Exercises

The following are recommended exercises.

Chapter 2

2.1 Let $X : \Omega \to \{1, 2, \cdots, k\}$ be a stochastic variable and $\sigma(X)$ be the sigma-algebra generated by X. Then X can be written,

$$X(\omega) = \sum_{\ell=1}^{k} \ell I_{\{X(\omega)=\ell\}} = \sum_{\ell=1}^{k} \ell I_{\{\omega \in X^{-1}(\ell)\}},$$

where I_A is the indicator function of the set A.

Assume that $Y \in L^2(\Omega, \sigma(X), P)$, express Y in terms of the indicator functions.

Extra:
Assume that $Z \in L^2(\Omega, \mathcal{A}, P)$, where $\sigma(X) \subset \mathcal{A}$, determine the conditional expectation $\mathrm{E}[Z|X]$.

2.2 Show that $\mathbf{A} \perp \mathbf{B}|\mathbf{X}$ implies $\mathrm{E}^{\mathbf{A}\vee\mathbf{X}}\beta = \mathrm{E}^{\mathbf{X}}\beta$ for all $\beta \in \mathbf{B}$.

2.3 The definition of a (Hilbert-)adjoint operator is as follows.
If $T : H_1 \to H_2$ is a bounded linear operator, where H_1 and H_2 are Hilbert spaces. Then the Hilbert-adjoint operator T^* *of T is the operator $T^* : H_2 \to H_1$ such that for all $x \in H_1$ and $y \in H_2$,*

$$\langle Tx, y\rangle = \langle x, T^* y\rangle.$$

We have claimed that the adjoint of the shift operator $U_t : \mathbf{H} \to \mathbf{H}$ is $U_t^* = U_t^{-1}$, and this is easy to verify from the definition. The adjoint of the adjoint U_t^* is then U_t.

If we instead consider this operator U_t^* on the invariant subspace $\mathbf{X}^-$, i.e. $U_t^* : \mathbf{X}^- \to \mathbf{X}^-$, it's adjoint is no longer U_t, why ?

On page 21 of the book, the compressed right shift

$$\begin{array}{rcl} T_t : \mathbf{X}^- & \to & \mathbf{X}^- \\ \xi & \mapsto & \mathrm{E}^{\mathbf{X}^-} U_t \xi \end{array}, \quad t \geq 0,$$

is introduced and it is claimed that this is the adjoint of U_t^* on the invariant subspace $\mathbf{X}^-$. Verify this from the definition.

The purpose of considering the compressed right shift is to characterize the Markov property in terms of invariance for the operator T as in Proposition 2.6.1.

Chapter 3

3.1 Consider example 3.2.2. Use the expression for $y(t)$ to derive the expression for $\Lambda(\tau)$. Then use the expression given for F to derive the expression for $\Lambda(\tau)$ by determining the integral in (3.2.1).

3.2 Consider Theorem 3.5.5.

First note the difference between the operators $\mathcal{I}_w$ and $\mathcal{I}_{\hat{w}}$. The first one is actually the sum defined in Theorem 3.5.1 and the second is the stochastic integral with respect to the stochastic measure $d\hat{w}$. (In continuous time (cor 3.5.2) it is an integral and the notation makes more sense)

Note that according to relation (3.5.6) arrows corresponding to the map $\mathcal{I}_w$ can be added between the spaces ℓ^2_m and $\mathbf{H}(w)$ in the commutative diagram of the theorem.

Take an $f(t) \in \ell^2$ and try to see how it is mapped around by the operators $\mathcal{I}_w$, $\mathcal{F}$, T and so on through the commutative diagram.

You could start with $f(t) = \delta_t$, i.e. let $f(0) = 1$ and $f(t) = 0$ for all other t.

