Infinitely iterated Brownian motion

Let $B_1, B_2, \ldots$ be independent one-dimensional Brownian motions parametrized by the whole real line such that $B_i(0) = 0$ for every $i \geq 1$. We consider the nth iterated Brownian motion $W_n(t) = B_n(B_{n-1}(\cdots(B_2(B_1(t)))\cdots))$. Such a process first appeared in a paper by Funaki as a probabilistic construction of the solution of higher order diffusion equations. The process received some attention by the probability community (Burdzy, Khoshnevisan, Shi, Eisenbaum, Bertoin) in relation to the pathwise properties of this non-semimartingale process. In this talk, we will review some of the fascinating properties of W_n such as Burdzy's result that, given a path of W_2, one can recover the paths B_1, B_2 up to a sign. We will then show that W_n has a limit, in a certain very weak sense, as $n \to \infty$, which is exchangeable and, therefore, by the de Finetti-Hewitt-Savage theorem, a conditionally independent collection of random variables. We identify the object, μ_∞, we condition on as being the limit of the random occupation measures, on $[0, 1]$, of W_n. It turns out that μ_∞ has almost surely finite support and continuous (random) density. The limiting marginal distributions have some rather curious properties, but only the 1-dimensional ones are explicitly known (from which we can obtain a new characterization of the exponential distribution). This talk is based on joint work with Nicolas Curien, Université Paris VI.

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