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Application of Mean Absolute Deviation Optimization in Portfolio Management

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Abstract

This thesis is an implementation project of a portfolio optimization model, with the purpose of creating a decision support tool. It aims to provide quantitative input to the portfolio construction process at Handelsbanken Fonder, by applying Konno & Yamazaki's Mean Absolute Deviation method, with a Feinstein & Thapa modification. Additionally, the Black-Litterman model is implemented to approximate the input of expected return. The linear optimization problem was then solved by the Simplex algorithm. The main deliverable is a model that can assist portfolio managers in making investment decisions. Back-testing of the model showed that it did not outperform the benchmark portfolios, which is likely a result of only allowing long positions in the model. Nevertheless, the model provides value by giving the user a second opinion on the efficient frontier, for any given investment decision.

Keywords: Portfolio Theory, Linear Programming, Mean Absolute Deviation, Black-Litterman.

Sammanfattning

Den här uppsatsen är ett implementationsprojekt av en portföljoptimerings-modell, med syftet att skapa ett beslutsstödjande verktyg. Den strävar efter att ge ett kvantitativt bidrag till portföljallokeringsprocessen på Handelsbanken Fonder, genom att använda Konno & Yamazaki's Mean Absolute Deviation-metod med en Feinstein & Thapa-modifiering. Vidare har Black-Litterman modellen implementerats för att approximera den förväntade avkastningen. Det linjära optimeringsproblemet löstes sedan med Simplex-algorithmen. Det huvudsakliga resultatet är en modell som kan assistera fondförvaltare i investeringsbeslut. Utförda utfallstest visade att modellen inte överträffade de använda benchmarkfonderna, vilket sannolikt är ett resultat av att modellen enbart tillåter långa positioner. Likväl, kan modellen vara värdefull genom att erbjuda användaren ett alternativ på den effektiva fronten, för ett givet investeringsbeslut.

Titel: Tillämpning av Mean Absolute Deviation inom Portföljförvaltning

Nyckelord: Portföljteori, Linjär Programmering, Mean Absolute Deviation, Black-Litterman.

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Gustav Rehnman & Nils Tesch

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1 Introduction

1.1 Problem Background

The mathematical fields of probability, statistics and optimization form a natural basis for quantitative analysis of investment decisions. This has lead to the development of a large amount of financial theories and models treating this process. The field of portfolio theory, was pioneered by Harry Markowitz in 1952, when he introduced a model that has become known as Modern Portfolio Theory (MPT). The model assumes that an investor aims to maximize a portfolio's expected return, contingent on a given amount of risk. Portfolios meeting this criteria are known as efficient portfolios, and any portfolio with the same expected return, but higher risk, are consequently sub-optimal. This concept caused investment professionals to rethink their asset allocations and made adopters of the model redistribute their holdings according the theories and models of Markowitz (1952) and his successors. As time has passed, several shortcomings of MPT have been brought to light, which has lead to the development of new models that attempt to overcome these shortcomings. Rom & Ferguson (1994) argued that the risk measure in MPT, the standard deviation of asset returns, was an inappropriate choice. They proposed the model of Post-Modern Portfolio Theory (PMPT), which instead used the standard deviation of negative asset returns as the risk measure, which tends to better capture reality. Konno & Yamazaki (1991) proposed a linear model (in contrast to Markowitz (1952) and Rom & Ferguson (1994), who used quadratic models), which used the mean-absolute deviation as risk measure. The model has been shown to perform in line with the preceding quadratic model but due to its linearity, it reduced the complexity of the mathematical operations considerably. Black & Litterman (1991) solved MPT's problem of requiring input in terms of asset expected return, and developed a model which lets the portfolio manager provide a relative view of certain sub-groups, rather than expected returns. Indeed, there have been many attempts to create improved models for portfolio optimization.

In addition to mathematical progress in the field of portfolio theory, digitalization has had an impact on the practical work of investment professionals. As computational power increased, optimization software entered the market and became widely used. Programs like Bloomberg, Axioma and Barra offer portfolio management, including the option to optimize your portfolio. However, these systems tend to be complex and have a high subscription fee. Both these features could make it difficult for a portfolio manager to motivate using the model, when he or she redistributes assets sparsely. This implies that there is a need for models that are customized to the client's needs, both in terms of cost and functionality.

1.2 Purpose and Problematization

In the daily business of the fund management of Handelsbanken Fonder AB, buying and selling securities is a given task for a portfolio manager. Decisions to invest, keep, decrease or dispose of holdings are based on information and conclusions derived from a collaborative fundamental process. The implementation of ideas and the portfolio construction process is however, within relevant constraints, delegated to the manager responsible for the portfolio. In this process a variety of tools are being used today to assist and support the portfolio managers, most prominently Bloomberg's portfolio function. This approach has historically worked well for the fund management company, based on their returns. The tools that are used today are however described as somewhat cumbersome and complicated to use. The company believes that quantitative input in the portfolio construction process is an important decision support for the portfolio managers and wants to find a more comprehensive tool to use in the day-to-day decisions.

Given this introduction, Handelsbanken Fonder AB, requested us, the authors, to develop a portfolio optimization tool to fill this aforementioned demand in their operative business. Consequently, our work and decisions in this thesis have been heavily influenced by the requests of the client. The key characteristics of the tool that have been requested are i) simplicity, ii) speed and iii) accuracy. To enhance simplicity, we have aimed towards producing a tool that has an intuitive user interface and a low amount of input parameters. To do so, we chose to create the optimization software in Microsoft Excel's VBA (Visual Basic for Applications), a program most portfolio managers at Handelsbanken are comfortable with. Furthermore, we created a custom ribbon in Microsoft Excel for input data, with the purpose of making the program more clear and to raise the level of usability. This is likely to make the optimization tool more valuable and consistently used. The request for speed has influenced our choice of optimization algorithm and the implementation of it, making quick calculations and data collection a high priority. In this regard, Bloomberg's Application Programming Interface (API) has been important, as it enables quick data requests to be retrieved from the Bloomberg server. Furthermore, Konno & Yamazaki's (1991) MAD-optimization model fulfills the speed-criterion as it is a linear model, in contrast to MPT and PMPT, which are quadratic and hence considerably more time consuming in terms of calculation time. For large portfolios, MAD reduces the computation time drastically (Konno & Yamazaki, 1991). The third and final requested characteristic, accuracy, raises the importance of using a robust and recognized model. Based on our literature review, MAD-optimization fulfills this criteria as well, as it has been proven to produce similar results to other optimization methods (Silva et al. 2017). Additionally, it is important that the model reflects the reality of the fund management at Handelsbanken Fonder. This has affected the developed model in two main ways. The first is that no short positions are allowed. This condition can cause optimization models to generate sparse portfolios that put large weights in some assets and 0 in others (Levy & Ritov, 2001). As a result, the portfolio becomes less diversified, compared to before the optimization. Secondly, the European Securities and Markets authorities "5/10/40-rule" is considered in the optimization (ESMA, 2009), which sets upper bounds in the optimized asset weights.

This master's thesis aims to apply and implement existing mathematical theory into a practical optimization tool, which aims to assists in the decision-making in the investment or disposal of assets in a given portfolio. It does not contribute with new findings to the field of mathematics. Instead and primarily, it contributes to the operative business of the fund management at Handelsbanken Fonder AB. Secondly, it contributes to the thousands of people or organizations who own shares in the mutual funds that Handelsbanken manage and distribute. By providing Handelsbanken with a tool that potentially improves the risk/reward profile of a given fund, value is added and society as a whole benefits. Furthermore, this thesis also has an empirical contribution in the sense that it documents the process of implementing a tool for portfolio optimization in VBA. Other portfolio-managing individuals or organizations could follow the method presented in this thesis, to implement a Bloomberg and VBA connected optimization tool in their own organization.

In this degree project, we aim to create a program that provides a recommendation on how decisions of the following type should be made:

- *Given that a portfolio manager wants to invest in security A, B and C while selling holdings in security D, E, and F, and having a cash level of K (%) of the AUM - how many (if any) shares of security A, B and C should the portfolio manager buy and how many (if any) shares of security D, E and F should the portfolio manager sell to have an optimal portfolio?*

Where AUM stands for Assets Under Management, and it this context it refers to the assets under management of a given portfolio. The above question can be expanded to treat any number of securities to buy, but for intuitivity we have chosen to formulate the question for three arbitrary securities A, B and C. The number of securities to sell can also be more than the securities D, E and F, but in contrast to the number of securities to buy, this number is limited to the number of securities present in the portfolio at the time of the optimization (since no short-sells are allowed). K is the total share of the AUM in currency.

Furthermore, the program should be able to provide additional information related to the above decision. An example is an issue of the type:

- *Given the answer to or result of the above question, how should the portfolio manager act in the holdings of security X, Y and Z?*

The number of securities in the above question is limited to the number of securities currently held in the portfolio, given that the security in this question is in fact present in the portfolio. If the security is not currently present in the portfolio the number of securities does not have an upper bound (other than securities available in Bloomberg and securities allowed by the fund rules).

1.3 Delimitation

Handelsbanken Fonder provides equity, fixed-income and mixed equity/fixed income funds. Upon request from the commissioner of this degree project, the model and the tool will however be delimited to only handle equities. It will also be delimited to only handle long positions, i.e. no short positions. Presently, only a limited number of Handelsbanken's mutual funds have a mandate to work with short positions, and that portfolio construction process has a number of significant differences compared to a long-only fund. This delimitation will have considerable impact on the model, since optimization algorithms typically allow for both long and short positions, to enable the resulting portfolio to benefit from both positive and negative asset returns. Handelsbanken Fonder's active equity funds do not per se take currency positions. Therefore, for the purpose of this study, the total amount of currency is aggregated into one single variable, that is left out of the optimization, except for the calculation of available cash that can be used for additional investments.

1.4 Limitation

Since the model retrieves data from Bloomberg, it demands that the computer you are using has access to the platform. It is also limited by what data is available in Bloomberg. Bloomberg is however the main provider of financial data on the market and it is unlikely that any asset that is present in any of the mutual funds, will not be available on Bloomberg's platform. The model uses Bloomberg's built-in API, that communicates with Excel through VBA-code.

1.5 Outline

In Section 1 - *Introduction*, the one above, we explain the problem description provided by our Commissioner, Handelsbanken Fonder. We also provide the purpose and the subsequent problematization that this degree project is supposed to answer. In addition, we provide the contribution to research, delimitation and limitation of this degree project. In Section 2 - *Theory*, we provide the reader with the necessary background of portfolio theory and corresponding mathematical theory, that is needed to completely understand the context in which this degree project is carried out. In Section 3 - *Literature Review*, we provide information of previous research and findings in portfolio theory, as well as other portfolio optimization software on the market. In Section 4 - *Methodology*, we provide our method of application of the aforementioned theory. In addition, we explain other aspects such as data handling, data collection and user interface. In section 5 - *Results*, we provide the reader with the results of the model. Primarily, the performance of the model compared to actual outcome of a selection of mutual funds. The results are carried out through back-testing. In section 6 - *Discussion*, we discuss and analyze the results and the implications and implementations of the model at Handelsbanken Fonder. In section 7 - *Conclusion*, we leave the reader with our conclusion of this degree project. In section 8 - *Further Research & Applications*, we discuss further research and suggest ideas of additional applications and implementations of portfolio theory.

