of multivariate models, parametric risk factor models and some of the risk measures used as risk analytics in portfolios. Furthermore it aims to give an introduction to the Black-Scholes model and market price of risk, as the latter is used to motivate parts of the modelling approaches.

4.1 Multivariate Models

Multi-asset portfolios where the risk factors are assumed to have a joint distribution can be represented by multivariate models.

4.1.1 Spherical Distributions

Y has a spherical distribution in $\mathbb{R}^d$ if

$$OY \stackrel{d}{=} Y,$$

for every orthogonal matrix O. This means that the distribution for Y is invariant under rotations and reflections [13].

4.1.2 Elliptical Distributions

A random vector X has an elliptical distribution if there exist a vector μ, a matrix A and a spherically distributed vector Y such that the following holds

$$X \stackrel{d}{=} \mu + AY$$

The multivariate normal distribution is used in this thesis. A random vector X has a $N_d(\mu, \Sigma)$- distribution if

$$X \stackrel{d}{=} \mu + AZ$$

where $AA^T = \Sigma$ and Z has a $N_d(0, I)$-distribution.

The following is an important property of the elliptical distribution and is useful for large portfolios. If $X = \mu + AZ$ is normally distributed with $AA^T = \Sigma$, then any linear combination of X is also normally distributed. Hence

$$w^T X \stackrel{d}{=} w^T \mu + (w^T \Sigma w)^{1/2} Z_1 \tag{4.1}$$

where w is a non-random vector with same dimension as X [13].

4.2 Value at Risk

Value at Risk (VaR) is a measure of the risk of investments and estimates how much a portfolio might lose during normal market conditions for a set time period and probability level. The VaR at level $p \in (0, 1)$ for a portfolio with value X at time 1 is

$$\mathrm{VaR}_p(X) = min\{m : P(mR_0 + X < 0) \leq p\}, \tag{4.2}$$

where R_0 is the percentage return of a risk-free asset.

If we let

$$L = \frac{-X}{R_0}$$

where X is the net gain of the portfolio and R_0 is the percentage return of a risk-free asset, we can express (4.2) as

$$\mathrm{VaR}_p(X) = min\{m : P(L \leq m) \geq 1 - p\},$$

Similarly, in statistical terms, it can be expressed as

$$\mathrm{VaR}_p(X) = F_L^{-1}(1 - p)$$

where, $F_L^{-1}(u)$ is the quantile value of F_L such that $F_L^{-1}(u)$ is the smallest value m for which $F_L(m) \geq u$, see [13].

4.2.1 VaR for Large Portfolios

Consider a portfolio consisting of a linear combination of different assets with an elliptical distribution. Let $X = V_T - V_0$ denote the gain of the portfolio at time T. Using (4.1), the Value at Risk of the portfolio can be expressed as

$$\mathrm{VaR}_p(X) = w^T \mu + (w^T \Sigma w)^{1/2} \Phi^{-1}(1 - p) \tag{4.3}$$

where w represent the portfolio weights and $\Phi^{-1}(\cdot)$ is the quantile of the normal cumulative distribution function.

In many cases, we will assume that $\mu = 0$, which reduces (4.3) to

$$\mathrm{VaR}_p(X) = (w^T \Sigma w)^{1/2} \Phi^{-1}(1 - p)$$

4.3 Parametric Linear VaR Models

The Parametric Linear VaR Model is a model that can be used to gain an oversight of the risk in a portfolio. A bottom-up-modelling approach is often used for the model which allows for risk measurement calculations at different levels in the portfolio, from the security level to the entire portfolio. The construction of the factors are specific to each risk model, which is why no effort will be made on describing the various risk factors.

The risk in a portfolio can be split into systematic and idiosyncratic risk. Systematic risk, also known as market risk, is the risk inherent to the entire market. Idiosyncratic or unsystematic risk is the opposite to systematic risk and refers to risk that can be attributed to a particular asset or instrument. As it does not affect every investment in the portfolio, this risk can partly be mitigated through diversification. The relationship between systematic risk, idiosyncratic risk and total portfolio risk is

$$Total\ Portfolio\ Risk = Systematic\ Risk + Idiosyncratic\ Risk \tag{4.4}$$

A normal distribution is assumed where the mean μ is set to zero and the volatility σ is estimated from historical data. The magnitude of VaR and other risk measures can be expressed in terms of absolute numbers or in terms of returns. The formulas below have primarily been taken from the books by Alexander, see [2] and [1]. Modifications have been done to some of the formulas.

4.3.1 Systematic Risk Factors

Systematic Linear VaR

The systematic return of a portfolio is the return that can be explained by the variation in risk factors. For a linear portfolio the systematic return is expressed by the following weighted sum

$$R_S = \sum_{i=1}^{n} e_i X_i \tag{4.5}$$

where e_i is the exposure in percentage term, and X_i is the return of risk factor i.

To be able to calculate the systematic linear VaR, the expectation and the variance of the portfolio's systematic returns over the specified time horizon is needed. To compute these, the factor sensitivities or exposures, e, and the covariance matrix Σ of the risk factor returns need to be estimated. In addition to adjusting the time period of estimation, various methods can be used to estimate the covariance matrix, such as applying a weighting schedule to the factor return data.

The expectation $E_S[R_S]$ of the systematic returns can be expressed as

$$E_S[R_S] = e^T \mu$$

where $\mu = (E_S[X_1], ..., E_S[X_n])^T$ is the expected systematic return of the factors and $e = (e_1, ..., e_n)$ is a vector of exposures to n risk factors.

The variance $V_S[R_S]$ of the systematic returns can be expressed as

$$V_S[R_S] = e^T \Sigma e$$

where Σ is the $n \times n$ covariance matrix for the risk factor returns and $e = (e_1, ..., e_n)$ is a vector of exposures to n risk factors.

Assuming that the risk factors follow a multivariate normal distribution, the systematic linear VaR, VaR_p^S, is given by

$$\mathrm{VaR}_p^S = \Phi^{-1}(1-p)(e^T \Sigma e)^{1/2} - e^T \mu$$

Assuming that the expected systematic return is equal to the discount rate, the expression above reduces to the following formula

$$\mathrm{VaR}_p^S = \Phi^{-1}(1-p)(e^T \Sigma e)^{1/2} \tag{4.6}$$

Often it is assumed that the risk factors are I.I.D. in respect of time dependence, which in combination with absence of auto correlation or heteroskedasticity, allows for application of the square-root-of-time rule. This result in the following property for the covariance matrix

$$\Sigma_t = t\Sigma_1$$

where Σ_t is the covariance matrix of the t-period risk factor returns and Σ_1 is the covariance matrix for the one-period return.

The systematic VaR for the t-period could therefore be expressed as

$$\mathrm{VaR}_{t,p}^S = \sqrt{t}\mathrm{VaR}_{1,p}^S \tag{4.7}$$

Stand-Alone Risk and Stand-Alone VaR

Stand-alone risk and stand-alone VaR is the systematic risk and systematic VaR respectively, isolated to a specific risk factor or block of risk factors. The sum of the stand-alone VaR for the risk factors is greater than or equal to the total systematic VaR. This is due to diversification effect between risk factors.

Assume that a portfolio can be mapped against risk factors for three assets; A, B and C. The exposure vector e can then be expressed as

$$e = (e_A^T, e_B^T, e_C^T)^T$$

where e_A^T, e_B^T and e_C^T represent the exposure vectors in percentage term for each of the assets. The method of estimation of exposure will differ for the assets.

The covariance matrix Σ can be expressed in terms of sub-covariance matrices, i.e. the covariance matrices for each asset's risk factor returns. Hence, Σ is given by

$$\Sigma = \begin{pmatrix} \Sigma_a & \Sigma_{a,b} & \Sigma_{a,c} \\ \Sigma_{b,a} & \Sigma_b & \Sigma_{b,c} \\ \Sigma_{c,a} & \Sigma_{c,b} & \Sigma_c \end{pmatrix}$$

In order to calculate the VaR for one of the assets, the exposure vectors for the other assets are set to zero. As an example, the stand-alone VaR for asset A is obtained by letting $e_b = e_c = 0$. The stand-alone risk or volatility for asset A is then given by

$$\sigma_a = (e_a^T \Sigma_a e_a)^{1/2}$$

The stand-alone VaR for asset A is then obtained by

$$\mathrm{VaR}_p^A = \Phi^{-1}(1-p)(e_a^T \Sigma_a e_a)^{1/2} = \Phi^{-1}(1-p)\sigma_a$$

Contribution to Risk

Contribution to risk is a measurement of the change in risk due to a small change in factor exposure. The sum of the contribution to risk of all factors in a portfolio is equal to the portfolio volatility.

The portfolio volatility σ_P is given by

$$\sigma_P = \sqrt{e^T \Sigma e + w^T \Psi w}$$

where Σ and Ψ are the covariance matrices for the systematic risk factor returns and the idiosyncratic risk term returns respectively, e is a vector of exposures to the risk factors and w is a vector with the market value weights of the securities in the portfolio.

