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Robust Control with Classical Methods – QFT

Per-Olof Gutman

- Review of the classical Bode-Nichols control problem
- QFT in the basic Single Input Single Output (SISO) case
- Fundamental Design Limitations
- Identification of Uncertain Transfer functions
- QFT for non-minimum phase and computer controlled systems
- QFT for cascaded systems, and for a class of non-linear plants
- QFT for Multi-Input Multi-Output (MIMO) plants
- A comparison between QFT and other robust and adaptive control

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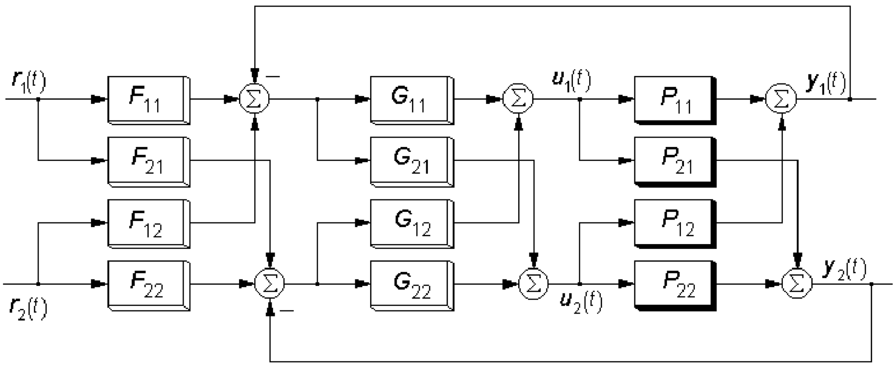
QFT in the Multi-Input Multi-Output (MIMO) case

- **The 2x2 servo problem**
- **The structure of $G(s)$**
- **The pre-compensator**
- **Equivalent SISO systems**
- **The equivalent plant $1/W_{jj}$**
- **Servo specifications**
- **The first design step**
  - **- Servo bounds**
  - **- Horowitz-Sidi bounds**
  - **- Pre-filter design**
- **The second design step - true SISO**
- **Example**

The 2x2 MIMO servo problem

- **Closed loop block diagram of TITO servo system w/o disturbances**



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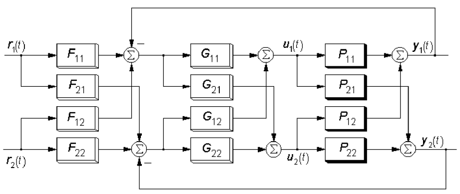
The 2x2 MIMO servo problem, cont'd

$$F = \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix}$$



$$G = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}$$

$$Y = TR$$

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}$$

$$T = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = (I + PG)^{-1} PGF$$

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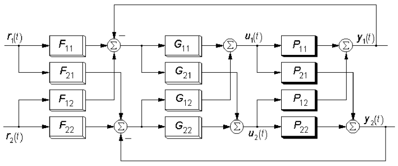
The structure of *G*(*s*)



- **Let the full matrix feedback compensator**

$$G = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix}$$

be decomposed as

$$G = P_0^{-1} L_0$$

with the *a priori* user chosen

- ***Pre-compensator***

$$P_0 = \begin{pmatrix} P_{011} & P_{012} \\ P_{021} & P_{022} \end{pmatrix}$$

and the to-be-designed

- ***Diagonal compensator***

$$L_0 = \begin{pmatrix} L_{10} & 0 \\ 0 & L_{20} \end{pmatrix}$$

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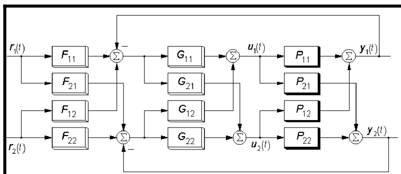
The pre-compensator $P_0(s)$



$$G = P_0^{-1} L_0$$

$$P_0 = \begin{pmatrix} P_{011} & P_{012} \\ P_{021} & P_{022} \end{pmatrix} \qquad L_0 = \begin{pmatrix} L_{10} & 0 \\ 0 & L_{20} \end{pmatrix}$$

$$\boldsymbol{T}(\boldsymbol{s}) = \left(\boldsymbol{I} + \boldsymbol{P}(\boldsymbol{s})\boldsymbol{P}_0^{-1}(\boldsymbol{s})\boldsymbol{L}_0(\boldsymbol{s})\right)^{-1} \boldsymbol{P}(\boldsymbol{s})\boldsymbol{P}_0^{-1}(\boldsymbol{s})\boldsymbol{L}_0(\boldsymbol{s})\boldsymbol{F}(\boldsymbol{s})$$

- **Choice of $P_0(s)$ connected to *input-output pairing***

- **Example:** $\boldsymbol{P}(\boldsymbol{s}) = \begin{pmatrix} 0 & \boldsymbol{P}_{12} \\ \boldsymbol{P}_{21} & 0 \end{pmatrix}$

  invites the choice $\boldsymbol{P}_0(\boldsymbol{s}) = \begin{pmatrix} 0 & \boldsymbol{P}_{012} \\ \boldsymbol{P}_{021} & 0 \end{pmatrix}$

  in order to make PP_0^{-1} diagonal.
- **Use RGA or dynamic RGA for pairing**
- **$V(s)=P(s)P_0^{-1}(s)$ = *de facto* plant**
- ***Decentralized control***
  $P_0(s)$ = I
- ***Nominally decoupling control***
  $P_0(s) = s^d P_{nom}(s)$

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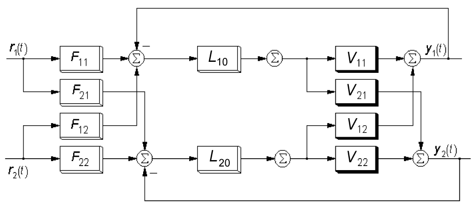
Equivalent SISO systems



$V(s)=P(s)P_0^{-1}(s)$ = *de facto* plant

$$T(s)=\left(I+V(s)L_0(s)\right)^{-1}V(s)L_0(s)F(s)$$

$$W_d=\begin{pmatrix}W_{11} & 0\\ 0 & W_{22}\end{pmatrix},\qquad W_b=\begin{pmatrix}0 & W_{12}\\ w_{21} & 0\end{pmatrix}$$

$$\left(I+V(s)L_0(s)\right)T(s)=V(s)L_0(s)F(s)$$

$$\left(V^{-1}(s)+L_0(s)\right)T(s)=L_0(s)F(s)$$

$$\left(W_d(s)+W_b(s)+L_0(s)\right)T(s)=L_0(s)F(s)$$

$$\left(W_d(s)+L_0(s)\right)T(s)=L_0(s)F(s)-W_b(s)T(s)$$

$$\left(I+W_d^{-1}(s)L_0(s)\right)T(s)=W_d^{-1}\left(L_0(s)F(s)-W_b(s)T(s)\right)$$

$$W=\begin{pmatrix}W_{11} & W_{12}\\ w_{21} & W_{22}\end{pmatrix}\overset{def}{=}V^{-1}=P_0P^{-1}$$