2 Theory

2.1 Portfolio Optimization

Portfolio optimization is the process of selecting asset weights in order to achieve an optimal portfolio, based on an objective function. Typically, the objective is to maximize expected return or to minimize financial risk. It can also be a combination of the two.

2.1.1 Modern Portfolio Theory

MPT (Modern Portfolio Theory), or mean-variance analysis, is a theory pioneered by Harry Markowitz in 1952. It assumes that investors make rational decisions and expect a higher return for increased risk. According to the theory, it is possible to construct a portfolio which maximizes the expected return, given a certain level of risk. Such portfolio is said to be on the "efficient frontier". An investor would not take on extra risk if it does not mean larger returns. Conversely, the investor must take on more risk if the goal is to achieve higher returns. A key insight in this theory is that the return and risk of an asset should not be viewed separately, since the two factors together affect a portfolios overall risk and return (Markowitz, 1952).

Despite its groundbreaking theories, MPT has faced criticism. To begin with, it requires the input of expected returns, which requires the investor to predict future outcomes. In practice, this is often done by extrapolating historical data. Such predictions often fail to take new circumstances into account, which results in predictions that are flawed. Also, as the risk measure of MPT is variance, the optimization model become quadratic, since variance is quadratic. For large portfolios, this implies heavy computations, which can make the model inefficient in a computational sense. Additionally, MPT assumes that the asset returns follow a Gaussian distribution, which has two serious implications. Firstly, it underestimates the probability of large and important movements in the price of an asset. Secondly, by relying on the correlation matrix, it fails to capture the relevant dependence structure among the assets. This limits the practical usefulness of MPT (Rachev & Mittnik, 2006).

Nevertheless, MPT has contributed with strong theoretical value. The findings of Markowitz can be formulated in three different ways. The three different views can be seen below, equation 1, 3 and 5. Here w_0 is the nominal cash allocated to a risk-free asset and $R_0 = 1/B_0$, where B_0 is the price of a zero-coupon bonds which at time 1 pays 1 of the chosen currency. $\boldsymbol{w}$ is the nominal cash allocated to risky assets. V_0 is the total cash amount available of the investor. $\boldsymbol{\mu}$ is the expected return for each asset and $\boldsymbol{\Sigma}$ is the asset covariance matrix. c is the trade-off parameter, which captures how risk averse the investor is. μ_0 and σ_0 are the weighted required expected return of the portfolio and the standard deviation, respectively.

Maximization of Expectation

The first formulation considers the objective to maximize the expected return, given a risk (variance) constraint. Here we assume that $\boldsymbol{\mu} \neq R_0\mathbf{1}$, which rules out an unrealistic degenerate form of the equation (Hult et al., 2012).

$$\begin{aligned} &\max\ w_0 R_0 + \boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{\mu}, \\ &\text{s.t. } \boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{\Sigma}\boldsymbol{w} \leq \sigma_0^2 V_0^2, \\ &w_0 + \boldsymbol{w}^{\boldsymbol{T}}\mathbf{1} \leq V_0. \end{aligned} \tag{1}$$

The above equation has the following solution, where the vector $\boldsymbol{w}$ is the allocation of weights corresponding to the optimal solution.

$$\boldsymbol{w} = \sigma_0 V_0 \frac{\boldsymbol{\Sigma^{-1}}(\boldsymbol{\mu} - R_0\mathbf{1})}{\sqrt{(\boldsymbol{\mu} - R_0\mathbf{1})^T\boldsymbol{\Sigma^{-1}}(\boldsymbol{\mu} - R_0\mathbf{1})}}, \tag{2}$$

provided that Σ^{-1} exists.

Minimization of Variance

The second Markowitz formulation has the objective function to minimize the variance of the portfolio, given a lower bound on expected value

$$\begin{aligned} &\min \frac{1}{2}\boldsymbol{w^T}\boldsymbol{\Sigma}\boldsymbol{w}, \\ &\text{s.t. } w_0R_0 + \boldsymbol{w^T}\boldsymbol{\mu} \geq \mu_0 V_0, \\ &w_0 + \boldsymbol{w^T}\mathbf{1} \leq V_0. \end{aligned} \tag{3}$$

The above equation has the following solution, where the vector $\boldsymbol{w}$ is the allocation of weights corresponding to the optimal solution (Hult et al., 2012).

$$\boldsymbol{w} = V_0(\mu_0 - R_0)\frac{\boldsymbol{\Sigma^{-1}}(\boldsymbol{\mu} - R_0\mathbf{1})}{\sqrt{(\boldsymbol{\mu} - R_0\mathbf{1})^T\boldsymbol{\Sigma^{-1}}(\boldsymbol{\mu} - R_0\mathbf{1})}}. \tag{4}$$

The Combination of Expected Return and Weighted Risk as Objective Function

Lastly, if we combine the maximization of expected return and the minimization of weighted risk in the objective function we get the following equation

$$\begin{aligned} &\max w_0R_0 + \boldsymbol{\mu^T}\boldsymbol{w} - \frac{c}{2V_0}\boldsymbol{w^T}\boldsymbol{\Sigma}\boldsymbol{w}, \\ &\text{s.t. } w_0 + \boldsymbol{w^T}\mathbf{1} \leq V_0. \end{aligned} \tag{5}$$

The above equation has the following solution, where the vector $\boldsymbol{w}$ is the allocation of weights corresponding to the optimal solution (Hult et al., 2012).

$$\begin{aligned} &\boldsymbol{w} = \frac{V_0}{c}\boldsymbol{\Sigma^{-1}}(\boldsymbol{\mu} - R_0\mathbf{1}), \\ &w_0 = V_0 - \boldsymbol{w^T}\mathbf{1}. \end{aligned} \tag{6}$$

2.1.2 The Efficient Frontier

The efficient frontier is a cornerstone in MPT and was introduced by Markowitz (1952). It is defined as the set of optimal portfolios that offers the highest expected return for a defined level of risk, or the lowest risk for a given level of expected return. By definition, portfolios that does not coincide with the efficient frontier are sub-optimal. The efficient frontier is typically illustrated as a hyperbola with the rate of return on the y-axis and the risk on the x-axis.

2.1.3 Post-Modern Portfolio Theory

In 1994, Rom & Ferguson proposed PMPT (post-modern portfolio theory) which builds on Markowitz's (1952) MPT-model, but counteracts some of its flaws. PMPT uses the standard deviation of the negative returns as the risk measure, whereas MPT uses the standard deviation of the returns. The overall standard deviation and variation of asset returns is a symmetric risk measure, since upside and downside deviations are treated equally. Rom & Ferguson argue that this is counter-intuitive for investors, as positive deviations are beneficial and should not be penalized. They conclude that from a practical standpoint, risk is severely skewed, with the greatest concern going to the downside. Furthermore, PMPT recognizes that investment risk should be tied to each individual investor's goals. They defined a target return, referred to as MAR (Minimum Acceptable Return). It represents the return that must be achieved in order to avoid failing an important financial objective. This measure is explicitly included in the calculation of PMPT efficient frontiers, which means that there is a unique efficient frontier for each MAR. This stands in contrast to MPT, where the investor's goals are never considered explicitly.

To represent the underlying uncertainty of asset forecasts, optimization procedures in both MPT and PMPT require a statistical return distribution to be specified for each asset. MPT only allows for the two-parameter normal and log-normal distributions, whereas PMPT permits a broader set of distributions, including asymmetrical distributions. Rom & Ferguson (1994) argue that PMPT optimization, will generally provide more accurate results, as it allows for a more accurate representation of an asset's true shape. This also means that PMPT allows for optimization of heavily skewed investment strategies.

2.1.4 MAD Optimization

To overcome the potential computational intractability of Markowitz's quadratic model (1952) and derivatives of such, some authors have proposed to use alternative, more efficient models. In 1991, Konno & Yamazaki introduced a model called Mean Absolute Deviation (MAD). The MAD model results in a linear programming model, which has proved to be equivalent to the aforementioned Markowitz's model, but with the tractable feature of being considerably more effective in terms of computation time. The MAD linear programming model, can be posed as follows:

$$
\begin{aligned}
&\min \sum_{t\in\mathcal{T}} p_t y_t, \\
&\text{s.t. } y_t + \sum_{j\in\mathcal{N}} w_j(r_{j,t} - r_j) \geq 0,\ t\in\mathcal{T}, \\
&y_t - \sum_{j\in\mathcal{N}} w_j(r_{j,t} - r_j) \geq 0,\ t\in\mathcal{T}, \\
&0 \leq w_j \leq u_j,\ j\in\mathcal{N}, \\
&y_t \geq 0,\ t\in\mathcal{T},
\end{aligned}
\tag{7}
$$

where p_t is the probability of scenario t. y_t is the new variable. $r_{j,t}$ is the average expected return of asset j in scenario t. w_j is the proportion of the investors capital allocated to asset $j = 1, \ldots, \mathcal{N}$. r_j is the average expected return of asset j. u_j is a limit set to ensure a greater portfolio diversification.

Suitable Reductions of the MAD Optimization

Konno & Yamazaki's (1991) model can be further reduced, as explained by Fox (2014) among others. Fox (2014) proposed the following model

$$
\begin{aligned}
&\min \frac{1}{T}\sum_{t\in\mathcal{T}} y_t,\\
&\text{s.t. } y_t + \sum_{j\in\mathcal{N}} w_j(r_{j,t}-r_j) \geq 0,\ t\in\mathcal{T},\\
&y_t - \sum_{j\in\mathcal{N}} w_j(r_{j,t}-r_j) \geq 0,\ t\in\mathcal{T},\\
&\sum_{j\in\mathcal{N}} r_j w_j \geq \rho,\\
&\sum_{t\in\mathcal{T}} w_j = 1,\\
&0 \leq w_j \leq u_j \leq 1,\ j\in\mathcal{N}.
\end{aligned} \tag{8}
$$

Here, the objective function is approximated such that $(1/T)$ describes the probabilities, p_t, which was previously nested in the sum in the objective function of equation 2.2.2. The scalar ρ represents the minimum rate of return required by an investor. Fox (2014) stated that this reduced model minimizes the risk function $(1/T)\sum_{t\in\mathcal{T}} \mid \sum_{j\in\mathcal{N}}(r_{jt}-r_j)w_j \mid$. This note shows that only one of the first and second constraint sets is required to find optimal solutions to the problem, meaning the other is redundant.