The risk contribution RC_f of factor f is defined as

$$RC_f = \frac{\partial \sigma_P(e)}{\partial e_f} e_f = \frac{\partial(\sqrt{e^T \Sigma e + w^T \Psi w})}{\partial e_f} e_f = \frac{(e^T \Sigma_f) e_f}{\sigma_P} \tag{4.8}$$

where e is the exposure vector, e_f is the exposure to factor f, $\frac{\partial \sigma_P}{\partial e_f}$ is the partial derivative of the portfolio volatility σ_P with respect to the exposure to factor f and Σ_f is the column for factor f in Σ.

Marginal VaR

Stand-alone VaR can be converted to marginal VaR for which the sum is equal to the total systematic VaR. The gradient vector is obtained by differentiating the formula for the systematic VaR, see (4.6), with the respect to each component in e. If one risk factor f is considered, the exposure to the other risk factors are set to zero.

The marginal VaR can then be estimated as

$$\text{Marginal VaR} \approx e^T \nabla(e)$$

where e is the exposure to the asset's factor returns and ∇. The gradient vector ∇ is given by

$$\nabla(e) = \Phi^{-1}(1-p)(\Sigma e)(e^T \Sigma e)^{-1/2} \tag{4.9}$$

where e is a vector of exposure, Σ is the covariance matrix for the returns, p is the confidence level, $\Phi^{-1}(\cdot)$ is the quantile of the normal cumulative distribution function and RC_f is the risk contribution of factor f as given in (4.8).

Hence, the marginal VaR can be expressed as

$$\text{Marginal VaR} \approx e^T \Phi^{-1}(1-p)(\Sigma e)(e^T \Sigma e)^{-1/2}$$

Incremental VaR

The formula for Marginal VaR can be modified as to assess the impact on VaR of a trade. This is also called incremental VaR and the first order approximation is given by

$$\text{Incremental VaR} \approx \Delta e^T \nabla(e) = \Delta e^T \Phi^{-1}(1-p)(\Sigma e)(e^T \Sigma e)^{-1/2}$$

where Δe is the change in risk factor exposure as result of a specific trade, e is the original exposure vector and ∇ is the gradient vector given in (4.9).

4.3.2 Idiosyncratic Risk Terms

A few risk measures for the idiosyncratic risk terms will be brought up in this section. More detailed descriptions of the risk measures can be found under Section 4.3.1. where the risk measures are brought up for the systematic risk factors.

For a linear portfolio, the idiosyncratic return of a portfolio is the return is represented by the following weighted sum

$$R_I = \sum_{j=1}^{m} w_j Y_j \tag{4.10}$$

where w_j is the security weight in percentage terms, and Y_j is the return of idiosyncratic risk term j.

The variance $V_I[R_I]$ of idiosyncratic risk can be expressed as

$$V_I[R_I] = w^T \Psi w \tag{4.11}$$

where w is a vector of the market value weights of the securities in the portfolio. The risk factors Y_j are I.I.D $\mathcal{N}(0, \Psi_{jj})$ and therefore, Ψ is a $m \times m$ diagonal matrix containing the variances of the idiosyncratic risk terms for the securities.

The idiosyncratic risk and the matrix Ψ can be estimated using the relationship stated in (4.4). The variance σ_P^2 of the portfolio is given by

$$\sigma_P^2 = V_S[R_S] + V_I[R_I]$$

where $V_S[R_S]$ and $V_I[R_I]$ is the variance of the returns of the systematic risk factors and the idiosyncratic terms respectively.

Rearrangement of the expression above, therefore gives that the idiosyncratic risk σ_I in a portfolio can be estimated as

$$\sigma_I = \sqrt{V_S[R_S] - \sigma_P^2}$$

Stand-Alone Risk and VaR

The stand-alone risk for the idiosyncratic terms of asset a is given by

$$\sigma_a = (w_a^T \Psi_a w_a)^{1/2}$$

where w_a is the market weights in asset a and Ψ_a is the diagonal covariance matrix of asset a.

The stand-alone VaR for the idiosyncratic terms of asset A is then obtained by

$$\mathrm{VaR}_p^A = \Phi^{-1}(1-p)(w_a^T \Psi_a w_a)^{1/2} = \Phi^{-1}(1-p)\sigma_a$$

where $\Phi^{-1}(\cdot)$ is the quantile of the normal cumulative distribution function.

Contribution to Risk

The risk contribution of a idiosyncratic risk term i is given by

$$RC_i = \frac{\partial \sigma_P(w)}{\partial w_i} w_i = \frac{\partial(\sqrt{e^T \Sigma e + w^T \Psi w})}{\partial w_i} w_i = \frac{w_i^2 \Psi_{i,i}}{\sigma_P}$$

where , w_i is the weight in risk term i, $\frac{\partial \sigma_P}{\partial w_i}$ is the partial derivative of the portfolio volatility σ_P with respect to the exposure to risk term i, and $\Psi_{i,i}$ is the variance of the returns of risk term i. The portfolio volatility σ_P is stated in (4.12).

4.3.3 Total Portfolio Risk and VaR

The return in a multi-factor model can then be expressed as

$$R = R_S + R_I = \sum_{i=1}^{n} e_i X_i + \sum_{j=1}^{m} w_j Y_j$$

As the systematic and idiosyncratic risk factors are independent, we get that the variance of the returns of systematic and idiosyncratic risk terms is

$$V[R] = V[\sum_{i=1}^{n} e_i X_i + \sum_{j=1}^{m} w_j Y_j] = e^T \Sigma e + w^T \Psi w$$

The total volatility σ_P of a portfolio with both systematic and idiosyncratic risk can therefore be expressed as

$$\sigma_P = \sqrt{e^T \Sigma e + w^T \Psi w} \tag{4.12}$$

The VaR for a portfolio subject to both systematic and idiosyncratic risk is therefore given by

$$\mathrm{VaR}_p = \Phi^{-1}(1-p)\sigma_P$$

Assuming that covariance matrices have been estimated for a specific period, the t-period VaR is given by

$$\mathrm{VaR}_{t,p} = \sqrt{t}\Phi^{-1}(1-p)\sigma_P$$

4.3.4 Linear VaR for Cash Flows

The following method is applicable for portfolios containing bonds, loans and swaps. Their common denominator is the ability to be expressed as cash flows. The risk factors in this case are yield curves. Yield curves are sets of fixed maturity interest rates of a specific credit rating.

In general, interest rates can be decomposed into a reference rate such as the LIBOR and a credit spread. The risk factors will then be the yield curve for the reference rate and the credit spread term structured for different credit ratings.

Using a linear approximation, the change in present value of the entire cash flow series is

$$\Delta PV = -e^T \Delta r$$

where e is a vector of risk factor sensitivities and element i represent the cash exposure for the ith index and Δr is a vector of changes in interest rates in basis points at the standard maturities.

4.3.5 Exposure/ Sensitivity vector

There are various ways of defining the exposure or sensitivity vector and it could either be defined as in nominal or percentage terms. Alexander [2] used PV01, the present value of a basis point change as a sensitivity measure. Using PV01, the assumption is that the covariance matrix Σ for the risk factors is expressed in basis points. PV01 is defined as

$$PV01_T = PV01(C_T, R_T) = PV(C_T, R_T - 0.01\%) - PV(C_T, R_T)$$

where $PV(C_T, R_T)$ is the present value of the cash flow at time t. Using a discretely compounded discount rate R_T in annual terms, $PV(C_T, R_T)$ can be computed as

$$PV(C_T, R_T) = C_T(1 + R_T)^{-T}$$

If we instead use a continuously compounded rate, we can express $PV(C_T, R_T)$ for any maturity T, not only integer values. The $PV(C_T, R_T)$ is then

$$PV(C_T, R_T) = C_T e^{(-r_T T)}$$

An approximation to PV01 using this expression is therefore

$$PV01_T \approx TC_T e^{(-r_T T)} 10^{-4}$$

Using PV01, the sensitivity vector in nominal terms can be expressed as

$$e = (PV01_1, PV01_2, ..., PV01_n)^T$$

where $PV01_i$ is the PV01 for index i.

To obtain the sensitivity in percentage terms, we simply divide by the total portfolio value. The percentage sensitivity therefore can be expressed as

$$e = P^{-1}(PV01_1, PV01_2, ..., PV01_n)^T, \tag{4.13}$$

see [2]. This is closely related to the concept modified duration, which in percentage terms is

$$Modified\ Duration = 100\frac{PV01}{PV} = -\frac{100}{PV}\frac{\partial PV}{\partial y}$$

where PV refers to the price of the bond, PV01 is the present value of a basis point change, and $\frac{\partial PV}{\partial y}$ is the partial derivative of the bond price with respect to the yield [11].

4.4 Black-Scholes Model

In this section, the Black-Scholes model will be brought up from a martingale point of view. From this point of view, the probability space is chosen as $(\Omega, F, P, \underline{F})$, and is carrying a P-Winer process W^P. The filtration $\underline{F}$ is generated by W^P.

The Black-Sholes model on this space is defined as

$$dS_t = \alpha_t S_t dt + \sigma_t S_t dW_t^P$$

$$dB_t = r_t B_t dt$$

where S_t can be seen as the stock price and B_t the bank account at time t. α_t, σ_t and r_t can be arbitrarily adapted but integrable processes with the condition $\sigma_t \neq 0$.