Let $W=W_d+W_b$

$$\boxed{T(s)=\left(I+W_d^{-1}(s)L_0(s)\right)^{-1}W_d^{-1}\left(L_0(s)F(s)-W_b(s)T(s)\right)}$$

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Equivalent SISO systems, cont'd

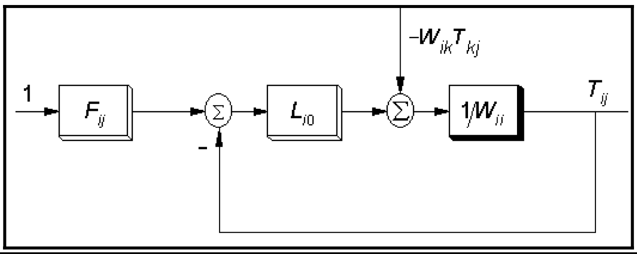
$$\boxed{T(s)=\left(I+W_d^{-1}(s)L_0(s)\right)^{-1}W_d^{-1}\left(L_0(s)F(s)-W_b(s)T(s)\right)}$$

$$T_{11}=\frac{\frac{L_{10}}{W_{11}}F_{11}-\frac{W_{12}}{W_{11}}T_{21}}{1+\frac{L_{10}}{W_{11}}}\qquad T_{12}=\frac{\frac{L_{10}}{W_{11}}F_{12}-\frac{W_{12}}{W_{11}}T_{22}}{1+\frac{L_{10}}{W_{11}}}$$

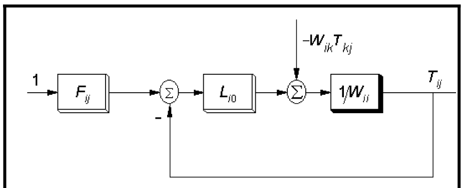
$$T_{21}=\frac{\frac{L_{20}}{W_{22}}F_{21}-\frac{W_{21}}{W_{22}}T_{11}}{1+\frac{L_{20}}{W_{22}}}$$

$$T_{22}=\frac{\frac{L_{20}}{W_{22}}F_{22}-\frac{W_{21}}{W_{22}}T_{12}}{1+\frac{L_{20}}{W_{22}}}$$

- **Horowitz: after the design, the specification would be satisfied. It would hold that**
  $|T_{kj}|<b_{kj}(\omega)$
- **Hence, $-W_{ik}T_{kj}$ is replaced by a worst case "disturbance"**
  $D_{ij}=\max|W_{ik}|b_{kj}e^{j\varphi}$



The equivalent plant $1/W_{jj}$



$V(s)=P(s)P_0^{-1}(s)$

$$W=\begin{pmatrix} W_{11} & W_{12} \\ w_{21} & W_{22} \end{pmatrix} \overset{def}{=} V^{-1}=P_0P^{-1}$$

$$W=W_d+W_b$$

$$W_d=\begin{pmatrix} W_{11} & 0 \\ 0 & W_{22} \end{pmatrix}, \qquad W_b=\begin{pmatrix} 0 & W_{12} \\ w_{21} & 0 \end{pmatrix}$$

- **Example: $V=PP_0^{-1}$ diagonal.**

$W=V^{-1}=\mathrm{diag}(1/V_{11}, 1/V_{22})$

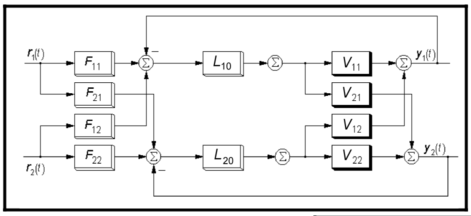
$\Rightarrow 1/W_{jj}=V_{jj}$

- ***Decentralized control*** **$P_0(s)=\mathrm{I}$**

$$W(s)=\frac{1}{P_{11}P_{22}-P_{12}P_{21}}\begin{pmatrix} P_{22} & -P_{12} \\ -P_{21} & P_{11} \end{pmatrix}$$

$$\frac{1}{W_{11}}=P_{11}-\frac{P_{12}P_{21}}{P_{22}}$$

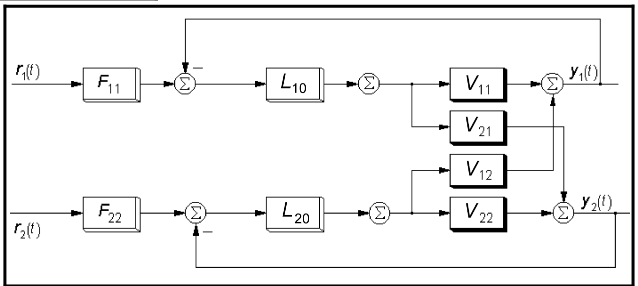
Basically non-interacting specifications



$$T(s)=\left(I+V(s)L_0(s)\right)^{-1}V(s)L_0(s)F(s)$$

- $a_{ij}(\omega)\le\left|T_{ij}(j\omega)\right|\le b_{ij}(\omega), \qquad i=j$
- $\left|T_{ij}(j\omega)\right|\le b_{ij}(\omega), \qquad i\ne j$

- **For the basically non-interacting servo problem, we *postulate* that**
- $F_{12}=F_{21}=0$



Selection of the first design step

$$T_{11} = \frac{\frac{L_{10}}{W_{11}} F_{11} - \frac{W_{12}}{W_{11}} T_{21}}{1 + \frac{L_{10}}{W_{11}}} \qquad T_{12} = \frac{\frac{L_{10}}{W_{11}} F_{12} - \frac{W_{12}}{W_{11}} T_{22}}{1 + \frac{L_{10}}{W_{11}}}$$

$$T_{21} = \frac{\frac{L_{20}}{W_{22}} F_{21} - \frac{W_{21}}{W_{22}} T_{11}}{1 + \frac{L_{20}}{W_{22}}} \qquad T_{22} = \frac{\frac{L_{20}}{W_{22}} F_{22} - \frac{W_{21}}{W_{22}} T_{12}}{1 + \frac{L_{20}}{W_{22}}}$$