2.1.5 The Feinstein-Thapa Modification

Feinstein and Thapa (1993) further reduced 8 by defining non-negative variables a_t and b_t and applying them to the first and second constraint in 8.

$$
\begin{aligned}
y_t + \sum_{j\in\mathcal{N}}(r_{j,t}-r_j)w_j = 2a_t,\ t\in\mathcal{T},\\
y_t - \sum_{j\in\mathcal{N}}(r_{j,t}-r_j)w_j = 2b_t,\ t\in\mathcal{T}.
\end{aligned} \tag{9}
$$

a_t and b_t are used to eliminate the y_t variables. The objective function and the first and second constraints in in 8 are transformed to

$$
\begin{aligned}
&\min\ Z = \sum_{t\in\mathcal{T}}(a_t+b_t),\\
&\text{s.t. } b_t + \sum_{j\in\mathcal{N}}(r_{j,t}-r_j)w_j = a_t.
\end{aligned} \tag{10}
$$

Now, substituting a_t into the new objective function leads to

$$
Z = \sum_{t\in T}(2b_t + \sum_{j\in\mathcal{N}}(r_{j,t}-r_j)w_j) = 2\sum_{t\in\mathcal{T}} b_t + \sum_{j\in\mathcal{N}} w_j \sum_{t\in\mathcal{T}}(r_{j,t}-r_j), \tag{11}
$$

but,

$$
\sum_{t\in\mathcal{T}}(r_{j,t}-r_j) = 0, \tag{12}
$$

since,

$$
r_j = \frac{1}{T}\sum_{t\in\mathcal{T}} r_{jt}. \tag{13}
$$

Finally, the objective function and the first and second constraints in 8 are replaced by

$$\begin{aligned} &\min\ Z = 2\sum_{t\in\mathcal{T}} b_t, \\ &\text{s.t. } b_t + \sum_{j\in\mathcal{N}} (r_{j,t} - r_j) w_j = a_t, \\ &t \in \mathcal{T}. \end{aligned} \tag{14}$$

a_t can now be dropped and the new formulation is equivalent to Konno & Yamazaki's (1991) model, but without the second constraint. Analogously, b_t can be substituted with a_t to drop the first constraint, instead of the second.

2.1.6 Mansini-Speranza Optimization

Mansini & Speranza (2005) also proposed extensions of Konno & Yamazaki's (1991) MAD-model. The purpose of the extensions was to consider additional characteristics of real portfolios, such as block-trades of shares and the incorporation of transaction costs. In addition, the authors weighed the expected return and portfolio risk in the objective function using a mean-risk model. They called it the Mansini-Speranza (or in short MS) model. See below for mathematical definition.

$$\begin{aligned} &\max\ \sum_{j\in\mathcal{N}} [(1-g) r_j s_j x_j - c_j z_j] - \sum_{t\in\mathcal{T}} p_t y_t, \\ &\text{s.t. } y_t + \sum_{j\in\mathcal{N}} (r_{jt} - r_j) s_j x_j \geq 0,\ t \in T, \\ &\sum_{j\in\mathcal{N}} [(1-g) r_j s_j x_j - c_j z_j] \geq \omega \sum_{j\in\mathcal{N}} s_j x_j, \\ &\sum_{j\in\mathcal{N}} s_j x_j \leq M_0, \\ &x_j \leq u_j z_j,\ j \in \mathcal{N}, \\ &y_t \geq 0,\ t \in \mathcal{T}, \\ &x_j \in \mathbb{Z}_+,\ j \in \mathcal{N}, \\ &z_j \in \{0,1\},\ j \in \mathcal{N}. \end{aligned} \tag{15}$$

g is the tax paid for the returns $r_j, j \in \mathcal{N}$. c_j is the fixed cost incurred only if there is investment in asset j. z_j is a binary decision variable, which equals 1 if asset j is selected, otherwise it is equal to 0. x_j is the integer value for the number of shares purchased in a block. Other notations follows the notations of the definition of the MAD-optimization.

2.1.7 The Beta Model

The Beta model was proposed by Albuquerque (2009) and is considered to be an extension of the MS-model by Mansini & Speranza (2005). The Beta model adds the factor of diversifiable and non-diversifiable risks. Diversifiable risk is considered by imposing a minimum number of assets in the composition of the optimal portfolio, whereas non-diversifiable risk is considered using the beta coefficient of the portfolio.

$$
\begin{aligned}
&\max \sum_{j\in\mathcal{N}} [(1-g)r_j s_j x_j - c_j z_j] - \sum_{t\in\mathcal{T}} p_t y_t,\\
&\text{s.t. the first - third and fifth - seventh MS constraints in 15 fulfill}\\
&\sum_{j\in\mathcal{N}} (\beta_{max} - \beta_j) s_j x_j \geq 0,\\
&\sum_{j\in\mathcal{N}} (\beta_j - \beta_{min}) s_j x_j \geq 0,\\
&\sum_{j\in\mathcal{N}} z_j \geq k\\
&l_j z_j \leq x_j \leq u_j z_j,\ j \in \mathcal{N}
\end{aligned}
\tag{16}
$$

Here l_j is the lower bound of the number of acquired assets j. k is the minimum number of assets that should make up the portfolio. β_{max} is the maximum value that the beta of the assets can take and β_{min} is the minimum value that the beta of the assets can take. β_j is the beta of asset j. Other notations follows the notations of the definitions of the MS and MAD-models.

2.2 Financial Risk Measures

2.2.1 Standard Deviation

A measure of risk is a metric of high importance in the field of portfolio theory. The choice of risk measure tends to vary depending on the purpose. The standard deviation of asset returns is a common risk measure, which measures the dispersion of the data from its expected value. The mathematical definition of a portfolio is as follows:

$$
\sigma_p = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}, \tag{17}
$$

where N is the number of assets in the portfolio, x_i is the sample outcome for each asset in the portfolio i and $\bar{x}$ is the mean of the outcome of the assets in the portfolio.

2.2.2 Mean Absolute Deviation

Konno & Yamazaki (1991) used the mean absolute deviation as the risk measure in their linear optimization model. This decision had the benefit of reducing the mathematical complexity and also removed the assumption of Gaussian asset returns.

$$
\text{MAD} = \frac{1}{N}\sum_{i=1}^{n} \mid x_i - m(X) \mid, \tag{18}
$$

where N is the sample size and x_i is the sample outcome of asset i. $m(X)$ is the measure of central tendency, most commonly chosen to be the mean of the median of the sample.

2.2.3 The Sharpe Ratio

The Sharpe Ratio is a method for the calculating risk-adjusted return of a portfolio. A typical application of the method is to compare the change in the overall risk and return of a portfolio, when adding a new asset to the portfolio.

$$\text{Sharpe Ratio} = \frac{\bar{x} - r_f}{\sigma_p}, \tag{19}$$

where $\bar{x}$ is the mean of the outcome of the assets in the portfolio, r_f is the risk-free rate and σ_p is the standard deviation of the portfolio.

2.3 Expected Return

The expected return of an investor is the profit (or loss) that an investor anticipates on an arbitrary investment, conditioned that there is either known or estimated returns of the investment. An intuitive example for three arbitrary assets with arbitrary returns, can be viewed below:

$$E[R_0] = p_1 r_1 + p_2 r_2 + p_3 r_3, \quad i = 1, 2, 3 \tag{20}$$

Where $E[R_0]$ is the expected return, p_i is the probability of the potential outcome of asset i and r_i is the corresponding return of asset i.

2.3.1 CAPM - The Capital Asset Pricing Model

The Capital Asset Pricing model can be expressed as:

$$E[r_i] = r_f + \beta_{im}(E[R_m] - r_f), \tag{21}$$

where,

$$\beta_{im} = \frac{Cov(r_i, r_m)}{Var(r_m)} \tag{22}$$

Where r_i is the return of the asset, $E[r_i]$ is the expected return of the asset, r_f is the risk-free rate, β_{im} is the correlation between the asset and the market, $E[R_m]$ is the expected return on the market. Here $(E[R_m] - r_f)$, is the expected return on the market subtracted by the risk-free rate, called the market risk premium.

2.3.2 The Black-Litterman Model

The Black-Litterman model, proposed by Fischer Black and Robert Litterman, uses a Bayesian approach to create a mixed estimate of expected returns. It does this by the combination of subjective views of the investor regarding expected return with the market equilibrium vector of expected returns. The market equilibrium vector is the neutral starting point, which is derived using a reverse optimization method in which the vector or implied excess equilibrium returns is extracted from known information, as displayed below.

$$\mathbf{\Pi} = \delta \mathbf{\Sigma} \boldsymbol{w_m}, \tag{23}$$

where $\mathbf{\Pi}$ is the implied excess equilibrium return vector and δ is the market risk aversion coefficient. It characterizes the expected risk-return trade-off. The investor will sacrifice return for less variance at the rate of this coefficient. $\mathbf{\Sigma}$ is an $\mathcal{N} \times \mathcal{N}$-matrix of covariances between assets. $\boldsymbol{w_m}$ is the market capitalization weight of the assets.

Now that market equilibrium vector $\mathbf{\Pi}$ is known, we can correct it for the views of the investor. The views can be either absolute or relative. An absolute view is only dependent on one type of asset, while a relative view is expressed with dependence to another asset.

$$\boldsymbol{E[R_0]} = [(\tau\mathbf{\Sigma})^{-1} + \boldsymbol{P}^T\mathbf{\Omega}^{-1}\boldsymbol{P}]^{-1}[(\tau\mathbf{\Sigma})^{-1}\mathbf{\Pi} + \boldsymbol{P}^T\mathbf{\Omega}^{-1}\boldsymbol{Q}]. \tag{24}$$

For $\mathcal{N}$ number of assets, $\boldsymbol{E}[\boldsymbol{R_0}]$ is an $\mathcal{N} \times \mathcal{N}$-matrix of new mixed estimates of expected returns. τ is the uncertainty ratio, a scalar that is usually set to a number between 0.01 and 0.05 (Lee, 2000). For K numbers of views, $\boldsymbol{P}$ is $\mathcal{K} \times \mathcal{N}$-matrix that identifies each asset with a view. $\boldsymbol{\Omega}$ is a $\mathcal{K} \times \mathcal{K}$-matrix of error terms, representing the error terms for the views. $\boldsymbol{Q}$ is a $\mathcal{K} \times 1$ vector of views (Idzorek, 2005).

To show how the views work, we provide an example of two arbitrary views, one absolute and one relative, in an example with four assets.

- View 1 (Absolute) - Asset 1 will have an absolute excess return of 1.00%. The confidence of the investor in this view is 50%, the investor believes that it is a 50/50 chance of the view happening.
- View 2 (Relative) - Asset 3 will outperform Asset 4 with 50 basis points. The confidence of the investor in this view is 25%, that is the investor believe that the probability of the view happening is 1/4.