The model is free of arbitrage only if there is a martingale measure Q for the model. Using the Girsanov Theorem, we can look for a Girsanov kernel process h such that the measure Q is in fact a martingale measure. The Q-dynamics of S is given by

$$dS_t = \{\alpha_{t+}\sigma_t h_t\} S_t dt + \sigma_t S_t dW_t^P$$

where h_t is the Girsanov kernel process.

For Q to be a martingale measure, the local rate of return under Q must equal the short rate. Hence, there is a need of a process h such that

$$\alpha_t + \sigma_t h_t = r_t$$

which has the solution

$$h_t = -\frac{\alpha_t - r_t}{\sigma_t}$$

The Girsanov kernel process h_t is stochastic and can be expressed in terms of market risk by the relation $h_t = -\lambda_t$.

The market price of risk λ_t is given by

$$\lambda_t = \frac{\alpha_t - r_t}{\sigma_t} \tag{4.14}$$

where α_t is the local mean rate of return, r_t is the risk-free rate and σ_t is the volatility. The numerator $\alpha_t - r_t$ could be seen as the rate of excess return over the risk-free rate, i.e. the risk premium [4]. The market price of risk can be interpreted as the excess return that is required by investors for a certain level of risk.

Chapter 5

Modelling Approaches

The intention with this chapter is to provide possible remedies to the risk modelling of private infrastructure debt. Two modelling approaches are proposed; a systematic approach presented in Section 5.4 and an idiosyncratic approach presented in Section 5.5.

5.1 Introduction

Key features and characteristics of private infrastructure debt have been identified based on existing research and interviews with professionals from the industry. Perceiving infrastructure as a contract rather than as an asset suggest new risk drivers that are different to those of other assets. With this view, the risk factors used for traditional assets are not sufficient for explaining the actual risk in private infrastructure debt investments. The research provided thoughts on several possible sets of risk factors. However, according to previous research, see [7], revenue risk seem to be a main driver of credit risk in private infrastructure debt.

Revenue risk is the risk occurring from the type of revenue model in the infrastructure project or company. In a previous study by Blanc-Brude and Ismail [7], the authors showed that the credit spread for private infrastructure debt varies with the sources of income for the project. The data set in this study also indicated that the spread level differ a lot between infrastructure debt investments categorised by revenue model. The two suggested modelling approaches therefore aim at incorporating revenue risk into the modelling of private infrastructure debt in a risk factor model.

The amount of data on private infrastructure debt deals is limited. The modelling approaches have therefore been designed to take use of public indices and estimates based on earlier studies and a small data set.

5.2 Baseline Model

The suggested modelling approaches could be seen as modifications to a baseline model. The baseline model represent a way of modelling private infrastructure debt and is assumed to be the way private infrastructure debt is modelled currently. Therefore, in order to be able to introduce the new modelling approaches, the baseline model will be presented.

In the baseline model, private infrastructure debt is modelled as a debt instrument, e.g. a corporate bond or a loan, with exposure to a infrastructure debt factor. The debt instrument is modelled with the available risk factors in the model. The infrastructure debt factor is assumed to be represented by a broad index which captures a general infrastructure debt specific risk and adjust the spread of the debt instrument accordingly. The spread or the credit spread is here defined as the difference between required return for the infrastructure debt instrument and the reference rate.

The infrastructure debt factor return is referred to as X_{ID} and therefore the return of this factor is given by

$$R_{ID} = e_{ID} X_{ID}$$

where e_{ID} is the linear exposure to the infrastructure debt factor and X_{ID} is the return of the infrastructure debt factor. The volatility of the factor returns is simply the volatility of the returns of the infrastructure debt index used to represent the factor.

The factor is represented by a spread and a commonly used sensitivity measure for spreads is the spread duration. As the factor is represented by a general infrastructure debt index, the exposure e_{ID} can be set as the spread duration of the index. The spread duration can either be obtained from information related to the index or be estimated. An example of a estimation of the exposure vector is given in (4.13).

5.2.1 Covariance Matrix

As the infrastructure debt factor only consist of one factor, the covariance matrix Σ_{ID} for the factor is simply the variance of the factor return. The covariance matrix is therefore given by

$$\Sigma_{ID} = \mathrm{Var}[X_{ID}] \tag{5.1}$$

where X_{ID} is the return of the infrastructure debt factor, i.e the return of the general infrastructure debt index.

5.2.2 Risk Measures

The following section brings up risk measures for the baseline model. The risk measures are solely focusing on the infrastructure debt factor. The risk measures for the asset private infrastructure debt are different because the underlying debt instrument is modelled with a range of factors not brought up in this thesis.

Stand-Alone Risk

The stand-alone risk for the infrastructure debt factor is given by

$$\sigma_{ID} = \sqrt{(e_{ID}^T \Sigma_{ID} e_{ID})} = e_{ID}\sqrt{\mathrm{Var}[X_{ID}]} \tag{5.2}$$

where Σ_{ID} is the covariance matrix for the returns of the factor, e_{ID} is the linear portfolio exposure to the infrastructure debt factor and $\mathrm{Var}[X_{ID}]$ is the variance of the factor's returns.

Risk Contribution

The risk contribution of the infrastructure debt factor is given by

$$RC_{ID} = \frac{\partial \sigma_P(e)}{\partial e_{ID}} e_{ID} = \frac{\partial(\sqrt{e^T \Sigma e + w^T \Psi w})}{\partial e_{ID}} e_{ID} = \frac{(e^T \Sigma_{ID}) e_{ID}}{\sigma_P} \tag{5.3}$$

where e is the exposure vector, e_{ID} is the exposure to infrastructure debt investments, $\frac{\partial \sigma_P}{\partial e_{ID}}$ is the partial derivative of the portfolio volatility σ_P with respect to the exposure to infrastructure debt and Σ_{ID} is the column for the infrastructure debt factor in Σ.

The new modelling approaches will now be introduced.

5.3 Revenue Risk

The suggested modelling approaches could be seen as an extension of the representation of the infrastructure debt factor. Instead of letting the infrastructure debt factor be represented by one factor and one proxy, it could be seen as a block of sub-factors. Ideally, the additional factors would capture the risk occurring from the revenue model in each private infrastructure debt investment and hence give a better representation of the risk in these investments.

The factors for revenue risk have been based on the revenue model categorisation by Blanc-Brude and Ismail [7], which consists of the categories merchant, contracted and regulated. Further details regarding these revenue models can be found under Section 3.3.3. The revenue factors and commonly associated descriptions for each of the revenue models can be found in Table 5.1. below.

Factor	Alternative Names
Merchant(M)	Commercial Scheme, Real Tolls
Regulated(R)	Partial Commercial Scheme, Partially Contracted, Shadow Tolls
Contracted(C)	Availability Payment Scheme

Table 5.1: Revenue Risk Factors

Revenue risk can be incorporated into the model as either idiosyncratic or systematic risk. The credit spread differ between the revenue risk model categories and the spread level changes over the time of the infrastructure project. From another perspective, the limited amount of private data for infrastructure debt investments might make one approach of estimating the revenue risk more suitable than the other. Possible approaches for modelling revenue risk as both idiosyncratic risk as well as systematic risk will therefore be discussed.

Both the idiosyncratic and the systematic approach is based on the idea of adding revenue factors that adjusts the volatility of the general infrastructure debt index for the type of revenue model in the infrastructure project. The adjustment is represented by a constant coefficient for the type of revenue model in the project and is here referred to as β_i, where the index i correspond to the revenue model merchant, contracted or regulated. There will therefore be a β coefficient for each revenue category. Details on how the β coefficients are incorporated into the modelling can be found under the sections for the systematic revenue risk factors and idiosyncratic revenue risk terms.

5.4 Systematic Revenue Risk Factors

The revenue risk factors could be seen as systematic risk factors where a credit spread adjustment is made for infrastructure debt investments where the project or company has a particular revenue model.

The return of the infrastructure debt factor X_{ID} in the baseline model is now replaced by X_M,X_C and X_R, which are the returns of the factors merchant, contracted and regulated respectively. Hence,

$$X_{ID} \Rightarrow X_M, X_C, X_R$$

The index used for representing the new revenue factors is the same as the index used for representing the previous infrastructure debt factor. In order for each of the revenue factors to capture the specific risk of the type of revenue model, the general infrastructure debt index is multiplied with a β coefficient specific to the revenue factor.

The return of a revenue factor i is given by

$$X_i = \beta_i X_{ID}$$

where $i = \{M, C, R\}$, β_i is the coefficient for revenue model i and X_{ID} is the return of the index used for the infrastructure debt factor in the baseline model.

The variance of the returns of revenue factor i is given by

$$\text{Var}[X_i] = \text{Var}[\beta_i X_{ID}] = \beta_i^2 \text{Var}[X_{ID}] \tag{5.4}$$

The introduced factors X_M,X_C and X_R are therefore perfectly correlated but with different volatilities. An explanation to why an adjustment of the volatility can be done with β_i is given in Section 5.7.

