DEGREE PROJECT IN MATHEMATICS,
SECOND CYCLE, 30 CREDITS
*STOCKHOLM, SWEDEN 2018*

# On model risk and interconnectedness in banks

**HARALD KIHLSTRÖM**

**KTH ROYAL INSTITUTE OF TECHNOLOGY**
**SCHOOL OF ENGINEERING SCIENCES**

# On model risk and interconnectedness in banks

**HARALD KIHLSTRÖM**

Degree Projects in Financial Mathematics (30 ECTS credits)
Degree Programme in Industrial Engineering and Management
KTH Royal Institute of Technology year 2018
Supervisor at Handelsbanken: Magnus Hanson
Supervisor at KTH: Boualem Djehiche
Examiner at KTH: Boualem Djehiche

*TRITA-SCI-GRU 2018:068*
*MAT-E 2018:20*

Royal Institute of Technology
*School of Engineering Sciences*
**KTH** SCI
SE-100 44 Stockholm, Sweden
URL: www.kth.se/sci

# Abstract

Nowadays banks are more reliant on the use of models and many of these models depend on each other. In this thesis techniques from graph theory are used to depict and study the network caused by these dependencies. This is done by creating a portfolio that corresponds to a simplified version of a bank and then selecting models appropriate to evaluate the portfolio. The importance of each model in the network is then measured using the centrality measures; degree centrality, Katz centrality and Page rank. The model risk associated with specific models can differ depending on the interested party's views. These views can be reflected in the Katz and Page rank measurements by slightly modifying them. In this thesis three perspectives representing possible views of interested parties are investigated. The perspectives were focused on the amount valued by each valuation model, the sensitivity and the complexity of the models. The results indicate that the connections between the models affect the centrality measures to a greater extent than the risk introduced dependent on the different perspectives. Moreover the results indicate that the centrality measures used are more appropriate to identify potential victims of contagion rather than sources of contagion.

# Om modellers risk och inbördes samband i banker

## Sammanfattning

På senare tid har banker blivit alltmer beroende av användandet av modeller och många av dessa modeller är ofta beroende av varandra. I den här uppsatsen används tekniker från grafteori för att framställa och undersöka det nätverk som uppstår på grund sambanden. Detta görs genom att skapa en portfölj som motsvarar en förenklad version av en bank och sedan välja modeller lämpliga för att utvärdera portföljen. Vikten av varje modell undersöks med hjälp av de följande centralitetsmåtten; gradcentralitet, Katz-centralitet och Page Rank. Modellrisken associerad med specifika modeller kan skilja sig åt beroende på den intresserade partens åsikter. Genom viss modifiering av Katz och Page rank måtten är det möjligt att spegla dessa åsikter. Tre perspektiv som kan motsvara olika intressenters åsikter undersöks och dessa fokuserar på hur mycket som värderas av värderingsmodellerna, känsligheten samt komplexiteten hos varje enskild modell. Resultaten tyder på att sambanden mellan de olika modellerna har större betydelse än den risk som introduceras med hjälp av de modifierade centralitetsmåtten. Resultaten tyder även på att de använda centralitetsmåtten är mer lämpliga för att identifiera modeller som blivit påverkade av andra modellers risk än de modeller som sprider risk i nätverket.

## Acknowledgements

I would like to thank my supervisor Magnus Hanson at Handelsbanken for the opportunity to write this thesis as well as many interesting discussions during its course. I would also like to thank my supervisor Boualem Djechiche at KTH and finally I would like to thank my family and friends for their support throughout my years of study.

# Contents

**1 Introduction** . . . **1**

**2 Techniques from Graph Theory** . . . **2**
2.1 Definition and terminology . . . 2
2.2 Adjacency Matrix . . . 2
2.3 Types of graphs . . . 2
2.3.1 Directed . . . 2
2.3.2 Acyclic . . . 2
2.3.3 Weighted . . . 2
2.4 Centrality . . . 3
2.4.1 Degree centrality . . . 3
2.4.2 Eigenvector centrality . . . 3
2.4.3 Katz centrality . . . 3
2.4.4 Page rank . . . 4

**3 An overview of model risk** . . . **5**
3.1 Model risk . . . 5
3.2 Interconnectedness . . . 6

**4 Methodology for creating and studying a network of models** . . . **7**
4.1 Creating the portfolio . . . 7
4.2 Deciding on models needed to asses the portfolio . . . 7
4.2.1 Inputs . . . 7
4.2.2 Scenario . . . 8
4.2.3 Bond valuation . . . 8
4.2.4 Swap valuation . . . 8
4.2.5 Option valuation . . . 9
4.2.6 Loan valuation . . . 9
4.2.7 Sensitivity measures . . . 9
4.2.8 Value-at-Risk . . . 10
4.2.9 Expected Shortfall . . . 11
4.2.10 Probability of Default . . . 11
4.2.11 Loss Given Default . . . 11
4.2.12 Exposure at Default . . . 11
4.2.13 Expected Loss . . . 12
4.3 Building the network . . . 12
4.4 Centrality measures . . . 12
4.5 Dependence on the perspective of the interested party . . . 12
4.5.1 Balance . . . 12
4.5.2 Sensitivity . . . 13
4.5.3 Complexity . . . 14

**5 Results** . . . **15**
5.1 Mapping the connections . . . 15
5.2 Centrality measures . . . 16
5.3 Different perspectives . . . 19
5.3.1 Balance . . . 19
5.3.2 Sensitivity . . . 21

5.3.3 Complexity . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 23
5.4 Comparing the results of the perspectives . . . . . . . . . . . . . . . . . . . . . 24

**6 Discussion** . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . **27**
6.1 Centrality measures . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 27
6.2 Contagion of model risk . . . . . . . . . . . . . . . . . . . . . . . . . . . 27
6.3 Conclusions . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 28
6.4 Future research . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 28

**7 References** . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . **30**

**8 Appendix** . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . **31**

# List of Figures


1 The visualised network of the chosen models. . . . . 16
2 Visualisation of the network resized using the degree centrality. . . . . 18
3 Visualisation of the network resized using centralities when $\alpha = 0.3$. . . . . 18
4 Visualisation of the network resized using centralities when $\alpha = 2.1$. . . . . 19
5 Visualisation of the network resized using centralities when all model are considered equally important. . . . . 25
6 Visualisation of the network resized using centralities dependent on the balance of the model. . . . . 25
7 Visualisation of the network resized using centralities dependent on the sensitivity to small changes in the inputs of the model. . . . . 26
8 Visualisation of the network resized using centralities dependent on the complexity of the model. . . . . 26
9 The sensitivity of the bond model given its different inputs. . . . . 31
10 The sensitivity of the swap model given its different inputs. . . . . 32
11 The sensitivity of the option model valuing a call option given its different inputs. 33
12 The sensitivity of the option model valuing a put option given its different inputs. 34
13 The sensitivity of the loan model given its different inputs. . . . . 35
14 The sensitivity of the value-at-risk given its different inputs. . . . . 36
15 The sensitivity of the expected shortfall given its different inputs. . . . . 37
16 The sensitivity of the probability of default given its different inputs. . . . . 37
17 The sensitivity of the expected loss given its different inputs. . . . . 38


# List of Tables


1 The values of the parameters used for estimation of the probability of default . . . 11
2 The connections between all the models. . . . . 15
3 Centrality measures when all models are considered equally important. . . . . 17
4 The balance of the portfolio. . . . . 19
5 The weights of the portfolio. . . . . 20
6 Centrality measures dependent on the balance of the portfolio. . . . . 20
7 Parameters for the normal distribution of the inputs. . . . . 21
8 The regression coeffiecents $\hat{\beta}_1$ for the inputs of each model. . . . . 22
9 Centrality measures dependent on the sensitvity of each model. . . . . 23
10 Centrality measures dependent on the complexity of each model. . . . . 24

# 1 Introduction

The interconnectedness between banks in a system has been a well known phenomenon for quite some time and after the financial crisis of 2008 where chain reactions between banks caused immense financial distress, the topics of interconnectedness and contagion became prevalent.

The usage of models in a modern bank is vast and models are used for everything from valuation to credit exposure. Among all these models there exist dependencies due to the fact that a model often depends on one or multiple other models, this leads to the occurrence of an intricate network. In 2011 the U.S Office of The Comptroller of the Currency (OCC) and the Board of Governors of the Federal Reserve System released a supervisory guidance letter regarding model risk management. The letter addressed the concept of risk for individual models as well as model risk caused by interconnectedness of models. This lead to the topic of interconnectedness discussed in networks of banks was raised for networks of models used in banks.

The interconnectedness of the network of models leads to a risk of contagion, if one model fails it could greatly impact the rest of the network which could lead to significant financial losses. Hence it is of interest for a bank to analyse the interconnectedness of their models in order to evaluate and manage the aggregate model risk. Since there are so many models with different levels of complexity used in a modern bank it is difficult to get an overview of all the dependencies and their magnitudes. Therefore it is difficult to determine which model has the most impact on the network and therefore has the largest potential to cause contagion.

The purpose of this thesis is to create a simplified version of a bank and evaluate the network of models used. Focus will be on the influence each model has on the network and how the influence might differ dependent on an interested party's view on risk.

The following research questions have been formulated with a network of models used in a bank in mind:

- What models are used in a simplified version of a bank and how are they connected?
- Which models are the most influential and how do they vary depending on different perspectives of an interested party?

This thesis will only study a smaller network of models which corresponds to simplified version of a bank. The data used will be simulated and will represent a fixed timespan. The reason behind this is that the focus of the study is the interactions between the models and given the scope of a master thesis the delimitation enables a more in depth study of these interactions.

# 2 Techniques from graph theory

## 2.1 Definition and terminology

A *graph* is defined by $G = (V, E)$ where $V$ and $E$ are two sets. The elements of $V$ are the graphs *vertices*, or *nodes*, and the elements of $E$ are its *edges*, or *lines*. $E$ is defined such that its elements are 2-element subsets of $V$ and an edge between the vertices $i$ and $j$ is defined as $(i, j)$. A graph is usually pictured with its vertices represented by dots and its edges represented by lines between the dots [3]. The rest of the definitions given in this section are given by Newman (2010) [9].

## 2.2 Adjacency Matrix

To represent a graph mathematically an *adjacency matrix* $\mathbf{A}$ can be constructed. The elements of the adjacency matrix are defined as follows

$$A_{ij} = \begin{cases} 1 & \text{if there is an edge between vertices } i \text{ and } j \\ 0 & \text{otherwise} \end{cases} \tag{2.1}$$

If there exists multiple edges, so called *multiedges*, between two vertices $i$ and $j$ then $A_{ij} = A_{ji}$ will be equal to the number of edges connecting the two vertices. If there exists a edge that goes from vertex $i$ to $i$, a so called *self-edge*, then $A_{ii} = 2$ since the edge has two ends which are both connected to vertex $i$.

## 2.3 Types of graphs

There are different types of graphs with different properties in this section the graphs types relevant to this thesis are presented.

### 2.3.1 Directed

In some situations the connection between two vertices does not go in both directions. In such a situation the edges are said to have a direction and the edges are often represented as lines with arrows on them. Graphs containing directed edges are referred to as *directed graphs*.

### 2.3.2 Acyclic

In a directed graph the term *cycle* refers to a closed loop of edges which have the same direction around the loop. If there does not exist a cycle in a directed graph it is called an *acyclic graph*. Worth noting is that since a self-edge meets the requirement of being a cycle, hence a graph containing a self-edge can not be acyclic.

### 2.3.3 Weighted

In some situations it is useful to represent edges as having a weight to them. This can be represented by giving the elements of the adjacency matrix values equal to the corresponding weight of the edges. These graphs are referred to as *weighted graphs*.

## 2.4 Centrality

The ability to deduce which vertices are the most influential in a graph is often desirable. The concept of which vertices are the most influential is referred to as *centrality*. In this section the measures of centrality used in this thesis will be presented.

### 2.4.1 Degree centrality

One of the simplest way to measure centrality in a graph is to look at how many edges are connected to each vertex, i.e the degree of each vertex and hence the name *degree centrality*. The downside of this measure is that it does not take into consideration the effect of vertices being connected to other influential vertices are more likely to be more influential themselves.

### 2.4.2 Eigenvector centrality

The philosophy of *Eigenvector centrality* is giving vertices a centrality score proportional to the sum of scores of its neighbours. Mathematically this is solved using eigenvector, hence the name, and for a given graph $G = (V, E)$ with adjacency matrix $\mathbf{A}$ the eigenvector centrality for the vertex $v$ is given by

$$x_v = \frac{1}{\lambda} \sum_t A_{v,t} x_t \tag{2.2}$$

where $\lambda$ is a constant and $A_{v,t}$ is defined by equation 2.1. The equation can also be written in matrix notation referred to as the eigenvector equation as follows

$$\mathbf{Ax} = \lambda \mathbf{x} \tag{2.3}$$

where $\lambda$ is the eigenvector.