3.3 We will consider a discrete time version of Example 3.6.2. So let x, y be a joint stationary process generated by the linear stochastic system

$$\begin{cases} x(t+1) &= Ax(t) + Bw(t), \\ y(t) &= Cx(t) + Dw(t), \end{cases}$$

where w is a normalized white noise process, i.e. it's spectral distribution function F_w satisfies $dF_w(\theta) = \frac{1}{2\pi}d\theta$. Assume that A is a stability matrix so the sum

$$x(t) = \sum_{k=0}^{\infty} A^k Bw(t-k-1)$$

converges. (and it will not matter how x was initiated)

Corresponding to the white noise w, there is a spectral measure $d\hat{w}$ such that

$$w(t) = \int_{-\pi}^{\pi} e^{i\theta t} d\hat{w}(\theta).$$

According to Theorem 3.3.1 there is an $\hat{f}$ such that $x(0)$ has a spectral representation

$$x(0) = \int_{-\pi}^{\pi} \hat{f}(e^{i\theta}) d\hat{w}(\theta) = \mathcal{I}_{\hat{w}}(\hat{f}),$$

and then

$$x(t) = \int_{-\pi}^{\pi} e^{it\theta} \hat{f}(e^{i\theta}) d\hat{w}(\theta) = \mathcal{I}_{\hat{w}}(e^{it\theta} \hat{f}),$$

Determine $\hat{f}$.

Since $x(0)$ is spanned by the white noise, it follows from Theorem 3.5.1 that

$$x(0) = \sum_{s=-\infty}^{\infty} f(-s)w(s) = \mathcal{I}_w(f),$$

where $f \in \ell_m^2$, and

$$x(t) = \sum_{s=-\infty}^{\infty} f(t-s)w(s) = \mathcal{I}_w(T^t f).$$

Determine f, and show that $\hat{f}$ is the Fourier transform of f.

Now we will consider the process y. First note that y has a spectral representation in terms of an orthogonal increment process $\hat{y}$ such that $d\hat{y}(\theta) = W(e^{i\theta})d\hat{w}(\theta)$, i.e.

$$y(t) = \int_{-\pi}^{\pi} e^{it\theta} d\hat{y}(\theta) = \mathcal{I}_{\hat{y}}(e^{it\theta}) = \int_{\pi}^{\pi} e^{it\theta} W(e^{i\theta}) d\hat{w}(\theta) = \mathcal{I}_{\hat{w}}(e^{it\theta} W),$$

for some function W. Determine W.

Finally, use Theorem 3.1.3 and the spectral representation of y to derive an expression for

$$\Lambda(\tau) = \mathrm{E}\{y(t)\overline{y(t+\tau)}\},$$

and determine the spectral distribution function F_y of y. (we can assume that y is scalar)

Chapter 4

4.1 Show that the rational function

$$F(z) = \frac{2z^3 - 11z^2 + 17z - 6}{5z^3 + 3z^2 + z + 2}$$

is in $\mathcal{H}^2$, and then determine the inner-outer factorization.

4.2 Assume that x and y are two jointly stationary second order processes, with spectral density given by

$$\Phi(z) = \begin{bmatrix} \Phi_x & \Phi_{xy} \\ \Phi_{yx} & \Phi_y \end{bmatrix} = \begin{bmatrix} \frac{200+40(z+z^{-1})}{3-(z+z^{-1})} & \frac{1}{z+1/2} \\ \frac{1}{z^{-1}+1/2} & e^{z+z^{-1}} \end{bmatrix}.$$

Determine a cascade implementation of the causal Wiener filter for estimating x based on the past values of y.

Answer by giving W and $\hat{F}$

Hint: If w is a process determined by filtering y with W^{-1}, then $\Phi_{xw} = \Phi_{xy}W^{-1}$.

Would the result be different if we exchange the expressions for Φ_{xy} and Φ_{yx} ?

4.3 Use the Beurling-Lax Theorem 4.6.4 to show that there exists an inner function Q such that

$$\mathcal{Y} := \{W(z) \in \mathcal{H}^2 | W(z_k) = 0, |z_k| > 1, k = 1, \cdots, n\} = \mathcal{H}^2 Q,$$

and determine Q.

Furthermore, show that if f_0 belongs to

$$\mathcal{J} := \{W(z) \in \mathcal{H}^2 | W(z_k) = \alpha_k, |z_k| > 1, k = 1, \cdots, n\},$$

then an arbitrary $f \in \mathcal{J}$ can be decomposed as $f = f_0 + g$ where $g \in \mathcal{H}^2 Q$.

For those interested, I mention here that the orthogonal complement $\mathcal{K}$ of $\mathcal{H}^2 Q$ in $\mathcal{H}^2$, i.e. $\mathcal{K} = \mathcal{H}^2 \ominus \mathcal{H}^2 Q$ plays a key role in interpolation theory. $\mathcal{K}$ is usually called the coinvariant subspace since it is invariant with respect to the compressed shift in $\mathcal{H}^2$, compare exercise 2.3, it is n-dimensional and it is spanned by the terms in the partial fraction representation of Q. This space is introduced in chapter 9 in the book.