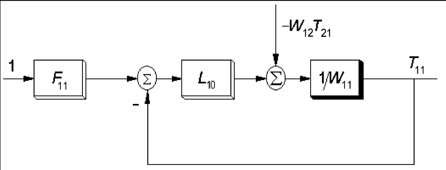
$$a_{ij}(\omega) \le \left|T_{ij}(j\omega)\right| \le b_{ij}(\omega), \qquad i = j$$

$$\left|T_{ij}(j\omega)\right| \le b_{ij}(\omega), \qquad i \ne j$$

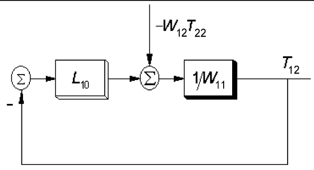
$$F_{12} = F_{21} = 0$$

- **Alternative I: Design L_{10}, F_{11} for {T_{11}, T_{12}} and L_{20}, F_{22} for {T_{21}, T_{22}} seperately, and independently. Disadvantage: Cross-coupling represented by worst-case ”disturbances” only.**
- **Alternative II: Design first the easiest of the two tasks. Then, the second design step becomes a true SISO problem with correct cross-coupling. Additional advantage: In a fully coupled system, the last loop to be closed in a sequential design, determines stability (Bode).**

The first design step



$$a_{11}(\omega) \le \left| \frac{\frac{L_{10}(j\omega)}{W_{11}(j\omega)} F_{11}(j\omega) - \frac{W_{12}(j\omega)}{W_{11}(j\omega)} T_{21}(j\omega)}{1 + \frac{L_{10}(j\omega)}{W_{11}(j\omega)}} \right| \le b_{11}(\omega)$$



$$\left| \frac{\frac{W_{12}(j\omega)}{W_{11}(j\omega)} T_{22}(j\omega)}{1 + \frac{L_{10}(j\omega)}{W_{11}(j\omega)}} \right| \le b_{12}(\omega)$$

- **T_{21} is replaced by b_{21}**
- **Mixed servo & disturbance rejection problem ⇒**
- **Design of L_{10} and F_{11} is not separated, as in SISO**

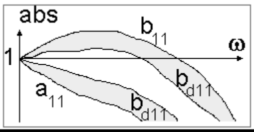
- **T_{22} is replaced by b_{22}**
- **Modify the high frequency roll-off of b_{22} or b_{12}, in order to ease the hf gain demand on L_{10} or L_{20}.**

The first design step: Servo bounds

$$a_{11}(\omega) \le \left| \frac{\frac{L_{10}(j\omega)}{W_{11}(j\omega)}F_{11}(j\omega) - \frac{W_{12}(j\omega)}{W_{11}(j\omega)}T_{21}(j\omega)}{1+\frac{L_{10}(j\omega)}{W_{11}(j\omega)}} \right| \le b_{11}(\omega)$$

$$a_{11}(\omega)+b_{d11}(\omega,\cdot) \le \left| \frac{\frac{L_{10}(j\omega)}{W_{11}(j\omega)}F_{11}(j\omega)}{1+\frac{L_{10}(j\omega)}{W_{11}(j\omega)}} \right| \le b_{11}(\omega)-b_{d11}(\omega,\cdot)$$

$$\left| \frac{\frac{W_{12}(j\omega)}{W_{11}(j\omega)}b_{21}(\omega)}{1+\frac{L_{10}(j\omega)}{W_{11}(j\omega)}} \right| \le b_{d11}(\omega,\cdot)$$



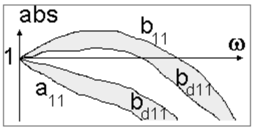
$$\left| \frac{\frac{L_{10}(j\omega)}{W_{11}(j\omega)}F_{11}(j\omega)}{1+\frac{L_{10}(j\omega)}{W_{11}(j\omega)}} \right| + \left| \frac{\frac{W_{12}(j\omega)}{W_{11}(j\omega)}b_{21}(\omega)}{1+\frac{L_{10}(j\omega)}{W_{11}(j\omega)}} \right| \le b_{11}(\omega)$$

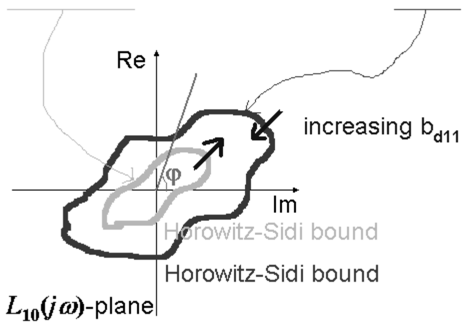
$$a_{11}(\omega) \le \left\| \frac{\frac{L_{10}(j\omega)}{W_{11}(j\omega)}F_{11}(j\omega)}{1+\frac{L_{10}(j\omega)}{W_{11}(j\omega)}} \right| - \left| \frac{\frac{W_{12}(j\omega)}{W_{11}(j\omega)}b_{21}(\omega)}{1+\frac{L_{10}(j\omega)}{W_{11}(j\omega)}} \right\|$$

- **For each ω and for each $\varphi = \arg(L_{10})$, find $b_{d11}(\omega,\varphi)$: $0 \le b_{d11}(\omega,\varphi) \le (b_{11}(\omega) - a_{11}(\omega))/2$ such that the Horowitz-Sidi bounds become equal.**

The first design step: Servo bounds, cont'd

$$a_{11}(\omega)+b_{d11}(\omega,\cdot) \le \left| \frac{\frac{L_{10}(j\omega)}{W_{11}(j\omega)}F_{11}(j\omega)}{1+\frac{L_{10}(j\omega)}{W_{11}(j\omega)}} \right| \le b_{11}(\omega)-b_{d11}(\omega,\cdot) \qquad \left| \frac{\frac{W_{12}(j\omega)}{W_{11}(j\omega)}b_{21}(\omega)}{1+\frac{L_{10}(j\omega)}{W_{11}(j\omega)}} \right| \le b_{d11}(\omega,\cdot)$$





- **For each ω and for each $\varphi = \arg(L_{10})$, find $b_{d11}(\omega,\varphi)$: $0 \le b_{d11}(\omega,\varphi) \le (b_{11}(\omega) - a_{11}(\omega))/2$ s. t. the bounds become equal.**
- **⇒ conservativeness due to Δ-inequality reduced.**
- **Modify the hf roll-off of $b_{21}(\omega)$, see below.**

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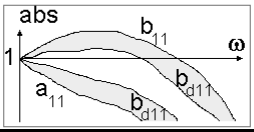
The first design step: Horowitz-Sidi bounds

Servo bounds from

$$a_{11}(\omega)+b_{d11}(\omega,\cdot)\le\left|\frac{\frac{L_{10}(j\omega)}{W_{11}(j\omega)}F_{11}(j\omega)}{1+\frac{L_{10}(j\omega)}{W_{11}(j\omega)}}\right|\le b_{11}(\omega)-b_{d11}(\omega,\cdot)$$

$$\left|\frac{\frac{W_{12}(j\omega)}{W_{11}(j\omega)}b_{21}(\omega)}{1+\frac{L_{10}(j\omega)}{W_{11}(j\omega)}}\right|\le b_{d11}(\omega,\cdot)$$