For arbitrary returns and covariance matrix, the matrices become:

$$\Sigma = \begin{bmatrix} 1 & 0.5 & 0.25 & -0.25 \\ 0.5 & 1 & 0.33 & 0.75 \\ 0.25 & 0.33 & 1 & -0.10 \\ -0.25 & 0.75 & -0.10 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & -0.5 \end{bmatrix},$$

$$Q = \begin{bmatrix} 1.00 \\ 0.50 \end{bmatrix} + \Omega, \quad \Pi = \begin{bmatrix} 0.03 \\ 0.04 \\ -0.01 \\ 0.07 \end{bmatrix}$$

2.4 The Simplex Algorithm

The simplex algorithm operates on linear programs in standard form:

$$\max \boldsymbol{c^T x}, \tag{25}$$

$$\text{s.t. } \boldsymbol{Ax = b}, \forall : x_i \geq 0, \tag{26}$$

$$\forall : x_i \geq 0, \tag{27}$$

where $\boldsymbol{c^T x}$ is the objective function, where the vector c is the coefficients of the corresponding x's that must be larger or equal to zero. $\boldsymbol{Ax = b}$ are the constraints expressed in matrix form, where A is an nxm, where $n \geq m$, with the coefficients of the corresponding x's. The objective of the Simplex Method is to solve the m variables by 26 and express these variables in terms of the $n - m$ remaining variables.

If one assumes that the first m rows in A are linearly independent we can, without loss of generality, write equation 26 as

$$A_B x_B + A_N x_N = b, \tag{28}$$

where x_B is a vector containing the m, assumed, linearly independent components and x_N contains the $n - m$ remaining components. We the multiply both sides with A_B^{-1}. B stands for basic and N for non-basic variables. Since A_B is an mxm-matrix it has full rank. Consequently, x_B can be solved uniquely. By multiplying both sides with A_B^{-1}, we obtain

$$\begin{aligned} x_B + A_B^{-1} A_N x_N &= A_B^{-1} b, \\ x_B &= A_B^{-1} - A_B^{-1} A_N x_N. \end{aligned} \tag{29}$$

With elementary linear algebra in mind, we know that we can choose x_N arbitrarily. An intuitive choice is $x_N = 0$, which by definition is a basic solution, since $x_B = A_B^{-1}b, X_N = 0$ is feasible under the condition that $x \geq 0$, i.e. $x_B = A_B^{-1}b \geq 0$. By dividing the c into $c^t = (c_B^t, c_N^t)$, can compute the objective function for this basic solution, $X_N = 0$ and $z = \bar{z}$

$$\begin{aligned} z &= c_B^t x_b + c_N^t x_N = c_B^t A_B^{-1} b - c_B^t A_B^{-1} A_N x_N + c_N^t x_N, \\ &= (c_N^t - c_B^t A_B^{-1} A_N) x_N + c_B^t A_B^{-1} b =: c_N^{-t} x_N + \bar{z}, \end{aligned} \tag{30}$$

where $c^{-t} = (c_N^t - c_B^t A_B^{-1} A_N)$ is called the reduced cost, since it shows how much the objective function varies when x_N varies.

We now denote $\bar{A}_N = A_B^{-1} A_N$ and $\bar{b}$ =$\mathrm{A}_B^{-1} b$

$$\begin{aligned} \min\ & z = c_N^{-t} x_N + \bar{z} = \bar{z} + \sum_{j \in N} \bar{c}_j x_j, \\ \text{s.t.}\ & x_B = \bar{b} - \bar{A}_N x_N, \\ & x_B,\ x_N \geq 0, \end{aligned} \tag{31}$$

where the corresponding basic solution is $(x_B^t, x_N^t) = (\bar{b}_t, 0)$. By assuming that this basic solution is feasible ($\bar{b} \geq 0$). If some of the components in $\bar{c_N}$ are negative we know that it is meaningful to increase x_j for that the corresponding j of c

$$x_B = \bar{b} - \bar{a}_j x_j, \tag{32}$$

where $\bar{a_j}$ is the column corresponding to x_j in $\bar{A_N}$. As long as $X_B \geq 0$, the solution is feasible and given the equation in 32, we see that

$$(x_B)_i = \bar{b} - \bar{a}_{i,j} x_j, \quad j \in N. \tag{33}$$

We now acknowledge the following two cases:

- If $\bar{a}_{i,j} \leq 0$, $(x_B)_i$ either increase or stays constant as x_j increase. Hence, $(x_B)_i \geq 0$ has no impact, since $(x_B)_i$ will never become zero.
- If $\bar{a}_{i,j} > 0$, $(x_B)_i$ decreases and it becomes zero when $x_j = \bar{b}_i / \bar{a}_{i,j}$. Hence, the solution is feasible if and only if $x_j \leq min_i(\frac{\bar{b}_i}{\bar{a}_{i,j}} | \bar{a}_{i,j} > 0)$.

Now $x_j > 0$ and it becomes the new basic variable, while the variable that became zero is the non-basic variable. This switch, is the core of the Simplex Method. If some component in $\bar{c}_N$ is less than zero, the feasible solution can always be improved. However, if all components in $\bar{c}_N$ are positive, the feasible solution can not be improved. Hence, we arrive at the following theorem:

Theorem Assume that $\bar{b} \geq 0$ and $\bar{c}_N \geq 0$ in the transformed LP problem 31. Then the corresponding basic solution is optimal.

The proof of this theorem is the following. Since $\bar{b} \geq 0$, the basic solution $(x_B^t, x_N^t) = (\bar{b}^t, 0)$ is, as previously shown, feasible. By relaxing the condition $x_B = \bar{b} - \bar{A}_N x_N$ in 31, we arrive at the relaxed problem

$$\begin{aligned}\min\ z = \bar{c}_N^t x_N + \bar{z} = \bar{z} + \sum_{j\in N} \bar{c}_j x_j, \\ \text{s.t. } x_B, x_N \geq 0.\end{aligned} \tag{34}$$

Since $\bar{c}_j \geq 0$, we can minimize for each x_j independently, it is easy to see that $(x_B^t, x_N^t) = (\bar{b}^t, 0)$ is the optimal solution to 34 and hence also to 31. Since both problems have the same objective function, both give the same value of the objective function (Zhou, 2011).

The Algorithm in Four Steps

1. `Start` with a feasible basic solution.
2. Transform the problem so that it is on the same form as 26.
3. If $\bar{c}_j \geq 0$, `stop` the algorithm, the optimal solution is found.
4. If $\bar{c}_j < 0$, increase the corresponding x_j until $(x_B)_{i)=0}$. This is the new basic solution. Return to `step 2`.

2.5 The 5/10/40 Rule

The 5/10/40 rule is a nickname for UCITS article 52. UCITS stands for Undertakings for Collective Investment in Transferable Securities and is a directive from the European Securities and Markets Authority of the European Union (ESMA, 2009). It states that

- No more than 10% of net assets of a UCITS fund can be invested in transferable securities or money market instruments issued by the same issuer.
- Where investments in transferable securities and money market instruments each represent more than 5% of net assets of a fund, these investments in aggregate must not exceed 40% of the total net assets of the fund.
- The 10% rule does not apply to:
  - Deposits and OTC FDI's made with financial institutions subject to prudential supervision.
  - Certain transferable securities.
  - Investments in other UCITS funds or UCI's.

3 Literature Review

3.1 Methods of Portfolio Optimization

The mathematical fields of probability theory, mathematical statistics and optimization theory form a natural basis for quantitatively analyzing investment decisions. This has lead to the development of many financial theories and models. One such model, in the field of portfolio theory, was pioneered by Harry Markowitz in 1952 and is known as Modern Portfolio Theory (MPT). It assumes that an investor aims to maximize a portfolio's expected return, contingent on a given amount of risk. Portfolios meeting this criteria are known as efficient portfolios, and any portfolio with the same expected return, but higher risk, are consequently sub-optimal. This concept caused investment professionals to rethink their asset allocations and made adopters redistribute their holdings according to Markowitz's (1952) and his successors' theories. As time has passed, several shortcomings of MPT have been brought to light, which has lead to the development of new models that attempt to overcome said flaws. Rom & Ferguson (1994) argued that the risk measure in MPT, the standard deviation of asset returns, was an inappropriate choice. They proposed the Post-Modern Portfolio Theory (PMPT), which used the standard deviation of negative asset returns as the risk measure, which tends to better capture reality. Konno & Yamazaki (1991) proposed a linear model, which used the (MAD) mean-absolute deviation as risk measure. Due to its linearity, it reduced the complexity of the mathematical operations. Feinstein & Thapa (1993) reduced the MAD-model and proved you could drop one of the constraints. Many others have built upon other existing models and added constraints to fit their needs. One example comes from Mansini & Speranza (2005) who built upon the MAD-model and added constraints to include characteristics of real portfolios, such as block-trades and transaction costs. Albuquerque (2009) took it further and divided risks into diversifiable and non-diversifiable. Indeed, there are many ways to construct a model for portfolio optimization. It is clear that new models have emerged over time and contributed to the improvement of portfolio optimization.

3.2 Comparison of Existing Portfolio Optimization Methods

As there are numerous models for portfolio optimization, it is of interest to investigate which one is the most appropriate. Bower & Wentz (2005) performed a comparison of Markowitz (1952) M-V (mean-variance) model and Konno & Yamazaki's (1991) MAD-optimization model. They did this comparison by creating 30 portfolios with 5 randomly selected S&P500 stocks and a six-month bond. They then assumed an investor wanted to invest for a six-month period and weighed the portfolios as suggested by respected optimization model. Results showed that the two methods yielded similar returns. 0.0423% for M-V and 0.0410% for MAD. Statistical analysis revealed no statistical significance in these findings. They concluded that the two models were similar in their ability to generate returns, but stated that MAD was less complicated to use and therefore preferable for small portfolios.

Bower & Wentz (2005) only used small portfolios in their study, which imply that we can not draw conclusions in the model's performance in larger portfolios. Furthermore, their study was set under a short time period, meaning it did not compare the two models performance during different market conditions. Silva et al. (2017) did a more comprehensive study and compared the performance of M-V, MAD, MS, Beta and their own model named Beta-CVaR. Beta-CVaR is an extension of the Beta-model and includes the conditional value at risk concept in order to take large losses in low probabilities into account. The study measured these models ability to generate efficient portfolios. It included 6 computational tests, where 3 of them had 34 assets in the time period 2004-2013 and 3 tests with 48 assets in 2007-2013. All 6 tests were in the highly volatile Brazilian market. They found that M-V and MAD generated diversified portfolios with lower risk. The MS, Beta and Beta-CVaR models showed good results in terms of effective returns. Silva et al. (2017) concluded that the choice of model should depend on investor preferences, as they yield different results. If the model results are deemed good or bad depends on the preferences. We consider these findings particularly interesting as their study includes larger portfolios than Bower & Wentz (2005), has a longer time horizon and is set in a volatile market.