Chapter 6

6.1 Consider the forward stochastic system

$$\Sigma \quad \begin{cases} x(t+1) &= Ax(t) + Bw(t) \\ y(t) &= Cx(t) + Dw(t) \end{cases},$$

where

$$A = \begin{bmatrix} 0 & 0 \\ 1/2 & 1/2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 2/3 \end{bmatrix}, D = 0;$$

and $w(t)$ is a normalized white noise.

(a) Determine the transfer function $W(z)$ for Σ.
Is Σ a minimal (deterministic) realization for W ?

(b) Determine the backward system $\bar{\Sigma}$ corresponding to the same state space $\mathbf{X}_t$ as for Σ.

(c) Determine the transfer function $\bar{W}(z)$ for $\bar{\Sigma}$.
Is $\bar{\Sigma}$ a minimal (deterministic) realization for $\bar{W}$?

(d) Is Σ a minimal (stochastic) realization for Σ ?

(e) Determine the structural function K of $(\Sigma, \bar{\Sigma})$.
What are the poles and zeros of K ?

6.2 During the derivation of the backward system in class we used the equations:

$$\mathrm{E}x(t)x(t+1)' = PA'$$

and

$$\mathrm{E}y(t)x(t+1)' = CPA' + DB'.$$

Prove them using the state equations and the stationary state covariance $P = \mathrm{E}x(t)x(t)'$.

6.3 Use the state equations to show the expression (6.6.3)

$$\Lambda_t = \begin{cases} CA^{t-1}\bar{C}' & \text{for } t > 0; \\ CPC' + DD' & \text{for } t = 0; \\ \bar{C}(A')^{|t|-1}C' & \text{for } t < 0, \end{cases}$$

for the output covariances.

6.4 As in proposition 6.9.1 we show here the expressions for the Kalman gain and the recursion satisfied by the covariance matrix of the state estimate $\hat{x}(t)$.

The Kalman gain is defined as the matrix that satisfies

$$\mathrm{E}^{[\tilde{y}(t)]}x(t+1) = K(t-\tau)\tilde{y}(t),$$

where $\tilde{y}(t) = y(t) - \hat{y}(t)$ and $\hat{y}(t) = C\hat{x}(t)$.

Then from (2.2.5) in the book, we have

$$K(t-\tau) = \mathrm{E}\{x(t+1)\tilde{y}(t)\} \left(\mathrm{E}\{\tilde{y}(t)\tilde{y}(t)'\}\right)^{-1}.$$

Now show (6.9.7) and (6.9.8) by completing the following steps:

(a) Use the orthogonality $\tilde{y}(t) \perp \hat{y}(t)$ to show that

$$\mathrm{E}\{\tilde{y}(t)\tilde{y}(t)'\} + \mathrm{E}\{\hat{y}(t)\hat{y}(t)'\} = \mathrm{E}\{y(t)y(t)'\}.$$

(b) Use the expression from (a) to show that

$$\mathrm{E}\{\tilde{y}(t)\tilde{y}(t)'\} = \Lambda_0 - C\Pi(t-\tau)C',$$

where $\Lambda_0 = \mathrm{E}y(t)y(t)'$ and $\Pi(t) = \mathrm{E}\{\hat{x}(t+\tau)\hat{x}(t+\tau)'\}$.

(c) Show that

$$\mathrm{E}\{x(t+1)\tilde{y}(t)'\} = \bar{C}' - A\Pi(t-\tau)C'$$

using the state equations $x(t+1) = Ax(t) + Bw(t)$, $y(t) = Cx(t) + Dw(t)$, where w is normalized white noise, i.e. $\mathrm{E}w(t)w(s) = \delta_{t,s}I$, the state covariance matrix $P = \mathrm{E}\{x(t)x(t)'\}$, and the definition $\bar{C} = CPA' + DB'$.

Hint: Show first that $\tilde{y}(t) = C(x(t) - \hat{x}(t)) + Dw(t)$.