Theoretically eigenvector centrality can be calculated for both undirected and directed graphs, however complications arise in the case of directed graphs. The adjacency matrix of a directed graph is asymmetric which leads to two sets of eigenvectors, a left and a right set. Most commonly the right set is used due to the fact that centrality is usually bestowed by other vertices pointing towards a vertex. Another complication is that a vertex with only outgoing edges will be given the eigenvector centrality zero, due to the formulation of equation 2.2. The case of vertices receiving the eigenvector centrality zero also occurs in the case of an acyclic graph since such a graph has no strongly connected components except one vertex.

### 2.4.3 Katz centrality

*Katz centrality* is a way to counteract the problem of vertices receiving zero in eigenvector centrality discussed in section 2.4.2. This is implemented by giving all vertices in a graph a small amount of centrality $\beta > 0$, which yields a slightly modified version of equation 2.2. Using the same notation as in equation 2.2 the modified version is given by

$$x_v = \alpha \sum_t A_{v,t} x_t + \beta \tag{2.4}$$

where $\alpha$ is a constant. In matrix notation the equation is defined as

$$\mathbf{x} = \beta \left(\mathbf{I} - \alpha \mathbf{A}\right)^{-1} \mathbf{1} \tag{2.5}$$

The parameter $\alpha$ governs the balance between the eigenvector term and a value must be chosen for the parameter. If $\alpha = 0$ then only the constant term is left which will lead to all vertices having the same centrality $\beta$. As $\alpha$ increases from zero the centralities $x_v$ will grow until a point at which they will diverge. This point occurs when $(\mathbf{I} - \alpha\mathbf{A})$ diverges and can be written as

$$\det\left(\mathbf{I} - \alpha^{-\mathbf{1}}\mathbf{A}\right) = 0 \tag{2.6}$$

which is the characteristic equation whose roots are equal to the eigenvalues of the adjacency matrix. With an increasing $\alpha$ the first time equation 2.6 occurs is when $\alpha = \lambda_{\text{max}}^{-1}$ where $\lambda_{\text{max}}$ is the largest eigenvalue of the adjacency matrix. From this reasoning it is clear that $0 < \alpha < \lambda_{\text{max}}^{-1}$. Beyond these restrictions $\alpha$ can be chosen freely however it is common to chose an $\alpha$ that is close to $\lambda_{\text{max}}^{-1}$ in order to place the maximum amount of weight on the eigenvector term.

### 2.4.4 Page rank

One feature of the Katz centrality that sometimes is undesirable is that a vertex with high Katz centrality that points to many other vertices will give all these vertices a high centrality. It is possible to remedy this by using a slightly modified version of Katz centrality where the centrality of each vertex is divided by their out-degree. This will reduce the centrality transferred from a vertex with high centrality and many adjacent vertices. This modified version of Katz centrality is called *Page rank* and using the same notation as in section 2.4.3 it is defined as

$$x_v = \alpha \sum_t A_{v,t} \frac{x_t}{k_t^{\text{out}}} + \beta \tag{2.7}$$

where $k_t^{\text{out}}$ is the out-degree of vertex $t$. If a vertex $t$ has no outgoing edges then $A_{v,t} = 0$ and more importantly $k_t^{\text{out}}$ which leads to division with 0. This is remedied by setting $k_j^{\text{out}} = 1$, or any other non-zero value. Since $A_{v,t} = 0$ the vertex $t$ will still not contribute with centrality to any other vertices, which is expected since it has no outgoing edges. In matrix notation the equation is defined as

$$\mathbf{x} = \alpha\mathbf{A}\mathbf{D}^{-1}\mathbf{x} + \beta\mathbf{1} \tag{2.8}$$

where $\mathbf{D}$ is the diagonal matrix with elements $D_{i,i} = \max\left(k_i^{out}, 1\right)$. To solve for $\mathbf{x}$ the equation is rearranged as follows

$$\mathbf{x} = \beta\left(\mathbf{I} - \alpha\mathbf{A}\mathbf{D}^{-1}\right)^{-1} \tag{2.9}$$

since $\beta$ is simply a overall multiplier for the centrality it can be set to 1 giving

$$\mathbf{x} = \mathbf{D}\left(\mathbf{D} - \alpha\mathbf{D}\right)^{-1}\mathbf{1} \tag{2.10}$$

# 3 An overview of model risk

## 3.1 Model risk

The risk coherent with models is often refered to as *model risk*. What model risk and its consequences entails is not always clear however a often used definition is the one given by the OCC and the Federal Reserve quoted below [12]:

> *"The use of models invariably presents model risk, which is the potential for adverse consequences from decisions based on incorrect or misused model outputs and reports. Model risk can lead to financial loss, poor business and strategic decision making, or damage to a bank's reputation."*

The guidelines given by the OCC and the Federal Reserve also the state that the primary reasons behind the occurrence of model risk is that the model has fundamental errors or the model may be used incorrectly/inappropriately. Generally quantitative models are based upon application of theory, choice of sample design, section of inputs, estimation and implementation. During any of these steps errors can occur which would be considered a fundamental error in the final model. Even if a model is fundamentally correct it will not give reliable outputs if it is not used properly or if the quality of the underlying data is comprised [12].

The causes behind model risk mentioned above are generally speaking difficult to quantify, however there are studies that breach the subject. The article *Model Risk - Daring To Open Up the Black Box* by Aggarwal et al. (2015) introduces the concept of model risk as well as different ways to analyse it [1]. In the article real life cases are presented and both qualitative and quantitative measures are discussed. Aggarwal et al. also discusses model risk associated with specific types of models. The article also found that model risk can be caused by both human and programming errors, which is in line with the OCC and the Federal Reserves views. The article specifies that different methods for quantifying the model risk is appropriate depending on which business line is concerned. In other words depending on which party is interested in the model risk, different measures are appropriate. The article also states that the correlation between the error in the model and the loss of a firm is not always clear, e.g a large error might not lead to a large loss. This depends on how the model risk is mitigate as well as how important the model is for the firm.

In the article *The quantification and aggregation of model risk: perspectives on potential approaches* by Jacobs (2015) the quantification process of model risk is discussed [6]. The article states that the models described in the OCC and Federal Reserves supervisory guidance letter [12] can be described by three components; inputs, processing apparatus and reporting component. Each of these components can cause model risk. According to the article the first step of quantifying model risk is to identify the model risk sources in the following categories;

- Data errors, missing values, insufficient sample.
- Estimation uncertainty or model error, computational complexity, invalid assumptions
- Incorrect use of model or execution error.

Some of the methods that can be used to quantify the model risk in these sources are measuring the output sensitivity to potential fluctuations in the inputs, statistical estimations and benchmarks [6].

## 3.2 Interconnectedness

In a modern bank the usage of models is vast and the models more often than not depend on each other. This phenomenon can be referred to as interconnectedness. The importance of this phenomenon for the aggregate model risk is mentioned by the OCC and the Federal Reserve and can be clearly shown with the following quote [12]:

> *"Aggregate model risk is affected by interaction and dependencies among models; reliance on common assumptions, data, or methodologies; and any other factors that could adversely affect several models and their outputs at the same time."*

The subject of interconnectedness between the models used in a bank is relatively new and therefore not a lot of research have been done. In the article *This tangled web: banks seek to contain systemic model risk* from risk.net the topic is discussed [13]. In the article different professionals from the banking industry are interviewed for their opinion about the subject. The interviewees present the importance as well as the difficulty of mapping and understanding the model risk caused by the interconnectedness of models. The article mentions that the company Wells Fargo has used graph theory and more specifically Katz centrality to map and evaluate their network of models.

As mentioned earlier the subject of interconnectedness between models is lacking in research, however the interconnectedness between banks is a relatively well studied area. Since these areas are based on the same phenomenon, methods used to study one of them might be applicable to the other. In articles by Gai & Kapdia (2010), Bianchi et al. (2015) and Kanno (2015) directed graphs were used to map the interconnectedness between banks [4][2][7]. Further Kanno (2015) uses centrality measures to investigate the implied systemic risk between banks in the Japanese banking market [7].

# 4 Methodology for creating and studying a network of models

In this section the process of creating and studying the network of models used in a simplified version of a bank will be discussed. Below all the steps of the process are listed in order.

1. Creating the portfolio corresponding to a simplified version of a bank.
2. Deciding on models that can appropriately evaluate the portfolio.
3. Creating the network given by the connections between the models.
4. Deciding on centrality measures that can be used to examine the influence each model has on the network.
5. Modifying the centrality measures in order to incorporate an interested party's view on model risk.

## 4.1 Creating the portfolio

In order to find a suitable network of models that can represent a simplified version of a bank a portfolio of instruments is created. The idea is that the weights of the portfolio should somewhat depict how much of each instrument a real bank holds. The idea behind having a portfolio with reasonable weights is to enable a more adequate assessment of the magnitude the models have on the bank. For example if the majority of the banks holding is in bonds the errors in the valuation model for bonds is going to have greater impact on the banks balance sheet than errors in models pertaining instruments which are a small part of the total holdings.

Real banks use somewhat different strategies for making profits. The strategy that is to be reflected in the simplified bank is a bank with a business model focused on net interest income. In other words the bank is focused on lending and borrowing, hence the chosen instruments and their balance should reflect this. Therefore basic instruments mainly focused on interest rate are chosen, in this case; loans, bonds, swaps and options. Options are included in order to increase the variety and to include a more exotic instrument.

## 4.2 Deciding on models needed to asses the portfolio

When choosing models the simplicity of the model is prioritized over the accuracy. The reasoning behind this is to avoid too complex models which generally are more difficult to analyse due to their intricacy. Moreover the data set that will be used will be simulated for a set timespan. Since the interaction between the models is the main focus of the thesis, not the models themselves, this reasoning is motivated.

In order to evaluate the portfolio certain types of models are needed. The models needed to asses the portfolio can be categorized in to the following areas; *economic situation*, *valuation*, *sensitivity*, *portfolio risk* and *counter-party risk*. In this section all the models decided upon are discussed.

### 4.2.1 Inputs

The models referred to as inputs are the models that depict the macroeconomic and market parameters that constitutes the current economic situation. These inputs are modelled as normally distributed stochastic variables with distribution $N(\mu_i, \sigma_i)$, where $\mu_i$ is the expected value and $\sigma_i$ the standard deviation for input $i$. These will be used to simulate the current economic situation.

### 4.2.2 Scenario

The scenario model is used to create different scenarios of the economic situation and the model used will be a copula. A copula is a way of modelling the dependencies between stochastic variables. Since the inputs are macroeconomic and market parameters they will in some way correlate and moreover they are modelled as normally distributed stochastic variables, hence a copula seems appropriate.

To model the dependencies a *Gaussian copula* is used. For a $d$-dimensional standard normal distribution with linear correlation matrix $\mathbf{R}$ the Gaussian copula $C_R^{Ga}$ is the distribution function of the random vector $(\Phi(X_1), ..., \Phi(X_d))$, where $\Phi$ is the univariate standard normal distribution function and $X$ is $N_d(0, \mathbf{R})$. Then the copula is defined as [5]

$$C_{\mathbf{R}}^{Ga} = \Phi_{\mathbf{R}}^d \left(\Phi^{-1}(u_1), ..., \Phi^{-1}(u_d)\right) \tag{4.1}$$

where $\Phi_{\mathbf{R}}^d$ is the distribution of function of $X$. This is applied using the function *normalCopula* from the *R* package *copula*.

### 4.2.3 Bond valuation

The bond on the asset side is considered to be a bond traded in a foreign currency and the bond on the liability side is traded in domestic currency. To value a bond in general a discounting of cash flows is used. For a bond with face value $FV$, fixed annual coupon payment $c$ and time to maturity $T$ the value $V_{fixed}$ is obtained by

$$V_{fixed} = \sum_{t=1}^{T} \frac{c}{(1+r_t)^t} + \frac{FV}{(1+r_T)^T} \tag{4.2}$$

where $r_t$ is the discount rate for year $t$. This model was applied in *R* using a flat discount rate curve. For the bond traded in foreign currency the money invested will be exchanged to foreign currency then the valuation of the bond will be made and finally the value will be exchanged to domestic currency again.

If a bond has floating payments, i.e the coupon payments $c_t$ depend on time $t$, the valuation is done by the same model as for a bond with fixed payments except the coupon $c_t$ is different for each time period $t$. This yields the following equation

$$V_{floating} = \sum_{t=1}^{T} \frac{c_t}{(1+r_t)^t} + \frac{FV}{(1+r_T)^T} \tag{4.3}$$

where floating amount $c_t$ is given by a forward curve of an Interbank Offer Rate (IBOR).