We can now introduce a new infrastructure debt factor which is a block of the three revenue factors. The return of the new infrastructure debt factor $\widehat{R_{ID}}$ is given by

$$\widehat{R_{ID}} = \sum_{i=\{M,C,R\}} e_i X_i = \sum_{i=\{M,C,R\}} e_i \beta_i X_{ID}$$

where e_i is the linear exposure to factor return X_i, β_i is the coefficient for revenue model i and X_{ID} is the return of the general infrastructure debt index. Note that one infrastructure debt investment can only have one revenue model and hence only have exposure to one of the revenue factors.

We will denote the exposure vector for the new infrastructure debt factor as

$$\hat{e}_{ID} = (e_M, e_C, e_R) \tag{5.5}$$

where e_M, e_C and e_R is the exposure to the revenue factors merchant, contracted and regulated respectively.

The exposure e_i to revenue factor i, $i = \{M, C, R\}$, is given by

$$e_i = e_{ID} \frac{w_i}{w_{ID}} \tag{5.6}$$

where w_{ID} is the portfolio weight for all infrastructure debt investments, w_i is the portfolio weight in infrastructure debt investments with revenue model i and e_{ID} is the linear exposure to all infrastructure debt investments as in the baseline model.

5.4.1 Covariance Matrix

In a portfolio with m number of assets, the covariance matrix Σ is a $m \times m$ matrix containing the sub-covariance matrices for each asset. The sub-covariance matrix is the covariance matrix for the asset's risk factor returns, i.e. the covariance between returns for the proxies used to represent the risk factors.

The covariance matrix Σ for the portfolio is therefore given by

$$\Sigma = \begin{pmatrix} \Sigma_{1,1} & \Sigma_{1,2} & \cdots & \Sigma_{1,m} \\ \Sigma_{2,1} & \Sigma_{2,2} & \cdots & \Sigma_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{m,1} & \Sigma_{m,2} & \cdots & \Sigma_{m,m} \end{pmatrix} \tag{5.7}$$

The covariance matrix for the asset infrastructure debt is represented by one of the diagonal sub-matrices in (5.7). The cross-covariance to the other assets is given by the sub-matrices with one index that corresponds to the index for infrastructure debt itself.

If we choose to represent the infrastructure debt factor with n specific factors, then the covariance matrix for the returns of the infrastructure debt factors is given by the following $n \times n$ matrix

$$\Sigma_{ID} = \begin{pmatrix} \Sigma_{1,1} & \Sigma_{1,2} & \cdots & \Sigma_{1,n} \\ \Sigma_{2,1} & \Sigma_{2,2} & \cdots & \Sigma_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma_{n,1} & \Sigma_{n,2} & \cdots & \Sigma_{n,n} \end{pmatrix}$$

In the proposed approach, we replace the infrastructure debt factor with three new revenue factors. We therefore obtain the following 3×3 covariance matrix $\widehat{\Sigma_{ID}}$ for the new infrastructure debt factor

$$\widehat{\Sigma_{ID}} = \begin{pmatrix} \Sigma_{X_M,X_M} & \Sigma_{X_M,X_C} & \Sigma_{X_M,X_R} \\ \Sigma_{X_C,X_M} & \Sigma_{X_C,X_C} & \Sigma_{X_C,X_R} \\ \Sigma_{X_R,X_M} & \Sigma_{X_R,X_C} & \Sigma_{X_R,X_R} \end{pmatrix} \tag{5.8}$$

where Σ_{X_i,X_j} for $i = j$ is the variance of the returns of factor i and Σ_{X_i,X_j} for $i \neq j$, is the covariance between the factors i and j. The index i and the index j, correspond to revenue models merchant, contracted and regulated.

For $i = j$ we get that

$$\Sigma_{X_i,X_j} = \Sigma_{X_i,X_i} = \mathrm{Var}[X_i] = \beta_i^2 \mathrm{Var}[X_{ID}]$$

where $\mathrm{Var}[X_i]$ is obtained from (5.4).

For $i \neq j$ we get that

$$\Sigma_{X_i,X_j} = Cov(X_i, X_j) = \sqrt{\mathrm{Var}[X_i]\mathrm{Var}[X_j]}\rho_{i,j}$$

where $\rho_{i,j}$ is the correlation between the factor returns X_i and X_i. We know that the factor returns X_M, X_R, X_C are represented by the same general infrastructure debt index. The returns of the factors are therefore perfectly correlated and $\rho_{i,j} = 1$.

Hence, for $i \neq j$ we get that

$$\Sigma_{X_i,X_j} = \sqrt{\mathrm{Var}[X_i]\mathrm{Var}[X_j]} = \sqrt{\beta_i^2 \mathrm{Var}[X_{ID}]\beta_j^2 \mathrm{Var}[X_{ID}]} = \beta_i \beta_j \mathrm{Var}[X_{ID}]$$

In order to be able aggregate the risk to a portfolio level, a number of other covariance matrices need to be estimated. These include a covariance matrix for the underlying debt instrument, and a covariance matrix for the asset infrastructure debt and the other securities in the portfolio.

The covariance and cross-covariance matrices need to be estimated in order to compute the risk measures. There are various ways of estimating the covariance matrices from historical data. Furthermore, there are possibilities to apply weighting schemes, typically to let more recent data have greater influence. The estimation method is chosen on a case to case basis and therefore the estimation of covariance matrices are left out in this thesis.

5.4.2 Risk Measures

In the following section, stand-alone risk and risk contribution on the revenue factor level and the infrastructure debt factor level will be presented.

Stand-Alone Risk

The factor volatility and the linear factor exposure are used to compute the stand-alone risk from all private infrastructure debt investments with a particular revenue model in the portfolio.

The stand-alone risk for revenue factor i is given by

$$\sigma_i = \sqrt{e_i \Sigma_{X_i,X_i} e_i} = e_i \beta_i \sqrt{\mathrm{Var}[X_{ID}]} \tag{5.9}$$

where $i = \{M, C, R\}$, e_i is the exposure to revenue factor i, β_i is the coefficient representing the estimated change in volatility due to the specific revenue model and $\mathrm{Var}[X_{ID}]$ is the variance of the returns of the general infrastructure debt index.

The stand-alone risk from the total infrastructure debt factor is received by letting every exposure element, except for those of the the revenue factors, be set to zero. The stand-alone risk from the total infrastructure debt factor is therefore

$$\widehat{\sigma_{ID}} = \sqrt{(\hat{e}_{ID}^T \widehat{\Sigma_{ID}} \hat{e}_{ID})} \tag{5.10}$$

where $\widehat{\Sigma_{ID}}$ is the covariance matrix for the new infrastructure debt factor given in (5.8) and $\hat{e}_{ID} = (e_M, e_C, e_R)$ is the vector of exposures to the revenue factors.

Risk Contribution

The risk contribution from revenue factor i is given by

$$RC_i = \frac{\partial \sigma_P(e)}{\partial e_i} e_i = \frac{\partial(\sqrt{e^T \Sigma e + w^T \Psi w})}{\partial e_i} e_i = \frac{(e^T \Sigma_i) e_i}{\sigma_P} \tag{5.11}$$

where e is the exposure vector, e_i is the exposure to infrastructure debt investments with revenue model i, $\frac{\partial \sigma_P}{\partial e_i}$ is the partial derivative of the portfolio volatility σ_P with respect to the exposure to revenue factor i and Σ_i is the column for revenue factor i in Σ. Furthermore, $i = \{M, C, R\}$.

Similarly, the risk contribution from the total infrastructure debt factor is given by

$$\widehat{RC_{ID}} = \frac{\partial \sigma_P}{\partial e_{ID}} e_{ID} = \frac{(e^T \Sigma_{ID}) e_{ID}}{\sigma_P} \tag{5.12}$$

where e is the exposure vector, e_{ID} is the exposure to infrastructure debt investments, $\frac{\partial \sigma_P}{\partial e_{ID}}$ is the partial derivative of the portfolio volatility σ_P with respect to the exposure to all infrastructure debt investments, Σ_{ID} is the column for the infrastructure debt block in the total covariance matrix Σ, $i = \{M, C, R\}$

5.5 Idiosyncratic Revenue Risk Terms

Revenue risk can also be seen as a type of idiosyncratic risk which can not be explained by factors or the market as the risk is specific to the security. In this approach, a constant spread is added for the type of revenue model in the infrastructure project. The infrastructure debt factor can therefore be represented by two parts; the systematic risk reflected by the infrastructure debt factor from the baseline model and a risk term representative for the risk due to the revenue model in a project.

The revenue risk terms are here referred to as idiosyncratic risk terms although this is not completely true as we now have risk terms that can explain variation in risk between infrastructure debt investments. The true idiosyncratic risk for the infrastructure debt instrument is in fact given by the subtracting the estimated idiosyncratic risk with the risk estimates for the revenue factor.

Each of the revenue risk categories merchant, contracted and regulated is attributed its own risk estimate based on the estimated β coefficient and the volatility of the general infrastructure debt index.
There are a number of assumptions and conditions underlying this approach:

> *One Revenue Factor.* It is assumed that an individual private infrastructure debt investment can only belong to one revenue risk category and hence only have exposure to one revenue factor.
>
> *Uncorrelated.* The revenue factors are assumed to be uncorrelated to other risk factors. Furthermore, the same revenue factor for different infrastructure debt investment are assumed to be uncorrelated.