(d) Use the equation

$$\hat{x}(t+1) = A\hat{x}(t) + K(t-\tau)(y(t) - C\hat{x}(t)), \quad \hat{x}(\tau) = 0,$$

to show the recursion $\Pi(0) = 0$, and

$$\Pi(t+1) = \Pi(t) + R(\Pi(t)), \quad t = 0, 1, \dots$$

i.e.

$$\Pi(t+1) = A\Pi(t)A' + (\bar{C}' - A\Pi(t)C')\left(\Lambda_0 - C\Pi(t)C'\right)^{-1}(\bar{C}' - A\Pi(t)C')'.$$

Hint: Show that $\hat{x}(t) \perp \tilde{y}(t)$ and use that $K(t-\tau) = (\bar{C}' - A\Pi(t)C')\left(\Lambda_0 - C\Pi(t)C'\right)^{-1}$ as shown in (b) and (c).

6.5 Let $\tilde{x}(t) = x(t) - \hat{x}(t)$. Show that

$$P - \Pi(t) = \mathrm{E}\{\tilde{x}(t+\tau)\tilde{x}(t+\tau)'\}.$$

This is used in Lemma 6.9.2 to show that $\Pi(t)$ is upper bounded by any $P \in \mathcal{P}$.

6.6 The (forward) system

$$\Sigma \begin{cases} x(t+1) &= \frac{4}{5}x(t) + \begin{bmatrix} 3\sqrt{6/5} & 0 \end{bmatrix} u(t), \\ y(t) &= \frac{1}{5}x(t) + \begin{bmatrix} \frac{1}{3}\sqrt{6/5} & 1/3 \end{bmatrix} u(t), \end{cases}$$

is given. This system generates a stationary stochastic process with the spectral density $\Phi(z)$. (which need not be determined in this problem)

(a) Determine the positive real part $\Phi_+(z)$ of the spectral density $\Phi(z)$.

(b) The Positive Real lemma (Kalman-Yakubovich-Popovs lemma) states that $\Phi_+(z)$ is positive real if and only if a certain set $\mathcal{P}$ is non-empty.

Determine the set $\mathcal{P}$ for $\Phi_+(z)$ from a).

(c) Determine the forward model Σ_- of the predictor space $H^{+/-}$ for the y-process.

(d) Determine the forward model Σ_+ of the backward predictor space $H^{-/+}$ for the y-process.

(e) Determine the backward model $\bar{\Sigma}_+$ of the backward predictor space $H^{-/+}$ for the y-process.

(f) The student "Ture Teknolog" has determined the Kalman filter for Σ and calculated the covariance for the state estimate $\hat{x}(4)$ given measurements $y(0), y(1), y(2), y(3)$. He got

$$\mathrm{E}\left\{\hat{x}(4)^2\right\} = 27.$$

Is the result reasonable ? Explain.
(The Kalman filter need not be determined)

(g) What is the difference between the Kalman filter for Σ and Σ_+ ?

6.7 Assume that

$$a(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n,$$
$$\sigma(z) = \sigma_0 z^n + \sigma_1 z^{n-1} + \cdots + \sigma_n,$$

and

$$\frac{\sigma(z)}{a(z)} = \sum_{k=0} w_k z^{-k}. \tag{1}$$

Show that, then

$$
\begin{bmatrix} w_1 & w_2 & \cdots & w_n & w_{n+1} \\ w_2 & w_3 & \cdots & w_{n+1} & w_{n+2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ w_n & w_{n+1} & \cdots & w_{2n-1} & w_{2n} \end{bmatrix} \begin{bmatrix} a_n \\ a_{n-1} \\ \vdots \\ a_1 \\ a_0 \end{bmatrix} = 0
$$

and

$$
\begin{bmatrix} \sigma_0 \\ \sigma_1 \\ \vdots \\ \sigma_{n-1} \\ \sigma_n \end{bmatrix} = \begin{bmatrix} w_0 & 0 & \cdots & 0 & 0 \\ w_1 & w_0 & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ w_{n-1} & w_{n-2} & \cdots & w_0 & 0 \\ w_n & w_{n-1} & \cdots & w_1 & w_0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \\ a_n \end{bmatrix}.
$$

Hint: Multiply both sides of (1) with $a(z)$ and identify coefficients.

6.8 An eigth order linear model has been used to generate the data sequence y that can be obtained from the course homepage.

Use that data to estimate covariances by the truncated ergodic sum:

$$
\hat{r}_k = \sum_{\ell=1}^{N-k} y_\ell y_{\ell+k}.
$$

Then use exercise 6.7 to find a realization of a linear model, with transfer function Φ_+, matching the covariances $\hat{r}_0/2, \hat{r}_1, \cdots, \hat{r}_{16}$.