### 4.2.4 Swap valuation

The swap considered is a simple fixed for floating, i.e one party pays a fixed amount for a floating amount or vice versa. This can be valued by valuing a fixed and a floating bond with the same face value then calculating the difference in value, i.e

$$V_{swap} = V_{fixed} - V_{floating} \tag{4.4}$$

or

$$V_{swap} = V_{floating} - V_{fixed} \tag{4.5}$$

depending on which side of the contract the party is.

### 4.2.5 Option valuation

The options on the asset side are considered to be calls and the options on the liabilities side are considered to be puts. The most common method to value both call and put options is the *Black-Scholes* formula, or a variation of it. In this thesis the standard formula will be used. The price of an European call option $C(S_t, t)$ and an European put option $P(S_t, t)$ with underlying stock with spot price $S_t$ at time $t$ is [8]

$$C(S_t, t) = N(d_1)S_t - N(d_2)Ke^{-r(T-t)} \tag{4.6}$$

$$P(S_t, t) = N(-d_2)Ke^{-r(T-t)} - N(-d_1)S_t \tag{4.7}$$

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left(ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right) \tag{4.8}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{4.9}$$

where $T$ is the maturity, $N(\cdot)$ is the cumulative distribution function of the standard normal distribution, $K$ is the strike price, $r$ is the risk-free rate and $\sigma$ is the volatility of the underlying stock.

### 4.2.6 Loan valuation

Depending on if the bank is creditor or debtor of loan a different models was applied. If the bank is the debtor the loan is value as a bond using the model described in section 4.2.3 where the loan amount is the face value, the interest payments are the coupon payments and the cost of debt is the discount rate.

When the bank is the creditor a different model is used in order to include counter-party risk. The majority of these loans are considered to be mortgage loans. The value of the loan $V_t$ at time $t$ is defined as

$$V_t = \frac{PD(1 - LGD) + (1 - PD)(c + r + \mathbf{E}[V_{t-1}])}{1 + r} \tag{4.10}$$

where $PD$ is the *probability of default* (see 4.2.10), $LGD$ is the *loss given default* (see 4.2.11), $c$ is the amortization rate, $r_t$ is the interest rate and $\mathbf{E}[V_{t-1}]$ is the expected value of the loan at time $t-1$. $PD$, $LGD$, $c$ and $r$ are assumed to be the same for all time periods. Since the loan at time $t = 0$ is worth the remaining loan amount after the yearly amortization the formula can be solved recursively.

### 4.2.7 Sensitivity measures

In finance different measurements representing the sensitivity of the price of a instrument to a change in a underlying parameter is often referred to as *Greeks*. In this thesis only the first order measurements delta $\Delta$ and vega $\nu$ will be used.

*Delta*, $\Delta$, is the sensitivity of the price with regards to a change in the price of the underlying. In this case the underlying will considered to be the interest rate. The definition of delta $\Delta_i$ for instrument $i$ used in this thesis is given by

$$\Delta_i = \frac{V_i(r + dr) - V_i(r)}{dr} \tag{4.11}$$

where $V_i(r)$ is the value of asset $i$ given the interest-rate $r$.

*Vega*, $\nu$, is the sensitivity of the price with regards to a change in the volatility $\sigma$ and is defined in the same way as $\Delta$ except with regards to the volatility $\sigma$ instead of the interest-rate $r$ i.e:

$$\nu_i = \frac{V_i(\sigma + d\sigma) - V_i(\sigma)}{d\sigma} \tag{4.12}$$

where $V_i(\sigma)$ is the value of asset $i$ given the volatility $\sigma$.

### 4.2.8 Value-at-Risk

There are different models to assess the risk of a portfolio, where one commonly used model is *value-at-risk* also known as VaR. In this thesis value-at-risk and the closely related model *Expected Shortfall* will be used.

The idea behind value-at-risk is giving an indication of how much a portfolio might lose during a specified time period with a given confidence level $\alpha$. Mathematically the value-at-risk for a portfolio with value $X$ at time 1 at level $\alpha \in (0, 1)$ is specified as [5]

$$VaR_\alpha(X) = min\{m : P(mR_0 + X < 0) \leq \alpha\} \tag{4.13}$$

where $R_0$ is the percentage return of the risk-free asset.

In practice value-at-risk can either be estimated through historical data or using parametric values. In this thesis a parametric value-at-risk will be used which for portfolio $X$ composed of assets $V_i$ with holding period $t = 1$ year is formulated as [11]

$$VaR_\alpha(X) = -\sum_{i=1}^{N} V_i \mu_i t + z\sqrt{t}\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} \rho_{ij} V_i V_j \sigma_i \sigma_j} \tag{4.14}$$

$$\approx z_\alpha \sqrt{t}\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} \rho_{ij} V_i V_j \sigma_i \sigma_j} \tag{4.15}$$

where $\mu_i$ is the growth rate for asset $i$, $z_\alpha$ is the $(1 - \alpha)$ quantile of $N(0, 1)$ and $\rho_{ij}$ is the correlation between asset $i$ and $j$. Note that $V_i$ is positive if $i$ is a long position and negative if $i$ is a short position.

In the case of instruments mostly dependent on interest-rate (i.e loans, bonds, swaps) the standard deviation $\sigma_i$ of instrument $i$ is estimated by using the Greek $\Delta_i$ and the standard deviation of the underlying interest rate $\sigma_r$. Using the same notation as in section 4.2.7 the standard deviation $\sigma_i$ of instrument $i$ is then defined as

$$\sigma_i = \frac{|\Delta_i|\sigma_r}{V_i(r)} \tag{4.16}$$

For the instruments not mainly dependent on interest-rate, in this case options, vega $\nu$ will be used instead of $\Delta$.

Since $\Delta$ reflects the sensitivity of the price to the interest-rate and most of the chosen instruments are mainly dependent on the interest-rate $\Delta$ can be used to model the correlation between the instruments. The correlation $\rho_{ij}$ between instrument $i$ and $j$ will be modelled as

$$\rho_{ij} = \frac{\Delta_i V_j}{\Delta_j V_i} \tag{4.17}$$

#### 4.2.9 Expected Shortfall

The weakness of value-at-risk is that it ignores the left tail beyond level $\alpha$ of the distribution of the portfolio $X$. A way to remedy this is to look at the average value-at-risk values below level $\alpha$. This risk measure is called *expected shortfall* (ES) and using the same notations as in 4.2.8 it is defined as follows [5]

$$ES_\alpha(X) = \frac{1}{\alpha}\int_0^\alpha VaR_u(X)du \tag{4.18}$$

#### 4.2.10 Probability of Default

The probability of default is simply the probability that the creditor can not pay the loan and hence the loan will default. In order to calculate the probability of default a model based upon a previously made regression will be used. This might not be exactly accurate for this network however it enables some reflection of the dependencies between probability of default and macroeconomic factors. The model is stated as

$$PD = N(\alpha + \beta_1 \text{GDPgrowth} + \beta_2 \text{unemployment}) \tag{4.19}$$

where $N(\cdot)$ is the cumulative standard normal distribution and the parameters $\alpha, \beta_1\beta_2$ are based on the article previously mentioned, see Table 1 below for the values.

| **Parameter** | **Value** |
|---|---|
| $\alpha$ | $-2.5$ |
| $\beta_1$ | $-1.4$ |
| $\beta_2$ | $0.08$ |

Table 1: The values of the parameters used for estimation of the probability of default

#### 4.2.11 Loss Given Default

The loss given default reflects how much the lender loses if the creditor defaults. Since the majority of the loans that are valued are considered to be mortgages the loss given default that will be used is an average loss given default for mortgages. Specifically $LGD = 25\%$ will be used and is based upon the average loss given default on American mortgages [10].

#### 4.2.12 Exposure at Default

The exposure at default (EAD) differs depending on which instrument is considered. However for this thesis only loans are considered and the EAD will be set to the loan amount, i.e what the banks is owed.

#### 4.2.13 Expected Loss

The expected loss (EL) is the amount a bank can expect to lose due to credit events and is defined as

$$EL = PD \cdot LGD \cdot EAD \tag{4.20}$$

## 4.3 Building the network

By mapping all the inputs and outputs of the chosen models it is possible to create a network using techniques from graph theory. However in order to do this it is needed to assert which other models the inputs and outputs of each model are connected to. Since the flow of most of the connections are limited to one direction a directed graph will be used. The visualisation of the network of models is done by using the *R*-package *igraph*.

## 4.4 Centrality measures

It is of interest to be able to identify which models are the most influential in the network. A way to find the most central vertices in a network is comparing all the vertices using centrality measures. Depending on the type of network different centrality measures are appropriate. The following three measures will be applied; degree centrality, Katz centrality and Page rank.

## 4.5 Dependence on the perspective of the interested party

Evaluating the importance of a model in a network can give an indication of how substantial the model risk associated with a model is to the network. In other word it gives an indication of the magnitude the model risk associated with a model has on the entire bank. However it is possible from the get go that certain models are considered more important to the bank. It is also possible that different parties in the bank may view the model risk in different ways. Hence there are many different views on what is the most important factor when considering the model risk of a model in a network, however in this thesis the focus will be on following three perspectives; *balance*, *sensitivity* and *complexity*.

To be able to introduce individual importance to a model when using centrality measures the measure needs to be able to be modified. For the three centrality measures used only Katz centrality and Page rank can be modified in such a way that it can reflect different importance of each vertex. This is due to the fact that degree centrality simply is the number of edges connected to each vertex, which does not change. In order to implement the score given to each model the added centrality given by the term $\beta$ for Katz centrality and Page rank in equation 2.4 and 2.7 respectively will be individual and dependent on the importance of each model instead of being constant.

#### 4.5.1 Balance

How much balance is related to a model simply depends on how much money is invested in the instrument valued by the model. This is of interest because model risk related to models with larger balance will likely have a larger impact on the bank then model risk related to models with small balances. In order to reflect the balance related to each model each model will be given a score depending on how much of the total balance is related to each model.

This will be done by increasing the $\beta$ of the the valuation models by an amount corresponding to the balance evaluated by that model, all other models will still receive a $\beta = 1$. The new $\beta$ will be given by

$$\beta_j^w = \beta(1 + \sum_{w_i \in W} w_i) \tag{4.21}$$

where $W$ is the set of all the instruments that have balance related to model $j$ and $w_i$ is the weight of instrument $i$ given by the following

$$w_i = b_i \left( \sum_{b_i \in \{A,L\}} b_i \right)^{-1} \tag{4.22}$$

where $b_i$ is the balance for instrument $i$, $A$ the set of balances for all the assets, $L$ the set of balances for all the liabilities. The formula can be interpreted as the balance of instrument $i$ divided by the sum of all assets and liabilities. The reason that assets and liabilities are not separated is since the valuation of the balance for both are important.

### 4.5.2 Sensitivity

The sensitivity of a model that is referred to here is how sensitive a model is to its inputs. This is relevant since a model with high sensitivity will also be more affected by the model risk of its input models. In other words if a model has a high sensitivity to its inputs an error term in the input stemming from model risk will be more noticeable.

In order to investigate how sensitive each model is to changes in each input a small change will be introduced to each model used as an input. This will be done one at a time to see how change in each individual input impacts the model. The small change that is added can either be relative to the inputs value or just be a specific amount. Here a relative change will be used since the values of the models vary a lot. The logic behind this decision can be shown with the following example: if the specific amount is set to 1 it will greatly change the value of an interest rate but not a loan amount.

To evaluate the impact the small changes has a benchmark case is needed. The benchmark case that will be used is when the inputs are set at their mean. Then a number of cases for each input $j$ will be tested. The cases are defined by $\delta \cdot input_j$ where $\delta \in \{0.99^{-10}, \dots, 1, \dots, 1.01^{10}\}$. In other words $\delta$ will be increased and decreased by $1\%$ 10 times respectively. Given the cases the value of the model dependent on all inputs $j$ can be calculated. This will then be repeated for all models in the network.

The changes of model $k$ when all models are given by the benchmark case except input $j$ will be calculated by

$$change_k^i = \frac{model_k(\delta_i \cdot input_j) - model_k(input_j)}{model_k(input_j)} \tag{4.23}$$

where $i \in \{1, \dots, 21\}$ are all the cases of $\delta_i$. These changes of model $k$ will then be plotted against each corresponding $\delta_i$. The changes will also be fitted to a linear regression model given by

$$change_k = \hat{\beta}_0 + \hat{\beta}_1 \delta \tag{4.24}$$

The coefficient $\hat{\beta}_1$ will then be an indication of how sensitive model $k$ is to input $j$. This coefficient will then be used in order to given each model $k$ a specific centrality $\beta_k$ given by

$$\beta_k = \frac{1}{N} \sum_{j=1}^{N} |\hat{\beta}_1^j| \tag{4.25}$$

where $j = 1, \ldots, N$ are all the inputs needed for model $k$ and $\hat{\beta}_1^j$ their corresponding regression coefficient.

### 4.5.3 Complexity

The complexity of the model could be of interest when judging the model risk of a network. The reason behind this is that a model that is complex mostly likely is harder to fully understand and evaluate, in other words there is more room for error. Depending on the views of the interested party the complexity of a model will be judged differently therefore each party should be able to decide on the complexity individually.