Cross-coupling bounds from

$$\left|\frac{\frac{W_{12}(j\omega)}{W_{11}(j\omega)}b_{22}(\omega)}{1+\frac{L_{10}(j\omega)}{W_{11}(j\omega)}}\right|\le b_{12}(\omega)$$

Sensitivity bounds from

$$\left|\frac{1}{1+L_{10}(j\omega)/W_{11}(j\omega)}\right|\le x(\omega)$$

Sensitivity bounds for

- **Disturbance rejection**
- **Loop stability margins**
- **Avoiding nmp plant in the second design step, see below.**

♡ **and now, design $L_{10}= L^*_{10}$**

⟶ recall, b_{ij} hf roll-off

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The first design step: a note on roll-off

Notice that to get a high frequency roll-off for the bounds emanating from (8.27), it is in general required that $b_{21}(\omega)$ rolls off faster than $b_{d11}(\omega,\cdot)$, which is achieved if $b_{21}(\omega)$ rolls off faster than $b_{11}(\omega)$. On the other hand, to get a high frequency roll-off for the bounds emanating from (8.28), it is in general required that $b_{22}(\omega)$ rolls off faster than $b_{12}(\omega)$. If one has symmetrical specifications (8.18), (8.19), i.e. $b_{12}(\omega)=b_{21}(\omega)$, and $b_{11}(\omega)= b_{22}(\omega)$, then this roll-off condition is impossible to achieve. Computationally the conflict may be solved either by ignoring the high frequency bounds from (8.27) and (8.28) , or better, by artificially giving $b_{12}(\omega)$ and $b_{21}(\omega)$ the required roll-off when computing bounds from (8.27) and (8.28). If $b_{11}(\omega)= b_{22}(\omega)$ roll off with -40 dB/dec, then let $b_{21}(\omega)$ roll off with e.g. -80 dB/dec, and let $b_{12}(\omega)$ have no high frequency roll off (0 dB/dec). This "trick" to get reasonable bounds will have no influence on the closed loop system behaviour, since $L_{10}(j\omega)/W_{11\mathrm{nom}}(j\omega)$ has to be designed with a reasonable high frequency roll-off, whether the bounds impose it or not.

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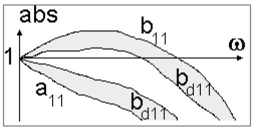
The first design step: Pre-filter design

$$a_{11}(\omega)+b_{d11}(\omega,\cdot)\le\left|\frac{\frac{L_{10}(j\omega)}{W_{11}(j\omega)}F_{11}(j\omega)}{1+\frac{L_{10}(j\omega)}{W_{11}(j\omega)}}\right|\le b_{11}(\omega)-b_{d11}(\omega,\cdot)$$



$$\left|\frac{\frac{W_{12}(j\omega)}{W_{11}(j\omega)}b_{21}(\omega)}{1+\frac{L_{10}(j\omega)}{W_{11}(j\omega)}}\right|\le b_{d11}(\omega,\cdot)$$

- **After the design of L^*_{10}, $b^*_{d11}(\omega)$ is computed:**

$$b^*_{d11}(\omega)=\max\left|\frac{\frac{W_{12}(j\omega)}{W_{11}(j\omega)}b_{21}(\omega)}{1+\frac{L^*_{10}(j\omega)}{W_{11}(j\omega)}}\right|$$

- **Then design $F_{11}(s)$ to satisfy**

$$a_{11}(\omega)+b^*_{d11}(\omega)\le\left|\frac{\frac{L^*_{10}(j\omega)}{W_{11}(j\omega)}F_{11}(j\omega)}{1+\frac{L^*_{10}(j\omega)}{W_{11}(j\omega)}}\right|\le b_{11}(\omega)-b^*_{d11}(\omega)$$

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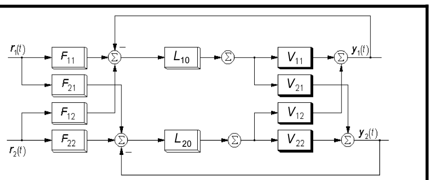
The second design step



$$T_{11}=\frac{\frac{L_{10}}{W_{11}}F_{11}-\frac{W_{12}}{W_{11}}T_{21}}{1+\frac{L_{10}}{W_{11}}}\qquad T_{12}=\frac{\frac{L_{10}}{W_{11}}F_{12}-\frac{W_{12}}{W_{11}}T_{22}}{1+\frac{L_{10}}{W_{11}}}$$

$$T_{21}=\frac{\frac{L_{20}}{W_{22}}F_{21}-\frac{W_{21}}{W_{22}}T_{11}}{1+\frac{L_{20}}{W_{22}}}\qquad T_{22}=\frac{\frac{L_{20}}{W_{22}}F_{22}-\frac{W_{21}}{W_{22}}T_{12}}{1+\frac{L_{20}}{W_{22}}}$$

- **With $L_{10}(s)$ and $F_{11}(s)$ designed, we get, with $F_{12}=F_{21}=0$**

$$T_{21}=\frac{\cancel{\frac{L_{20}}{W^e_{22}}F_{21}}-\frac{W_{21}}{W^e_{22}}\cdot\frac{L_{10}}{1+(L_{10}/W_{11})}F_{11}}{1+\frac{L_{20}}{W^e_{22}}}$$

$$T_{22}=\frac{\frac{L_{20}}{W^e_{22}}F_{22}-\cancel{\frac{W_{21}}{W^e_{22}}\cdot\frac{L_{10}}{1+(L_{10}/W_{11})}F_{12}}}{1+\frac{L_{20}}{W^e_{22}}}$$

$$W^e_{22}=W_{22}\left(1-\frac{\gamma}{1+(L_{10}/W_{11})}\right)\qquad\gamma=\frac{W_{12}W_{21}}{W_{11}W_{22}}$$

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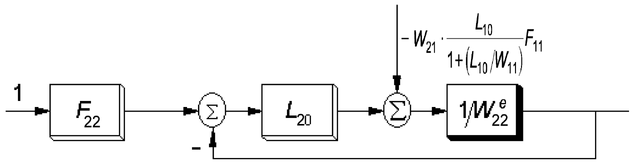
The second design step: true SISO



$$W_{22}^e = W_{22}\left(1-\frac{\gamma}{1+(L_{10}/W_{11})}\right)$$

$$\gamma = \frac{W_{12}W_{21}}{W_{11}W_{22}}$$

Servo bounds from

$$a_{22}(\omega)\le\left|\frac{\frac{L_{20}(j\omega)}{W_{22}^e(j\omega)}}{1+\frac{L_{20}(j\omega)}{W_{22}^e(j\omega)}}F_{22}(j\omega)\right|\le b_{22}(\omega)$$