3.3 Methods of Estimating Expected Returns

One of the most, if not the most, important inputs when executing optimization methods, where the objective function, more or less aims to maximize expected return, is the vector of expected returns for each asset. A small increase in the expected return of just one of a portfolio's assets can potentially force half of the assets from the resulting optimal portfolio (Best, 1991). The most widely known method for the estimation of expected returns is the CAPM - Capital Asset Pricing Model. The model was introduced independently by Jack Trevnor in 1961 and 1962 (French, 2003), William F. Sharpe (Sharpe, 1964), John Lintner (Lintner, 1965) and John Mossin (Mossin, 1966). The aforementioned scholars built their work on Markowitz's earlier work from 1952. Fischer-Black (1972) added the constraint that the model does not assume that there exists a risk-less asset. This model is called the Black CAPM or the zero-Beta CAPM. Their model proved to perform more in line with empirical testing and this breakthrough added to the broad adoption of the CAPM.

In 1990, Fischer Black and Robert Litterman, who both worked at Goldman Sachs at the time, started the development of the Black-Litterman Model (Black & Litterman, 1991). It was later published in full in 1992 (Black & Litterman, 1992). The model is a Mixed Estimation Model, a type of model that was first introduced by Henri Theil in 1961 (Theil, 1992). The model was however first applied on financial data by Black and Litterman. The model aims to overcome the problem of finding appropriate estimates for expected returns as input in portfolio optimization. In its initial stage, the model uses the assumption that the asset allocation of the investor should be proportional to the equilibrium market values of the available assets. In a second stage, the model modifies the estimates from the first stage, by taking the investor's view on each assets into account. The model hence results in a posterior estimate, that is the product of both prior and conditional estimates. The benefits of the model include the option to use both absolute and relative views that are added to the prior estimate to generate a posterior estimate that include all views. The new estimate has been shown to be closer to the unknown mean and with lower variance. That is, a higher precision in estimating the unknown expected return.

3.4 Effects of Long Only Optimization

Typically, optimization algorithms allow for both long and short positions, as the main purpose is finding the optimal asset weights. However, this approach is often not feasible in practice, since many fund managers only have mandate to invest in long positions. Levy & Ritov (2001) studied the effects of the long only constraint on mean-variance optimization in large portfolios. They concluded that the number of assets held short converges to 50% as the number of assets in a portfolio increases. They further noticed that investment proportions are extreme, meaning several assets are held in large position and the weight of several other assets are 0. The effect of no short-selling on the Sharpe ratio was also deemed to be high. For large portfolios, the Sharpe ratio can be more than doubled by relaxing this constraint. According to their findings, the effects of having a long only constraint are indeed significant on mean-variance optimization. Levy & Ritov (2001) do not define how many assets a "large" portfolio include, but they mention dealing with a portfolio of 200 assets. Similar studies to Levy & Ritov (2001) but for MAD-optimization and other models have not been found.

3.5 Methods of Applying the 5/10/40-Rule

In practice, portfolios on the efficient frontier tend to be more concentrated than their corresponding benchmark portfolio, if there are no upper bound constraints. This could possibly render the optimized portfolio useless, as a portfolio manager would be hesitant to implement the allocation in practice

(Demey et al. 2010). The 5/10/40-rule, is therefore a beneficial restriction for scholars who strive to implement optimal portfolio approaches, since it imposes constrictions that are both uniform and more importantly, connected to reality. All UCITS funds on the EU markets must comply with the policy. Since the inception of the 5/10/40-rule in 2009, various scholars have proposed methods on how to handle the implications of the rule, in terms on diversification constraints on portfolios in portfolio theory. The 5/10/40 rule is a hard constraint, i.e., it leads to a non-convex search space. In the same year as the inception of the policy, Branke et. al (2009) proposed a method to integrate an active set algorithm optimized for portfolio selection into a multi-objective evolutionary algorithm (MOEA). Branke et. al. adapt the decoding and repair mechanism of the hybrid binary/real-valued encoding algorithm, introduced by Streichert (2004). A method successfully applied also by Chang (2000). This results in a 7-step algorithm that impose the constrictions of the 5/10/40-rule a portfolio. The efficient frontier is then solved by the Critical Line Algorithm, in line with the work of Markowitz (1987).

3.6 Linear Programming Algorithms

The first known academic contribution to the problem of solving linear inequalities was done by Jean-Baptiste Joseph Fourier, when he in 1827 published a paper with a first solution to the problem (Sierksma, 2001). The method Fourier-Motzkin elimination, is named after Fourier to show gratitude for this contribution. Linear programming is a method for the optimization of a (linear) objective function, subject to a number of (also linear) equality and inequality constraints. The linear constraints define the feasible region, a convex polyhedron. The objective function is in turn a real-valued function defined in the feasible region. What a linear programming algorithm does, is that it finds the point the feasible region where the objective is either smallest or largest (depending on the approach), given that such a point exists. In 1947 George Dantzig invented the groundbreaking Simplex Method (Murty, 2000). The method tests adjacent vertices of the feasible set, a convex polytope. The algorithm performs this in sequence so that at each new vertex, the objective function either improves or remain unchanged. The algorithm was groundbreaking in terms of efficiency, generally finding the optimal solution in only 2 to 3 times the number of constraints, iterations. Moreover, the algorithm converges in expected polynomial time for certain distributions of random inputs (Nocedal and Wright 1999, Forsgren 2002). In worst case, the complexity of the algorithm is exponential (Klee and Minty 1972).

3.7 Existing Optimization Software

There are numerous optimization software's with different methods, applications and benefits. Some well known company names within the field are Axioma, Barra, Bloomberg and Northfield. Axioma has been used by Handelsbanken Fonder and they offer a product that can handle high complexity, is flexible and has many options (Axioma 2016). They do not announce the underlying optimization model, but Infanger (2011) states that Axioma offers M-V and MAD. Barra is an optimization software integrated in the financial information system Factset. It is delivered to Factset customers without extra charge. In turn, Factset is owned by MSCI, which is an American provider of financial indexes and equity portfolio tools (MSCI 2014). Barra has been used previously by Handelsbanken Fonder, but is not present at the moment. Infanger (2011) states that Barra also uses M-V and MAD optimization. Similar to MSCI's Barra, Bloomberg also offers an integrated tool for portfolio optimization (Bloomberg 2018) without extra charge. Their portfolio optimizing product is similar to previous companies. Bloomberg is currently used by most, if not all, portfolio managers at Handelsbanken Fonder. Therefore they currently have access to this optimization tool. The fourth and final example of portfolio optimizing software is Northfield. They also state that they solve complex optimization problems (Northfield 2018), and according to Infanger (2011) they offer both M-V and MAD. Based on the available information for these companies, their product offering in terms of portfolio optimization software is quite similar. They are tools that solve optimization problems and return efficient portfolios based on given assumptions. Mean-variance analysis or MAD optimization appears to be the chosen models for these software, but it is possible that they have options and variations which are not explained in detail.

4 Methodology

4.1 Implementation

4.1.1 Program of Choice

Upon initiation of the project, we considered several programs to code in. One important aspect was the possibility to communicate with Bloomberg through an API (Application Programming Interface), which has the required database for this project. Matlab is strong for mathematically complicated projects such as ours and has several relevant built-in functions. However, our client, Handelsbanken Fonder, does not have access to this program, which makes it a poor option. C#, C and C++ were also considered. These programs can communicate with the Bloomberg API and are likely to have been feasible coding languages, but we lack experience in these. We also considered Python which is likely to have worked as well. However, we concluded that the Microsoft Excel built-in program VBA (Visual Basic for Applications) was the best option. It has the required mathematical operations, has access to Bloomberg API and the portfolio managers are experienced in this program. To fulfill Handelsbanken Fonder's criteria of having a simple and intuitive tool, using Excel and VBA makes perfect sense.

4.1.2 Data Collection and Handling

Handelsbanken Fonder makes portfolio data available for their portfolio managers via the Bloomberg portfolio management system, PORT. Consequently, we use the Bloomberg API in VBA to retrieve data of the portfolios. We learned how to use the API by consulting Bloomberg support personnel, who supplied us with VBA examples. The WAPI-command in a Bloomberg terminal brings up information on this subject, which was also helpful.

To handle the data we receive from the user and the Bloomberg API, we use a custom class and a collection. The custom class is for the assets, where each asset is defined as an object with 5 attributes. These attributes are name, weight, activity, sector and returns. The name is a string-type and is simply the Bloomberg ticker, which is required to request data through the Bloomberg API. The weight-attribute is a double-type (also known as float) and is the percentage weight of the asset in the portfolio, before the optimization. The activity is a string-type and defines what will happen with the asset in the optimization. There are four different activities: `BUY`, `SELL`, `OPEN` and `KEEP`. Buy implies that new weight $\geq$ old weight. If the asset to be bought did not exist previously in the portfolio, old weight is set to 0. `SELL` means new weight $\leq$ old weight. Short-selling is not allowed so you must sell an existing asset in the portfolio. `OPEN` is the option if you want to change the weight of the asset but you do not know in which direction. `KEEP` means that new weight = old weight. This activity is automatically given to all assets which does not have any of the other three activities. Sector is a string-type and is the sector that the asset is in. Sector data is only needed if Black-Litterman is used. The returns-attribute is a variant-type (also known as array). It stores the asset returns, which are required to calculate the MAD. Each object (asset) is added and stored in a collection.

The process of gathering data starts with letting the user choose which portfolio to optimize through a drop-down menu. We have connected each portfolio name to a numerical Bloomberg ID manually, which is used to request data from the Bloomberg API. After the choice of portfolio, we request the data of the assets included in the portfolio and the respective weights. An object is created for each asset and the name and weight is added to a collection in VBA. Thereafter the user needs to define which stocks to `BUY`, `SELL` or keep `OPEN`. Each asset has its activity-attribute defined according to this data. If an asset is not defined here, it is automatically a `KEEP`. We have included an "optimize all" alternative, which sets all assets to `OPEN`. Thereafter, we request asset sector and monthly share price data over a one year period for each asset. From the price data, we calculate asset returns, which is used to calculate the risk measure, MAD. See section 2.2.2 for the mathematical definition. After these requests, we have the information required for the MAD optimization, except for the cash position and the expected return of the portfolio. Since the portfolio typically always have some amount of cash available, the user inputs a remaining cash percentage. When it comes to expected return, this is a notoriously difficult variable to estimate. We decided to give two options regarding this matter. The first is simply letting the user choose this variable, which requires no communication with the Bloomberg API. The second option is to use the Black-Litterman model. This option involves collecting data over the returns of different sectors. Consequently, we used the API to request data of ETF's which represent these sectors. Thereafter we consider the sector-attribute of each asset in order to get the sector weight of the portfolio. By using the mathematics presented in section 2.3.2, we could thereafter use the Black-Litterman model to calculate the weighted expected return and use it in our optimization.