Rescaling. Depending on the size of the portfolio, the risk estimate from the revenue factor will be scaled down to be representative for the portfolio.

The constant risk estimates $\tilde{\sigma}_i$ for each revenue category are constructed as

$$\tilde{\sigma}_i = (\beta_i - 1)\sigma_{index} \tag{5.13}$$

where β_i is the estimated coefficient representing the difference in volatility compared to the infrastructure debt index and σ_{index} is an estimate of the volatility of the index representing the infrastructure debt factor. Methods for estimating β_i are given in section 5.7.

The return of the infrastructure debt factor X_{ID} is now exchanged for the return of the same factor X_{ID} and three additional terms Y_M, Y_C, Y_R representing the returns of the revenue risk terms. Hence

$$X_{ID} \Rightarrow X_{ID}, Y_M, Y_C, Y_R$$

The return R_i of the idiosyncratic term i is given by

$$R_i = w_i Y_i$$

where w_i is the market weight for infrastructure debt investments with revenue model i and Y_i is the return of the idiosyncratic term.

There is nothing systematic in the idiosyncratic terms and it is therefore not appropriate to determine estimates of a constant return Y_i based on historical returns. The returns of the idiosyncratic terms are a residual of the return that can not be explained by the systematic risk factors. The idiosyncratic return for the asset infrastructure debt can therefore be estimated as the return of the asset subtracted with the return for the asset explained by the systematic factors that asset has exposure to. Hence the total idiosyncratic return R^I_{ID} for the asset infrastructure debt can be estimated as the residual

$$R^I_{ID} = R_{ID} - R^S_{ID} = R_{ID} - \sum_i e_i X_i = \sum_j w_j Y_j$$

where R_{ID} is the total return of infrastructure debt and $R^S_{ID} = \sum_i e_i X_i$ is the systematic return of all the factors that the asset infrastructure debt has exposure to.

The idiosyncratic return of the asset is then $\sum_j w_j Y_j$ which represent the weighted return of all idiosyncratic terms for the asset. However, although we can determine the total return of the idiosyncratic terms for the asset, we can not determine the individual return of the idiosyncratic terms.

The vector of market weights in the revenue risk terms is denoted

$$w = (w_M, w_C, w_R)$$

where w_i is the market weight in the idiosyncratic revenue risk term i, $i = \{M, C, R\}$. The exposure to the systematic infrastructure debt factor with return X_{ID} is

$$e = (e_{ID})$$

where e_{ID} is the linear portfolio exposure to the infrastructure debt factor, just as in the baseline model.

5.5.1 Covariance Matrix

In this modelling approach, we have one systematic risk factor and three idiosyncratic risk terms. There is therefore a need for two covariance matrices in order to calculate risk measures for the new infrastructure debt block.

The covariance matrix Σ_{ID} for the returns of the systematic factor X_{ID} is the same as in the baseline model. The covariance matrix is a 1×1 matrix consisting of the variance of X_{ID}. Hence, the covariance matrix for X_{ID} is given by

$$\Sigma_{ID} = \mathrm{Var}[X_{ID}] \tag{5.14}$$

where X_{ID} is the return of the infrastructure debt factor.

The covariance matrix Ψ_{ID} for the returns of the idiosyncratic risk terms is a 3×3 diagonal containing the variance of idiosyncratic risk terms. Hence

$$\Psi_{ID} = \begin{pmatrix} \Psi_{Y_M,Y_M} & 0 & 0 \\ 0 & \Psi_{Y_C,Y_C} & 0 \\ 0 & 0 & \Psi_{Y_R,Y_R} \end{pmatrix} \tag{5.15}$$

where Ψ_{Y_M,Y_M},Ψ_{Y_C,Y_C} and Ψ_{Y_R,Y_R} are the variances of the idiosyncratic revenue risk terms merchant, contracted and regulated respectively.

The variance of the idiosyncratic revenue risk terms is simply the squared risk estimates given in (5.13) and the variance is therefore constant. The covariance matrix for the returns of the idiosyncratic revenue risk terms is therefore given by

$$\Psi_{ID} = \begin{pmatrix} \tilde{\sigma}_M^2 & 0 & 0 \\ 0 & \tilde{\sigma}_C^2 & 0 \\ 0 & 0 & \tilde{\sigma}_R^2 \end{pmatrix} \tag{5.16}$$

where $\tilde{\sigma}_i^2 = ((\beta_i - 1)\sigma_{index})^2$ from (5.13) and $i = \{M, C, R\}$.

5.5.2 Risk Measures

The risk measures stand-alone risk and risk contribution are presented in the following sections. The risk measures are given at the following levels; the systematic infrastructure debt factor, the revenue factor and the block infrastructure debt.

Stand-Alone Risk

The stand-alone risk from the revenue factor is given by the exposure to only one of the revenue factors and in isolation to the rest of the factors in the portfolio. It is computed as the risk estimate scaled down to be representative for the portfolio. The stand-alone risk for the revenue factor is therefore

$$\sigma_i = \sqrt{w_i \Psi_{Y_i,Y_i} w_i} = w_i \tilde{\sigma}_i = w_i(\beta_i - 1)\sigma_{index} \tag{5.17}$$

where w_i correspond to the market value weight of all the private infrastructure debt investments with revenue model i in the portfolio and $\tilde{\sigma}_i$ is the risk estimate for the corresponding revenue model, $i = \{M, C, R\}$.

The stand-alone risk from all revenue factors is

$$\sigma_{R_{Tot}} = \sqrt{w^T \Psi_{ID} w} = \sqrt{\sigma_M^2 + \sigma_C^2 + \sigma_R^2} \tag{5.18}$$

where $\sigma_M, \sigma_C, \sigma_R$ is the stand-alone risk of the revenue factors merchant, contracted and regulated given in (5.17) respectively.

The stand-alone risk for the systematic infrastructure debt factor is the same as in the baseline model, i.e.

$$\sigma_{ID} = \sqrt{(e_{ID}^T \Sigma_{ID} e_{ID})} = e_{ID}\sqrt{\mathrm{Var}[X_{ID}]} \tag{5.19}$$

The stand-alone risk from the new infrastructure debt block is a function of the systematic infrastructure debt factor used in the baseline model and the new revenue risk terms.

The stand-alone risk of the new infrastructure debt block $\widehat{\sigma_{ID}}$ is therefore given by

$$\widehat{\sigma_{ID}} = \sqrt{(e_{ID}^T \Sigma_{ID} e_{ID}) + w^T \Psi_{ID} w} = \sqrt{\sigma_{ID}^2 + \sigma_{R_{Tot}}^2} \tag{5.20}$$

where σ_{ID} is the stand-alone risk from the systematic infrastructure debt factor given in (5.19) and $\sigma_{R_{Tot}}$ is the stand-alone risk from the revenue risk terms as given in (5.18).

Risk Contribution

The risk contribution from revenue factor i is given by

$$RC_i = \frac{\partial \sigma_P}{\partial w_i} w_i = \frac{\partial(\sqrt{e^T \Sigma e + w^T \Psi w})}{\partial w_i} w_i = \frac{w_i^2 \Psi_{i,i}}{\sigma_P} \tag{5.21}$$

where $\frac{\partial \sigma_P}{\partial w_i}$ is the partial derivative of the portfolio volatility σ_P to the market weight w_i of the infrastructure debt investments with revenue model i, $\Psi_{i,i}$ is the variance of risk term i as given in (5.16) and $i = \{M, C, R\}$.

The risk contribution of all revenue terms is

$$RC_{R_{Tot}} = RC_M + RC_C + RC_R \tag{5.22}$$

where RC_M, RC_C, RC_R are given by (5.21).

The risk contribution of the systematic infrastructure debt factor is the same as for the baseline model. Hence

$$RC_{ID} = \frac{\partial \sigma_P(e)}{\partial e_{ID}} e_{ID} = \frac{\partial(\sqrt{e^T \Sigma e + w^T \Psi w})}{\partial e_{ID}} e_{ID} = \frac{(e^T \Sigma_{ID}) e_{ID}}{\sigma_P} \tag{5.23}$$

where e is the exposure vector, e_{ID} is the exposure to infrastructure debt investments, $\frac{\partial \sigma_P}{\partial e_{ID}}$ is the partial derivative of the portfolio volatility σ_P with respect to the exposure to infrastructure debt and Σ_{ID} is the column for the infrastructure debt factor in Σ.

The risk contribution of the total infrastructure debt factor or block is now given by

$$\widehat{RC_{ID}} = RC_{ID} + RC_{R_{Tot}} \tag{5.24}$$

where RC_{ID} is given by (5.23) and $RC_{R_{Tot}}$ is given by (5.22).

5.6 Portfolio Level Risk Measures

The aim of the modelling approaches is also to be able to compute standard risk measures on a portfolio level.

Value at Risk

The total portfolio volatility can be computed as the sum of the risk contribution from the total infrastructure debt factor RC_{ID} and the risk contribution from all remaining factors and securities in the portfolio. Hence

$$\sigma_P = \widehat{RC_{ID}} + \sum RC_j \tag{5.25}$$

where $\widehat{RC_{ID}}$ is defined as (5.12) in the systematic approach and (5.24) in the idiosyncratic approach.

