On t-testing with bootstrap quantiles

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Abstract

We consider an i.i.d. sample from a probability distribution with mean μ and variance σ^2 and denote by $\hat{\mu}$ and $\hat{\sigma}^2$ sample mean and sample variance. Classical statistical theory, assuming normal distribution sampling, gives studentized confidence bounds $\hat{\mu} - q\,\hat{\sigma}/\sqrt{n}$ for μ, with q being a suitable quantile of a suitable t-distribution. We however consider, in order to avoid the normal assumption, replacing q by a "sample-adjusted" $\hat{q}$.

Several $\hat{q}$-methods have been suggested over the past decades of statistical theory, with recent additions and synthesis provided by methods from the more general framework of bootstrap confidence theory. But is there really any agreement on whether methods from bootstrap theory can outperform older methods based on third and fourth order sample moments and edgeworth expansion theory? And how well do the methods really perform?

We present results from an empirical investigation (computer simulations) attempting to reveal and explain accuracy behaviour (with respect to the desired level of confidence) of the main methods on sample size $15, 30, 50, 75$ for some interesting spectra of underlying distributions. Some general conclusions will be attempted.