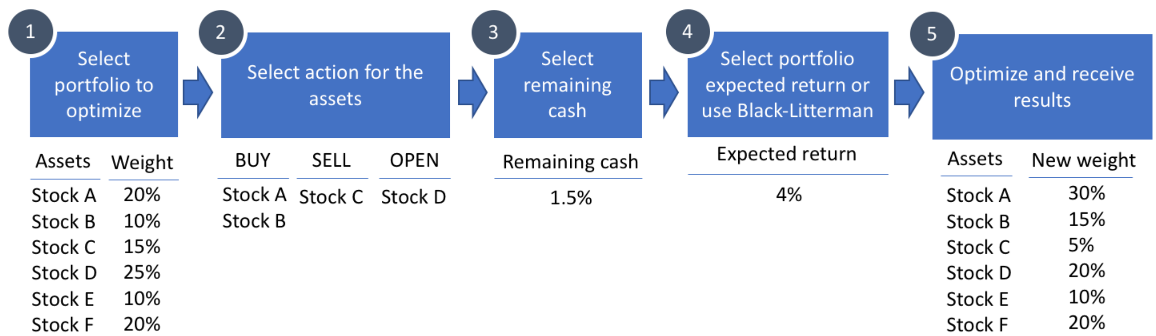

Figure 1: Illustrative representation of the optimization process

4.1.3 User Interface

A factor of considerable importance in this project, is the user interface. The portfolio managers, who will be using the program, requested something intuitive and quick, that will be able to assist them in the decision-making process. If the user interface would end up too complex and unintuitive, the program is unlikely to be used regularly. After discussions with our supervisor Staffan Lindfeldt, we concluded that it would be appropriate to construct a custom Ribbon for the user input. We used a third-party add-in for Microsoft Excel, the Custom UI Editor, for this task. By doing so, the user interface became more user-friendly, than if we would have had the user input in a worksheet. Output is received in the main worksheet, where we print the name of the assets, the activity i.e. `BUY`, `SELL`, `OPEN` or `KEEP`, its new weight (which is the same as the old if the activity is `KEEP`), the old weights of each asset, the difference and if it has been recommended to be bought or sold. In addition, we also print the remaining cash balance.

4.1.4 Implementation at Handelsbanken Fonder

To facilitate the implementation of this optimization software, the authors will provide the portfolio managers with opportunity to attend seminars. At these occasions, the mathematical model will be explained and its usability will be showcased. Additionally, the optimization tool will be delivered together with a description on how to use it. All code will be left open to Handelsbanken Fonder if they wish to review it or make any changes.

4.2 Mathematical Method

4.2.1 MAD Optimization

The Feinstein & Thapa (1993) modification of Konno & Yamazaki's (1991) MAD (Mean Absolute Deviation) was chosen as the optimization method. This decision was based on a number of factors. Firstly, an important reason to use a MAD-based model is that it allows for linear optimization. Compared to quadratic portfolio optimization theories, such as Markowitz (1952), the computation time is reduced drastically when dealing with large portfolios. As such, our model becomes quick and flexible. Secondly, MAD does not assume asset returns to be normally distributed, which is in contrast to Markowitz optimization. Again, this increases the flexibility of the model. Thirdly, according to our literature review, Markowitz (1952) model is a solid, well-known method, which laid the foundation for portfolio theory as we know it today. However, it has been proven that MAD produces similar portfolio returns as Markowitz (Bower, Wentz 2005) (Silva et al. 2017), which makes us confident in MAD's optimization ability. Lastly, renowned optimization software Axioma, Barra and Northfield are known to have used MAD-based optimization (Infanger 2011), which implies the model is appropriate for commercial use.

4.2.2 The Simplex Algorithm

The actual optimization is executed by first gathering the data and then making the necessary calculations of the Mean Absolute Deviation, see section 2.2.2. Thereafter, the resulting optimization problem is solved by the Simplex Algorithm to solve the linear system and receive the new weights of the assets, see section 2.4. A great benefit of the Simplex Algorithm is the constraint that all feasible solutions must be larger or equal to zero. This adds the requested constraint of long-only positions to our model.

4.2.3 Expected return

Expected portfolio return is an input required in the MAD optimization. We decided to give two options to our users in regard to this variable.

Required Expected Return

The first option is to give the user the opportunity to put in the percentage required as the minimum return. If the user has an idea of how much they believe the portfolio will return, this option has the benefit of being simple, quick and direct. However, estimating this number is generally difficult and some portfolio managers refrain from doing this.

The Black-Litterman Model

One method to overcome the cumbersome nature of the expected return is to use the Black-Litterman model, which we decided to include in this optimization software as it is simple to understand and has input that can be retrieved from Bloomberg.

To calculate the expected returns of the Black-Litterman model, we first retrieve the returns and standard deviation through the Bloomberg API of all sectors defined by MSCI (2016), except for Information Technology and Telecommunications, which we bundle into one sector called Technology. The sectors and the corresponding replicating ETF's and Bloomberg Tickers can be viewed in table 1. All of these cover over a 1000 assets (SPDR, 2018) worldwide. The ETF's are issued by State Streets Global Advisors and are exchange-traded funds incorporated in the USA. The ETF's track the performance of the Select Sector Index for each corresponding sector. Following a series of calculations, as explained in section 2.3.2, you receive output in terms of expected return of each sector in the model. Thereafter we multiply the sector weights with the expected return of each sector to get the weighted expected return of the portfolio and proceed with the MAD Optimization.

Sectors	**Replicating ETF**	**Bloomberg Ticker**
Consumer Discretionary	Consumer Disretionary Select Sector SPDR Fund	`XLY US Equity`
Consumer Staples	Consumer Staples Select Sector SPDR Fund	`XLP US Equity`
Energy	Energy Select Sector SPDR Fund	`XLE US Equity`
Financials	Financial Select Sector SPDR Fund	`XLF US Equity`
Health Care	Health Care Select Sector SPDR Fund	`XLV US Equity`
Materials	Materials Select Sector SPDR Fund	`XLB US Equity`
Industrials	Industrials Select Sector SPDR Fund	`XLI US Equity`
Real Estate	Real Estate Select Sector SPDR Fund	`XLRE US Equity`
Technology	Technology Select Sector SPDR Fund	`XLK US Equity`
Utilites	Utilites Select Sector SPDR Fund	`XLF US Equity`

Table 1: Sectors, replicating ETF's and corresponding Bloomberg tickers

4.2.4 Implementation of the 5/10/40-Rule

To make the optimization compatible with the 5/10/40-rule, we defined conditions which iterates the optimization algorithm until the rule is fulfilled. The assets which are responsible of breaking the rule receive an upper bound in the next optimization iteration. When the asset allocations are in-line with the 5/10/40-rule, the program is finished. As such, we receive the MAD efficient frontier, while simultaneously adhering to this important rule. The algorithm works in the following manner:

1. Run the optimization.
2. Check if the resulting portfolio fulfill the criteria of the 5/10/40-rule. If this is the case, then you have arrived at your optimal portfolio, `Exit` the algorithm. Otherwise, continue to `step 3`.
3. Assign a ranking to all non-zero assets corresponding to the portfolio weights, in a ascending order.
4. If any asset that is not set to `KEEP`, violated the condition of having a weight over 10%, set an upper bound of 10%.
5. If the portfolio, after the aforementioned correction, violates the rule that the total sum of all assets that are over 5%, sum up to more than 40%, give the asset with the lowest rank, that is not `KEEP` an upper bound of 5%.
6. Run the optimization. Return to `step 1`.

4.2.5 Computational Restrictions

In general portfolio management, certain investors may be prohibited to invest in certain assets due to fund rules or that the fund has to uphold a certain criteria, a sustainability criteria for example. We have not installed a function that takes this into account, which makes it possible to optimize portfolios for prohibited stocks as well. The result can be that the rendered optimal portfolio, may not be feasible to acquire. We leave the responsibility of only optimizing for relevant assets to the user.

A vital restriction to implement has been to not allow for short-selling. Presently, only a limited number of Handelsbanken's mutual funds have a mandate to work with short positions. If the program allowed for short-selling, it would be useless, as most portfolio managers would not be able to implement the allocations in practice.

The model does not take transaction costs or taxes into account. There are optimization models that take this into account, such as the one proposed by Mansini & Speranza (2005) as seen in section 2.1.6. We decided to not implement this aspect as we deemed it difficult to implement and it seems like a niche option which does not fit this optimization tool, which is supposed to be relevant for several different portfolio managers and many different portfolios.

Furthermore, asset liquidity is not taken into aspect. The model assumes that it is possible to buy or sell any number of shares at any time. In practice, this may not be the case.

4.3 Back-testing

To test our models performance we studied historic investment data of three different Handelsbanken funds; fund A, B and C. This process requires approved access from Handelsbanken Fonder, as complete fund holdings are not visible in Bloomberg without the right access. For each fund we gathered monthly holding data over a one year period. Thereafter, we rebalanced the portfolio at the beginning of each month and compared the market value of the rebalanced optimized portfolio with the actual portfolio, assuming no additional changes were made to the portfolio during this month. In these tests, we had expected return set to 4%.

Based on the data we received from this back-testing we calculated a number of values for the optimized portfolio, versus the actual portfolio. The first value we look at is number of wins. The optimized portfolio receive a "win" if its monthly return is greater than the actual portfolio's monthly return and vice versa. Thereafter we compare average monthly return, aggregated monthly return and standard deviation of monthly returns. We also report the average number of assets in the different portfolios, as the optimized portfolio can set weights to 0%. We also compare the difference in monthly returns for the funds against a volatility index to investigate if the market volatility affects the model's performance. The back-testing time frame was chosen to be the length of a year, from 2017-04-03 to 2018-04-03, with 13 data points, corresponding to the first business day of each month. The choice of the number of data points, was chosen due to the time-consuming process of the actual back-testing, which was characterized by several manual procedures. The number of data points was hence, chosen as a balance between yielding a long a enough time series to draw reasonably good conclusions and time consumption.

5 Results

5.1 The Portfolio Optimization Tool

5.1.1 Input

In this master's thesis, the main deliverable is a tool for portfolio optimization to Handelsbanken Fonder. To run the tool, a number of input parameters is required and the input is provided by the user in a custom made Microsoft Excel ribbon. Firstly, the user has to choose which fund to optimize. After choosing this option, all holdings are given to the user in the main worksheet, in the form of the Bloomberg Tickers and the corresponding portfolio weights. All cash positions are bundled into a single variable called cash positions. Furthermore, a drop-down menu is filled with the assets of the portfolio. This allows the user to enter further input in terms of which assets they want to either `BUY`, `SELL` or `OPEN`. After choosing the assets you wish to include in the optimization and its activity, you proceed to enter how much cash you wish to have left in the portfolio. Thereafter, you have to consider the expected return parameter. You can either choose to manually put in the required expected return of the portfolio, or you can use the Black-Litterman option. This alternative lets to user put in a view of how much certain sectors will perform relative others. Based on this, the Black-Litterman model will deliver an expected return percentage that is used in the optimization. Finally, after selecting the parameters explained above, you can run the simulation. Alternatively, you can choose to optimize the whole portfolio. In this scenario you select only the cash and the expected return as all assets are set to `OPEN`.