Cross-coupling bounds from

$$\left|\frac{\frac{W_{21}(j\omega)}{W_{22}^e(j\omega)}\cdot\frac{L_{10}(j\omega)}{1+(L_{10}(j\omega)/W_{11}(j\omega))}\cdot F_{11}(j\omega)}{1+\frac{L_{20}(j\omega)}{W_{22}^e(j\omega)}}\right|\le b_{21}(\omega)$$

Sensitivity bounds from $\left|\frac{1}{1+L_{20}(j\omega)/W_{22}^e(j\omega)}\right|\le y(\omega)$

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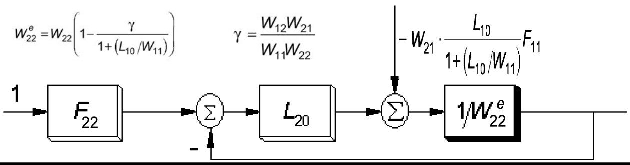
The second design step: true SISO

$$W_{22}^e = W_{22}\left(1-\frac{\gamma}{1+(L_{10}/W_{11})}\right)\qquad \gamma = \frac{W_{12}W_{21}}{W_{11}W_{22}}$$



Servo bounds from

$$a_{22}(\omega)\le\left|\frac{\frac{L_{20}(j\omega)}{W_{22}^e(j\omega)}}{1+\frac{L_{20}(j\omega)}{W_{22}^e(j\omega)}}F_{22}(j\omega)\right|\le b_{22}(\omega)$$

Cross-coupling bounds from

$$\left|\frac{\frac{W_{21}(j\omega)}{W_{22}^e(j\omega)}\cdot\frac{L_{10}(j\omega)}{1+(L_{10}(j\omega)/W_{11}(j\omega))}\cdot F_{11}(j\omega)}{1+\frac{L_{20}(j\omega)}{W_{22}^e(j\omega)}}\right|\le b_{21}(\omega)$$

Sensitivity bounds from

$$\left|\frac{1}{1+L_{20}(j\omega)/W_{22}^e(j\omega)}\right|\le y(\omega)$$

- **As in SISO, design L_{20} to satisfy the Horowitz-Sidi bounds, and such that the closed loop is stable, which will ensure MIMO closed loop stability if the plant is fully connected.**
- **Design F_{22}.**
- **Simulate in the frequency and time domains. Check closed loop stability**
- **Re-design, if necessary.**

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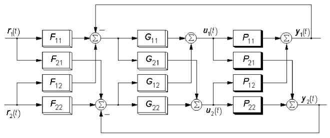
Example



$$P = \begin{pmatrix} k_{11}/s & k_{12}/s \\ k_{21}/s & k_{22}/s \end{pmatrix},$$

$$k_{11} \in [2,6], k_{12} \in [0.5, 1.5]$$

$$k_{21} \in [0.5, 1.5] \quad k_{22} \in [2,6]$$

$$P_{nom} = \begin{pmatrix} 2/s & 1/s \\ 1/s & 2/s \end{pmatrix}$$

```
function   [Par,w_tpl,w_nom,method,P_num,P_den, ...
            n_dif,Uns_Par] = p11

% MIMO Plant:  p11.m

% Definition of the parameters
% ====================
   Par = ['k11=[2,6,2,4]'];
   Uns_Par=[];

% Definition of the frequency vectors [rad/sec]
% ====================================
  w_tpl = [0.1 0.2 0.5 1 2 5 10 20 50 70 100];
  w_nom = logspace(-2,3,200);

% Definition of the template computation method
% ==========================================
  method = 'rff [1,1]';

% Plant definition
% ===============
  P_num='(gain,k11)';
  P_den='[1  0]';
```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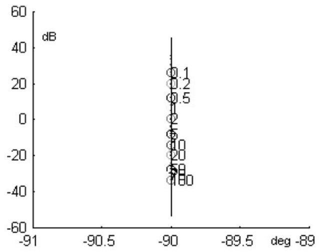
Example, cont'd: templates and pre-compensator

```
ctpl('p11',[],'grid');
ctpl('p12',[],'grid');
ctpl('p21',[],'grid');
ctpl('p22',[],'grid');
```



templates of $P_{11}(s)$ *in* `p11.tpl` *displayed*

- **Here: *nominally decoupling* design with precompensator chosen as $P_0(s)=P_{nom}(s)$.**

```
function [P0] = p0(s)
% p0.m  precompensator transfer function for 2x2 MIMO

P0 = [2./s 1./s
      1./s 2./s];
```

•Exercise: carry out a *decentralized* design with $P_0(s)$=I.

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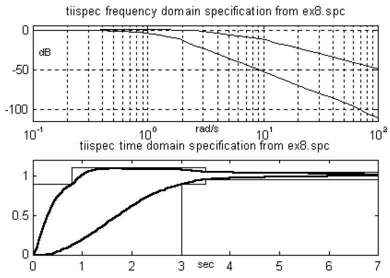
Example, cont'd: specifications

Servo specification

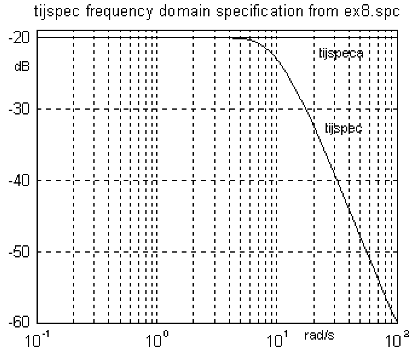



```
rsrs('ex8','tiispec',[3 0.8],10,3.5,[],logspace(-1,2),[],3)
showspc('ex8','tiispec');                                   % Fig
```

6 dB sensitivity specification

```
add2spc('ex8','sens',logspace(-1,2),6);
```

Cross coupling specification




```
add2spc('ex8','tijspeca',logspace(-1,2),-20);  % b12 with roll-off=0
add2spc('ex8','tijspec',logspace(-1,2),[.1],[.01 .14 1]);% -40 dB/dec
showspc('ex8','tijspec');showspc('ex8','tijspeca',[],[],gcf));
```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Example, cont'd: equivalent plants

$$P = \begin{pmatrix} k_{11}/s & k_{12}/s \\ k_{21}/s & k_{22}/s \end{pmatrix},\quad P_{\text{nom}} = \begin{pmatrix} 2/s & 1/s \\ 1/s & 2/s \end{pmatrix}$$