The evaluation of the complexity will be done by a scoring system. The idea is that interested party will give a score from 0 to 5 on how complex they believe each model is. This will then allow for the interested party's views to be incorporated in to the centrality measures. In order to keep roughly the same magnitude of the increases to $\beta$ for all the different perspectives the complexity score $cs_i$ for model $i$ will be translated to corresponding $\beta_i$ using the following formula

$$\beta_i = 1 + \frac{cs_i}{5} \tag{4.26}$$

In other words a complexity score $\{0, 1, ...\}$ corresponds to a $\beta = \{1, 1.2, ...\}$

## 5 Results

In this section all the results given from using the methods described previously will be presented. First the connections between all the models will be mapped in order to create the network which in turn will be investigated using centrality measures. Then the different perspectives will be discussed and their corresponding network investigated using centrality measures.

### 5.1 Mapping the connections

By mapping the inputs and outputs given by each model chosen to asses the portfolio all the connections between the models can be found. In Table 2 below all the connections for each model are listed. For model $i$ the upstream models are models that model $i$ need results from in order to function and downstream refers to the models which need the results from model $i$.

| **Model** | **Abbreviation** | **Upstream** | **Downstream** |
|---|---|---|---|
| Exchange-rate | FX | - | Scen |
| Stock price | Stock | - | Scen |
| Risk-free rate | IR | - | Scen |
| Interbank offered rate | IBOR | - | Scen |
| GDP growth rate | GDP | - | Scen |
| Unemployment rate | Unemp | - | Scen |
| Volatility | Vol | - | Scen |
| Scenario | Scen | FX, Stock, IR, IBOR, GDP, Unemp, Vol | Bond, Swap, Option, Loan, VaR, Greeks, PD, LGD, EAD |
| Bond valuation | Bond | Scen | VaR, Greeks, Swap |
| Swap valuation | Swap | Scen, Bond | VaR, Greeks |
| Option valuation | Option | Scen | VaR, Greeks |
| Loan valuation | Loan | Scen, PD, LGD | VaR, Greeks |
| Sensitivity measures | Greeks | Scen, Bond, Swap, Option, Loan | VaR |
| Value-at-risk | VaR | Scen, Bond, Swap, Option, Loan, Greeks | ES |
| Expected shortfall | ES | VaR | - |
| Probability of default | PD | Scen | Loan, EL |
| Loss given default | LGD | Scen* | Loan, EL |
| Exposure at default | EAD | Loan | EL |
| Expected loss | EL | PD, LGD, EAD | - |

Table 2: The connections between all the models.

* *Even though LGD is set to a specific number it will be considered to be dependent on the scenario since that number is based on the economic situation.*

When all the connections between the models are mapped it is possible to model and visualise the network. This visualised network can be seen in Figure 1 where the inputs are coloured orange, the scenario yellow, the valuation models green and the risk models blue.

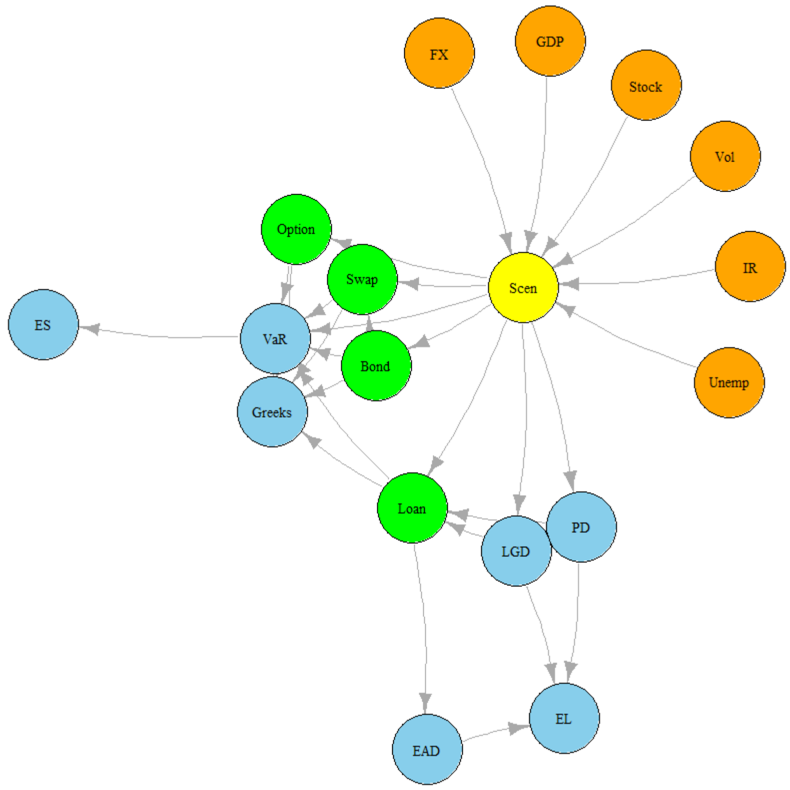



Figure 1: The visualised network of the chosen models.

### 5.2 Centrality measures

Using the modelled network it is possible to calculate centrality measures for each model in order to evaluate the influence each model has on the network. When calculating degree centrality no parameter choices are required. However when Katz centrality and Page rank is to be calculated the parameters $\alpha$ and $\beta$ needs to be chosen. The restrictions for $\alpha$ is $0 < \alpha < \lambda_{\text{max}}^{-1}$ and $\beta$ can be chosen freely. For this network the largest eigenvalue of the adjacency matrix is $\lambda_{\text{max}} \approx 0.408$ hence $0 < \alpha < 2.45$. Since there is no exact science to which $\alpha$ should be chosen two cases will be tested, one in the lower end of the interval and one in the upper end, specifically $\alpha \in \{0.3, 2.1\}$. Since $\beta$ is a certain amount given to all nodes and at this stage the centralities will be calculated without any thoughts on the importance of each model $\beta = 1$ will be used for simplicity. The centralities of each model is presented in Table 3, note that Page rank is abbreviated PR.

From Table 3 it is noticeable that all inputs have low centralities, this is expected due to the fact that they only have one edge which is downstream to the scenario model. For all three centralities the scenario, loan valuation, sensitivity measures and value-at-risk models are considered influential. Worth noting is that the scenario model is less significant when looking at the Katz centrality and Page rank, this is expected since a large portion of the edges connected to the scenario are from the inputs which as mentioned earlier have low centralities.

| | | $\alpha = 0.3$ | | $\alpha = 2.1$ | |
|---|---|---|---|---|---|
| **Model** | **Degree** | **Katz** | **PR** | **Katz** | **PR** |
| Exchange-rate | 1 | 1.00 | 1.00 | 1.00 | 1.00 |
| Stock price | 1 | 1.00 | 1.00 | 1.00 | 1.00 |
| Risk-free rate | 1 | 1.00 | 1.00 | 1.00 | 1.00 |
| Interbank offered rate | 1 | 1.00 | 1.00 | 1.00 | 1.00 |
| GDP growth rate | 1 | 1.00 | 1.00 | 1.00 | 1.00 |
| Unemployment rate | 1 | 1.00 | 1.00 | 1.00 | 1.00 |
| Volatility | 1 | 1.00 | 1.00 | 1.00 | 1.00 |
| Scenario | 13 | 2.80 | 1.26 | 13.6 | 2.80 |
| Bond valuation | 4 | 1.84 | 1.13 | 29.6 | 2.96 |
| Swap valuation | 4 | 2.39 | 1.36 | 91.6 | 7.05 |
| Option valuation | 3 | 1.84 | 1.19 | 29.6 | 3.94 |
| Loan valuation | 6 | 2.94 | 1.19 | 154 | 8.48 |
| Sensitivity measures | 5 | 3.70 | 2.51 | 640 | 48.1 |
| Value-at-risk | 7 | 5.66 | 3.64 | 2014 | 155 |
| Expected shortfall | 1 | 2.70 | 2.09 | 4230 | 326 |
| Probability of default | 3 | 1.84 | 1.19 | 29.6 | 3.94 |
| Loss given default | 3 | 1.84 | 1.19 | 29.6 | 3.94 |
| Exposure at default | 2 | 1.88 | 1.88 | 324 | 18.8 |
| Expected loss | 3 | 2.67 | 2.14 | 805 | 57.0 |

Table 3: Centrality measures when all models are considered equally important.

In Table 3 it is noticeable that the models with the largest centralities receive a significantly larger centrality when $\alpha = 2.1$. It is also noticeable that the models downstream from influential models receive higher centralities than their influential upstream neighbours. This is due to the fact that when $\alpha > 1$ the centrality received from of a neighbour is multiplied, which is especially apparent when a neighbour has a substantial centrality.

In order to get a better comparison of the centralities they will be visualised by resizing each vertex in network in accordance with its centrality. See Figure 2 for the visualisation of the degree centrality and Figure 3 and 4 for the visualisation of the Katz centrality and Page rank. The text for each model is omitted to avoid clutter and since the main goal of the visualisation is simply to get a more concrete comparison between the more and less influential vertices.

In Figure 4 the effects of the multiplication property when $\alpha = 2.1$ is clearly visible. In the future only $\alpha = 0.3$ will be used in order to avoid the multiplication property as well as to keep centralities in a smaller range which makes a comparison both in graphs and numbers more perspicuous.

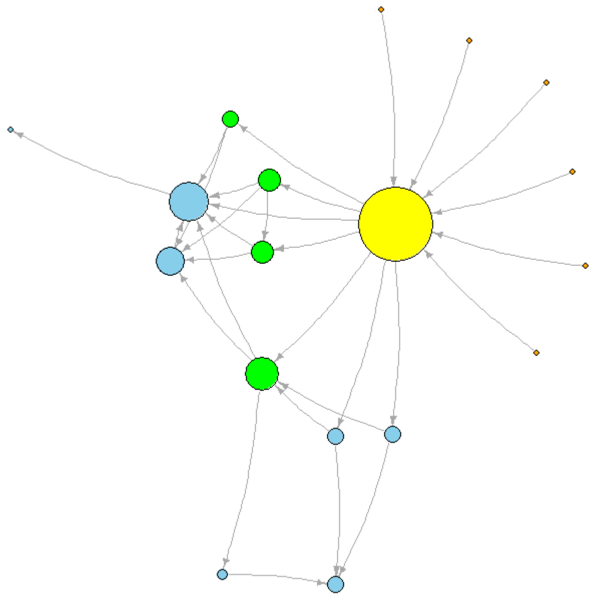
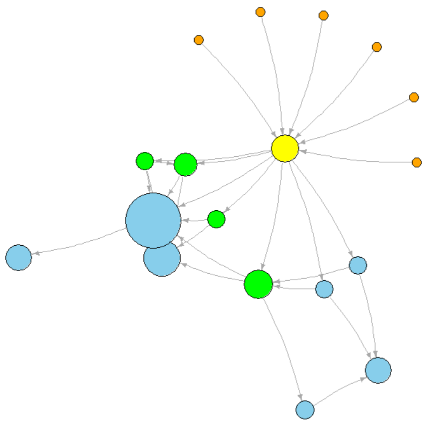
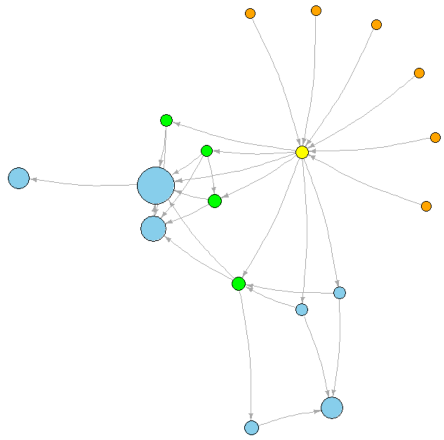


Figure 2: Visualisation of the network resized using the degree centrality.

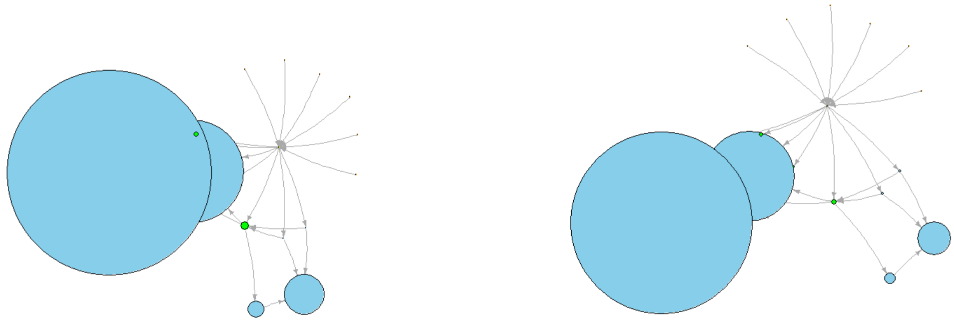


(a) Katz centrality



(b) Page rank

Figure 3: Visualisation of the network resized using centralities when $\alpha = 0.3$.