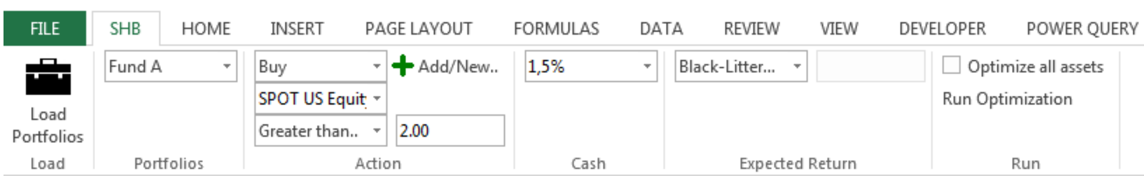

Figure 2: The ribbon in Excel, created and customized for the model

5.1.2 Output

The output is delivered in two steps. Firstly, after selecting the portfolio to optimize, the current holdings and weights are output in the worksheet. Secondly, after the optimization has been run, the assets which were selected to be bought, sold or "open" are presented. Together with the names of these assets, we output the previous portfolio weight, the current portfolio weight and the difference. We also output the remaining cash balance. See figure 3 for an example of the output of an actual optimization, but where the names have been changed and a number of `KEEP`-assets are not displayed. Here we chose one asset to `BUY`, two assets to have `OPEN` and two assets to `SELL`. The rest were automatically assigned to `KEEP`. Remaining cash balance was set to 1%. As we can see, the tool assigned 2.29% to buy in stock A. Everything was sold in stock B and an additional 0.4% was bought in stock C. The selected stocks to sell kept the same weight. As explained in section 4.1.2, the `SELL` means that new weight $\leq$ old weight, which means that sometimes the algorithm suggests to keep the same weight.

Asset	Activity	New weight	Old weight	Difference
Stock A	BUY	2,29%	0,00%	2,29%
Stock B	OPEN	0,00%	1,85%	-1,85%
Stock C	OPEN	4,39%	3,99%	0,40%
Stock D	SELL	4,35%	4,35%	0,00%
Stock E	SELL	4,15%	4,15%	0,00%
Stock F	KEEP	4,88%	4,88%	0,00%
Stock G	KEEP	3,97%	3,97%	0,00%
Cash Position	KEEP	1,00%	1,85%	-0,85%

Figure 3: Example of how the output is displayed after running the portfolio optimization tool

5.1.3 Portfolio Optimization Model

The mathematics behind the portfolio optimization model is Feinstein & Thapa's (1993) reduction of Konno & Yamazaki's (1991) MAD-optimization. Additional conditions to fit the 5/10/40-rule has been added. To solve this linear optimization problem, we use the simplex algorithm. Additionally, there is the option to use the Black-Litterman model to estimate expected return.

5.2 Back-testing

5.2.1 Mutual Funds

For the purpose of back-testing, three different mutual funds of various orientation were chosen by the supervising portfolio Manager Staffan Lindfeldt. These three funds have the placeholder names A, B and C.

Fund A

The back-testing of Fund A shows that the actual portfolio allocation would yield better result than the optimized. While the optimized portfolio had higher returns more often, the average monthly return and the 12 month aggregated return was lower than the actual portfolio allocation. Additionally, the standard deviation was higher for the optimized scenario. We also note that the optimized model set a lot of asset weight's to 0%, as the number of assets is a lot lower in this scenario.

Allocation	Number of wins	Average monthly gated returns	1 year aggre- returns	Standard deviation	Average number of assets
Optimized	7	1.32%	15.8%	3.70%	23
Actual	6	1.46%	17.5%	2.57%	90

Table 2: Results of back-testing of Fund A

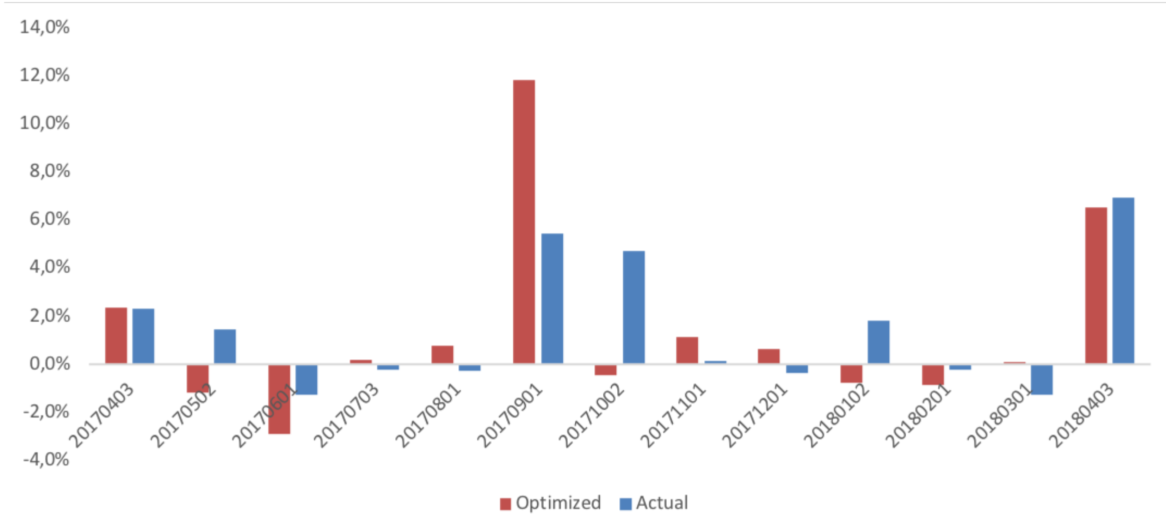

Figure 4: Monthly returns of the optimized portfolio versus the actual portfolio for Fund A

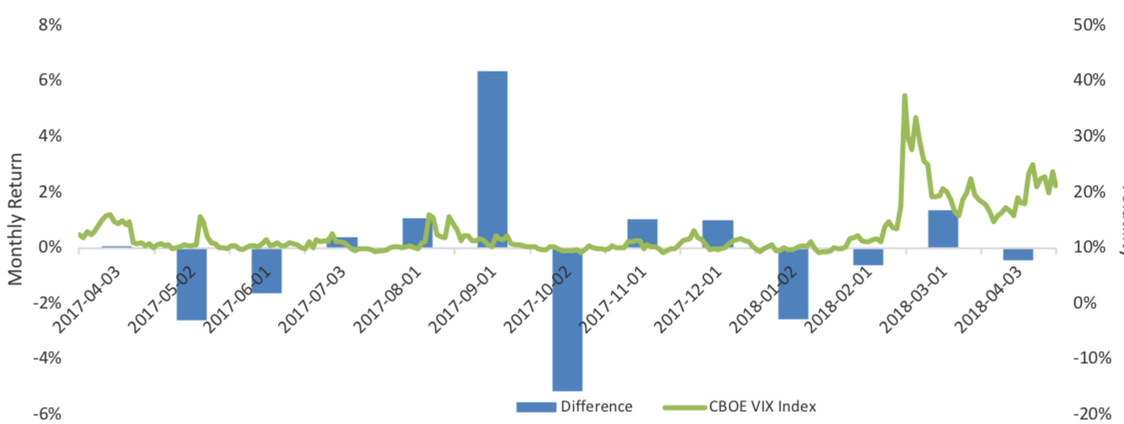

Figure 5: Difference in monthly return plotted against CBOE Volatility Index for fund A

Fund B

Results reveal that Fund B, which typically consists of 20-25 different stocks, would not have performed better if our optimization model was used. Average monthly return was higher in the actual asset allocation and the standard deviation over the monthly returns was lower in the actual asset allocation.

Allocation	Number of wins	Average monthly gated returns	1 year aggre- returns	Standard deviation	Average number of assets
Optimized	6	1.24%	14.9%	2.59%	18
Actual	7	1.38%	16.6%	2.56%	23

Table 3: Results of back-testing of Fund B

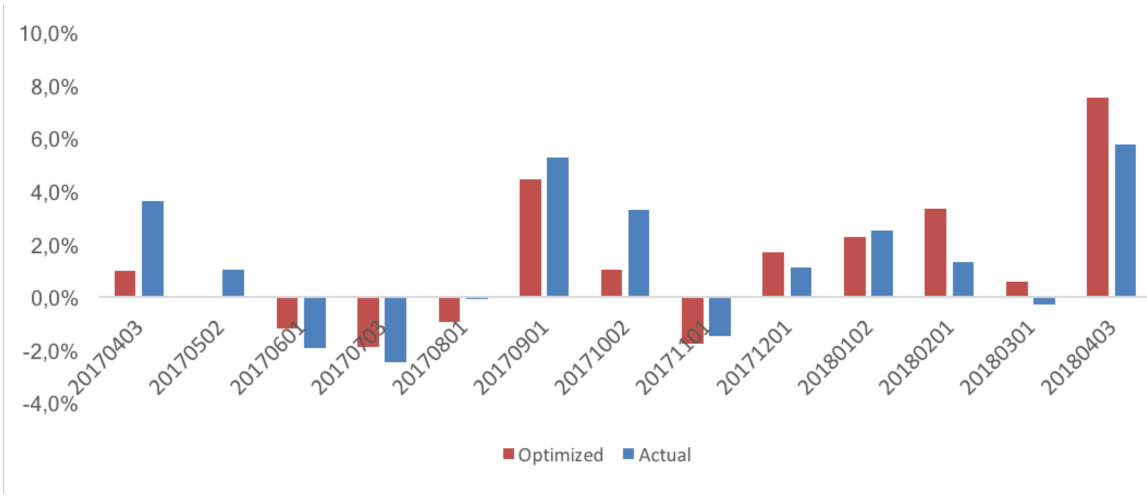

Figure 6: Monthly returns of the optimized portfolio versus the actual portfolio for Fund B

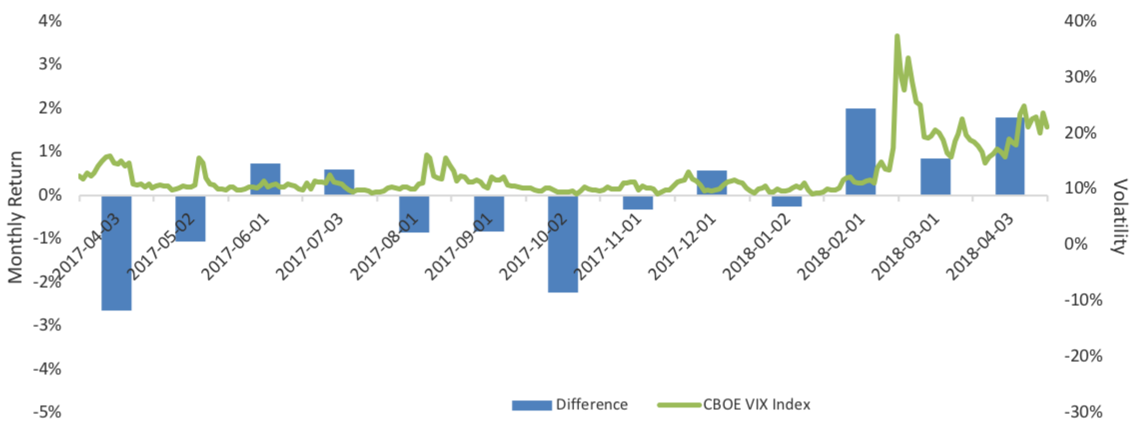

Figure 7: Difference in monthly return plotted against CBOE Volatility Index for fund B

Fund C

The optimized performance of Fund C was worse than the actual portfolio as well. This applies both to standard deviation and expected return. Similar to fund A, the number of assets was greatly reduced.