Alternatively, the total portfolio volatility can be computed as the sum of the volatility of the systematic risk factors and the volatility of the idiosyncratic terms as in (4.12), i.e. as

$$\sigma_P = \sqrt{e^T \Sigma e + w^T \Psi w} \tag{5.26}$$

where e is the exposure vector, Σ is the covariance matrix for the systematic returns, w is the market value weights and Ψ is the covariance matrix for the idiosyncratic factors.

Using the total portfolio volatility, the VaR for the portfolio can be estimated. Assuming that the covariance matrix have been estimated over a specific time period, the T-period VaR at level α is given by

$$\mathrm{VaR}_{\alpha}^{T} = \sqrt{T}\Phi^{-1}(1-\alpha)\sigma_P \tag{5.27}$$

where $\Phi(\cdot)$ is the standard normal distribution function.

5.7 β Coefficients

The β coefficients can be estimated from the spread levels of the infrastructure debt investments belonging to the different revenue categories. Adjusting the volatility of the credit spread for infrastructure debt with β is justified with market price of risk λ_t, see (4.14). As an example, infrastructure debt for a project with high demand risk, here classified as merchant, is expected to have a higher credit spread than an infrastructure project with a regulated or contracted revenue model. Market price of risk gives us that a higher credit spread implies a higher volatility.

As a reminder, the market price of risk λ_t is given by

$$\lambda_t = \frac{\alpha_t - r_t}{\sigma_t} = \frac{s_t}{\sigma_t} \tag{5.28}$$

where α_t is the local mean rate of return, r_t is the risk-free rate, s_t is the spread and σ_t is the volatility if the instrument. The numerator $s_t = \alpha_t - r_t$ can be seen as the risk premium or in this case, the credit spread for a private infrastructure debt investment. As λ_t is constant, a larger numerator or a wider credit spread s_t, must imply a higher higher volatility σ_t. Furthermore, this relationship is proportional.

The formula above can be rearranged. Assume a portfolio P representative for the general infrastructure debt index at time t. The volatility σ_t^P for the portfolio, i.e. the volatility for the infrastructure debt instruments or the index is then represented by

$$\sigma_t^P = \frac{s_t^P}{\lambda_t^P}$$

An infrastructure debt investment with revenue risk model i is then added to the portfolio. The credit spread for this investment is different to the spread of the portfolio due to the revenue model in the infrastructure project. The volatility $\sigma_t^{P'}$ of the portfolio is now given by

$$\sigma_t^{P'} = \beta_i \sigma_t^P$$

The relationship is linear and β_i represent the impact of revenue model i on the volatility of the general infrastructure debt index. This follows from

$$\sigma_t^{P'} = \frac{s_t^{P'}}{\lambda_t^{P'}} = \{\lambda_t^{P'} = \lambda_t^P = \frac{s_t^P}{\sigma_t^P}\} = \frac{s_t^{P'}}{s_t^P}\sigma_t^P = \frac{s_t^P\beta_i}{s_t^P}\sigma_t^P = \beta_i\sigma_t^P \tag{5.29}$$

In the earlier example, the infrastructure debt investment with the merchant revenue model will most likely have a positive β_i, implying and a higher volatility than the average infrastructure debt investment. The interpretation of (5.29) is that the volatility of a infrastructure debt investment with revenue model i can be represented by the adjusted volatility of the general infrastructure debt index.

5.7.1 Estimation of β coefficients

The suggested modelling approach is heavily based on the estimation of the β coefficient for each of the three revenue model categories. The following sections give an overview of results from a previous study and a studied data set. Furthermore, the aim is to provide a guideline for the construction of the β_i's in the systematic and idiosyncratic modelling approach. A similar approach could be performed with the use of internal data.

Formula for β

From (5.29), we can obtain the following relationship

$$s_t^{P'} = s_t^P \beta_i \tag{5.30}$$

where s_t^P is the spread for the general infrastructure debt portfolio and $s_t^{P'}$ is the same portfolio with added infrastructure debt investments with revenue model i.

Rearrangement of (5.30) gives that the β coefficients can be estimated as

$$\beta_i = \frac{s_t^{P'}}{s_t^P}$$

The spread within the revenue model categories merchant, contracted and regulated vary. A more reliable estimate of the spread for a category is therefore given by the average spread. Hence β_i can be estimated as

$$\beta_i = \frac{\bar{s_i}}{s_{index}} \tag{5.31}$$

where s_{index} is the spread for the general infrastructure debt index and $\bar{s_i}$ is the mean spread for the revenue category and $i = \{M, C, R\}$.

5.7.2 Previous Study

Blanc-Brude and Ismail [7] performed an empirical study using two data sets in order to determine the most important factors for the level of credit spreads. These factors included the revenue factors merchant, contracted and regulated and showed significant impact of the revenue model on the credit spread level. This study is the main inspiration for suggesting that revenue risk should be incorporated to the risk model of private infrastructure debt. Parts of the statistics for the data set will therefore be used to get risk estimates of the β coefficients.

The amount of internal data might be limited in terms of the number of deals and information regarding the projects. Sample A in the study by Blanc-Brude and Ismail [7] contained approximately 5500 deals. The β coefficients based on the results of this study might therefore be more reliable than the β coefficients estimated from a much smaller internal data set.

Figure 5.1 shows the the distribution for the deals in Sample A that Blanc-Brude and Ismail [7] used in their study.

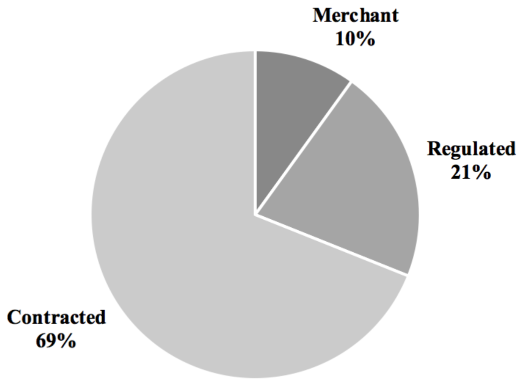

Figure 5.1: Distribution of deals in the study by Blanc-Brude and Ismail.

The mean and the standard deviation of the spread for each revenue category is shown in the Table 5.2 below.

Revenue Category	**Mean**	**SD**
Merchant	164.97	80.67
Regulated	157.10	78.39
Contracted	150.4	69.49

Table 5.2: Mean and Standard Deviation(SD) for the spreads in Sample A in the study by Blanc-Brude and Ismail [7].

Based on the mean spreads for each category, the β coefficients were estimated. The coefficient β_i should be computed as in (5.31) using the spread level of the general infrastructure debt index and the mean spread for the revenue category. In this study, the spreads of the index and the data are defined differently, leading to unrealistic values of the coefficient. The constituents of the general infrastructure debt index primarily have a regulated revenue model. The mean spread of the revenue category regulated could therefore serve as an approximation of the spread for the general infrastructure debt index. The β estimates from the mean spreads in the study can be found in Table 5.3. below.

Revenue Factor	$\boldsymbol{\beta_i}$	$\boldsymbol{\beta_i}$ **Estimate**
Merchant	β_M	1.05
Regulated	β_R	1
Contracted	β_C	0.96

Table 5.3: β coefficients based on a previous study.

5.7.3 Data Set

The β_i for each of the revenue factors can be estimated from data on previous deals and investments in private infrastructure debt. The estimates can be obtained by the following steps:

1. Collect data on private infrastructure debt investments. Important attributes are the credit spread and the source of revenue for the project or company. If there is no information regarding the revenue model, collect deal specific information which can be used for searching the deal on internet.

2. Categorise the data into the revenue groups merchant, contracted and regulated, based on the revenue model.

3. Compute the mean and the standard deviation of the credit spread for each revenue category.

4. Compute the β_i coefficient for each revenue category as in (5.31).

5. Repeat the process when new data is obtained in order to improve the β_i estimates.

The process of obtaining the β_i estimates is very straightforward, however, it requires that the data has a attribute with the source of revenue in the project which can be obtained by adding the attribute to new data. Current data on private infrastructure debt deals typically have no attribute for the type of revenue model in the project. Therefore alternative methods have to be used for the categorisation of the data into revenue groups. An alternative method was used for the data set in this thesis.

The data set is firstly modified as described in Section 3.2.1. It is thereafter split into the categories merchant, regulated and contracted. Certain industrial sectors are known to have typical revenue models which makes

the categorisation easier. A quick search on each deal is however performed in order to confirm the source of revenue. In the cases where the industrial sector is not able to explain the revenue model, a more in depth search on the deal and the company is performed.
The resulting distribution of the revenue risk categories is shown in Figure 5.2 below.

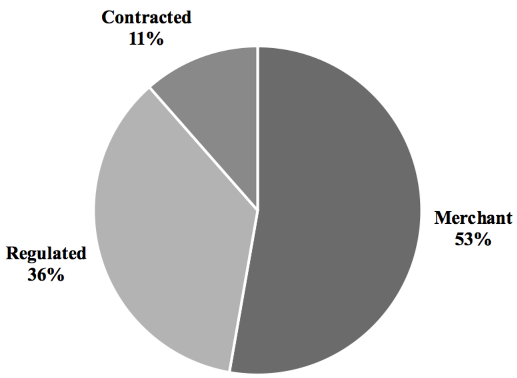

Figure 5.2: Distribution of deals in the data set.