(a) Katz centrality (b) Page rank

Figure 4: Visualisation of the network resized using centralities when $\alpha = 2.1$.

### 5.3 Different perspectives

Now that the the network has been evaluated with all models considered equally important the network will be evaluated using different perspectives to evaluate the importance of the models. The perspectives that will be analysed are *balance, sensitivity* and *complexity*. The methodology used can be seen in section 4.5.

#### 5.3.1 Balance

In order to evaluate the network dependent on the balance of the portfolio a balance needs to be assumed. Since the portrayed bank is focused on profits from the interest rate spread the balance of the bank should mainly consist of interest related instruments. The balance is chosen such that the bank at the moment has more assets than liabilities, this could be considered as previous profits have been invested in different instruments. The decided balance for each instrument is displayed in Table 4 below.

| **Instrument** | **Assets** | **Liabilities** |
|---|---|---|
| Loans | 80 | 40 |
| Bonds | 5 | 30 |
| Swaps | 5 | 8 |
| Options | 10 | 2 |

Table 4: The balance of the portfolio.

As visible in Table 4 the majority of the balance is invested in instrument associated with interest rates, in line with previous statements, however the balance for options on the asset side is not unsubstantial. The reason behind this is to introduce a little more volatility to the assets which seems reasonable since the bank currently has more assets than liabilities and therefore has the possibility to take on more risk.

Given the decided balance of the portfolio in Table 4 the weights are calculated and can be seen in Table 5.

| Instrument | Weight |
|---|---|
| Loans as creditor | 0.444 |
| Loans as debtor | 0.222 |
| Bonds | 0.194 |
| Swaps | 0.0722 |
| Options | 0.0667 |

Table 5: The weights of the portfolio.

Using the weights it is possible to calculate the importance $\beta_i^w$ given to each model $i$ as well as the Katz centrality and Page rank, see Table 6 for the results.

| Model | $\beta$ | Katz | PR |
|---|---|---|---|
| Exchange-rate | 1 | 1.00 | 1.00 |
| Stock price | 1 | 1.00 | 1.00 |
| Risk-free rate | 1 | 1.00 | 1.00 |
| Interbank offered rate | 1 | 1.00 | 1.00 |
| GDP growth rate | 1 | 1.00 | 1.00 |
| Unemployment rate | 1 | 1.00 | 1.00 |
| Volatility | 1 | 1.00 | 1.00 |
| Scenario | 1 | 2.80 | 1.26 |
| Bond valuation | 1.416 | 2.26 | 1.54 |
| Swap valuation | 1.0722 | 2.58 | 1.49 |
| Option valuation | 1.0667 | 1.91 | 1.26 |
| Loan valuation | 1.444 | 3.39 | 1.81 |
| Sensitivity measures | 1 | 4.04 | 2.83 |
| Value-at-risk | 1 | 6.09 | 4.05 |
| Expected shortfall | 1 | 2.83 | 2.22 |
| Probability of default | 1 | 1.84 | 1.19 |
| Loss given default | 1 | 1.84 | 1.19 |
| Exposure at default | 1 | 2.02 | 1.54 |
| Expected loss | 1 | 2.71 | 2.18 |

Table 6: Centrality measures dependent on the balance of the portfolio.

### 5.3.2 Sensitivity

To implement the sensitivity methodology the parameters for the normal distributions of the inputs needs to be assumed. The parameters are chosen to be somewhat realistic and are displayed in Table 7.

| **Input** | $\mu$ | $\sigma$ |
|---|---|---|
| Exchange-rate | 8.4 | 0.42 |
| Stock price | 5 | 0.5 |
| Risk-free rate | 0.015 | 0.0025 |
| Interbank offered rate | 0.03 | 0.009 |
| GDP growth rate | 0.02 | 0.01 |
| Unemployment rate | 0.07 | 0.014 |
| Volatility | 0.15 | 0.03 |

Table 7: Parameters for the normal distribution of the inputs.

The sensitivity to small changes in the inputs of each model will now be evaluated using the method described in section 4.5.2. In Table 8 the regression coefficients for each model are displayed and all the graphs depicting the correlation between the changes are displayed in the appendix. Note that the inputs are set values and the scenario model is simply a way to variate the inputs and therefore they are not included.

Looking at the sensitivity plots in the appendix it is clear that all the relations are linear for the interval of the different cases. Also worth noting is that for models with coefficient 1 the change in model and change in input is linear with proportions $1 : 1$. This could be viewed as scaling proportionate to the error in the input and by looking at some of the models it is easily understandable that is the case. For example in the expected shortfall model looking at the formulation given by equation 4.18 it is clear that a percentage change in value-at-risk is simply a constant that can be moved outside the integral sign, hence the proportion.

Using the coefficients in Table 8 the centrality $\beta$ that is to be added to each model can be calculated. All the $\beta$:s and the corresponding Katz centrality and Page rank for each model is shown in Table 9.

| **Model** | **Inputs** | **Coefficient $\hat{\beta}_1$** |
|---|---|---|
| Bond valuation | IR | -0.0704 |
| | FX | 1.00 |
| Swap valuation | IR | -0.0439 |
| | IBOR | 6.00 |
| | Bond | 1.00 |
| Option valuation | IR | 0.0821 |
| | Vol | 0.913 |
| | Stock | 1.00 |
| Loan valuation | IR | 0.00159 |
| | PD | -0.0117 |
| | LGD | -0.0117 |
| Sensitivity measures | Bond | 1.00 |
| | Swap | 1.00 |
| | Option | 1.00 |
| | Loan | 1.00 |
| Value-at-risk | Bond | 0.423 |
| | Swap | 0.0148 |
| | Option | -0.0260 |
| | Loan | 0.588 |
| | Greeks | 1.00 |
| Expected shortfall | VaR | 1.00 |
| Probability of default | GDP | -0.0796 |
| | unemp | 0.0159 |
| Loss given default | - | - |
| Exposure at default | - | - |
| Expected loss | PD | 1.00 |
| | LGD | 1.00 |
| | EAD | 1.00 |

Table 8: The regression coeffiecents $\hat{\beta}_1$ for the inputs of each model.

| **Model** | $\beta$ | **Katz** | **PR** |
|---|---|---|---|
| Exchange-rate | 1 | 1.00 | 1.00 |
| Stock price | 1 | 1.00 | 1.00 |
| Risk-free rate | 1 | 1.00 | 1.00 |
| Interbank offered rate | 1 | 1.00 | 1.00 |
| GDP growth rate | 1 | 1.00 | 1.00 |
| Unemployment rate | 1 | 1.00 | 1.00 |
| Volatility | 1 | 1.00 | 1.00 |
| Scenario | 1 | 2.80 | 1.26 |
| Bond valuation | 1.54 | 2.38 | 1.66 |
| Swap valuation | 3.35 | 4.90 | 3.79 |
| Option valuation | 1.67 | 2.51 | 1.85 |
| Loan valuation | 1.01 | 2.97 | 1.38 |
| Sensitivity measures | 2 | 5.82 | 4.60 |
| Value-at-risk | 1.41 | 7.82 | 5.77 |
| Expected shortfall | 2 | 4.35 | 3.73 |
| Probability of default | 1.05 | 1.89 | 1.24 |
| Loss given default | 1 | 1.84 | 1.19 |
| Exposure at default | 1 | 1.89 | 1.41 |
| Expected loss | 2 | 3.69 | 3.15 |

Table 9: Centrality measures dependent on the sensitvity of each model.

#### 5.3.3 Complexity

Judging the complexity of each model could be considered quite difficult and can depend heavily on who the judge is. The goal here is not to try and achieve as accurate judgement of each model as possible but rather show how the concept could be used. Therefore the complexity scores will be chosen somewhat arbitrarily. If this approach is implemented in reality it is probably wise to either let each model developer or the department responsible for model development be the judge. All the complexity scores, their corresponding $\beta$ and the Katz centrality and Page rank is displayed in Table 10.

| Model | Complexity | $\beta$ | Katz | PR |
|---|---|---|---|---|
| Exchange-rate | 0 | 1 | 1.00 | 1.00 |
| Stock price | 0 | 1 | 1.00 | 1.00 |
| Risk-free rate | 0 | 1 | 1.00 | 1.00 |
| Interbank offered rate | 0 | 1 | 1.00 | 1.00 |
| GDP growth rate | 0 | 1 | 1.00 | 1.00 |
| Unemployment rate | 0 | 1 | 1.00 | 1.00 |
| Volatility | 0 | 1 | 1.00 | 1.00 |
| Scenario | 5 | 2 | 3.80 | 2.26 |
| Bond valuation | 2 | 1.4 | 2.54 | 1.63 |
| Swap valuation | 1 | 1.2 | 3.10 | 1.78 |
| Option valuation | 5 | 2 | 3.14 | 2.34 |
| Loan valuation | 3 | 1.6 | 4.26 | 2.17 |
| Sensitivity measures | 3 | 1.6 | 5.51 | 3.98 |
| Value-at-risk | 4 | 1.8 | 8.51 | 6.05 |
| Expected shortfall | 3 | 1.6 | 4.15 | 3.41 |
| Probability of default | 2 | 1.4 | 2.54 | 1.74 |
| Loss given default | 2 | 1.4 | 2.54 | 1.74 |
| Exposure at default | 0 | 1 | 2.28 | 1.65 |
| Expected loss | 1 | 1.2 | 3.41 | 2.74 |

Table 10: Centrality measures dependent on the complexity of each model.

### 5.4 Comparing the results of the perspectives

In order to compare all the results from the different perspectives the network will be visualised using the different centralities. The resized networks are displayed in Figure 5, 6, 7 and 8.

Viewing the networks for the perspective it seems that both Katz centrality and Page rank yields similar results. However when the scenario model is given a higher individual centrality the centralities for most of the network increases more noticeably when using Katz centrality instead of Page rank. The reason behind this is that the scenario model has many outgoing edges and is therefore heavily penalized in the Page rank centrality formula.

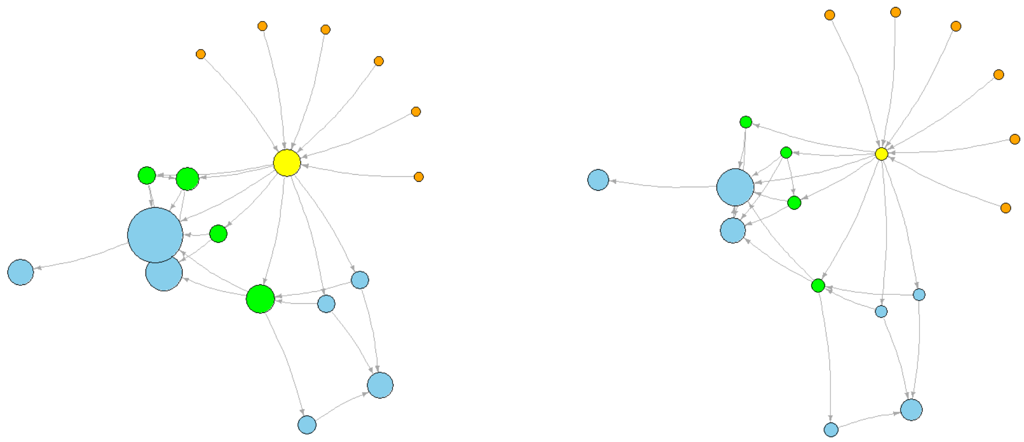
In Figure 5, 6, 7 and 8 as well as Tables 6, 9 and 10 it is clearly displayed that the centralities of the input models do not change for the different perspectives. The reason behind this is that no balance is connected to them directly and since they are chosen parameters they are not sensitive nor considered complex models. The same argument can be used for the loss given default model.

Worth noting is that all the networks for the different perspectives are relatively similar. The influence of certain models changes however the models with the highest centralities are often the same. As a group of models the valuation models centralities seems to slightly increase for the three different perspectives.

The models with a high centrality across all perspectives is the value-at-risk and the loan valuation model. One likely reason for this is that both models have many connections to other models.