Allocation	**Number of wins**	**Average monthly returns**	**1 year aggregated returns**	**Standard deviation**	**Average number of assets**
Optimized	5	1.06%	12.7%	3.74%	23
Actual	8	1.75%	21.0%	3.28%	58

Table 4: Results of back-testing of Fund C

Figure 8: Monthly returns of the optimized portfolio versus the actual portfolio for Fund C

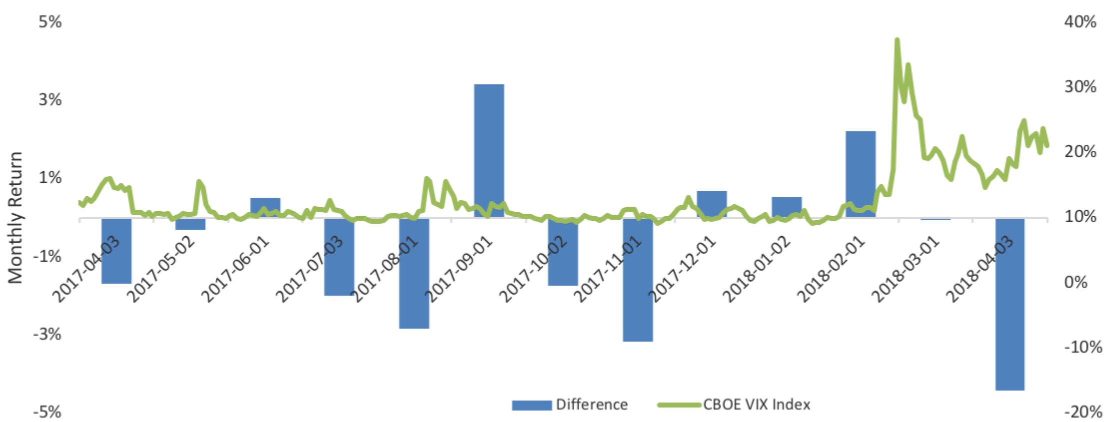

Figure 9: Difference in monthly return plotted against CBOE Volatility Index for fund C

6 Discussion

6.1 The Choice of Portfolio Optimization Model

The choice of optimization model is a vital decision in this thesis as a different model would have presented the user with different asset allocations. The decision of using Feinstein & Thapa's (1993) modification of Konno & Yamazaki's (1991) MAD-model, ultimately came down to the characteristics requested by Handelsbanken, i.e. simplicity, speed and accuracy. We argue that the model we decided to use fit these criteria. We further presented the reasoning behind our choice in section 4.2.1. However, we recognize that there are pros and cons with all models and one may argue that we should have implemented several portfolio optimization models and let the user decide for themselves which one to use. Silva et al. (2017) raised this particular point. On the other hand, this would require the user to be very knowledgeable within portfolio theory, and it would make the program more complex and less friendly to inexperienced users. The processes of implementing several models would also be extremely time consuming. Consequently, we decided to implement only one optimization model and put additional focus on the data-gathering process and the user-friendliness of the software. Instead, we took advantage of previous studies and compared various portfolio optimization models that best fit the requirements put forward by Handelsbanken. This process was in line with purpose of this thesis, to add value to the investment process at Handelsbanken.

6.2 Back-testing

The time frame was chosen to be the length of a year, from 2017-04-03 to 2018-04-03, with 13 data points, corresponding to the first business day of each month. It can be argued that more data points would have led to different results. However, given the results we can with a relatively high level draw the conclusion that an arbitrary fund, would not have performed better if the model was used. However, the results of the back-test could have come out differently if we had chosen another time period. The period from 2017-04-03 to 2018-04-03 has been characterized as a bull market, with relatively low volatility until the beginning of 2018, when the volatility rose. An idea would be to back-test the model during a period characterized by a bear market, e.g. the financial crisis of 2008.

What we noticed during the optimization of these different funds is that the optimization model tend to allocate 0% weight to a number of assets, and instead put larger weight in other assets. This finding is similar to what Levy & Ritov (2001) covered in their study. They discussed the implications of having a long only constraint in mean-variance optimization and found that large portfolios tend to allocate large weights to some assets and 0 weight to others when under this constraint. Releasing this constraint generates very different results that generates a better Sharpe ratio. Our study uses Konno & Yamazaki's (1991) MAD-model, not Markowitz (1952) mean-variance model, but we have similar findings as Levy & Ritov (2001). Many asset weights are indeed 0 and others hit the upper bound of 10%, as imposed from the 5/10/40-rule. A complication of this is that the portfolio becomes less diversified, even though it has the correct weights from a mathematical standpoint. Consequently, we reported a higher standard deviation in monthly returns for all three funds A, B and C in the optimized scenario. The difference in standard deviation in fund A and C was particularly large. This is probably because these funds are quite large (typically around 90 and 60 securities respectively), which made the number of assets heavily reduced in the optimized scenario for these funds. Fund B had low difference in standard deviation, which makes sense since it has the smallest difference in number of assets. In fact, the back-testing against fund B showed promising results as the optimized scenario was close to the actual one, both in terms of expected return and standard deviation, while having 5 our of the 23 assets weighted to 0. From this information we can conclude that if the user wants to optimize all assets of the portfolio, it is best done on a portfolio with a small amount of assets, as the number of assets weighted to zero will be smaller in the optimization.

As a result of the findings in this back-testing, we can not confidently advise the portfolio manager to follow the suggested weighting in our software, since it would mean that they would have to sell off a large portion of the portfolio. This is likely not a feasible course of action in practice. As Levy & Ritov (2001) discussed, the long-only constraint has large effects on the weights of the optimized portfolio and that is apparent in our study as well. However, the results can still be valuable as it provides the user with information on the assets that are preferred by the optimization model. This information can help the portfolio manager in the decision-making, as it becomes clear which assets are preferred over others, from a mathematical standpoint. However, the MAD-optimized portfolio with the long-only constraint should not be considered "optimal". It should be noted that the aim of the model has never been to out-perform the actual asset allocation. The purpose of the tool is to complement the portfolio manager in a small subset of the investment process, where a robust mathematical approach can add to the experience and sound investment strategies that have helped Handelsbanken add value to their customers historically.

For smaller sets of consecutive data points under periods of high volatility, especially for fund B, we can identify an overall better performance of the asset allocation suggested by the optimization model. The result suggests, that for portfolios with a lower total number of assets, the optimization model finds an asset allocation that perform well under times of high volatility. Given the rather low sample size, we can however not with certainty conclude that this is not just due to chance.

6.3 The Application of the Model

During the back-testing, all assets of the portfolio were given the `OPEN`-activity, meaning all assets are included in the optimization. This approach made sense from a testing standpoint, but it is not the only way the model can be used. In fact, its main purpose is to compare fewer alternatives. To provide the reader with a sense of the model's usability, we will provide a few examples.

- If the portfolio manager has spare cash and is considering buying 3 different stocks, but is not sure how much weight to put in each, then this tool can provide the portfolio manager with the weighting of these 3 assets that is mathematically optimal, which could serve as basis for an investment decision.
- If the portfolio manager is uncertain what to do with some of the assets, the user can select the `OPEN`-alternative for these assets to let the model allocate the weights, without defining if it should buy or sell.
- As performed in the back-testing, if the portfolio manager wants to get a new view of the assets of a portfolio should be allocated, then the user can select the "optimize all" alternative. This will allocate all assets and give the user a sense of the efficient frontier allocation.

7 Conclusion

The purpose of this thesis has been to create and deliver a decision support tool to provide quantitative input to the portfolio construction process at Handelsbanken Fonder. The model was created using Konno & Yamazaki's Mean Average Optimization method, with a Feinstein & Thapa modification. Additionally, the Black-Litterman model was implemented to approximate the input of expected return. The linear optimization problem was then solved by the Simplex algorithm. The delivered model was programmed in VBA and utilizes the Bloomberg API to collect data.

The results from the back-test showed that, generally, none of the funds would have performed better if the optimization model would have been used to freely choose the asset allocation of the whole portfolio. The actual historical asset allocation provided both higher average monthly return and lower standard deviation. That is, higher return for lower risk. This is a consequence of only allowing long-positions, which caused the optimized portfolios to become more sparse and consequently less diversified. It should however be noted that the context in which the model was back-tested, is not the environment for which the model was primarily created. The tool is not, in any sense, supposed to replace fund managers. The objective of the tool is instead to give portfolio managers an alternative view and assist them in finding allocations along the efficient frontier. For this purpose, we argue that the delivered optimization model is likely to be valuable to the portfolio managers at Handelsbanken Fonder.

8 Further Research & Applications

One interesting topic of discussion in this thesis would be to compare the results of our model with other models. However, we considered this to be out of the scope of this thesis as this has mainly been an implementation project. Research which compare optimization models has been done previously, such as by Bower & Wentz (2005) and Silva et al. (2017). However, we argue there is room for additional empirical research within this field. Bower & Wentz (2005) based their research on small portfolio's consisting of five S&P500 stocks and one bond. Silva et al. (2017) had larger portfolios, but based their study on the Brazilian market. While it is likely there are additional studies than these two, we see that there is possibility for empirical research that could add value to the field of portfolio theory. Testing different portfolio models with varying size, time period and market would indeed be interesting. This could further reveal the robustness of the models.

Another interesting topic would be to compare different software in portfolio optimization. In section 3.7 we mentioned examples of optimization software on the market. It would have been interesting to compare them against each other and our own model, but here we are limited by our lack of access to these systems. Based on Infanger (2011), it appears like the underlying mathematical model used in these software is either a form of MPT or MAD. If the underlying mathematics and the input is the same, then the software should produce the same results. If so, then the cheapest alternative should be preferable to an asset manager, assuming we disregard other aspects. This analysis could certainly provide value to portfolio managers as many optimization software are burdened by high costs.

The back-testing of our model was performed over a recent time period. Considering that equity markets generally have been generating strong returns over the last few years, it would have been interesting to test our model in a market trending downwards. The financial crisis of 2007-2008 and the following years would have been a good example to see if different results would have been achieved in this time period. We did not do this analysis since the funds we back-tested against do not have data over this time period.

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