$k_{11} \in [2,6],\quad k_{12} \in [0.5, 1.5]$

$k_{21} \in [0.5,1.5]\ k_{22} \in [2,6]$

- **Note: $k_{11}\, k_{22}\, k_{12}\, k_{21} > 0$**

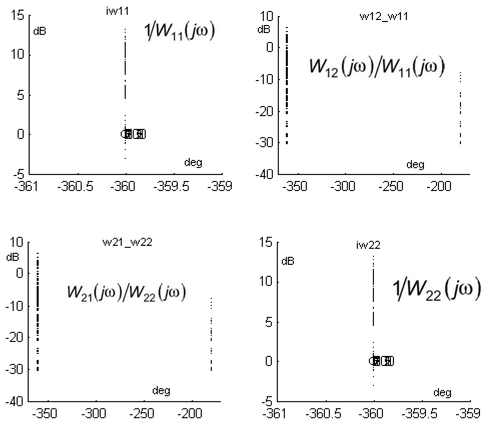
$$P_0^{-1} = P_{\text{nom}}^{-1} = \frac{s}{3}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

$$V = PP_0^{-1} = \frac{1}{3}\begin{pmatrix} 2k_{11}-k_{12} & 2k_{12}-k_{11} \\ 2k_{21}-k_{22} & 2k_{22}-k_{21} \end{pmatrix}$$

$$W = V^{-1} = \frac{1}{k_{11}k_{22}-k_{12}k_{21}}\begin{pmatrix} 2k_{22}-k_{21} & k_{11}-2k_{12} \\ k_{22}-2k_{21} & 2k_{11}-k_{12} \end{pmatrix}$$

$$W_{\text{nom}} = V_{\text{nom}}{}^{-1} = I$$

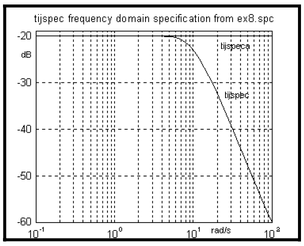
```
mtpl1('iw11','iw22','w12 w11','w21 w22','p11','p12','p21','p22','p0');
showtpl('iw11'); showtpl('w12 w11'); % Figure 8.11
showtpl('w21 w22'); showtpl('iw22'); % Figure 8.11
```

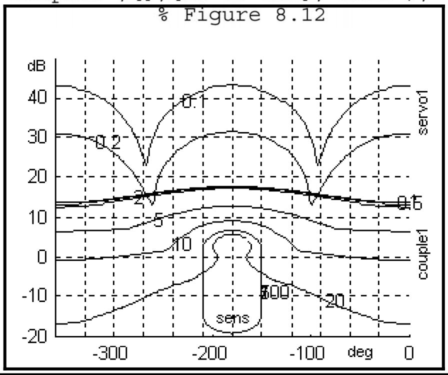


Example, cont'd: bounds for $L_{10}/W_{11\mathrm{nom}}$

```
mcbnd11('ex8','tiispec','tijspec',[],[],'ex8','iw11',' w12 w11');
                                           % Default bound name: servo1
mcbnd11('ex8','tijspeca','tiispec',[],[],'ex8','iw11',' w12_w11');
                                           % Default bound name: couple1
cbnd('ex8','sens',[],[],'ex8','iw11','sens','fodsrs');
showbnd('ex8',[],[.1 .2],'servo1',[], ...
            [.5 1 2 5 10 20 ],'couple1',[],[50 70 100],'sens');
axis([-360 0 -20 50 ]),mgrid(12,7)           % Figure 8.12
```

- **mcbnd11** also computes the b_{d11}-vector for each servo bound into default matrix **servo1bd11**





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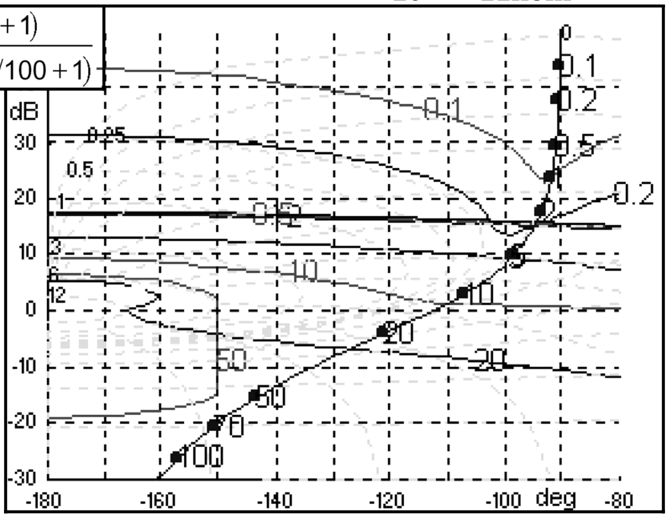
Example, cont'd: design of $L_{10}/W_{11\mathrm{nom}}$

$$L_{10}(s)=\frac{15(s/80+1)}{s(s/30+1)(s/100+1)}$$



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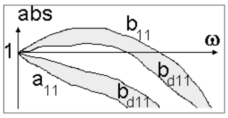
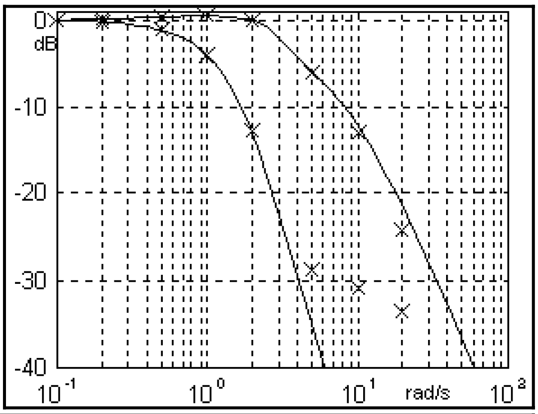
Example, cont'd: modified servo specification



After the design of L^*_{10}, $b^*_{d11}(\omega)$ is computed from

$$b^*_{d11}(\omega)=\max\left|\frac{\frac{W_{12}(j\omega)}{W_{11}(j\omega)}b_{21}(\omega)}{1+\frac{L^*_{10}(j\omega)}{W_{11}(j\omega)}}\right|$$



Compute $L_{10}(j\omega)/W_{11}(j\omega)$

```
tplfop('L10_w11','*',[],'iw11',1,'L10');
```

Compute the modified servo specification

$$\left[a_{11}(\omega)+b^*_{d11}(\omega),\ b_{11}(\omega)-b^*_{d11}(\omega)\right]$$

```
bd11spc('ex8','tiispec','servo1bd11',[],'L10_w11');
            % default name for new servo spec: tiispecm in ex8.spc
showspc('ex8','tiispec','freq')
showspc('ex8','tiispecm','freq','cx',gcf)        % Figure 8.15
```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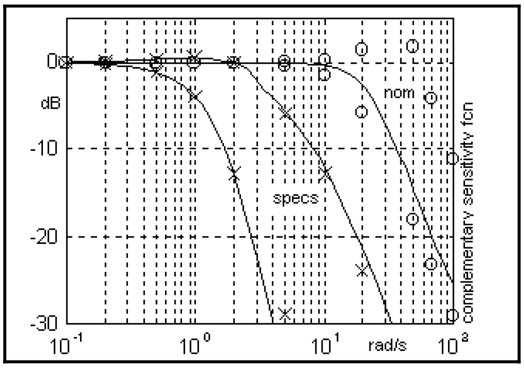
Example, cont'd: closing the first loop