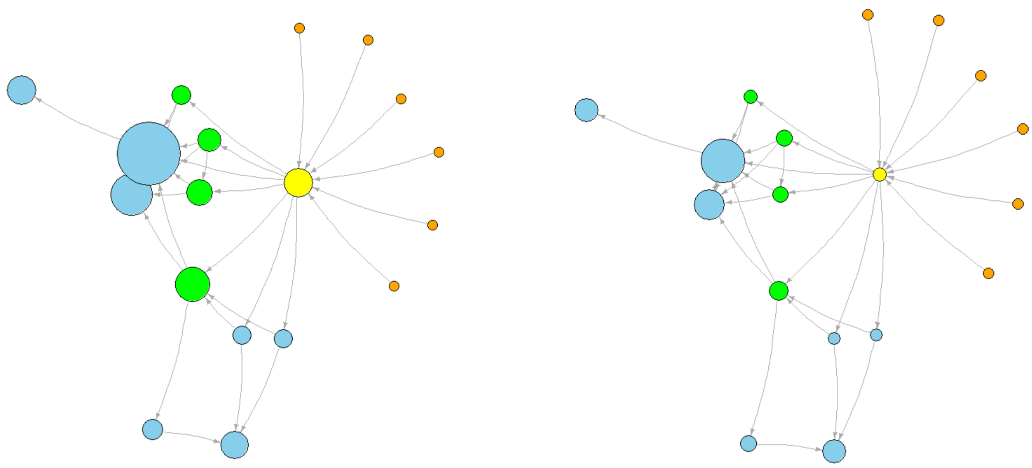
Moreover the value-at-risk model evaluates portfolio risk and therefore is connected to the entire portfolio while the loan valuation model values a large portion of the balance.

A distinct special case is the centrality of the swap valuation model for the sensitivity perspective. The value of the swap is given by the difference in value of a fixed and a floating bond therefore the swap value changes drastically if the value of only one of the bonds changes. In this case the sensitivity comes from changes in the IBOR since only the floating bond is dependent on the IBOR whilst both of the bonds are dependent on the interest rate used for discounting.



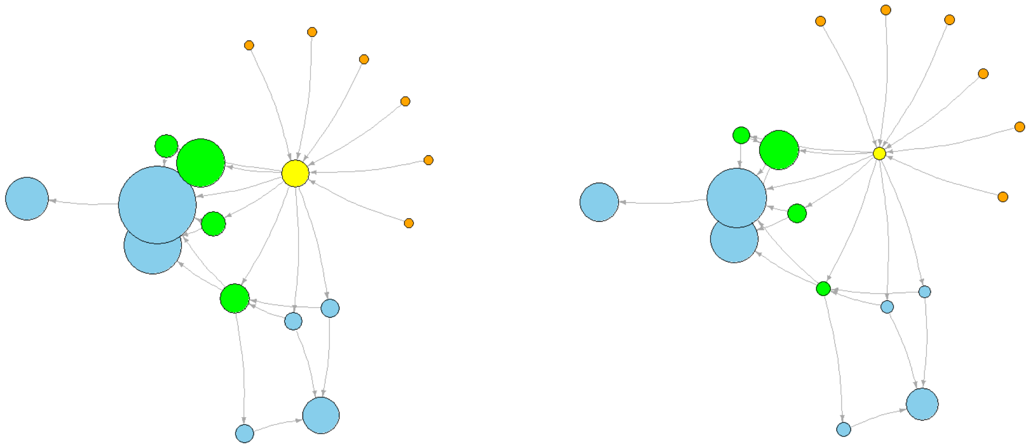
(a) Katz centrality (b) Page rank

Figure 5: Visualisation of the network resized using centralities when all model are considered equally important.



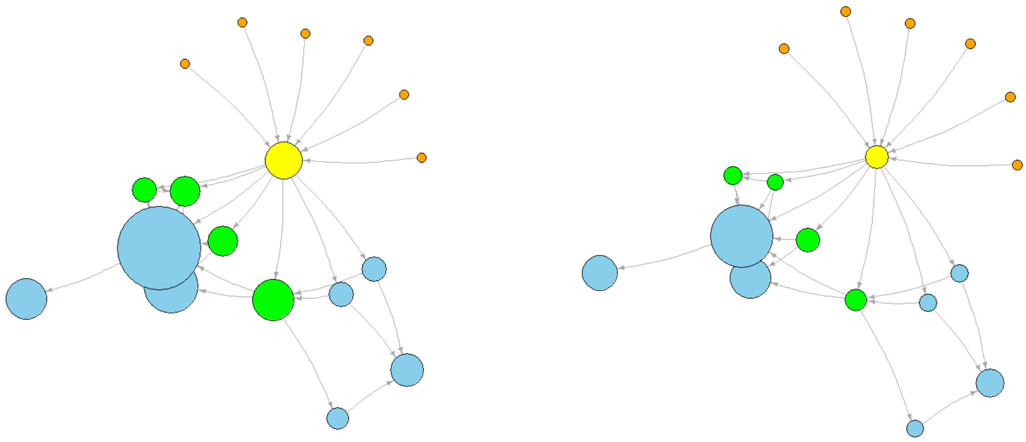
(a) Katz centrality (b) Page rank

Figure 6: Visualisation of the network resized using centralities dependent on the balance of the model.



(a) Katz centrality (b) Page rank

Figure 7: Visualisation of the network resized using centralities dependent on the sensitivity to small changes in the inputs of the model.



(a) Katz centrality (b) Page rank

Figure 8: Visualisation of the network resized using centralities dependent on the complexity of the model.

# 6 Discussion

## 6.1 Centrality measures

When comparing the visualisations of the resized networks using the models centrality measures for all the different perspectives the proportions between the vertices seemed fairly similar. Since the difference between the perspectives is the individual centrality $\beta$ given to each model this give some indication that the connections between models, i.e the eigenvector part of the centralities, are more important than the individual centrality given to each model. The balance between how much weight should be given to the eigenvector centrality over the individual centralities is given by the parameter $\alpha$. For the different perspectives $\alpha = 0.3$ was used which could be considered fairly low and in spite of that the eigenvector centrality seemed to affect the centrality measures more than the individual centrality $\beta$. This indicates that the factor that is the main predictor of a models centrality is its connections. One way to test this in the future could be to increase the magnitude of the individual $\beta$ given to each model dependent on the chosen perspective and investigate if this causes a different outcome.

Since the connections between models seems to be the most important factor in the centrality given for each model it is worth noting the importance of model choice and formulation. The importance of this is due to the fact that use different models and model formulations will result in different connections between the models. For example the value-at-risk model used in this thesis is a parametric approach dependent on the sensitivities of each instrument, if the value-at-risk was instead estimated using historical data the connections to the model would be different thus the centralities would likely be different. In other words the centralities for a specific network may significantly vary dependent on the choice of models.

Using centrality measures to evaluate a network of models will for some models give somewhat misleading results. For example the centrality measures for the expected shortfall model across all perspectives are relatively high. However the model is almost only what could be considered a reformulation of the value-at-risk model. Therefore one might argue that the model itself does not have any real model risk on its own and all of the risk is inherited from the value-at-risk model. In other words when looking at the centralities the model looks interesting and worth investigating when in reality it is the value-at-risk model that should investigated. This mostly shows that this approach of judging a network of models requires some fundamental understanding of the models contained by the network. However these results could be modified to remedy the misleading results in order to present it to a more general audience. However their is a clear downside of this since it introduces more bias to the results which in itself can causes errors.

## 6.2 Contagion of model risk

From the results it is apparent that models with many upstream connections generally receive a high centrality. This is somewhat expected given how the selected centrality measures work. When the centrality measures act this way a model with high centrality probably is more likely to be affected by an overall contagion of model risk since the model risk from each upstream model will affect the model. By knowing which models are highly affected by contagion it is possible to inform the decision makers of the increased uncertainty of the results given by those models. However it does not remedy the cause of the problem which is the contagion.

The first step to minimize the contagion in a network would be identifying the models that are the main sources of it. Likely sources of contagion are models which results are used by many other models, simply put models with many downstream connections. In order to find these models one possible solution could be to find and use centrality measures that are based on the reverse idea of the centrality measure used in this thesis, in other words models with many downstream connections receive a high centrality. Another approach to identifying the sources of contagion could be using a similar approach as the one used in the sensitivity perspective except varying the outputs from models further upstream. Doing this it would be possible to see how an error in a model affects other models further downstream.

When looking at the network used in this thesis the scenario model would most likely be given a significantly higher centrality when the number of downstream connections are valued higher. That assessment would probably hold in reality as well since the scenario model relays information to a lot of other models. In reality the input models would most likely also be considered more interesting than they have been deemed in this thesis. The reason behind this is that in reality the inputs are generally based on market data and their values play an important role for many models while in this thesis the inputs for the most part been considered numbers that are needed use all the other models and focus have not been on the accuracy of these numbers.

If it was possible to completely remove the contagion there would be no need for finding the models mostly affected by it. However in reality achieving this seems highly improbable. Therefore being able to identify the cause of contagion as well as the victims of the contagion are both important ways to enable management of model risk.

## 6.3 Conclusions

When analysing the connections between models used in a bank techniques from graph theory are proficient in creating a perspicuous overview of the corresponding network. In graph theory centrality measures are used to identify influential nodes in a network and can be applied to a network of models used in a bank. The centrality measures can be used to identifying the models mostly affected by the contagion of model risk and it might also be possible to use a similar approach to identify the sources of said contagion. Further it is possible to add an individual's view on the model risk associated with each model to the centrality measures. However the used centrality measures seem to mainly depend on the connections between the models rather than individual centrality given based on the views of said individual. Therefore being diligent in choosing models as well as mapping the connections of each model is important to receive adequate result.

## 6.4 Future research

Since this thesis was done without the use of real data a similar approach would be interesting to further investigate using real data. It would also be interesting to apply this data to the models more focused on accuracy than simplicity. Using real life data would also enable a more realistic study, especially when considering the different perspectives. It would be particularly enlightening to do a similar study using the network of models used in a modern day bank.

It would also be interesting to investigate how the dynamics of the network and centrality measures changes with time. This might enable identification of how the contagion spreads over time. If done with real data it would also present a better opportunity to study how changes in the market situation caused by time affects the network.

Another interesting possibility for future studies is finding centrality measures, or similar methods, to better enable the identification of the sources of contagion. Since the first step of avoiding contagion is identifying it such a study would be a start to minimize the aggregate model risk in a network.

# 7 References


[1] Aggarwal, A., Beck, M. B., Cann, M., Ford, T., Georgescu, D., Morjaria, N., Smith, A., Taylor, Y., Tsanakas, A., Witts, L., Ye, I., *Model Risk - Daring To Open Up the Black Box*, British Actuarial Journal, Vol. 21, part 2, pp. 229-296, 2015.

[2] Bianchi, D., Billio, M., Casarin, R., Guidolin, M., *Modelling Contagion and Systemic Risk*, SYRTO, 2015.

[3] Diestel, R. *Graph Theory*, Springer, 5th edition, 2017.

[4] Gai, P., Kapadia, Sujit., *Contagion in financial networks*, Bank of England 2010.

[5] Hult, H., Lindskog, F., Hammarlind, O., Rehn, C.J., *Risk and Portfolio Analysis: Principle and Methods*, Springer, 2012.

[6] Jacobs, M. Jr., *The quantification and aggregation of model risk: perspectives on potential approaches*, Int. J. Financial Engineering and Risk Management, Vol. 2, No. 2, 2015.

[7] Kanno, M., *The network structure and systemic risk in the Japanese interbank market* Elsevier, 2015.

[8] Lang, H., *Lectures on Financial Mathematics*, KTH Mathematics, 2012.

[9] Newman, M., *Networks: An Introduction*, Oxford University Press, 2010.

[10] Qi, M., Yang, X., *Loss Given Default of High Loan-to-Value Residential mortgages*, Office of the Comptroller of the Currency, 2007.

[11] Straja, S. R., *Value at Risk for Long/Short Positions*, Montgomery Investment Technology, 2014.

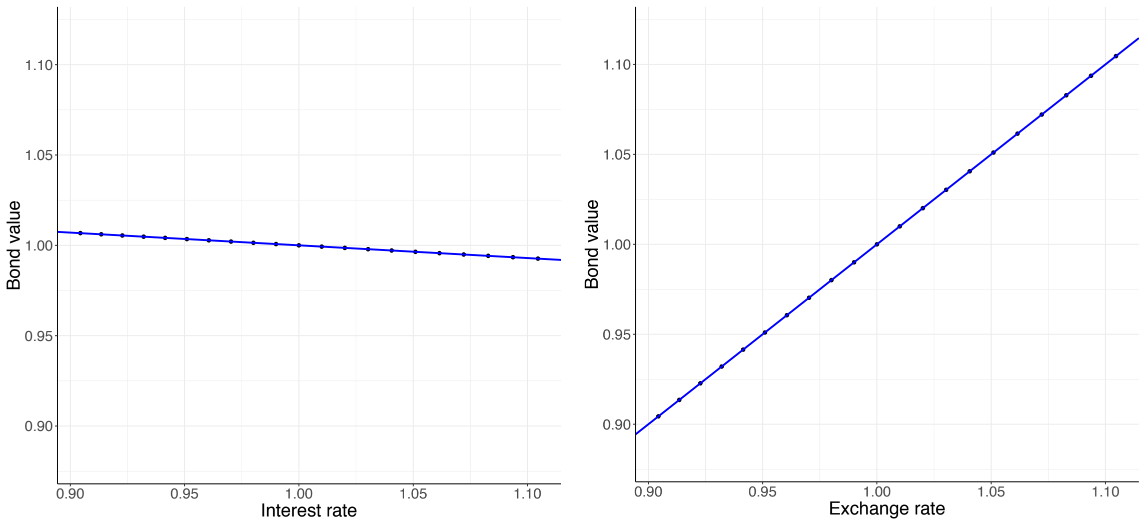
[12] The U.S Office of The Comptroller of the Currency (OCC) and the Board of Governors of the Federal Reserve System (BOG-FRB), *SR 11-7: Supervisory Guidance on Model Risk Management*, Washington, DC, 2011.