Show that the function Φ_+ is positive real. Then determine the spectral factor W_- of $\Phi(z) = \Phi_+(z) + \Phi_+(z^{-1})$ corresponding to the forward predictor space.

Hint: To do the last step, it could be useful to iterate equation (6.9.8), or to use the command dare *in Matlab.*

The spectral density of the data sequence y can be estimated in Matlab using the command “psd(y)”. Compare this estimate with the spectral density corresponding to W_- and the spectral density of the generating model. The spectral density of W_- can be plotted using the program “modelmag.m” available at the homepage. The data for the spectral density of the generating model is included in the data file “y_data.mat”.

What happens if you try to match a fourth order model instead ?

Chapter 7

7.1 Assume that $\mathbf{H}^-$ and $\mathbf{H}^+$ are conditionally orthogonal with respect to $\mathbf{X}$, i.e. $\mathbf{H}^- \perp \mathbf{H}^+ | \mathbf{X}$.

Show that
$$\left\langle \mathrm{E}^{\mathbf{X}}\lambda, \mathrm{E}^{\mathbf{X}}\mu \right\rangle = \langle \lambda, \mu \rangle \,,$$
for all $\lambda \in \mathbf{H}^-$ and $\mu \in \mathbf{H}^+$.

Hint: It may be useful to use the decomposition $\xi = \mathrm{E}^{\mathbf{X}}\xi + \mathrm{E}^{\mathbf{X}^\perp}\xi$.

7.2 Let $\mathbf{X}$ be a splitting subspace.

Applying the decomposition $A = \mathrm{E}^A B \oplus (A \cap B^\perp)$ on $\mathbf{X}$ together with $\mathbf{H}^-$ and $\mathbf{H}^+$ we get the two equations (7.3.12):

$$\mathbf{X} = \mathrm{E}^{\mathbf{X}}\mathbf{H}^+ \oplus \left(\mathbf{X} \cap (\mathbf{H}^+)^\perp\right)$$
$$\mathbf{X} = \mathrm{E}^{\mathbf{X}}\mathbf{H}^- \oplus \left(\mathbf{X} \cap (\mathbf{H}^-)^\perp\right)$$

Since $\mathbf{X} \cap (\mathbf{H}^+)^\perp$ is the unobservable part and $\mathbf{X} \cap (\mathbf{H}^-)^\perp$ is the unconstructible part one could suspect that $\mathbf{X}$ is still a splitting subspace if these parts are removed.

In fact, Lemma 7.3.4 say that if $\mathbf{X}$ is a splitting subspace, then $\mathrm{E}^{\mathbf{X}}\mathbf{H}^+$ and $\mathrm{E}^{\mathbf{X}}\mathbf{H}^-$ are splitting subspaces.

Use the two equations above and Lemma 7.3.3 to prove Lemma 7.3.4.

7.3 This example will illustrate why $(\mathbf{H}^-)^\perp$ is not a subspace of $\mathbf{H}^+$, even if $\mathbf{H}^- \vee \mathbf{H}^+ = \mathbb{H}$.

Let (x, y, z) be coordinates in $\mathbb{R}^3$. Now introduce the following subspaces. Let $\mathbf{H}^-$ be the subspace $z = 0$, which is a plane in $\mathbb{R}^3$. Let $\mathbf{H}^+$ be the subspace $z = x$, which also is a plane in $\mathbb{R}^3$. Then $\mathbb{R}^3 = \mathbf{H}^- \vee \mathbf{H}^+$.

Determine $(\mathbf{H}^-)^\perp$ and show that it is not a subspace of $\mathbf{H}^+$.

What is $(\mathbf{H}^- \vee \mathbf{H}^+) \ominus \mathbf{H}^-$?

7.4 Assume that $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ are subspaces of some Hilbert space $\mathbf{H}$.

Show that $(\mathbf{A} \vee \mathbf{B}) \cap \mathbf{C} = (\mathbf{A} \cap \mathbf{C}) \vee (\mathbf{B} \cap \mathbf{C})$ does not hold in general.

Hint: It is easy to find a counter example with $\mathbf{H} = \mathbb{R}^2$.