- **Compute the complementary sensitivity function** $\bar{S}_1(j\omega)=\dfrac{L_{10}(j\omega)/W_{11}(j\omega)}{1+\left(L_{10}(j\omega)/W_{11}(j\omega)\right)}$

```
tplfop('cosens1','idsrs',[],'L10_w11');
h0=fdesign('cosens1.tpl',[],'new');
showspc('ex8','tiispecm','freq','cx',gcf);
showspc('ex8','tiispec','freq',[],gcf);
```



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Example, cont'd: design of F_{11}

- **Design the prefilter, and display the first loop servo transfer function**

```
h1=fdesign('cosens1.tpl','F11.m', h0);
```

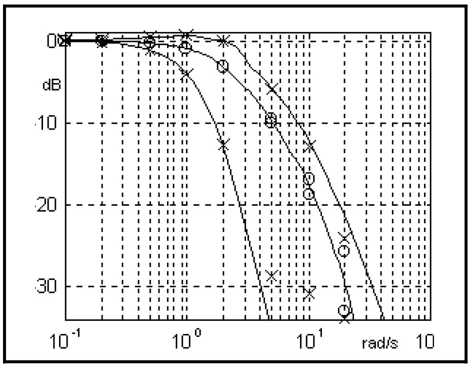
$$F_{11}(s) = \frac{1}{(s/2+1)(s/10+1)}$$

```
function [F]=F11(s)
% F11.m  is the prefilter file for the first step

F = (1)./( (s/2 + 1).*(s/10 + 1) );
```

$$T_1(j\omega) = F_{11}(j\omega)\frac{L_{10}(j\omega)/W_{11}(j\omega)}{1+\left(L_{10}(j\omega)/W_{11}(j\omega)\right)}$$

```
tplfop('closed1','rsrs',[],'iw11',1,'L10','F11');
```



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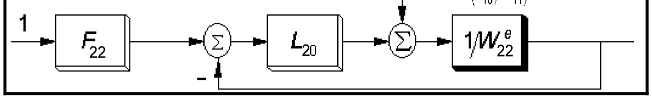
Recall: The second design step

$$W_{22}^e = W_{22}\left(1-\frac{\gamma}{1+(L_{10}/W_{11})}\right) \qquad \gamma = \frac{W_{12}W_{21}}{W_{11}W_{22}} \qquad -W_{21}\cdot\frac{L_{10}}{1+(L_{10}/W_{11})}F_{11}$$



Servo bounds from

$$a_{22}(\omega) \le \left|\frac{\dfrac{L_{20}(j\omega)}{W_{22}^e(j\omega)}}{1+\dfrac{L_{20}(j\omega)}{W_{22}^e(j\omega)}}F_{22}(j\omega)\right| \le b_{22}(\omega)$$

Cross-coupling bounds from

$$\left|\frac{\dfrac{W_{21}(j\omega)}{W_{22}^e(j\omega)}\cdot\dfrac{L_{10}(j\omega)}{1+(L_{10}(j\omega)/W_{11}(j\omega))}\cdot F_{11}(j\omega)}{1+\dfrac{L_{20}(j\omega)}{W_{22}^e(j\omega)}}\right| \le b_{21}(\omega)$$

Sensitivity bounds from

$$\left|\frac{1}{1+L_{20}(j\omega)/W_{22}^e(j\omega)}\right| \le y(\omega)$$

- **As in SISO, design L_{20} to satisfy the Horowitz-Sidi bounds, and such that the closed loop is stable, which will ensure MIMO closed loop stability if the plant is fully connected.**
- **Design F_{22}.**
- **Simulate in the frequency and time domains. Check closed loop stability**
- **Re-design, if necessary.**

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Example, cont'd: second design step templates

$$S_1(j\omega)=\frac{1}{1+\left(L_{10}(j\omega)/W_{11}(j\omega)\right)}$$

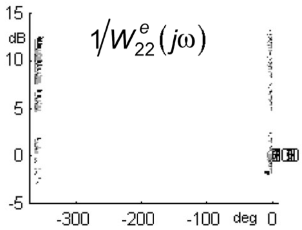
```
tplfop('sens1','odsrs',[],'iw11',1,'L10');
```

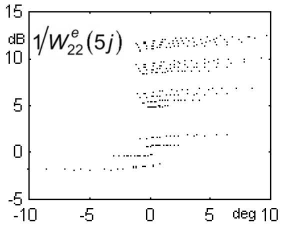
Compute templates of x-coupling tf, `wwc1=`

$$\frac{W_{21}(j\omega)}{W_{22}^e(j\omega)}\cdot\frac{L_{10}(j\omega)}{1+\left(L_{10}(j\omega)/W_{11}(j\omega)\right)}\cdot F_{11}(j\omega)$$

and of "equivalent plant"
`iw22e =` $1/W_{22}^e(j\omega)$

```
mtpl2('iw22e','wwc1','iw22','w12_w11','w21_w22','sens1','closed1');
```





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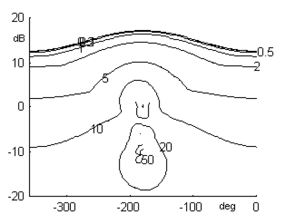
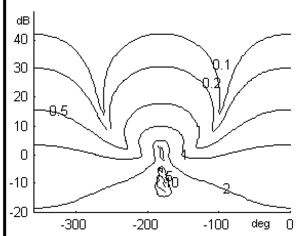
Example, cont'd: second design step bounds

```
cbnd('ex8_2','tijspeca',[],[],'ex8','iw22e','couple2a',...
                        'fcouple2',[],[],[],'wwc1','mimo2');
cbnd('ex8_2','tiispec',[],[],'ex8','iw22e','servo2','frsrs');
cbnd('ex8 2','sens',[],[],'ex8','iw22e','sens2','fodsrs');
```

