SF2524

Gram-Schmidt variants

Problem:

Given orthonormalized vectors

$q_1, q_2, \dots q_k \in \mathbb{R}^m$ and $w \in \mathbb{R}^m$

with $w \notin \operatorname{span}(q_1, \dots q_k)$

Compute $h_1, h_2, \dots h_k, \beta \in \mathbb{R}$

and $q_{k+1} \in \mathbb{R}^m$ such that

$\|q_{k+1}\| = 1$ and q_{k+1} orthogonal

against $q_1, \dots q_k$ and

$$w = h_1 q_1 + \dots h_k q_k + \beta q_{k+1}$$

Classic Gram-Schmidt CGS

"single GS"

$h = Q^T w$

$y = w - Q h$

$\beta = \text{norm}(y)$

$q_{k+1} = y / \beta$

Modified Gram-schmidt MGS

$y = w$

for $i = 1, \dots k$

$h_i = q_i^T y$

$y = y - q_i h_i$

end

Version 1

Double Gram-Schmidt $\checkmark$ =

Repeated Gram-Schmidt

$$h = Q^T w$$

$$y = w - Qh$$

$$g = Q^T y$$

$$y = y - Qg$$

$$h = h + g$$

$$\beta = \| y \|$$

$$q_{k+1} = y / \beta$$

Double Gram-Schmidt version 2.

$h = 0$

$\beta = 0$

$y = w$

for $k = 1:2$

$\quad g = Q^T y$

$\quad y = y - Qg$

$\quad \gamma = \| y \|$

$\quad \beta = \beta + \gamma$

$\quad h = h + g$

$\quad y = y / \gamma$

end

$q_{k+1} = y$