7.5 Show that

$$\mathrm{E}^{\mathbf{H}^-}\mathbf{H}^{\Box} = \mathbf{X}_-,$$

and

$$\mathrm{E}^{\mathbf{H}^+}\mathbf{H}^{\Box} = \mathbf{X}_+.$$

Hint: Proposition 7.4.13 could be useful

7.6 Is the Frame space $\mathbf{H}^{\Box}$ a minimal Markovian splitting subspace ?

(In general? / In some special situation ?)

We know that $\mathbf{H}^{\Box} \sim (\mathbf{S}, \bar{\mathbf{S}})$ for $\mathbf{S} = (\mathbf{N}^+)^{\perp}$ and $\bar{\mathbf{S}} = (\mathbf{N}^-)^{\perp}$.

Use Theorem 7.4.3 to determine $(\mathbf{S}_1, \bar{\mathbf{S}}_1)$, and then $\mathbf{X}_1 = \mathbf{S}_1 \cap \bar{\mathbf{S}}_1$.

Chapter 8

8.1 Use the same data as in exercise 6.8.

An approach to realizing a system from the data is to do something like this:

We would like to define a state. Let

$$x(t) = \begin{bmatrix} y(t-1) \\ y(t-2) \\ \vdots \\ y(t-n) \end{bmatrix},$$

and we will assume that $\mathbf{X} = \{a'x(0) | a \in \mathbb{R}^n\}$ is a Markovian splitting subspace. Is $\mathbf{X}$ an internal state ?

We could estimate a dynamics matrix A by minimizing

$$\left\| \sum_{t=n}^{N-1} x(t+1) - Ax(t) \right\|^2.$$

To solve this least-squares problem it can be useful to form the matrices:

$$H_0 = \begin{bmatrix} y(n) & y(n+1) & \cdots & y(N-1) \\ y(n-1) & y(n) & \cdots & y(N-2) \\ \vdots & \vdots & \ddots & \vdots \\ y(1) & y(2) & \dots & y(N-n-1) \end{bmatrix},$$

and

$$H_1 = \begin{bmatrix} y(n+1) & y(n+2) & \cdots & y(N) \\ y(n) & y(n+1) & \cdots & y(N-1) \\ \vdots & \vdots & \ddots & \vdots \\ y(2) & y(3) & \dots & y(N-n) \end{bmatrix}.$$

This method for determining A should be equivalent to approximating (8.3.19) with truncated ergodic sums, i.e.

$$\begin{aligned} A &= \mathrm{E}\{x(1)x(0)'\}P^{-1} \\ &\approx \frac{1}{N-n-1}\sum_{t=n+1}^{N-1} x(t+1)x(t)' \left(\frac{1}{N-n}\sum_{t=n+1}^{N} x(t)x(t)'\right)^{-1}. \end{aligned}$$

Use one of these methods to estimate A for the case $n = 8$.

To estimate B, we could again proceed in different ways. First, define $w(t) = x(t+1) - Ax(t)$. Assuming that the system is driven by normalized white noise, B can be estimated by factoring

$$\mathrm{E}w(t)w(t)' \approx \frac{1}{N-n}\sum_{t=n+1}^{N} w(t)w(t)'.$$

Finally, C and D can be obtained by noting that $y(t)$ is the first component of $x(t+1)$, and extracting the corresponding parts of A and B.

As in exercise 6.8, the spectral density of the data sequence y can be estimated in Matlab using the command "psd(y)". Compare this estimate with the spectral density corresponding to the model determined here and the spectral density of the generating model.

8.2 Consider a state space (forward) system with matrices

$$A = \begin{bmatrix} 0 & -2/3 & -1/3 \\ -1/3 & 0 & -2/3 \\ -2/3 & -1/3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 5/3 \\ -1/3 \\ -4/3 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & -1 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 1 \end{bmatrix}$$

Generate a stationary stochastic process y by feeding white noise through this system. This can be acchieved by using "dlsim(A,B,C,D,U,X0)" in

Matlab, where U is the white noise that can determined by “randn(1,1000)”, and X0 is the initial condition that can be chosen as “randn(3,1)”.

Plot the output y.

Then determine the system in a basis adapted to the decomposition of the signal y into one p.d and one p.n.d part as in Theorem 8.4.8.

Then plot $y_0(t)$ and $y_\infty(t)$ in separate subplots.

Determine the corresponding backward model.

8.3 Repeat exercise 8.2 for the system defined by the data matrices A, B, C and D in the file “model83.mat” available at the homepage.