Computation of mean and sample standard deviation of the spreads for each of the three revenue categories is thereafter performed. The β coefficients were then computed using (5.31). As in the previous approach, the mean of the regulated revenue category served as a proxy for the general infrastructure debt index.

The β coefficients are shown in Table 5.4 below.

Revenue Factor	$\boldsymbol{\beta_i}$	$\boldsymbol{\beta_i}$ **Estimate**
Merchant	β_M	1.66
Regulated	β_R	1
Contracted	β_C	1.37

Table 5.4: β coefficients for the data set.

As illustrated in the table, the coefficient is one for the category regulated which is a consequence of the approximation of the spread of index with the mean spread of the regulated revenue category. In comparison to the deals where the income is regulated, deals that have merchant risk or are contracted have a positive β, indicating that the volatility should be greater for these categories. The result for the β of the merchant category is in line with previous studies which also suggest that these type of deals should have the highest risk out of the three revenue categories. The result for the contracted category does however not coincide with earlier research where the contracted projects had the lowest credit spread out of the three categories.

5.7.4 Results and Notes

The estimates for the β coefficients vary significantly between the two data sets. The estimates of the coefficients are not meant to be used for modelling the revenue factors, the process of obtaining the estimates should rather serve as guidelines for how the coefficients can be estimated outside of this study.

For every data set, different estimates of the β coefficients will be obtained. It is therefore important that the estimates are based on a representable data set. The larger the data set, the more reliable the estimates should be. An internal data set may give a better reflection of the risk in the private infrastructure debt investments. The best estimations of the β coefficients can however only be determined by backtesting of the model.

The general infrastructure debt index was approximated with the mean spread of the revenue category regulated, which result in a coefficient with value 1 for the revenue factor regulated. When estimating the β's, the spread level of the index should be used as shown in (5.31). It is however important that the definitions of the spread for the index and the categories are the same.

The β_i's were estimated using yearly credit spreads. Although they were computed using yearly credit spreads, the coefficients can be applied to the volatility for any time period. This is because the spreads are set as a ratio and therefore any time adjustment will cancel out. It is however important that the two spreads used in the coefficient estimations reflect the spread over the same length of the time periods. If not, the spreads need to adjusted be adjusted in order to be able to compute the coefficients.

5.8 Backtesting

The two modelling approaches were constructed with the intention of incorporating revenue risk, a risk driver that was identified from earlier studies. Although there are variations in credit spread between infrastructure debt investment with certain revenue models, it is not sure whether one and/or both of the proposed modelling approaches actually manages to capture these differences. Therefore, extensive backtesting of the model using the two modelling approaches need to be performed.

Backtesting could be used to assess the level of risk and return of a trading strategy or to be able to determine how good a model is. Both the systematic and idiosyncratic approach need to be evaluated in order to determine if both or any of the approaches enable better modelling of the risk in private infrastructure debt investments.

Furthermore, the β coefficients have a significant impact on the results for both of the modelling approaches and therefore the data used for estimating the coefficients need to be analysed and categorised correctly into the revenue categories in order to give reliable estimates of the β coefficients. When assessing the two modelling approaches, changes in the β coefficients also need to be included and tested.

5.8.1 Systematic Risk Factor Approach

Briefly, backtesting of the systematic risk factor model could be done in the following way:

- Use an existing portfolio with fixed portfolio weights in the holdings.
- Introduce the new factors to the model and estimate covariance matrices using historical data.
- Apply a historical scenario to the portfolio where the historical scenario includes changes in the prices of the securities as of that event.
- Compare the returns and systematic returns with those that would have been estimated by the old model in order to assess whether the new factors are able to explain more of the return.
- Compare the risk estimates given by the new model and the old model. Moreover, compare the actual stand-alone risks and risk contributions for the portfolio with those estimated by the old and the new model.

The process above is very brief. In reality, the backtesting needs to be much more comprehensive and detailed in order to properly assess the models.

5.8.2 Idiosyncratic Risk Term Approach

The modelling approach with idiosyncratic risk terms is more difficult to backtest. This is because of the unsystematic nature of the introduced risk terms for revenue risk.

The risk terms for each revenue model are constant and therefore the aim should be to evaluate whether the constructed risk estimates are of a realistic magnitude. The sum of the revenue risk terms can not be greater than the estimated idiosyncratic risk for the infrastructure debt instrument.

Chapter 6

Analysis and Conclusion

In this chapter the proposed modelling approaches and the results from the data sets are analysed. Moreover, a conclusion is stated in regards to the problem formulation and the purpose of this study.

6.1 Analysis of Modelling Approaches

The modelling approach for private infrastructure debt was to be suggested for a risk factor model based on analytical VaR. Therefore a parametric factor model with a range of factors , including a infrastructure debt factor, was assumed to be available.

There are many possible ways of modelling private infrastructure debt in a factor model. In order to restrict the range of possible modelling approaches as well as the comprehensiveness of the suggested approach, it was assumed that private infrastructure debt currently is modelled as a debt instrument with exposure to a infrastructure debt factor. This was referred to as the baseline model.

The baseline model is a strong assumption but necessary in order to restrict the modelling to that of the infrastructure debt factor. Private infrastructure debt does not have to be modelled as a debt instrument in the bottom. However, this assumption can be justified as private infrastructure debt has exposure to similar factors as traditional debt instruments and additional factors for the infrastructure specific risk. Suggesting a complete modelling approach for private infrastructure debt would therefore most likely be redundant.

The concentration of the modelling to that of the infrastructure debt factor is justified by the fact that the return on a infrastructure debt instrument can be represented as a benchmark rate and a credit spread which allows for focus on the credit spread attributable to infrastructure. Factors such as interest rate risk and credit rating have been ignored as these factors are captured in the modelling of the underlying debt instrument.

As the modelling was restricted to the infrastructure debt factor, the options were to either adjust the current infrastructure debt factor and/or to add new factors to the model. Infrastructure debt investments have risk drivers that are not typical to other assets. The infrastructure debt factor in the baseline model manage to catch some of the infrastructure debt specific risk. However, there were some variations between private infrastructure debt investments that this factor or the debt specific factors could not capture. Hence, there was a need for more infrastructure specific factors.

6.1.1 Revenue Risk

From the literature review, revenue risk was identified as a risk driver that could possibly explain the variations between private infrastructure debt investments. It could be argued that the credit rating should reflect the risk in these investments. Studying the deals in the merchant category of the obtained data set, there are investments with credit rating and other attributes that are similar to those of the regulated category and contracted category but have a higher spread. This suggest that there might be reasons to look at other factors, such as the revenue model in the project.

Revenue risk can be seen as both idiosyncratic and systematic risk, depending if it is viewed at a cross-sectional basis or if it is viewed as time series. The suggested approaches were therefore to model revenue risk as either idiosyncratic or systematic risk. Nevertheless, modelling some revenue factors as systematic and some as idiosyncratic might be an alternative.

The revenue factors were based on the categorisation of revenue models into three groups, which might not be the best representable categorisation of the different revenue models for infrastructure projects and companies. An alternative categorisation would have been to group the investments with regulated and contracted revenue models into one category, characterised by a less risky revenue model and to let projects with merchant revenue risk by represented by one category, characterised by a much riskier revenue model. This categorisation could be motivated by the fact that the investments in the merchant category show higher spreads than the rest of infrastructure debt deals.

In order to obtain good estimates of the factor volatilities, time series that represent the factors well are needed. As the amount of private data is limited and seldom tracked over time, public indices need to be used to represent the returns of the revenue risk factors. Ideally there would exist a infrastructure debt index where the constituents are divided into sub-indices that reflect each revenue model. To the knowledge of the author, there are currently no indices that track infrastructure debt investments with a certain revenue risk profile. The approaches therefore relied upon adjusting existing public indices with data on private infrastructure debt investments. These adjustments were referred to as β coefficients and the estimation of these are discussed under Section 6.1.4.

If there was a public index for each revenue category, the modelling approach would have simplified to representing revenue risk as systematic risk factors where the factor returns are represented by individual indices. Such approach would probably have given more reliable estimates of the volatility of each revenue category. In order to be able to realise this, it is required that we start to view infrastructure investments from a different perspective than we do today. Furthermore, the collecting and sharing of data on infrastructure debt investments and the creation of relevant infrastructure debt indices and benchmarks needs to be stepped up.

6.1.2 Systematic Approach

In the systematic risk factor approach, three new infrastructure debt specific factors were added. The returns of the revenue factors were given by a β coefficient multiplied with the return of the infrastructure debt factor in the baseline model, i.e. the return of the general infrastructure debt index.

The main advantages with this approach include that it is easy to implement as we just need to adjust our results from the baseline model with the estimated β coefficients. As the revenue factors in the bottom are represented by a general infrastructure debt index, whenever the volatility of the index changes, so does the the volatility of the introduced revenue risk factors. The change in volatility of the revenue risk factors over time can be argued as both positive and negative features of the modelling approach.