[13] Woodall, L., *This tangled web: banks seek to contain systemic model risk*, Risk.net, 2017.

[14] Zsigraiová, M., *Probability of default modelling using macroeconomic factors*, Charles University in Prague, 2014.

# 8 Appendix

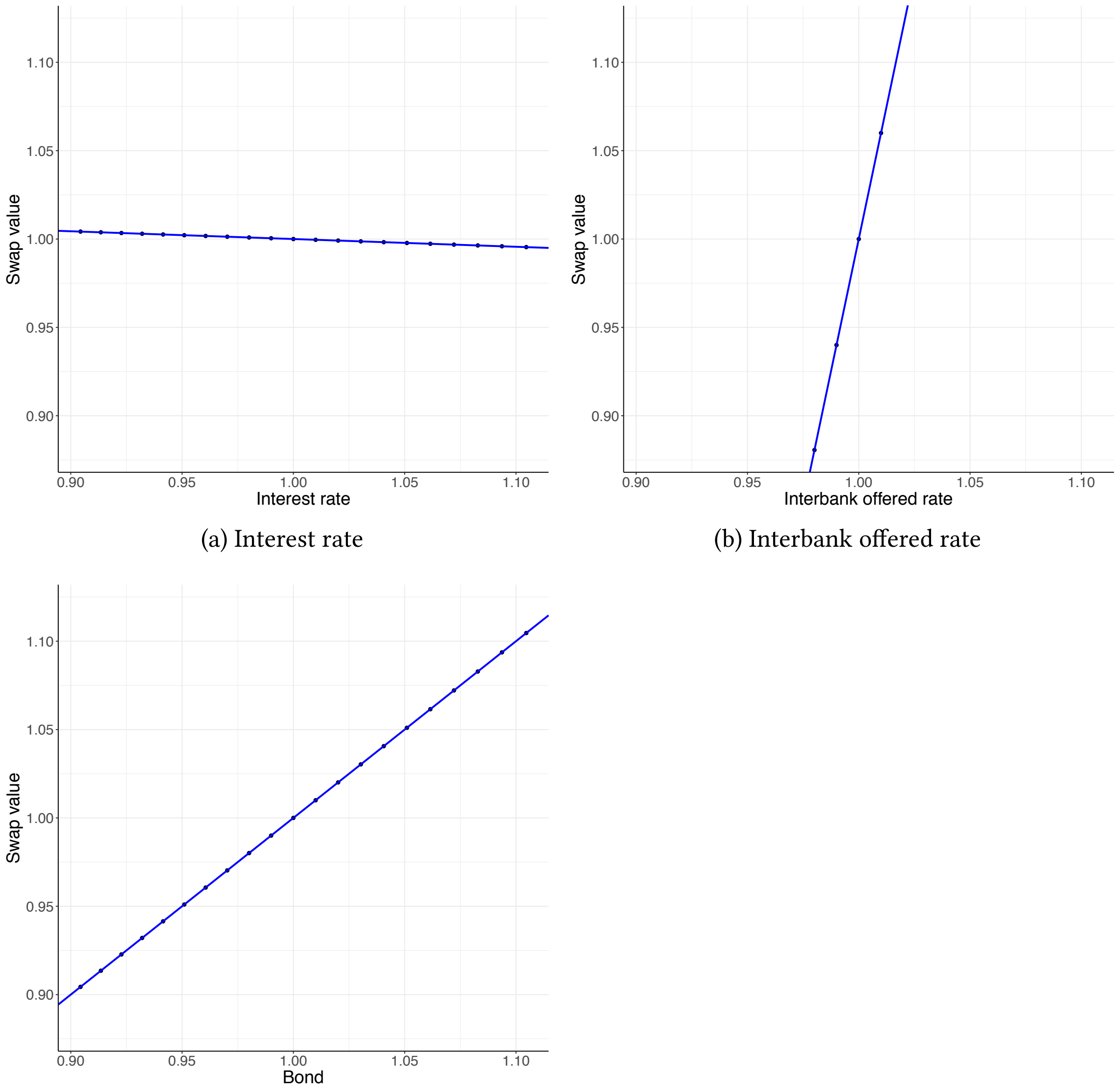
The graphs depicted here represent how small deterministic changes in each of the inputs of a model affects the output. The purpose of the graphs are to visualise the sensitivity of the models as well as the relation between input and output. See section 4.5.2 for further details.




(a) Interest rate

(b) Exchange rate

Figure 9: The sensitivity of the bond model given its different inputs.

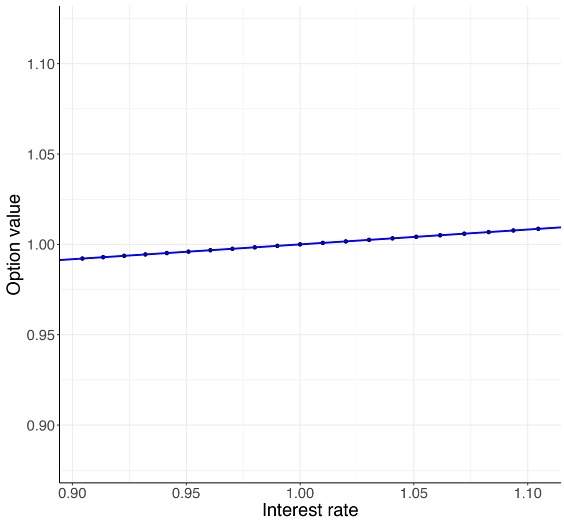



(a) Interest rate

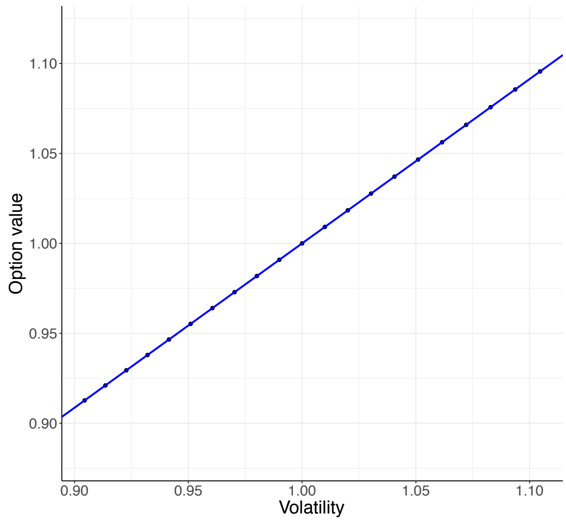
(b) Interbank offered rate

(c) Bond value

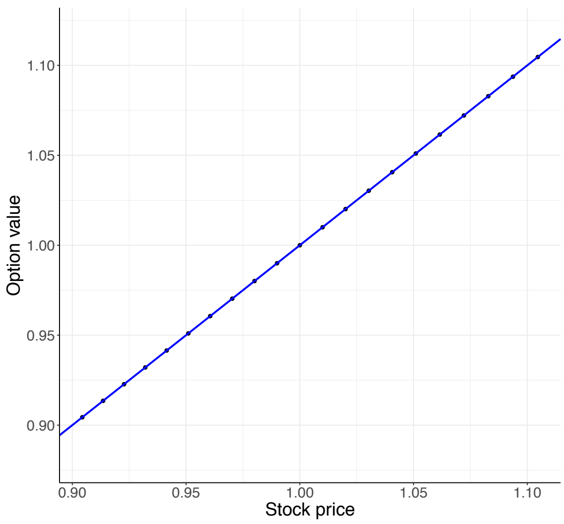
Figure 10: The sensitivity of the swap model given its different inputs.




(a) Interest rate

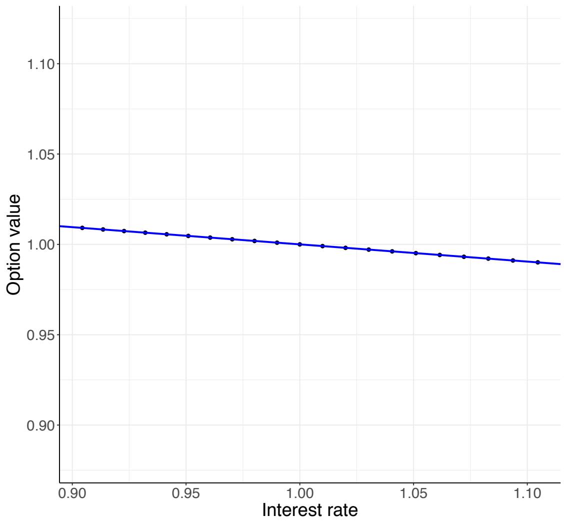



(b) Volatility

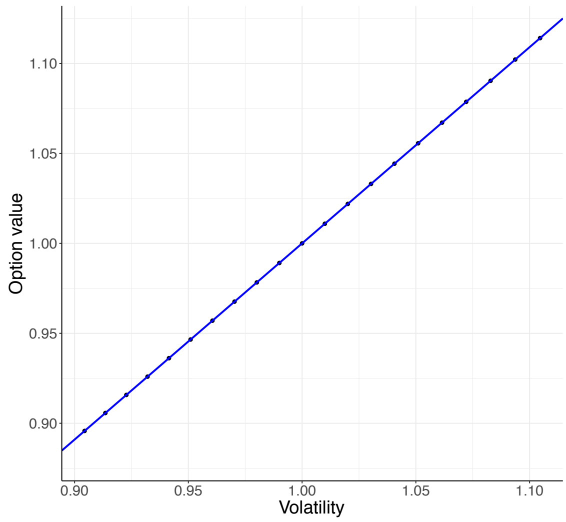



(c) Stock price

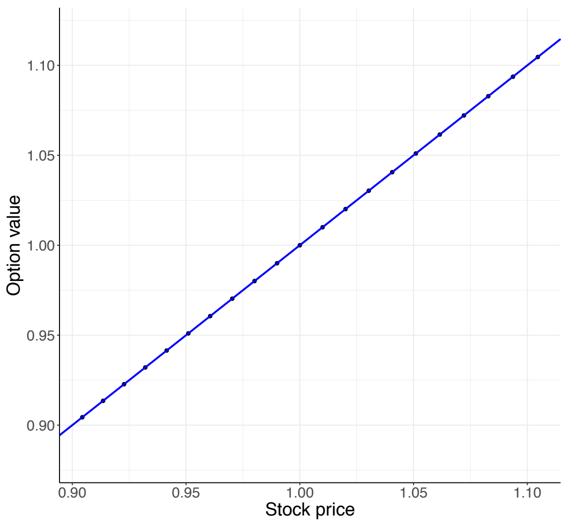
Figure 11: The sensitivity of the option model valuing a call option given its different inputs.




(a) Interest rate

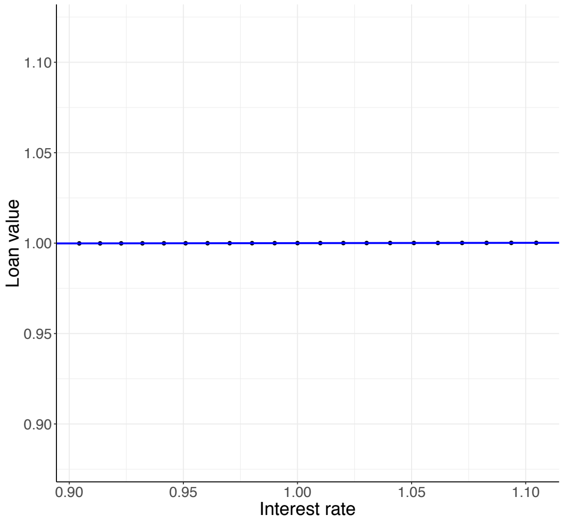



(b) Volatility

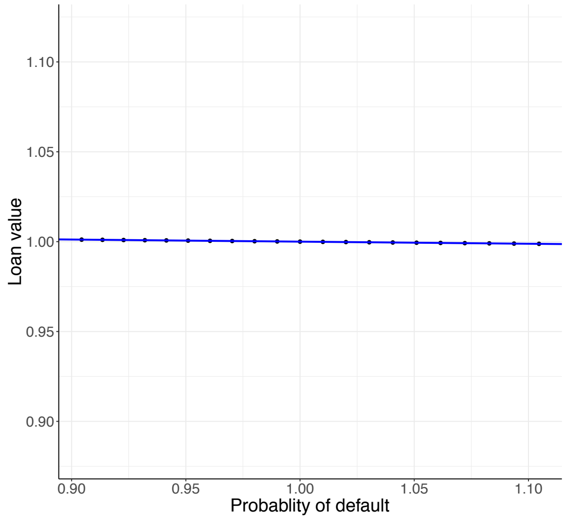



(c) Stock price

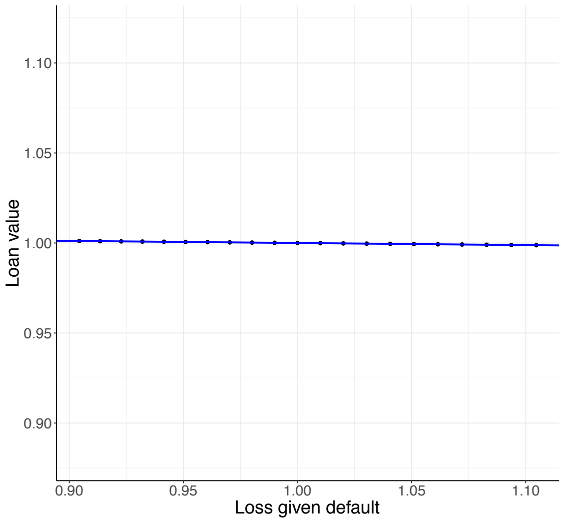
Figure 12: The sensitivity of the option model valuing a put option given its different inputs.