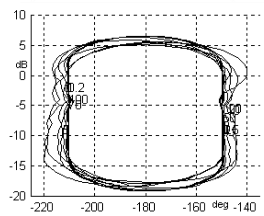


x-coupling (`'couple2a'`**)**



servo (`'servo2'`**)**

$$\left|\frac{1}{1+\left(L_{20}(j\omega)/W_{22}^e(j\omega)\right)}\right|\le 2$$



sensitivity (`'sens2'`**)**

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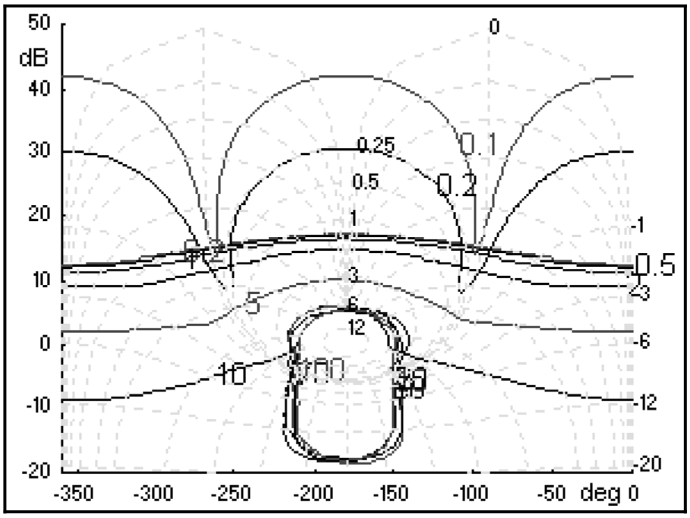
Example, cont'd: second design step dominant bounds



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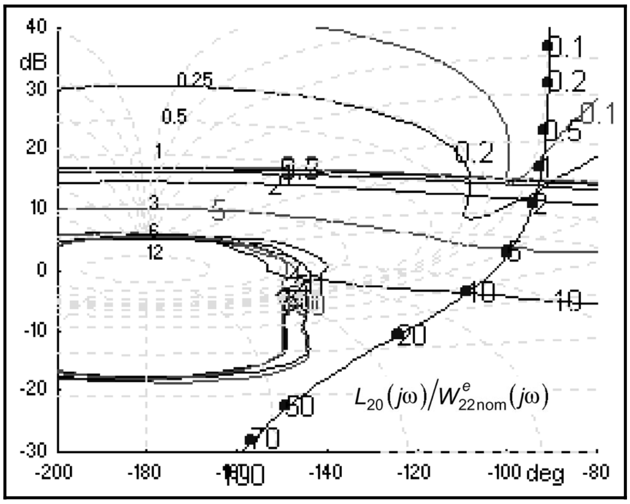
Example, cont'd: 2nd step feedback compensator

$$L_{20}(s) = \frac{7}{s(s/30+1)}$$



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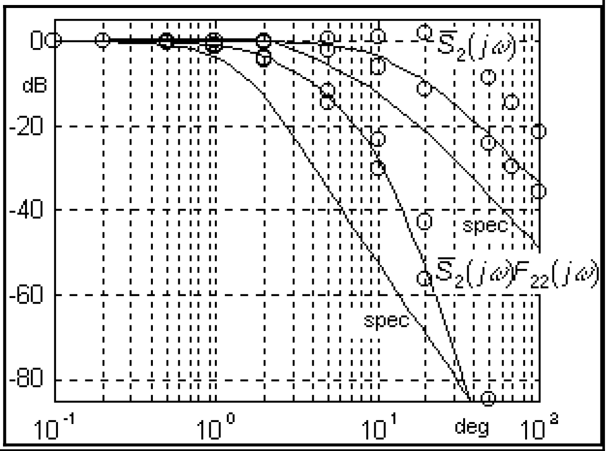
Example, cont'd: 2nd step pre-filter design

```
tplfop('cosens2','iosrs',[],'iw22e',1,'L20');
```

```
h0=fdesign('cosens2.tpl');
showspc('ex8','tiispec','freq',[],gcf);
h1=fdesign('cosens2.tpl','F22.m');
```

$$\bar{S}_2(j\omega) = \frac{L_{20}(j\omega)/W_{22}^{e}(j\omega)}{1+\left(L_{20}(j\omega)/W_{22}^{e}(j\omega)\right)}$$

$$F_{22}(s) = \frac{1}{(s/2+1)(s/5+1)\left(s^2/100+1.4s/10+1\right)}$$



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Example, cont'd: Design summary

$$P_{\text{nom}} = \begin{pmatrix} 2/s & 1/s \\ 1/s & 2/s \end{pmatrix}$$

$$\boldsymbol{P}_0^{-1} = \boldsymbol{P}_{\text{nom}}^{-1} = \frac{\boldsymbol{s}}{3}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

$$\boldsymbol{L}_0 = \begin{pmatrix} \dfrac{15(\boldsymbol{s}/80+1)}{\boldsymbol{s}(\boldsymbol{s}/30+1)(\boldsymbol{s}/100+1)} & 0 \\ 0 & \dfrac{7}{\boldsymbol{s}(\boldsymbol{s}/30+1)} \end{pmatrix}$$

$$G = P_0^{-1}L_0 = \begin{bmatrix} \dfrac{10(s/80+1)}{(s/30+1)(s/100+1)} & -\dfrac{7/3}{(s/30+1)} \\ -\dfrac{5(s/80+1)}{(s/30+1)(s/100+1)} & \dfrac{14/3}{(s/30+1)} \end{bmatrix}$$

$$F(s) = \begin{bmatrix} \dfrac{1}{(s/2+1)(s/10+1)} & 0 \\ 0 & \dfrac{1}{(s/2+1)(s/5+1)\left(s^2/100+1.4s/10+1\right)} \end{bmatrix}$$

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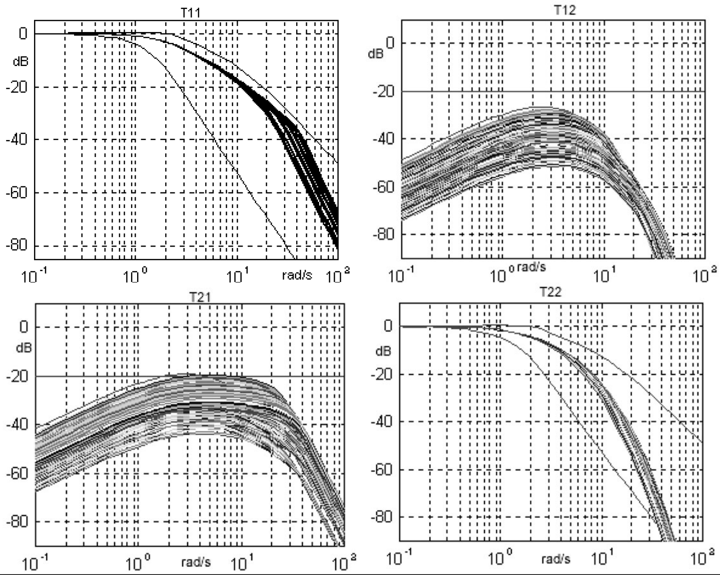
Example cont'd: Frequency domain simulation



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Example end: Time domain simulation