Arguments on the positive side include that it might be more realistic to assume that the volatility of infrastructure debt investments change over time, just as the volatility for most other assets do. The negative feature is the magnitude of the change in the volatility of the revenue risk factors. Depending on the size of the β coefficients, a volatility change in the infrastructure debt index might largely overestimate or underestimate the volatility of the revenue factors. On the other hand, the infrastructure debt index is quite stable over time and therefore it is not likely that there will be surprises in terms of the magnitude of the volatility for the revenue factors.

The β coefficients become crucial as they have a great impact on the return and volatility of the systematic revenue factors. This leads to the question of what magnitudes of the β coefficients that are realistic and when they should be updated. These are questions that can not be answered in this thesis as the modelling approach was not implemented and tested.

Ideally, each revenue factor should be represented by its own index as the volatility of these may be very different to that of the general infrastructure, not only in magnitude, but also in terms of its behaviour in relation the the infrastructure debt index over time. Varying β coefficients can not be estimated from the differences in spread levels and therefore is lacking in the proposed modelling approach.

6.1.3 Idiosyncratic Approach

In the idiosyncratic approach, three idiosyncratic revenue risk terms are added in addition to the systematic infrastructure debt factor in the baseline model. In this case, a risk estimate is computed as the β coefficient multiplied with a constant estimate of the volatility of the infrastructure debt index.

The big difference to the systematic approach is that the volatility of the revenue terms are constant rather than changing over time. On the positive side, we could argue that the risk solely due to the revenue model in a infrastructure project is a constant spread that could be added to a more general infrastructure debt spread. It might be more intuitive with having a constant risk estimate and we get a larger control over the magnitude of the added risk. On the negative side is the question of when the β coefficients and the estimate of the volatility should be updated.

Furthermore, this approach might be more difficult to implement than the systematic approach as it might not be possible to add non-systematic terms in some factor models. The approach is also more suitable for risk analysis rather than estimating returns of the idiosyncratic terms.

6.1.4 Estimation of β coefficients

The β coefficients were computed according to (5.31). The general infrastructure debt index was approximated with the mean spread for the regulated category. The reason for this is that the spread of the index was defined differently to that of the data set. The β coefficients would have made more sense if they were computed as the ratio of mean spread of the revenue category and the spread for the index. This is also the approach that is suggested if the coefficients are to be estimated outside this study.

(5.31) was obtained by assuming a fixed market price of risk across similar instruments. The market price of risk does not necessarily have to be constant over time. It is time-varying due to the change of aggregated risk aversion, which in turn can be explained by economic variables. The change over time does not have an impact on the estimation of the β coefficients, unless the spread levels are measured at different time periods. The spread levels in this study were for different time periods and therefore a fixed market price of risk may not have be an appropriate assumption. However, as the the average of the spread levels were used for all revenue categories when estimating the β coefficients, it could be argued that the impact of changes in market price of risk over time could partly be neglected.

Using a mean spread does not explain variation in spread in isolation to other factors. An alternative approach would therefore have been to use a regression to estimate the coefficients. In the study by Blanc-Brude and Ismail [7], regression and analysis were performed. Excluding other factors showed larger differences between the revenue model categories in comparison to just looking at the mean spread of the categories.

6.2 Discussion on Data

The estimates of the β coefficients are dependent on the the data used to compute the coefficients. In this study, there was shown to be large differences in the spread levels between the revenue categories for the data from an earlier study and the data set used in this thesis. This was reflected in the estimated β coefficients as they differed a lot between the approaches, shown in Table 5.3 and Table 5.4. The large dependence on the data is a major drawback with the suggested modelling approaches as whenever a new data set is used, the estimates will change. Whether the β coefficients of one of the two data sets in this study are more realistic can not be concluded as the models were not tested.

From the earlier study by Blanc-Brude and Ismail [7], it was suggested that for infrastructure debt investments categorised into the three revenue models, the spread level, from lowest to highest, should be contracted, regulated, merchant. The results are very intuitive as a project with riskier revenue model should have a higher credit spread. In contrast, the mean spreads from the obtained data set contradicts the hypothesis that contracted revenue models should have the lowest mean spread out of the three categories.

An explanation to the large difference in spread levels between the two data sets might be their very different distributions in terms of the number of deals in each revenue category. The distributions of the samples from the previous study and the data sets were illustrated in Figure 5.1 and Figure 5.2. The obtained data set contained deals that were mainly subject to merchant risk and the data set used in the previous study mainly consisted of deals with a contracted revenue model. A revenue category with a few number of deals may not be representable for the average spread level and as a consequence, give biased estimates.

The obtained data set had a small concentration of projects that were classified as contracted and these deals were mainly greenfield projects. Greenfield projects are known to have a higher credit spread than brownfield projects due to construction risk. The mean spread may therefore have been biased upwards due to higher concentration of greenfield projects in this category. In order to avoid bias in the spread levels, larger data sets with more deals in each revenue category would be preferred.

Furthermore, the categorisation of the data into the revenue categories may have affected the mean spread levels of the categories. The data was categorised to the best knowledge of the author by using public information. The very same project or company can however change its revenue model over time, especially in the constantly changing regulatory environment. Therefore what was identified as the revenue model might not have been the revenue model at the time of the infrastructure debt deal.

Outside of this study, it will be important that the data set is categorised into the revenue categories such that mean spread of each category is representable for infrastructure debt investments with a specific revenue model. As mentioned earlier, a way of obtaining this is to use a large data set with a fair distribution of the deals between the revenue categories.

6.3 Analysis of Method

The idea of incorporating revenue risk into the modelling of private infrastructure debt stems from results from previous research. There were other risk drivers that could have explained the risk in the investments better or could have serves as good complements to these factors. However, the aim was to propose a modelling approach that could be implemented easily.

An alternative method would have been to perform regressions and regression analysis on the obtained data set in order to determine the most significant risk drivers. The coefficients in the regression would need to be set arbitrarily or be constructed based on possible risk drivers identified in the literature review. As Blanc-Brude and Ismail [7] performed a regression analysis and found that revenue risk was one of the main and significant revenue factors, it was assumed that these results could serve as a substitute to performing a similar study in this thesis.

The two proposed modelling approaches have not been implemented and tested and it is therefore impossible to draw any conclusions regarding the feasibility of these approaches and whether they are better than the baseline model. For the same reason, no other factors sets except for the revenue model factors and terms were suggested as there was no possibility to evaluate and compare the different models. Therefore, this is certainly a weak point of the used method.

However, the modelling approaches have been described thoroughly in order to make them easy implement in a factor model. The modelling approaches could therefore be evaluated by managers of the model, which is also more accurate as the results may vary depending on the factors in the model and the index used to represent infrastructure debt investments in general.

6.3.1 Alternative Infrastructure Factors

The literature review brought up many possible factors within the categories loan characteristics and macro-level factors, and project-level risk factors. In earlier studies, most of the factors within these categories showed to have a small impact on the credit spread of infrastructure debt and these factors were therefore assumed to be less appropriate for the suggested modelling

approaches. Furthermore, some of the risk factors, such as political risk, are difficult to translate into a single set of factors, if even possible.
In comparison, for revenue model risk, the credit spread for infrastructure debt varied between the different revenue model categories. For a infrastructure debt investment, the revenue model of the project or the company can be found out and therefore the revenue risk factors seemed like a more realistic set of factors to be incorporated to the model. Nevertheless, there was no given way for how the revenue risk models should be translated into factors.

The research also showed that for a infrastructure debt investment, the credit spread systematically changes over the life cycle. In order to properly assess the changes in credit spread over time, an approach would be to split each infrastructure debt instrument into different phases/sub instruments. This would however require continuously updated information on each infrastructure debt instrument which probably is not the case in large portfolios where we only know which instruments that we hold and some of their characteristics. Therefore the idea of credit spread changes over time seemed less relevant for a factor model.

6.4 Further Studies

An extension of this study which includes the implementation, backtesting and evaluation of the modelling approaches would be interesting as a conclusion could be drawn in regards to the performance of the models.

It would also have been interesting to study the difference in credit spread due to the type of revenue model in isolation to other variables that might affect the credit spread. This could either confirm or reject the idea of including revenue risk factors or terms in the modelling of infrastructure debt.

Further studies on how to construct a broader range of indices and subindices for infrastructure debt would be desirable. In this thesis, the optimal representation for the systematic revenue factors would have been indices for infrastructure debt with particular revenue models.

6.5 Final Conclusion

The aim of this study was to propose an alternative modelling approach for private infrastructure debt in a risk factor model. From earlier research and a studied data set, it was indicated that the risk due to the revenue model in a project or company is one of the main risk drivers in these investments.

Approaches for incorporating revenue risk as idiosyncratic and systematic risk into the current modelling of private infrastructure debt were suggested. The revenue risk is integrated by adding three revenue risk specific factors or terms; merchant, contracted and regulated. The volatility of these factors are represented by an adjusted volatility of a general infrastructure debt factor. The magnitudes of these adjustments are estimated from data on private infrastructure debt investments and the process of obtaining these estimates was illustrated using two samples in this study.

The incorporation of revenue factors should give a better representation of the risk of private infrastructure debt investments in a multi-asset portfolio. However, implementation, backtesting and evaluation of the two approaches need to be performed before deciding whether one, both or none of the approaches give a better representation of the risk in private infrastructure debt investments.

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