# Convergence and restart of Arnoldi method for eigenvalue problems

## SF2524 - Matrix Computations for Large-scale Systems

2015-11-05

* Illustrate convergence with matlab demo *

Example of convergence theory of the Arnoldi method for eigenvalue problems:

## Theorem (Jia, SIAM J. Matrix. Anal. Appl. 1995)

Let $Q_n$ and $H_n$ be generated by the Arnoldi method and suppose $\lambda_i^{(n)}$ is an eigenvalue of $H_n$. Assume that $\ell_i = 1$ and the associated value $\|(I - Q_n Q_n^T)x_i\|$ is sufficiently small. Let $P_i^{(n)}$ be the spectral projector associated with $\lambda_i^{(n)}$. Then,

$$|\lambda_i^{(n)} - \lambda_i| \leq \|P_i^{(n)}\|\gamma_n \frac{\|(I - Q_n Q_n^T)x_i\|}{\|Q_n Q_n^T x_i\|} + \mathcal{O}\left(\frac{\|(I - Q_n Q_n^T)x_i\|^2}{\|Q_n Q_n^T x_i\|^2}\right)$$

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The theorem is not a part of the course. In this course we will gain qualitative understanding by bounding

$$\|(I - Q_n Q_n^T)x_i\|.$$