(a) Interest rate

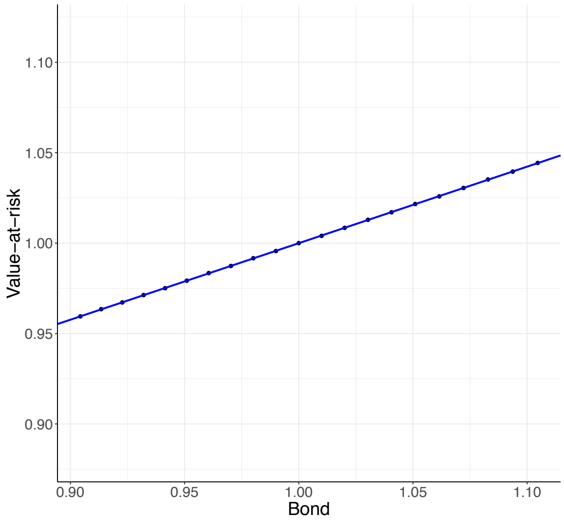



(b) Probability of default

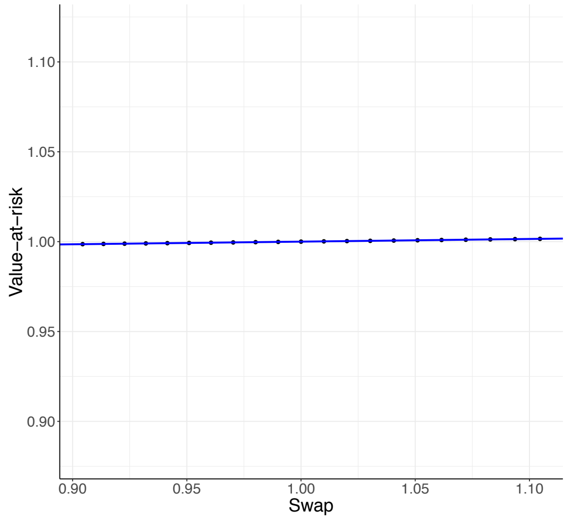



(c) Loss given default

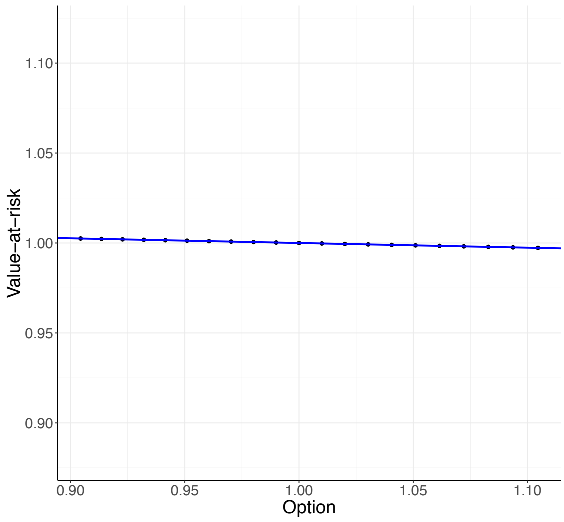
Figure 13: The sensitivity of the loan model given its different inputs.



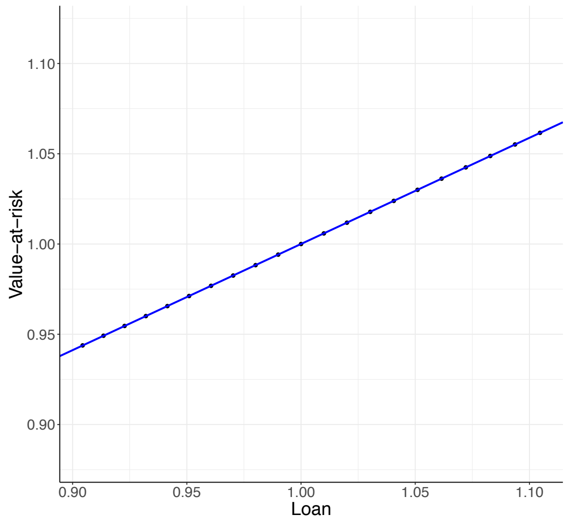

(a) Bond value



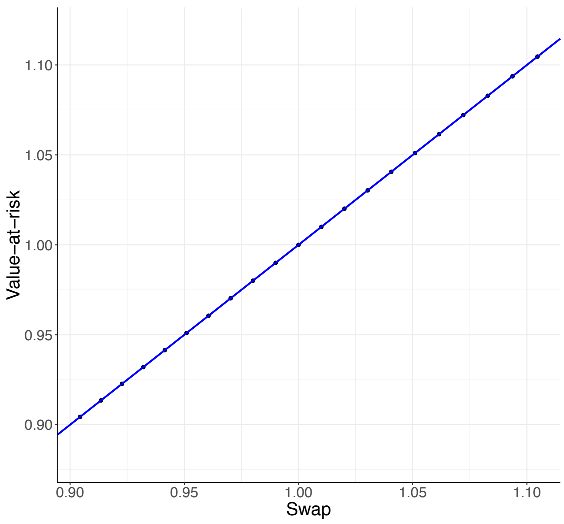

(b) Swap value




(c) Option value

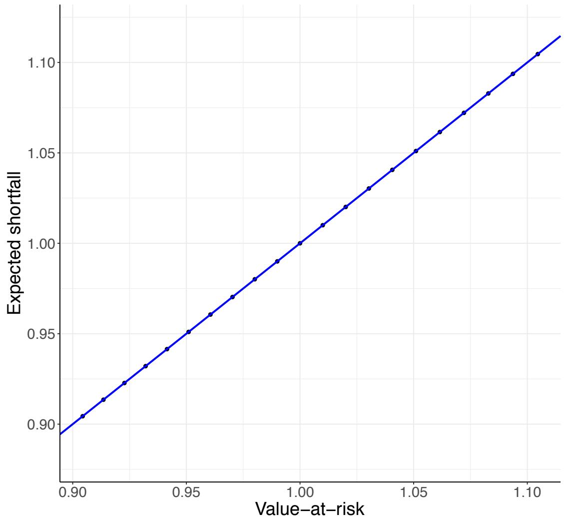



(d) Loan value



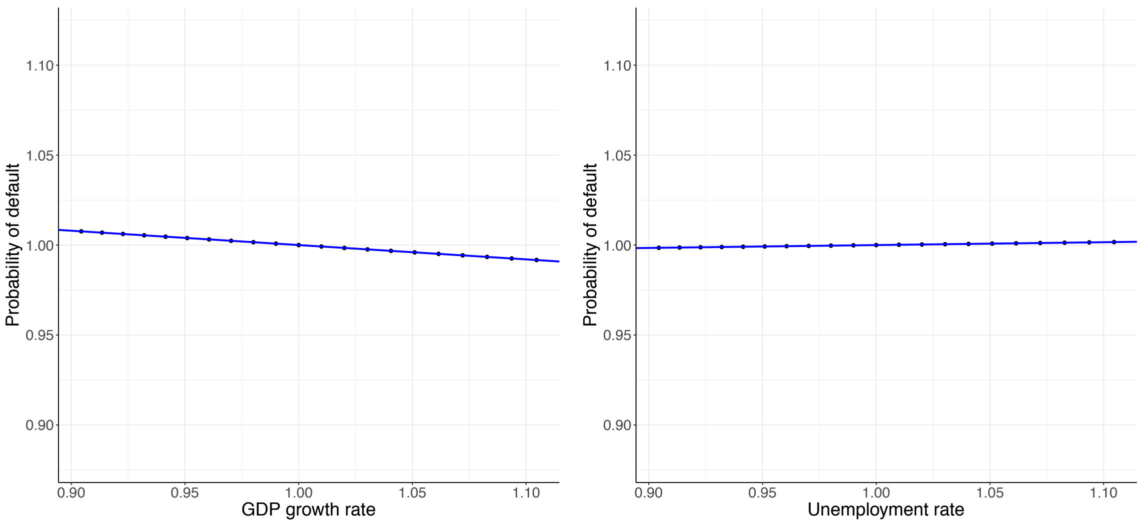

(e) Sensitivity measure

Figure 14: The sensitivity of the value-at-risk given its different inputs.




(a) Value-at-risk

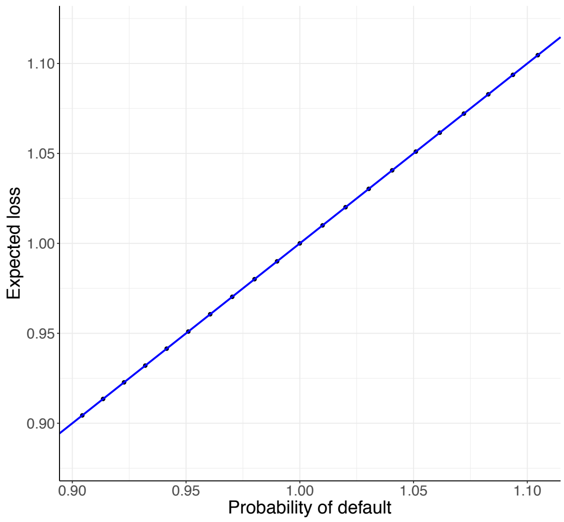
Figure 15: The sensitivity of the expected shortfall given its different inputs.



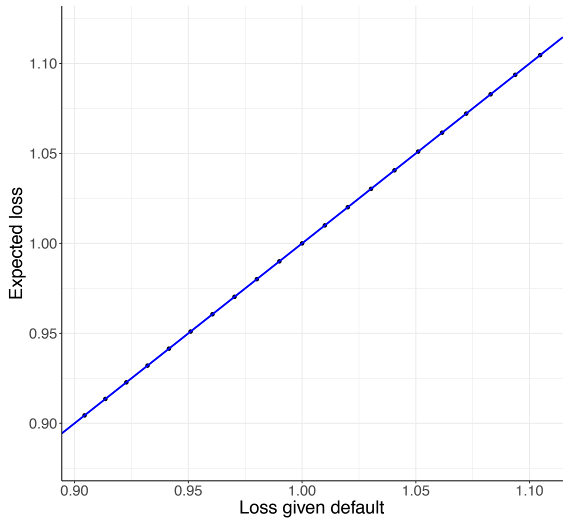

(a) GDP growth rate

(b) Unemployment rate

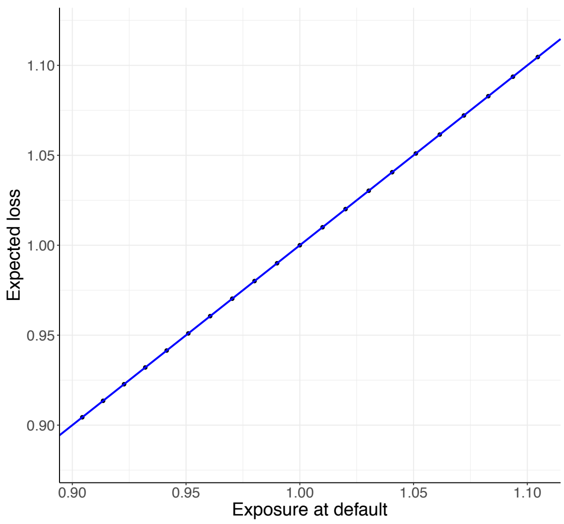
Figure 16: The sensitivity of the probability of default given its different inputs.




(a) Probability of default




(b) Loss given default




(c) Exposure at default

Figure 17: The sensitivity of the expected loss given its different inputs.

TRITA -SCI-GRU 2018:068